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Weak K–Amplitudes in the Chiral and $1/N_c$–Expansions

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Abstract

It is shown that there exist symmetry constraints for non–leptonic weak amplitudes which emerge when the $1/N_c$–expansion restricted to the leading and next–to–leading approximations only is systematically combined with χPT limited to the lowest non–trivial order. We discuss these constraints for the couplings $\mathbf{g}_8$ and $\mathbf{g}_{27}$ of $\Delta S = 1$ transitions and the $\mathbf{B}_K$–parameter of $K^0 - \bar{K}^0$ mixing.

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1. Chiral perturbation theory (χPT) provides a useful framework to study K–decays in the Standard Model. To lowest order in the chiral expansion: $\mathcal{O}(p^2)$, there are only two coupling constants $\mathbf{g}_8$ and $\mathbf{g}_{27}$ which govern non–leptonic K–decays. Once these couplings are fixed phenomenologically from $K \to \pi\pi$ decays, there follow a wealth of predictions for $K \to \pi\pi\pi$ and some other radiative K–decays*. Unfortunately, the study of chiral corrections to these lowest order predictions generally brings many new local terms in the chiral effective Lagrangian of $\mathcal{O}(p^4)$ [4–7]. The number of terms is too large to make a systematic phenomenological determination of the new couplings similar to what has been done in the purely strong interaction sector [8]. The problem here is not the complexity of the calculations; it is simply that the experimental information we have on $\Delta S = 1$ and $\Delta S = 2$ transitions is too limited when compared to the large number of possible $\mathcal{O}(p^4)$ couplings. Except for a few remarkable predictions which have been made, one is obliged in most cases to resort to chiral power counting arguments and/or models in order to make numerical estimates.

Ideally, one would like to develop well controlled approximation methods starting at the level of the Standard Model Lagrangian. In that sense the $1/N_c$–expansion in QCD [9, 10], where $N_c \to \infty$ with $\alpha_s \times N_c$ fixed, is a good candidate. Keeping only the leading contributions in the large–N_c limit for non–leptonic K–decays is however a bad approximation because, in that limit, many of the four–quark operators of the effective Lagrangian which emerge after integrating out the fields of heavy particles in the presence of gluon interactions are suppressed. One has to go to the next–to–leading order in the $1/N_c$–expansion before the complete set of possible four–quark operators appears.

The purpose of this note is to show that there exist symmetry constraints for non–leptonic weak amplitudes which emerge when the $1/N_c$–expansion, restricted to the leading and next–to–leading approximations only, is systematically combined with χPT at the lowest non–trivial order. Here we shall limit ourselves to spell out these constraints for the couplings $\mathbf{g}_8$ and $\mathbf{g}_{27}$ and the $\mathbf{B}_K$–parameter of $K^0 - \bar{K}^0$ mixing and to the discussion of their phenomenological implications. There are other interesting applications of the same type for other processes; in particular for the decay $K_1^0 \to \pi^0 e^+ e^-$ which at this approximation can be calculated in terms of known physical parameters, and which of course has interesting implications for $K_L^0 \to \pi^0 e^+ e^-$ and the possibility of observing direct CP–violation in this process. These other applications will be discussed elsewhere.

2. In the conventional formulation of χPT the octet of low–lying pseudoscalar states (π, K, η) are the Nambu–Goldstone bosons associated to the "broken" axial generators of chiral–$SU(3)$. The Nambu–Goldstone fields are collected in a unitary 3×3 matrix $U(x)$ with $\det U = 1$, which under $SU(3)_L \otimes SU(3)_R$ transformations (V_L, V_R) transforms linearly: $U \to V_R U V_L^\dagger$. In order to describe non–leptonic weak interactions it is useful to introduce the 3×3 flavour matrix vector field

*For recent reviews see e.g. refs. [1–3]

$$\mathcal{L}_\mu(x) \equiv -i\frac{f^2}{2}U(x)^\dagger D_\mu U(x), \tag{1}$$

where D_μ denotes the covariant derivative in the presence of external $SU(3)_L$ and $SU(3)_R$ gauge field sources, and f the f_π–coupling in the chiral limit ($f \simeq 86$ MeV). Under chiral–$SU(3)$ transformations: $\mathcal{L}_\mu \to V_L \mathcal{L}_\mu V_L^\dagger$. In terms of $\mathcal{L}_\mu$, and to lowest order in the chiral expansion, the operators with the same chiral transformation properties as those of the effective four–quark Lagrangian can then be readily obtained. They are:

$$\mathcal{L}_8(x) = \sum_i (\mathcal{L}_\mu)_{2i}(\mathcal{L}^\mu)_{i3}\,; \tag{2}$$

and

$$\mathcal{L}_{27}(x) = \frac{2}{3}(\mathcal{L}_\mu)_{21}(\mathcal{L}^\mu)_{13} + (\mathcal{L}_\mu)_{23}(\mathcal{L}^\mu)_{11}\,, \tag{3}$$

which transform respectively like $(8_L, 1_R)$ and $(27_L, 1_R)$ under $SU(3)_L \otimes SU(3)_R$. To lowest order in the chiral expansion, the effective Lagrangian of the Standard Model which describes $\Delta S = 1$ transitions between pseudoscalar states has then the following form:

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\, V_{\text{ud}} V_{\text{us}}^* \; [\mathbf{g}_8\, \mathcal{L}_8 + \mathbf{g}_{27}\, \mathcal{L}_{27}] + \text{h.c.}\,, \tag{4}$$

with $\mathbf{g}_8$ and $\mathbf{g}_{27}$ coupling constants which are not fixed by chiral symmetry arguments. The phenomenological determination of these couplings from $K \to \pi\pi$, to lowest order in the chiral expansion, gives[†] [11]

$$|\mathbf{g}_8 + \frac{1}{9}\mathbf{g}_{27}| \simeq 5.1\,, \qquad |\mathbf{g}_{27}| \simeq 0.29\,. \tag{5}$$

The decays $K \to \pi\pi$ and $K \to \pi\pi\pi$ have also been analyzed in the presence of chiral $\mathcal{O}(p^4)$ corrections [12]. The fitted value for $\mathbf{g}_8$ decreases then by 30% to $\mathbf{g}_8 \simeq 3.6$ while $\mathbf{g}_{27}$ is only slightly modified.

It is useful to go one step backwards in the theory and to analyze the combinatorics which in the Standard Model leads to the effective Lagrangian above. With one virtual W–field emitted and reabsorbed, and to lowest order in the chiral expansion, there are three possible chiral invariant configurations which give rise to the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{eff}} \;=\; -\frac{G_F}{\sqrt{2}}\,4\,\Big[&\mathbf{a}\,\text{tr}(Q_L^{(-)}\mathcal{L}_\mu)\text{tr}(Q_L^{(+)}\mathcal{L}^\mu) + \mathbf{b}\,\text{tr}(Q_L^{(-)}\mathcal{L}_\mu Q_L^{(+)}\mathcal{L}^\mu) \\ &+ \mathbf{c}\,\text{tr}(Q_L^{(-)}Q_L^{(+)}\mathcal{L}_\mu\mathcal{L}^\mu)\Big]\,, \end{aligned} \tag{6}$$

[†]Notice that $\mathcal{L}_{27}$ generates both $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions.

where $Q_L^{(\pm)}$ are the flavour matrices

$$Q_L^{(-)} = \begin{pmatrix} 0 & 0 & 0 \\ V_{\rm ud}^\star & 0 & 0 \\ V_{\rm us}^\star & 0 & 0 \end{pmatrix} \qquad \text{and} \qquad Q_L^{(+)} = \begin{pmatrix} 0 & V_{\rm ud} & V_{\rm us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{7}$$

which under chiral rotations transform like $Q_L^{(\pm)} \to V_L Q_L^{(\pm)} V_L^\dagger$. The underlying functional integral over quark and gluon fields which gives rise to the effective couplings in (6) is represented diagrammatically in Fig. 1. The solid lines correspond to quark fields propagating in a gluon background (the dots in the figure,) which is subsequently integrated down to the scales where the chiral Lagrangian of the Goldstone modes becomes effective. The $Q_L^{(\pm)}$–operators represent the emission and absorption of the virtual W–field. The restriction to lowest order in the chiral expansion implies that at most two $\mathcal{L}_\mu$ insertions are allowed. When further restricted to $\Delta S = 1$ transitions, the effective Lagrangian in (6) coincides with the conventional one in (4) with

$$\mathbf{g}_8 = \frac{3}{5}(\mathbf{a}+\mathbf{b}) - \mathbf{b} + \mathbf{c} \qquad \text{and} \qquad \mathbf{g}_{27} = \frac{3}{5}(\mathbf{a}+\mathbf{b})\,. \tag{8}$$

3. Let us now examine the behaviour of the coupling constants **a**, **b**, and **c** from the point of view of the $1/N_c$–expansion. It appears that the configuration which leads to the **a**–type coupling in (6) is $\mathcal{O}(N_c^2)$, while those leading to the **b**– and **c**– type couplings are non–leading $\mathcal{O}(N_c)$. To leading order in the $1/N_c$–expansion the coupling **a** can be calculated because in this limit the four–quark operators factorize into current density operators and their chiral effective realization is known from low–energy strong interaction physics to $\mathcal{O}(p^4)$. With the factor f^2, which is $\mathcal{O}(N_c)$, included in the definition of $\mathcal{L}_\mu(x)$ in (1), the coupling constant **a** is of $\mathcal{O}(1)$ in the $1/N_c$–expansion. The interesting observation is that the factorization result which emerges, $\mathbf{a} = 1$, can only be modified by gluonic configurations which are at least next–to–next–to–leading order in the $1/N_c$–expansion, as illustrated by the diagram in Fig. 2. Colour matrices are traceless, which implies that a minimum of two gluons exchanged from one fermion loop to the other are required to modify the factorization property, and this leads to a relative correction of $\mathcal{O}(1/N_c^2)$. We then conclude that:

$$\mathbf{a} = 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)\,. \tag{9}$$

The configuration which leads to the **c**–type coupling corresponds to the so called penguin–like diagrams in the effective four–quark Hamiltonian formulation. It is well known [13] that their contribution to the coupling **c** to leading order in the $1/N_c$–expansion which in this case is $\mathcal{O}(1/N_c)$ can also be calculated in terms of known phenomenological parameters, with the result[‡]:

[‡]A detailed discussion of this calculation can be found e.g. in ref. [2]

$$\mathbf{c} = \mathrm{Re}C_4 - 16\, L_5\, \mathrm{Re}C_6(\mu^2) \left[\frac{<\bar{\psi}\psi>}{f_\pi^3}\right]^2 \simeq 0.3 \pm 0.2\,, \tag{10}$$

where in this expression L_5 is one of the $\mathcal{O}(p^4)$ couplings of the strong effective chiral Lagrangian, and C_4, C_6 are the Wilson coefficients of the Q_4, Q_6 four–quark operators in standard notation. To $\mathcal{O}(1/N_c)$ the scale dependence in C_6 cancels with the one in $<\bar{\psi}\psi>$, while C_4 is scale-independent (below the charm threshold). The numerical result in (10) comes from using the chiral limit value $f_\pi \simeq f = 86$ MeV; $L_5 \simeq 1.4\times 10^{-3}$ and $<\bar{\psi}\psi>(1\mathrm{GeV}^2) = -(0.013\pm 0.003)\,\mathrm{GeV}^3$ [14]. The Wilson coefficients have been evaluated using the perturbative QCD two–loop expressions [15,16] restricted to $\mathcal{O}(N_c)$ with[§] $\Lambda_{\overline{\mathrm{MS}}} \simeq 300\,\mathrm{MeV}$. The error in (10) is partly due to the present error in the determination of $<\bar{\psi}\psi>$, partly to short–distance uncertainties in $C_{4,6}$.

The configuration which leads to the **b**–type coupling is also $\mathcal{O}(N_c)$. However, unlike the case of the **c**–coupling, we do not know at present how to evaluate **b** in a model independent way, even its leading $\mathcal{O}(1/N_c)$ contribution. There is however an important correlation which appears at the order of approximations which we are considering: the **b**–type configuration contributes with *opposite sign* to $\mathbf{g}_8$ and $\mathbf{g}_{27}$. This correlation of signs is in fact fully respected in the effective action model calculation of ref. [17], where it is found that **b** is negative (see also the recent work of refs. [34,35]). It is not quite respected in the model of refs. [18,19], inspired by the $1/N_c$–expansion; but this is due to the fact that in their approach some terms of higher $\mathcal{O}(1/N_c^2)$ have also been included.

A qualitative picture towards the understanding of the underlying physics begins to emerge at this simple level of approximations $\mathcal{O}(p^2)$ and $\mathcal{O}(1/N_c)$ which we are considering. With **a**= 1 and fixing for example **b** to the value $\mathbf{b} \simeq -0.52$, which is the one which follows from the phenomenological determination $|\mathbf{g}_{27}| \simeq 0.29$, implies $\mathbf{g}_8 \simeq 1.1$, still too low compared to the phenomenological number to be explained (which once corrected by the enhancement already provided by the $\mathcal{O}(p^4)$ chiral corrections is $\mathbf{g}_8 \simeq 3.6$), but in the right direction.

We are now in the position to bring in as well the discussion of the $\mathbf{B}_K$–parameter which governs $K^0 - \bar{K}^0$ mixing. By analogy with the previous analysis of $\Delta S = 1$ transitions, the short-distance $\Delta S = 2$ Hamiltonian can be visualized as a convolution of two $\bar{s} \to \bar{d}$ transitions, with two virtual W–fields being emitted and reabsorbed. Each transition results in a flavour matrix factor $(Q_{32})_{ij} = \delta_{i3}\delta_{j2}$ times calculable short–distance loop functions from the integration of the heavy fields. Therefore, the effective $\mathcal{O}(p^2)$ $\Delta S = 2$ Lagrangian has also the structure given in eq. (6), but with the matrices $Q_L^{(\pm)}$ replaced by Q_{32}. In this case the configuration **c** is identically zero because $(Q_{32})^2 = 0$ while **a** and **b** generate the same structure $(\mathcal{L}_\mu)_{23}(\mathcal{L}^\mu)_{23}$. Chiral symmetry guarantees that the coefficients **a** and **b** appearing in the $\Delta S = 1$ and $\Delta S = 2$ effective Lagrangians at $\mathcal{O}(p^2)$ are the same [20] (once the known short-distance factors

[§] More precisely, we have taken $\Lambda_{\overline{\mathrm{MS}}}^{N_c\to\infty} \simeq 400\,\mathrm{MeV}$, which for $N_c = N_f = 3$ corresponds to $\Lambda_{\overline{\mathrm{MS}}} \simeq 300\,\mathrm{MeV}$.

have been appropriately reabsorbed in the global normalizations of $\mathcal{L}_{\text{eff}}^{\Delta S=1}$ and $\mathcal{L}_{\text{eff}}^{\Delta S=2}$). The expression which emerges for $\mathbf{B}_K$ to lowest $\mathcal{O}(p^2)$ in the chiral expansion , but taking into account the chiral corrections which bring the chiral limit f–coupling to the physical f_K, is then

$$\mathbf{B}_K = \frac{3}{4}(\mathbf{a}+\mathbf{b}), \tag{11}$$

where, at the level of approximation in the $1/N_c$–expansion that we are considering [i.e., $\mathcal{O}(1)$ and $\mathcal{O}(1/N_c)$], $\mathbf{a}=1$. Using $\mathbf{b}=-0.52$ as before, one gets $\mathbf{B}_K=0.36$, a number which is compatible with the results of the effective action model calculations of ref. [17] as well as with various phenomenological QCD sum rule determinations [21–23]. Within errors, it is also compatible with the results of the $1/N_c$–approach calculations of ref. [19], but not with the most recent numerical estimates of $\mathbf{B}_K$ obtained by the lattice QCD simulations [24–28].

4. In order to get some insight into the underlying QCD dynamics we shall next examine the short–distance behaviour of the two–point function correlators

$$\Psi^{\Delta S=1,2}(q^2) \equiv i\int d^4x\, e^{iq\cdot x}\,\langle 0|T\left(\mathcal{H}_{\text{eff}}^{\Delta S=1,2}(x),\,\mathcal{H}_{\text{eff}}^{\Delta S=1,2}(0)^\dagger\right)|0\rangle \tag{12}$$

in perturbation theory and within the $1/N_c$–expansion. Here $\mathcal{H}_{\text{eff}}^{\Delta S=1,2}(x)$ denote the standard $\Delta S=1$ or 2 four–quark effective Hamiltonians. The spectral functions associated to these correlators describe in an inclusive way transitions from the vacuum to physical states with total strangeness $S=1$ or 2. They have been calculated in perturbation theory to next–to–leading logarithmic order in refs. [17, 29]. The results of these calculations give gluonic corrections of rather normal size for the $(27_L, 1_R)$ correlators (i.e., for $\Delta S=2$ transitions and $\Delta S=1$ transitions with $\Delta I=3/2$) and a big enhancement in the $(8_L, 1_R)$ correlator. The enhancement disappears completely when only the large–N_c limit component of the gluonic corrections is retained.

To simplify the discussion to the essential point let us restrict ourselves to the non–penguin operators $Q_\pm \equiv Q_2 \pm Q_1$ and consider the spectral functions associated with the $C_\pm(\mu^2)Q_\pm$ terms in the $\Delta S=1$ Hamiltonian in the absence of penguin–like contributions. The corresponding results from ref. [29] can then be written as follows:

$$\frac{1}{\pi}\text{Im}\Psi_{\pm\pm}(t) = \theta(t)\frac{2}{45}N_c^2(1\pm\frac{1}{N_c})\frac{t^4}{(4\pi)^6}\alpha_s(t)^{-2a_\pm}\,C_\pm^2(M_W^2)\left[1+\frac{3}{4}\frac{\alpha_s(t)N_c}{\pi}\mathcal{K}_\pm\right], \tag{13}$$

where $a_\pm = \pm\frac{9}{11N_c}\frac{1\mp 1/N_c}{1-6/11N_c}$ and

$$\mathcal{K}_+ = 1-\frac{30587}{3630}\frac{1}{N_c}+\frac{164936}{19965}\frac{1}{N_c^2}-\frac{51591}{14641}\frac{1}{N_c^3}+\frac{440193}{322102}\frac{1}{N_c^4}+\cdots = -\frac{3649}{3645}, \tag{14}$$

$$\mathcal{K}_- = 1 + \frac{30587}{3630}\frac{1}{N_c} + \frac{169706}{19965}\frac{1}{N_c^2} + \frac{70335}{14641}\frac{1}{N_c^3} + \frac{1810209}{322102}\frac{1}{N_c^4} + \cdots = +\frac{18278}{3645}. \quad (15)$$

A very revealing pattern emerges when the coefficients $\mathcal{K}_\pm$ of the $\mathcal{O}(\alpha_s)$ corrections are expanded in powers of $1/N_c$ as shown above. In the large N_c limit $\mathcal{K}_- = \mathcal{K}_+$ and the two spectral functions coincide. The $\mathcal{O}(1/N_c)$ corrections to these coefficients are enormous, and modify the spectral functions by the same amount but in opposite directions: $\frac{1}{\pi}\mathrm{Im}\Psi_{--}$ gets a large enhancement while $\frac{1}{\pi}\mathrm{Im}\Psi_{++}$ is strongly suppressed. Although the higher order $1/N_c$–corrections are smaller than those to next–to–leading order, they still have an important overall numerical effect when compared to the exact results. This is because in $\mathcal{K}_+$, the alternating signs of the first five terms of the series in powers of $1/N_c$ produce a compensating effect, while in $\mathcal{K}_-$ all the terms have the same positive sign which results in an important further enhancement.

We propose to compare the <u>relative</u> $1/N_c$–dependence of the spectral functions $\frac{1}{\pi}\mathrm{Im}\Psi_{\pm\pm}$ calculated in perturbation theory with those obtained to lowest order in χPT in the chiral limit [30], and in the $1/N_c$–expansion. We denote by $\frac{1}{\pi}\mathrm{Im}\Psi_{8,27}$ the spectral functions associated to the effective chiral Lagrangians $\mathcal{L}_{8,27}$ in eqs. (2) and (3). Then, with $\mathbf{g}_8 = \mathbf{g}_8^- + \mathbf{g}_8^+$, the "equivalent" spectral functions are:

$$|\mathbf{g}_8^-|^2 \mathrm{Im}\Psi_8 \sim \mathrm{Im}\Psi_{--}, \quad (16)$$

$$|\mathbf{g}_8^+|^2 \mathrm{Im}\Psi_8 \sim \left(\frac{1}{5}\right)^2 \mathrm{Im}\Psi_{++} \qquad \text{and} \qquad |\mathbf{g}_{27}|^2 \mathrm{Im}\Psi_{27} \sim \left(\frac{6}{5}\right)^2 \mathrm{Im}\Psi_{++}. \quad (17)$$

We find that the terms of relative $\mathcal{O}(1)$ and $\mathcal{O}(1/N_c)$ in both types of spectral functions i.e., those obtained from the effective chiral Lagrangian and those obtained in perturbation theory, have exactly the same correlation of signs as the one implied by eqs. (8) in the limit where $\mathbf{a} = 1$ and in the absence of penguins.

We shall use this comparison of relative $1/N_c$–dependence of spectral functions as a way to suggest a plausible pattern of the $\mathcal{O}(1/N_c)$ and $\mathcal{O}(1/N_c^2)$ contributions to the couplings $\mathbf{a}$ and $\mathbf{b}$. Setting

$$\mathbf{a} = 1 + \alpha\frac{1}{N_c^2} + \mathcal{O}(\frac{1}{N_c^3})\,; \qquad \mathbf{b} = \beta\frac{1}{N_c} + \beta'\frac{1}{N_c^2} + \mathcal{O}(\frac{1}{N_c^3})\,, \quad (18)$$

results then in the following equivalence relations:

$$\alpha \sim \frac{9}{22}\ln\left[\frac{\alpha_s(t)}{\alpha_s(M_W^2)}\right]\left\{1 + \frac{30587}{4840}\frac{\alpha_s(t)N_c}{\pi}\right\} + \frac{81}{242}\ln^2\left[\frac{\alpha_s(t)}{\alpha_s(M_W^2)}\right] + \frac{30257}{19360}\frac{\alpha_s(t)N_c}{\pi} - \frac{1}{8}\,, \quad (19)$$

$$\beta \sim -\frac{9}{11}\ln\left[\frac{\alpha_s(t)}{\alpha_s(M_W^2)}\right] - \frac{30587}{9680}\frac{\alpha_s(t)N_c}{\pi} + \frac{1}{2}\,, \quad (20)$$

$$\beta' \sim -\frac{54}{121}\ln\left[\frac{\alpha_s(t)}{\alpha_s(M_W^2)}\right] - \frac{477}{10648}\frac{\alpha_s(t)N_c}{\pi}\,. \tag{21}$$

We insist on the fact that these relations are not equalities. The derivation of quantitative relations would require the use of dispersion relations and a precise knowledge of the hadronic spectral functions at intermediate energies which unfortunately is not available. The relations above only show the type of $1/N_c$– corrections in the effective couplings which emerge if one assumes that the $1/N_c$–behaviour of the short–distance correlators is a universal feature of the full hadronic spectral function. The pattern suggested by this comparison is nevertheless rather interesting. As expected, the term corresponding to β is large and negative. It also shows that the $\mathcal{O}(1/N_c^2)$ term which corresponds to α contributes with positive corrections, which tend to cancel in the combination $\mathbf{a}+\mathbf{b}$. The $\mathcal{O}(1/N_c^2)$ corrections to $\mathbf{b}$ corresponding to β' have a much smaller size.

5. There are some conclusions we can draw from the previous analyses:

The phenomenological result $|\mathbf{g}_{27}| \simeq 0.29$ can be easily digested to lowest order in the chiral expansion and to next–to–leading order in the $1/N_c$–expansion. It requires a negative value for the coupling constant $\mathbf{b}$ to $\mathcal{O}(1/N_c)$. This also helps to explain part of the $\Delta I = 1/2$ enhancement, but a quantitative understanding of the phenomenological result $|\mathbf{g}_8| \simeq 3.6$ obtained with inclusion of the $\mathcal{O}(p^4)$ chiral corrections is still lacking at this level. As already mentioned, a negative $\mathbf{b}$–coupling is a common result of various model calculations. However, in order to explain *both* $|\mathbf{g}_{27}| \simeq 0.29$ and $|\mathbf{g}_8| \simeq 3.6$ one still needs sizable higher $\mathcal{O}(1/N_c^2)$ positive contributions to the $\mathbf{a}$–coupling constant which partly compensate in the sum $\mathbf{a}+\mathbf{b}$, the large and negative $\mathbf{b}$–coupling which is needed to get the $\Delta I = 1/2$ enhancement. No model so far has been produced which shows this convincingly; but it is interesting that both requirements appear to be compatible with the pattern of short–distance inclusive calculations discussed above.

The early chiral symmetry prediction [20] $\mathbf{B}_K \sim 0.35$ appears then as a natural result within this scenario, but the discrepancy of this prediction with the numerical estimates of $\mathbf{B}_K$ obtained by the lattice QCD simulations pose a serious puzzle which requires further comments on our part.

If one interprets the large lattice results as chiefly due to the fact that $\mathbf{b}$ is a very small negative quantity or even a positive one then, from the analysis above, it follows that the bulk of the $\Delta I = 1/2$ enhancement has to come from penguin–like configurations i.e., a large and positive value for the $\mathbf{c}$ coupling constant¶. If that is the case we have then to understand why here the $1/N_c$–expansion, at its first non–trivial level, breaks down so dramatically. The QCD perturbative calculation of the asymptotic spectral function associated to the penguin Q_6–operator made in ref. [17] shows in fact little difference between the leading result and the one including subleading terms in the $1/N_c$–expansion. There is also another well known problem in this case, which is that the predicted value for $\Delta I = 3/2$ transitions comes out

¶ Notice that such large corrections would also imply a large ϵ'/ϵ value.

too large: $\mathbf{b} \geq 0$, results in $\mathbf{g}_{27} \geq 0.6$ i.e., at least a factor of two bigger than the phenomenological determination in eq. (5).

If on the other hand we assume that the estimate $\mathbf{b} \simeq -0.5$ is in the right ballpark, then the large lattice results for the $\mathbf{B}_K$–factor imply that the chiral corrections to $\Delta S = 2$ transitions have to be as large as $\mathcal{O}(100\%)$! Where could such an enormous correction come from? The chiral loop corrections to the $K^0 - \bar{K}^0$ transition amplitude have been evaluated by several groups [19, 31–33] and it is now known that, once the terms which renormalize the f coupling in the chiral limit to the physical f_K are factorized, the rest of the corrections do not have large chiral logarithmic terms. Possible large corrections can then only come from the local $\mathcal{O}(p^4)$ terms of the $\Delta S = 2$ effective chiral Lagrangian. The model calculations of these couplings which so far have been made [32, 33] give results which are still controversial. These calculations are impressive but difficult to interpret. For example, the results of the $\mathbf{B}_K$–factor obtained in ref. [33] turn out to be too dependent on the choice of the cut–off which in their approach is supposed to separate long– and short–distances contributions. Further progress in this direction is indeed possible and hopefully will be made; but we are not there yet.

In the mean time, it seems fair to conclude that there is still, unfortunately, a large theoretical uncertainty in our knowledge of the $\mathbf{B}_K$–parameter. We do not understand the physics behind sufficiently well as yet to restrict the error bars to those of our favourite calculation as it is done in many phenomenological analyses of the unitarity triangle constraints. It is important to keep in mind that the ultimate purpose of these analyses is to *test the Standard Model* and <u>not</u> some particular QCD estimate of a hadronic matrix element.

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Figure Captions

Fig. 1: Diagrammatic representation of the three effective couplings in eq. (6). The solid lines represent quark fields propagating in a gluon background simulated by the dotted lines. The $Q_L^{(\pm)}$–operators represent the emission and absorption of the virtual W–field. The restriction to lowest order in the chiral expansion implies that at most two $\mathcal{L}_\mu$ insertions are allowed.

Fig. 2: Two gluons exchanged from one fermion loop to the other are at least required to modify their factorization and this leads to a relative correction of $\mathcal{O}(1/N_c^2)$.

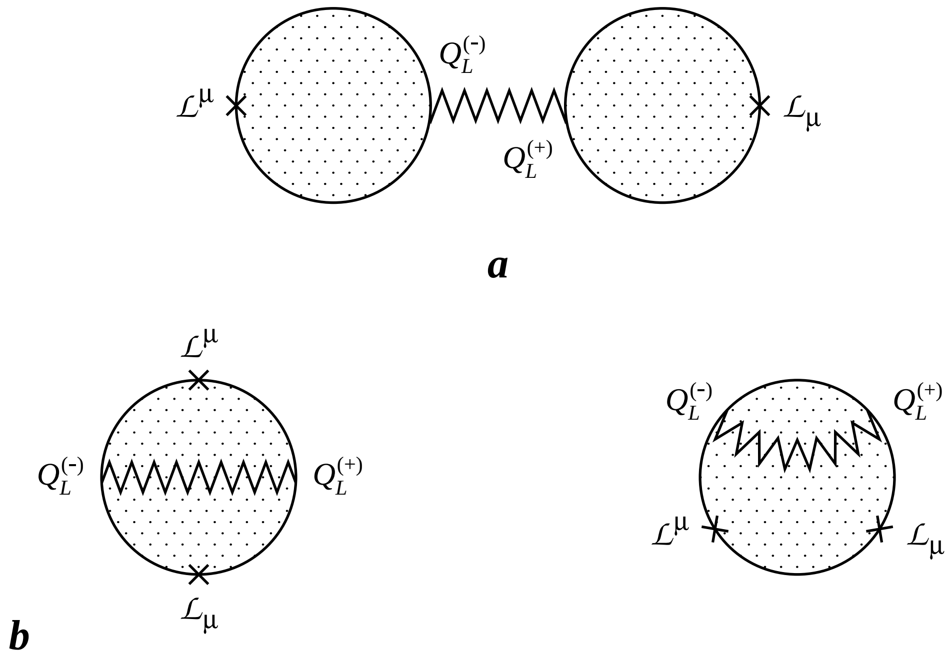
$\mathcal{L}^{\mu}$
$Q_L^{(-)}$
$Q_L^{(+)}$
$\mathcal{L}_{\mu}$
a
$\mathcal{L}^{\mu}$
$Q_L^{(-)}$
$Q_L^{(+)}$
$\mathcal{L}_{\mu}$
b
$Q_L^{(-)}$
$Q_L^{(+)}$
$\mathcal{L}^{\mu}$
$\mathcal{L}_{\mu}$
c

Fig. 1

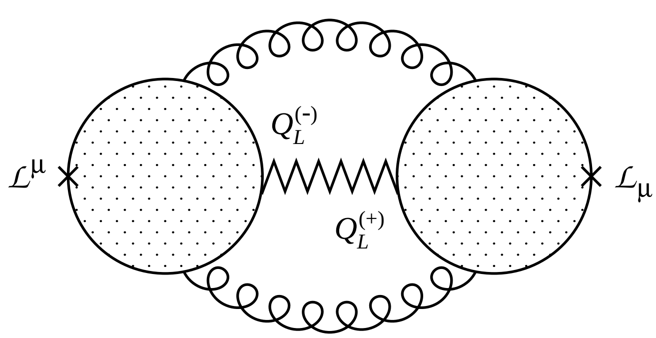
$\mathcal{L}^{\mu}$
$Q_L^{(-)}$
$Q_L^{(+)}$
$\mathcal{L}_{\mu}$

Fig. 2