Electromagnetic Decays of Heavy Baryons

M.C. Bañuls, A. Pich, I. Scimemi

Departament de Física Teòrica, IFIC, Universitat de València – CSIC
E-46100 Burjassot (València), Spain

Abstract

The electromagnetic decays of the ground state baryon multiplets with one heavy quark are calculated using Heavy Hadron Chiral Perturbation Theory. The M1 and E2 amplitudes for $S^* \to S\gamma$, $S^* \to T\gamma$ and $S \to T\gamma$ are separately computed. All M1 transitions are calculated up to $\mathcal{O}(1/\Lambda_\chi^2)$. The E2 amplitudes contribute at the same order for $S^* \to S\gamma$, while for $S^* \to T\gamma$ they first appear at $\mathcal{O}(1/(m_Q\Lambda_\chi^2))$ and for $S \to T\gamma$ are completely negligible. The renormalization of the chiral loops is discussed and relations among different decay amplitudes are derived. We find that chiral loops involving electromagnetic interactions of the light pseudoscalar mesons provide a sizable enhancement of these decay widths. Furthermore, we obtain an absolute prediction for $\Gamma(\Xi_c^{0'(*)} \to \Xi_c^0\gamma)$ and $\Gamma(\Xi_b^{-'(*)} \to \Xi_b^-\gamma)$. Our results are compared to other estimates existing in the literature.

1 Introduction

In some kinematical regions, which are not far from the chiral and heavy quark limits, both Chiral Perturbation [1] and Heavy Quark Effective Theories (HQET) [2] can be simultaneously used. In the $m_Q \to \infty$ limit, baryons containing a heavy quark, can emit and absorb light pseudoscalar mesons without changing its velocity v. In Heavy Hadron Chiral Perturbation Theory (HHCPT) one constructs an effective Lagrangian whose basic fields are heavy hadrons and light mesons [3]-[6]. In ref. [7], the formalism is extended to include also electromagnetism.

We use this hybrid effective Lagrangian to calculate the electromagnetic decay width of the ground state baryons containing a c or a b quark. We consider the decays $S^* \to S\gamma$ and $S^{(*)} \to T\gamma$. For most of these decays the available phase space is small, so that the emission of a pion is suppressed or even forbidden and the electromagnetic process becomes relevant. Some of these decays are starting to be measured [8], which makes necessary to perform a detailed theoretical analysis.

Some theoretical calculations of these decays can be already found in the literature. The $\mathcal{O}(1/\Lambda_\chi)$ amplitudes were first computed in ref. [7], using HHCPT. A more detailed analysis was presented in ref. [9], where the widths $\Gamma(S_c \to T_c\gamma)$ are estimated using heavy–quark and chiral symmetries implemented within the non-relativistic quark model. A similar procedure is followed in ref. [10], where the heavy–quark symmetry is supplemented with light–diquark symmetries to calculate the widths $\Gamma(\Sigma_c^+ \to \Lambda_c^+\gamma)$ and $\Gamma(\Sigma_{c,b}^* \to \Sigma_{c,b}\gamma)$. The authors of ref. [11] apply the relativistic quark model to predict the electromagnetic decays $\Gamma(S_c^{(*)} \to T_c\gamma)$ and $\Gamma(\Sigma_c^{+*} \to \Sigma_c^+\gamma)$. In ref. [12], $\Gamma(\Sigma_b^* \to \Sigma_b\gamma)$ and $\Gamma(\Sigma_b^{0(*)} \to \Lambda_b^0\gamma)$ are computed with light cone QCD sum rules at leading order in HQET. All these references consider only transitions of the M1 type. Finally, ref. [13] estimates the ratio of the E2 and M1 amplitudes for $\Gamma(\Sigma_c^{+*} \to \Lambda_c^+\gamma)$.

Here, we study all possible $S^{(*)} \to T\gamma$ and $S^* \to S\gamma$ decays in the context of HHCPT, considering both M1 and E2 transitions. Section 2 collects the HHCPT formalism as introduced in ref. [7]: the effective fields representing S and T baryons, the lowest order chiral Lagrangian and the $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(1/\Lambda_\chi)$ terms. In order to renormalize the resulting chiral loops, the introduction of higher–order operators with unknown couplings is required. In the case of $S^* \to S\gamma$, we calculate all contributions up to $\mathcal{O}(1/\Lambda_\chi^2)$ for M1 and E2 transitions. We find that all divergences and scale dependence can be absorbed in the redefinition of one $\mathcal{O}(1/\Lambda_\chi)$ coupling for each type of process (M1, E2). These results are presented in section 3. Section 4 describes the analogous calculation for $S^* \to T\gamma$; in this case, the E2 contribution has to be computed up to $\mathcal{O}(1/m_Q\Lambda_\chi^2)$, which requires two additional couplings. The decays $S \to T\gamma$ are analyzed in section 5; as in the previous cases the M1 amplitude is calculated up to $\mathcal{O}(1/\Lambda_\chi^2)$, while the E2 contribution is found to be $\mathcal{O}(1/m_Q^3\Lambda_\chi^2)$ and thus extremely suppressed. In each section we derive relations among amplitudes for different baryons within the same multiplet and between charm and bottom baryons. These relations are valid at lowest order in HHCPT and we prove that they still hold after one–loop chiral corrections are included. Comparing our expectation for the widths to the leading order HHCPT estimate, we find that the infrared effect due to electromagnetic interactions of

1

light pseudoscalar mesons can greatly enhance these widths. This is particularly true for the E2 contributions which are found to be infrared divergent in the exact chiral limit. We also give some comments on results existing in the literature. Finally, section 6 summarizes our conclusions.

2 HHCPT formalism for magnetic moments

The light degrees of freedom in the ground state of a baryon with one heavy quark can be either in a $s_l = 0$ or in a $s_l = 1$ configuration. The first one corresponds to $J^P = \frac{1}{2}^+$ baryons, which are annihilated by $T_i(v)$ fields transforming as a $\bar{\mathbf{3}}$ under the chiral subgroup $SU(3)_{L+R}$ and as a doublet under the HQET $SU(2)_v$. In the second case, $s_l = 1$, the spin of the heavy quark and the light degrees of freedom combine together to form $J^P = \frac{3}{2}^+$ and $J = \frac{1}{2}^+$ baryons, which are degenerate in mass in the $m_Q \to \infty$ limit. The spin-$\frac{3}{2}$ ones are annihilated by the Rarita–Schwinger field $S_\mu^{*ij}(v)$, while the spin-$\frac{1}{2}$ baryons are destroyed by the Dirac field $S^{ij}(v)$. It is very useful to combine both operators into the so-called superfield [14, 15]

$$S_\mu^{ij}(v) = \sqrt{\frac{1}{3}}\,(\gamma_\mu + v_\mu)\,\gamma^5\,S^{ij}(v) + S_\mu^{*ij}(v)\,,$$

$$\bar{S}_{ij}^\mu(v) = -\sqrt{\frac{1}{3}}\,\bar{S}_{ij}(v)\,\gamma^5\,(\gamma^\mu + v^\mu) + \bar{S}_{ij}^{*\mu}(v)\,, \tag{1}$$

which transforms as a $\mathbf{6}$ under $SU(3)_{L+R}$ and as a doublet under $SU(2)_v$ and is symmetric in the i, j indices.

The particle assignment for the $J = 1/2$ charmed baryons of the $\bar{\mathbf{3}}$ and $\mathbf{6}$ representations is

$$(T_1, T_2, T_3) = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)\,, \tag{2}$$

$$S^{ij} = \begin{pmatrix} \Sigma_c^{++} & \sqrt{\frac{1}{2}}\Sigma_c^+ & \sqrt{\frac{1}{2}}\Xi_c^{+'} \\ \sqrt{\frac{1}{2}}\Sigma_c^+ & \Sigma_c^0 & \sqrt{\frac{1}{2}}\Xi_c^{0'} \\ \sqrt{\frac{1}{2}}\Xi_c^{+'} & \sqrt{\frac{1}{2}}\Xi_c^{0'} & \Omega_c^0 \end{pmatrix}\,, \tag{3}$$

and the corresponding bottom baryons are

$$(T_1, T_2, T_3) = (\Xi_b^-, -\Xi_b^0, \Lambda_b^0)\,, \tag{4}$$

$$S^{ij} = \begin{pmatrix} \Sigma_b^+ & \sqrt{\frac{1}{2}}\Sigma_b^0 & \sqrt{\frac{1}{2}}\Xi_b^{0'} \\ \sqrt{\frac{1}{2}}\Sigma_b^0 & \Sigma_b^- & \sqrt{\frac{1}{2}}\Xi_b^{-'} \\ \sqrt{\frac{1}{2}}\Xi_b^{0'} & \sqrt{\frac{1}{2}}\Xi_b^{-'} & \Omega_b^- \end{pmatrix}\,. \tag{5}$$

The $J = 3/2$ partners of the baryons in Eq. (3) and Eq. (5) have the same $SU(3)_V$ assignments in S_μ^{*ij}.

2

Goldstone bosons are parametrized as

$$
\Phi = \begin{pmatrix} \sqrt{\tfrac{1}{2}}\pi^0 + \sqrt{\tfrac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\tfrac{1}{2}}\pi^0 + \sqrt{\tfrac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\tfrac{2}{3}}\eta \end{pmatrix} , \tag{6}
$$

and appear in the Lagrangian via the exponential representation $\xi \equiv \exp(i\Phi/\sqrt{2}f_\pi)$, being $f_\pi \sim 93$ MeV the pion decay constant.

The lowest–order chiral Lagrangian describing the soft hadronic and electromagnetic interactions of these baryons in the infinite heavy quark mass limit is given by [7]

$$
\begin{aligned}
\mathcal{L}^{(0)} &= -i\, \bar{S}^\mu_{ij}\,(v \cdot D) S^{ij}_\mu + \Delta_{ST}\, \bar{S}^\mu_{ij}\, S^{ij}_\mu + i\, \bar{T}^i\,(v \cdot D) T_i \\
&\quad + i\, g_2\, \varepsilon_{\mu\nu\sigma\lambda}\, \bar{S}^\mu_{ik} v^\nu (\xi^\sigma)^i_j (S^\lambda)^{jk} + g_3 \left[\epsilon_{ijk}\, \bar{T}^i(\xi^\mu)^j_l S^{kl}_\mu + \epsilon^{ijk}\, \bar{S}^\mu_{kl}(\xi_\mu)^l_j T_i \right] .
\end{aligned} \tag{7}
$$

In this formula, the heavy–baryon covariant derivatives are

$$
\begin{aligned}
D^\mu S^{ij}_\nu &= \partial^\mu S^{ij}_\nu + (\Gamma^\mu)^i_k S^{kj}_\nu + (\Gamma^\mu)^j_k S^{ik}_\nu - ie\mathcal{A}^\mu [Q_Q S^{ij}_\nu + Q^i_k S^{kj}_\nu + Q^j_k S^{ik}_\nu] , \\
D^\mu T_i &= \partial^\mu T_i - T_j(\Gamma^\mu)^j_i - ie\mathcal{A}^\mu [Q_Q T_i - T_j Q^j_i] ,
\end{aligned} \tag{8}
$$

where $\mathcal{A}^\mu$ is the electromagnetic field, Q_Q the heavy–quark charge, the light–quark charge matrix Q is given by

$$
\mathbf{Q} = \begin{pmatrix} \tfrac{2}{3} & & \\ & -\tfrac{1}{3} & \\ & & -\tfrac{1}{3} \end{pmatrix} , \tag{9}
$$

and the Goldstone mesons appear through axial–vector, ξ_μ, and vector, Γ_μ, fields

$$
\xi_\mu = \frac{i}{2}\left(\xi D_\mu \xi^\dagger - \xi^\dagger D_\mu \xi\right) , \qquad \Gamma_\mu = \frac{1}{2}\left(\xi D_\mu \xi^\dagger + \xi^\dagger D_\mu \xi\right) , \tag{10}
$$

with $D^\mu \xi = \partial^\mu \xi - ie\mathcal{A}^\mu[Q, \xi]$.

Because of the different spin configuration of the light degrees of freedom there is an intrinsic mass difference, $\Delta_{ST} \equiv M_S - M_T$, between the sextet and triplet baryon multiplets. Notice that a direct coupling of the pseudo-Goldstone bosons to the $\bar{\mathbf{3}}$ baryons is forbidden at lowest order in $1/\Lambda_\chi$.

The first contributions to the transitions we are considering come from:

1) the next order $(D = 5)$ in the baryon chiral Lagrangian [7]

$$
\begin{aligned}
\mathcal{L}^{(long)} &= \frac{e}{\Lambda_\chi}\Big\{ i\, c_S\, \mathrm{tr}\left[\bar{S}_\mu Q S_\nu + \bar{S}_\mu S_\nu Q\right] F^{\mu\nu} \\
&\quad + c_{ST}\left[\epsilon_{ijk}\, \bar{T}^i v_\mu Q^j_l S^{kl}_\nu + \epsilon^{ijk}\, \bar{S}_{\nu,kl} v_\mu Q^l_j T_i\right] \tilde{F}^{\mu\nu} \Big\} ,
\end{aligned} \tag{11}
$$

where c_S and c_{ST} are unknown chiral couplings and $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$. We will take $\Lambda_\chi = 4\pi f_\pi \simeq 1.2$ GeV, which fixes the normalization of these couplings. A long–distance magnetic

moment interaction for just the T baryons does not exist, since their light quarks are in a $s_l = 0$ configuration.

2) terms of order $1/m_Q$ in the heavy quark expansion which break both spin and flavor symmetries [7]

$$\mathcal{L}^{(short)} = -\frac{1}{2m_Q}\,\bar{S}^\lambda_{ij}(iD)^2 S^{ij}_\lambda - \frac{eQ_Q}{4m_Q}\,\bar{S}^\lambda_{ij}\sigma_{\mu\nu}S^{ij}_\lambda\,F^{\mu\nu} + \frac{1}{2m_Q}\,\bar{T}^i(iD)^2 T_i + \frac{eQ_Q}{4m_Q}\,\bar{T}^i\sigma_{\mu\nu}T_i\,F^{\mu\nu}\;;$$
(12)

3) chiral loops of Goldstone bosons coupled to photons, as described by the lowest–order Lagrangian.

3 Results for $S^* \to S\gamma$ decays

We will decompose our results in two different amplitudes

$$\mathcal{A}\,(B^* \to B\,\gamma)\;=\;A_{M1}\,\mathcal{O}_{M1} + A_{E2}\,\mathcal{O}_{E2}\,,$$
(13)

where the corresponding M1 and E2 operators are defined by

$$\begin{aligned}
\mathcal{O}_{M1} &=& e\,\bar{B}\gamma_\mu\gamma_5 B^*_\nu\,F^{\mu\nu},\\
\mathcal{O}_{E2} &=& i\,e\,\bar{B}\gamma_\mu\gamma_5 B^*_\nu\,v_\alpha\,(\partial^\mu F^{\alpha\nu} + \partial^\nu F^{\alpha\mu})\,,
\end{aligned}$$
(14)

The leading contributions to M1 transitions come from the light– and heavy–quark magnetic interactions which are of $\mathcal{O}(1/\Lambda_\chi)$ and $\mathcal{O}(1/m_Q)$, respectively. We have computed the next-to-leading chiral corrections of $\mathcal{O}(1/\Lambda_\chi^2)$, which originate from the loop diagrams shown in fig. 1.
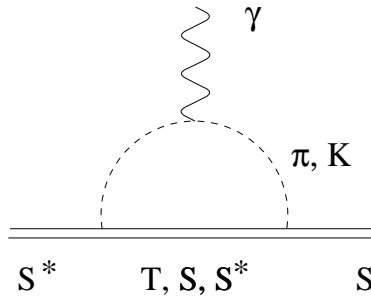

Figure 1: Meson loops contributing to $S^* \to S\gamma$.

The resulting M1 amplitudes can be written as:

$$A_{M1}(B^*) = \frac{1}{\sqrt{3}}\left(-\frac{Q_Q}{m_Q} - \frac{2c_s}{3\Lambda_\chi}\,a_\chi(B^*) + g_2^2\,\frac{\Delta_{ST}}{4(4\pi f_\pi)^2}\,a_{g_2}(B^*) + g_3^2\,\frac{m_K}{4\pi f_\pi^2}a_{g_3}\,(B^*)\right)\;.$$
(15)

4

c quark	b quark	a_χ	a_{g_2}	a_{g_3}
$\Sigma_c^{++*} \to \Sigma_c^{++}\gamma$	$\Sigma_b^{+*} \to \Sigma_b^{+}\gamma$	2	$I_\pi + I_K$	$1 + m_\pi/m_K$
$\Sigma_c^{+*} \to \Sigma_c^{+}\gamma$	$\Sigma_b^{0*} \to \Sigma_b^{0}\gamma$	$1/2$	$I_K/2$	$1/2$
$\Sigma_c^{0*} \to \Sigma_c^{0}\gamma$	$\Sigma_b^{-*} \to \Sigma_b^{-}\gamma$	-1	$-I_\pi$	$-m_\pi/m_K$
$\Xi_c^{0'*} \to \Xi_c^{0'}\gamma$	$\Xi_b^{-'*} \to \Xi_b^{-'}\gamma$	-1	$-(I_\pi + I_K)/2$	$-(1 + m_\pi/m_K)/2$
$\Xi_c^{+'*} \to \Xi_c^{+'}\gamma$	$\Xi_b^{0'*} \to \Xi_b^{0'}\gamma$	$1/2$	$I_\pi/2$	$m_\pi/(2m_K)$
$\Omega_c^{0*} \to \Omega_c^{0}\gamma$	$\Omega_b^{-*} \to \Omega_b^{-}\gamma$	-1	$-I_K$	-1

Table 1: Contributions to M1 amplitudes for $S^* \to S\gamma$.

In Table 1 we show the values of the coefficients $a_i(B^*)$ for the decays of baryons containing one charm or bottom quark. In the table,

$$I_i \equiv I(\Delta_{ST}, m_i) = 2\left(-2 + \log \frac{m_i^2}{\mu^2}\right) + 2\frac{\sqrt{\Delta_{ST}^2 - m_i^2}}{\Delta_{ST}} \log\left(\frac{\Delta_{ST} + \sqrt{\Delta_{ST}^2 - m_i^2}}{\Delta_{ST} - \sqrt{\Delta_{ST}^2 - m_i^2}}\right) . \quad (16)$$

Due to flavor symmetry, all contributions are equal for charm and bottom baryons, with the only exception of the term proportional to the heavy quark electric charge ($Q_c = +2/3$, $Q_b = -1/3$). The calculation of the decay amplitudes closely follows the one reported in ref. [16] for the magnetic moments of the $S^{(*)}$ baryons. Thus, we list the arguments common to both calculations:

1. contributions of $\mathcal{O}(1/(m_Q\Lambda_\chi))$ can be neglected for the b baryons. For the c baryons, they are expected to be smaller than 15% [16].

2. the corrections proportional to g_2^2 are obtained performing a one–loop integral (fig. 1 with an S baryon running in the loop) that has to be renormalized. The divergent part of the integral does not depend on the pion or kaon masses and is instead proportional to the mass of the baryon running in the loop. If one considers both pion and kaon loops the divergent part respects the $SU(3)$ structure of the chiral multiplet and can be canceled with an operator of the form

$$i\frac{e}{\Lambda_\chi^2} \text{tr}\left[\bar{S}_\mu (v \cdot D S_\nu) Q - \left(v \cdot D \bar{S}_\mu\right) S_\nu Q\right] F^{\mu\nu} . \quad (17)$$

This is the most general dimension–6 chiral– and Lorentz–invariant operator, constructed out from S_μ^{ij} and $Q F^{\mu\nu}$, preserving parity and time–reversal invariance, which contributes to the M1 amplitudes. When the equation of motion $i (v \cdot D) S_\mu = \Delta_{ST} S_\mu$ is applied, its contribution is of the same form as the term proportional to c_s in Eq. (11). Thus, the local contribution from the operator in Eq. (17) can be taken into account, together with the lowest–order term in Eq. (11), through an effective coupling $c_S(\mu)$. The scale μ dependence of the loop integrals is exactly canceled by the corresponding dependence of the coefficient $c_S(\mu)$;

3. the contribution proportional to g_3^2 involves a loop integral with a baryon of the T multiplet running in the loop. Since the Lagrangian does not have any mass term for T baryons, the result of the integral is convergent and proportional to the mass of the light mesons.

In order to see the behavior of $I(\Delta_{ST}, m)$ with the meson mass we have plotted it in fig. 2, for $\mu = 1$ GeV and Δ_{ST} as in Table 2. We see that the value of $I(\Delta_{ST}, m)$ raises considerably in the limit of zero Goldstone-boson mass.
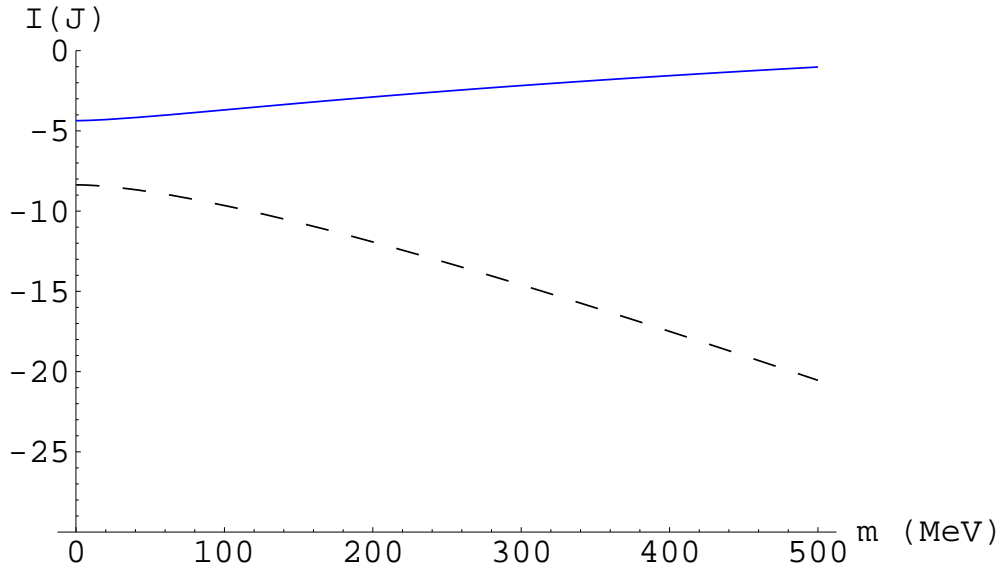

Figure 2: The scaling of the functions $I(\Delta_{ST}, m)$, Eq. (16), and $J(\Delta_{ST}, m)$, Eq. (23) as a function of the meson mass m. The dashed line is $I(\Delta_{ST}, m)$ and the continuous line is $J(\Delta_{ST}, m)$. The scale μ is fixed at 1 GeV and $\Delta_{ST} = 168$ MeV.

f_π	93 MeV
m_π	140 MeV
m_K	496.7 MeV
Δ_{ST}	168 MeV
m_c	1.3 GeV
$\alpha_{em}(m_\tau)$	1/133.3
m_b	4.8 GeV

Table 2: Constants used in numerical estimates.

From Table 1, one can derive the following linearly–independent relations for the M1 amplitudes of the $S^* \to S\gamma$ decays containing a charm quark:

$$A_{M1}(\Sigma_c^{++*}) = 2A_{M1}(\Sigma_c^{+*}) - A_{M1}(\Sigma_c^{0*}) = 2A_{M1}(\Xi_c^{+'*}) - A_{M1}(\Omega_c^{0*})$$

$$A_{M1}(\Sigma_c^{++*}) + 2A_{M1}(\Xi_c^{0\prime*}) \;\; = \;\; A_{M1}(\Sigma_c^{0*}) + 2A_{M1}(\Xi_c^{+\prime*}) = -\frac{2}{\sqrt{3}m_c} \;. \tag{18}$$

The $\mathcal{O}(1/\Lambda_\chi)$ and $\mathcal{O}(1/\Lambda_\chi^2)$ contributions cancel in the sum of the six $S^* \to S\gamma$ M1 amplitudes. Therefore, the average over the baryon sextet measures the $O(1/m_Q)$ contribution.

We can write four analogous relations for the bottom baryons:

$$A_{M1}(\Sigma_b^{+*}) \;\; = \;\; 2A_{M1}(\Sigma_b^{0*}) - A_{M1}(\Sigma_b^{-*}) = 2A_{M1}(\Xi_b^{0\prime*}) - A_{M1}(\Omega_b^{-*})$$

$$A_{M1}(\Sigma_b^{+*}) + 2A_{M1}(\Xi_b^{-\prime*}) \;\; = \;\; A_{M1}(\Sigma_b^{-*}) + 2A_{M1}(\Xi_b^{0\prime*}) = \frac{1}{\sqrt{3}m_b} \tag{19}$$

Two additional equations relate b and c baryons:

$$A(\Sigma_c^{+*}) - A(\Sigma_c^{++*}) \;\; = \;\; A(\Sigma_b^{0*}) - A(\Sigma_b^{+*})$$

$$A(\Sigma_c^{+*}) - A(\Xi_c^{+\prime*}) \;\; = \;\; A(\Sigma_b^{0*}) - A(\Xi_b^{0\prime*}) \;. \tag{20}$$

The same diagram in fig. 1 generates the leading contributions to E2 transitions. The graph is of $\mathcal{O}(1/\Lambda_\chi^2)$ and one has to include all chiral counterterms up to this order. There is only one operator with these features,

$$\frac{i}{4} \frac{e \, c_S^{E2}}{\Lambda_\chi^2} \, \mathrm{tr}\left[\bar{S}_\mu Q S_\nu + \bar{S}_\mu S_\nu Q \right] v_\alpha \, (\partial^\mu F^{\alpha\nu} + \partial^\nu F^{\alpha\mu}) \;, \tag{21}$$

and so only one new unknown constant (c_S^{E2}) appears. The E2 amplitudes can be written analogously to the M1 case:

$$A_{E2}(B^*) \;\; = \;\; \frac{1}{6\sqrt{3}} \left(\frac{c_S^{E2}}{\Lambda_\chi^2} \, b_\chi(B^*) - \frac{g_2^2}{4(4\pi f_\pi)^2} \, b_{g_2}(B^*) - \frac{g_3^2}{4\pi f_\pi^2} \, b_{g_3}(B^*) \right) \;. \tag{22}$$

The coefficients b_i are shown in table 3, where

$$J_i \;\; \equiv \;\; J(\Delta_{ST}, m_i) = \frac{\partial}{\partial \Delta_{ST}} \left[\Delta_{ST} I(\Delta_{ST}, m_i) \right] \;,$$

$$J_i^0 \;\; = \;\; \lim_{\Delta \to 0} J_i = -1 - i\pi + \log m_i/\mu \;. \tag{23}$$

The scale dependence of $c_S^{E2}(\mu)$ cancels the one coming from the loop calculation. While the behavior of $J(\Delta_{ST}, m_i)$ does not change much when one varies the meson mass (see fig. 2), J_i^0 is infrared divergent in the exact chiral limit. This divergence can be responsible for a considerable enhancement of the electric dipole effects.

The M1 and E2 amplitudes have identical $SU(3)$ structure. Therefore, we can construct for the E2 amplitudes exactly the same relations as in the M1 case [Eq. (18-20)]. However as there are no $1/m_Q$ terms contributing to E2, the last equations in (18) and (19) must be replaced by

$$A_{E2}(\Sigma_c^{++*}) + 2A_{E2}(\Xi_c^{0\prime*}) = A_{E2}(\Sigma_c^{0*}) + 2A_{E2}(\Xi_c^{+\prime*}) = 0 \;, \tag{24}$$

and

$$A_{E2}(\Sigma_b^{+*}) + 2A_{E2}(\Xi_b^{-\prime*}) = A_{E2}(\Sigma_b^{-*}) + 2A_{E2}(\Xi_b^{0\prime*}) = 0 \;. \tag{25}$$

7

c quark	b quark	b_χ	b_{g_2}	b_{g_3}
$\Sigma_c^{++*} \to \Sigma_c^{++}\gamma$	$\Sigma_b^{+*} \to \Sigma_b^{+}\gamma$	2	$J_\pi + J_K$	$J_\pi^0 + J_K^0$
$\Sigma_c^{+*} \to \Sigma_c^{+}\gamma$	$\Sigma_b^{0*} \to \Sigma_b^{0}\gamma$	$1/2$	$J_K/2$	$J_K^0/2$
$\Sigma_c^{0*} \to \Sigma_c^{0}\gamma$	$\Sigma_b^{-*} \to \Sigma_b^{-}\gamma$	-1	$-J_\pi$	$-J_\pi^0$
$\Xi_c^{0'*} \to \Xi_c^{0'}\gamma$	$\Xi_b^{-'*} \to \Xi_b^{-'}\gamma$	-1	$-(J_\pi + J_K)/2$	$-(J_\pi^0 + J_K^0)/2$
$\Xi_c^{+'*} \to \Xi_c^{+'}\gamma$	$\Xi_b^{0'*} \to \Xi_b^{0'}\gamma$	$1/2$	$J_\pi/2$	$J_\pi^0/2$
$\Omega_c^{0*} \to \Omega_c^{0}\gamma$	$\Omega_b^{-*} \to \Omega_b^{-}\gamma$	-1	$-J_K$	$-J_K^0$

Table 3: Contributions to E2 amplitudes for $S^* \to S\gamma$.

The electromagnetic decay widths are given by

$$\Gamma(S^* \to S\gamma) = \frac{4\alpha_{em}}{3} \frac{E_\gamma^3 M_S}{M_{S^*}} \left(|A_{M1}|^2 + 3E_\gamma^2|A_{E2}|^2 \right) \ , \tag{26}$$

where M_{S^*} and M_S are the masses of the initial and final baryons and E_γ the energy of the outgoing photon.

The E2 amplitudes come at higher chiral order with respect to the M1 ones. Therefore, the E2 contribution to the total width is suppressed by a factor $(E_\gamma/\Lambda_\chi)^2 \sim 5\%$. In principle, it should be possible to determine experimentally the ratio A_{E2}/A_{M1} by studying the angular distribution of photons from the decay of polarized baryons [13, 17, 18]. The Fermilab E-791 experiment has reported [19] a significant polarization effect on the production of Λ_c baryons, which perhaps could be useful in future measurements of these electromagnetic decays.

In order to provide an absolute theoretical prediction for all the decay widths, it is necessary to have an estimate of the couplings c_S, g_2 and g_3 (we neglect for the moment the small E2 contamination). The couplings g_2 and g_3 have been calculated theoretically [5, 20, 21, 22]; we report the results of these computations in Table 4.

There exists an experimental measurement of g_3 from CLEO coming from the decay $\Sigma_c^* \to \Lambda_c\pi$ [23, 24], $g_3 = 0.99 \pm 0.17$. The direct measurement of g_2 is not possible at present. However, the quark model relates its value to g_3 [24], yielding $g_2 = 1.40 \pm 0.25$.

Model	g_2	g_3
Large N_c [20]	1.88	1.53
Quark model [5]	1.5	1.06
Short–distance QCD sum rule [21]	0.83 ± 0.23	0.67 ± 0.18
Light–cone QCD sum rules [22]	$1.56 \pm 0.3 \pm 0.3$	$0.94 \pm 0.06 \pm 0.2$

Table 4: Theoretical estimates of g_2 and g_3.

The constant c_S is *a priori* unknown and its value should be extracted from the experiment or predicted by some more fundamental model. This coupling appears also in the

calculation of the magnetic moments of $S^{(*)}$ baryons [16]. Thus, the determination of its numerical value via the measurement of any of these electromagnetic decays, would also provide an absolute prediction for the magnetic moments.

Having a numerical determination of the couplings g_2 and g_3, it is possible to derive a scale independent relation between any couple of M1 (E2) amplitudes. The combinations

$$A_{M1}(B_1^*) - \frac{a_\chi(B_1^*)}{a_\chi(B_2^*)} A_{M1}(B_2^*), \quad A_{E2}(B_1^*) - \frac{b_\chi(B_1^*)}{b_\chi(B_2^*)} A_{E2}(B_2^*) \tag{27}$$

are independent of the unknown coupling $c_S(\mu)$ and can then be predicted. For instance

$$A_{M1}(\Sigma_c^{++*}) + 2\, A_{M1}(\Sigma_c^{0*}) = \frac{1}{\sqrt{3}} \frac{g_3^2}{4\pi f_\pi^2} (m_K - m_\pi) + \frac{\Delta_{ST}}{4\sqrt{3}} \frac{g_2^2}{(4\pi f_\pi)^2} (I_K - I_\pi) - \frac{2}{\sqrt{3} m_c} . \tag{28}$$

In order to get a numerical estimate of the left–hand side of Eq. (28) we set $g_2 = 1.5 \pm 0.3$, $g_3 = 0.99 \pm 0.17$ and the rest of the constants as in Table 2. We find then

$$A_{M1}(\Sigma_c^{++*}) + 2A_{M1}(\Sigma_c^{0*}) = 0.57 \pm 0.67 \text{ GeV}^{-1} \tag{29}$$

The analogous relation for b baryons reads

$$A_{M1}(\Sigma_b^{+*}) + 2A_{M1}(\Sigma_b^{-*}) = 1.58 \pm 0.66 \text{ GeV}^{-1} \tag{30}$$

The main contribution to these values corresponds to the chiral loop, with a much smaller correction coming from the $1/m_Q$ term. These sums would be zero if none of the previous contributions were included. The large errors in Eq. (29) and Eq. (30) come from the present uncertainties on $g_{2,3}$ ($\sim 20\%$). The same consideration holds for all numerical results in this and in the following sections.

A further comment is now in order. To estimate the importance of the effect of one loop HHCPT we define the ratio (see Eq. (15)),

$$R(B^*) = \frac{3}{2(4\pi f_\pi)} \frac{g_2^2 \, \Delta_{ST} \, a_{g2}(B^*)/4 \, + 4\pi \, m_K \, g_3^2 \, a_{g3}(B^*)}{|c_S(\mu)| \, a_\chi(B^*)} . \tag{31}$$

We find for $\Lambda_\chi/2 < \mu < \Lambda_\chi$,

$$R(\Sigma_c^{++*}) = R(\Sigma_b^{+*}) = R(\Xi_c^{0'*}) = R(\Xi_b^{-'*}) = \frac{3.2 \pm 1.9}{|c_S(\mu)|},$$

$$R(\Sigma_c^{+*}) = R(\Sigma_b^{0*}) = R(\Omega_c^{0*}) = R(\Omega_b^{-*}) = \frac{5.5 \pm 2.9}{|c_S(\mu)|},$$

$$R(\Sigma_c^{0*}) = R(\Sigma_b^{-*}) = R(\Xi_c^{+'*}) = R(\Xi_b^{0'*}) = \frac{1.0 \pm 1.1}{|c_S(\mu)|} . \tag{32}$$

The scale dependence of this result is not very strong and in any case within the errors. Naïvely one expects $|c_S(\mu)| \sim \mathcal{O}(1)$. Thus, from Eq. (32), we can deduce that the infrared effect due to the coupling of the photon to light mesons, is large on these electromagnetic

decays. This affirmation can be sustained also comparing our results with some estimates existing in the literature (so far there are no experimental data on $S^* \to S\gamma$ decays). In ref. [12] the three decays $\Sigma_b^* \to \Sigma_b\gamma$ are predicted, using light cone QCD sum rules; these results respect the HQET and chiral symmetries and agree with the first of our relations in Eq. (19), provided the proper relative signs among the amplitudes are chosen, namely

$$\frac{\sqrt{\Gamma(\Sigma_b^{*+} \to \Sigma_b^+ \gamma)} - \sqrt{\Gamma(\Sigma_b^{*-} \to \Sigma_b^- \gamma)}}{2\sqrt{\Gamma(\Sigma_b^{*0} \to \Sigma_b^0 \gamma)}} = 0.98 \; . \tag{33}$$

In order to derive this number from the results of ref. [12], we have made use of the baryon masses in Table 5. However in ref. [12] all coupling constants are determined at leading order in HQET. Writing

$$c_S(\mu)_{\overline{MS}} = c_S^0 + \frac{(c_S^1(\mu))_{\overline{MS}}}{\Lambda_\chi} \tag{34}$$

we derive (consistently with Eq. (33)) from ref. [12]

$$-1.6 < c_S^0 < -1.2, \quad \text{or,} \quad 1.3 < c_S^0 < 1.7 \tag{35}$$

depending on the overall sign of the amplitudes[1]. Thus ref. [12], obtains the expected order of magnitude of c_S^0, however, the important chiral effect due to the photon–meson coupling is not taken into account . Thus, choosing the sign between the amplitudes consistently with Eq. (33), it is impossible to deduce Eq. (30) from their calculation. Ref. [10] estimates these same decay rates and its results are consistent with ref. [12] so that the same comments are valid also for this reference. These considerations apply also if one considers the computation of the decays $\Sigma_c^* \to \Sigma_c\gamma$ of ref. [24]. In this case the first of our relations in Eq. (18) is exactly fulfilled and we can derive $|c_S^0| = 1 \pm 1$. We note however that the predictions of ref. [10, 11] and ref. [24] for $\Gamma(\Sigma_c^{*+} \to \Sigma_c^+ \gamma)$ are in desagreement as a much higher rate is predicted in the first two references.

4   Results for $S^* \to T\gamma$ decays

The M1 and E2 operators for these decays are defined as in Eq. (14). Similarly to what we have done in the previous paragraph, we write the M1 amplitude for $S^* \to T\gamma$ decays as

$$A_{M1}(B^*) = -\sqrt{2}\,\frac{c_{ST}}{\Lambda_\chi}\,a_\chi(B^*) + g_2\,g_3\,\frac{\Delta_{ST}}{2\sqrt{2}(4\pi f_\pi)^2}\,a_g(B^*) \; . \tag{36}$$

The values of the coefficients a_i are written in Table 6.

The first term in Eq. (36) comes from the Lagrangian (11), while the second one corresponds to the diagram of fig. 3. As in the case of $S^* \to S\gamma$, when all Goldstone boson loops are included, the scale μ dependence of the loop diagram is canceled by the corresponding

---

[1]The difference in the absolute value of positive and negative interval in Eq. (35) is due to the heavy quark term in Eq. (15).

c-baryons	M (MeV)	b-baryons	M (MeV)
Ξ_c^0	2470.3 ± 1.8	Ξ_b^-	5805.7 ± 8.1
Ξ_c^+	2465.6 ± 1.4	Ξ_b^0	5805.7 ± 8.1
Λ_c^+	2284.9 ± 0.6	Λ_b^0	5624 ± 9
Σ_c^{++}	2452.8 ± 0.6	Σ_b^+	5824.2 ± 9.0
Σ_c^+	2453.6 ± 0.9	Σ_b^0	5824.2 ± 9.0
Σ_c^0	2452.2 ± 0.6	Σ_b^-	5824.2 ± 9.0
$\Xi_c^{0'}$	2577.3 ± 3.2	$\Xi_b^{-'}$	5950.9 ± 8.5
$\Xi_c^{+'}$	2573.4 ± 3.1	$\Xi_b^{0'}$	5950.9 ± 8.5
Ω_c^0	2704.0 ± 4.0	Ω_b^-	6068.7 ± 11.1
Σ_c^{++*}	2519.4 ± 1.5	Σ_b^{+*}	5840.0 ± 8.8
Σ_c^{+*}	2518.6 ± 2.2	Σ_b^{0*}	5840.0 ± 8.8
Σ_c^{0*}	2517.5 ± 1.4	Σ_b^{-*}	5840.0 ± 8.8
$\Xi_c^{0'*}$	2643.8 ± 1.8	$\Xi_b^{-'*}$	5966.1 ± 8.3
$\Xi_c^{+'*}$	2644.6 ± 2.1	$\Xi_b^{0'*}$	5966.1 ± 8.3
Ω_c^{0*}	2760.5 ± 4.9	Ω_b^{-*}	6083.2 ± 11.0

Table 5: Masses of charm and bottom baryons. All masses of b baryons (except Λ_b^0) and the ones of Σ_c^{+*}, Ω_c^{0*} have been estimated theoretically in ref. [25]. The measured masses are taken from [26].

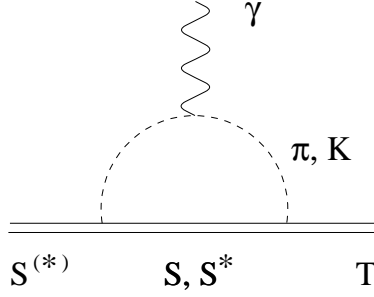| c quark	b quark	a_χ	a_g
$\Sigma_c^{+*} \to \Lambda_c^+ \gamma$	$\Sigma_b^{0*} \to \Lambda_b^0 \gamma$	1	$2I_\pi + I_K/2$
$\Xi_c^{+'*} \to \Xi_c^+ \gamma$	$\Xi_b^{0'*} \to \Xi_b^0 \gamma$	1	$I_\pi/2 + 2I_K$
$\Xi_c^{0'*} \to \Xi_c^0 \gamma$	$\Xi_b^{-'*} \to \Xi_b^- \gamma$	0	$-I_\pi/2 + I_K/2$

Table 6: Contributions to M1 amplitudes for $S^* \to T\gamma$.

Figure 3: Meson loops contributing to $S^{(*)} \to T\gamma$.

dependence of an effective $c_{ST}(\mu)$. After applying the equations of motion, the effective coupling $c_{ST}(\mu)$ contains all contributions to the M1 amplitude coming from $\mathcal{O}(1/\Lambda_\chi^2)$ counterterms, namely

$$
\begin{aligned}
i\,\epsilon_{ijk}\,Q_l^j\,\left(\bar{T}^i(v\cdot DS_\nu^{kl}) - (v\cdot D\bar{S}_\nu^{kl})T^i\right)v_\mu\tilde{F}^{\mu\nu} & \quad, \\
i\,\epsilon_{ijk}\,Q_l^j\,\left(\bar{T}^i(D_\mu S_\nu^{kl}) - (D_\mu\bar{S}_\nu^{kl})T^i\right)\tilde{F}^{\mu\nu} & \quad.
\end{aligned}
\tag{37}
$$

Our result in Eq. (36) does not depend on the heavy quark charge or mass. We thus obtain the same predictions for charm and bottom baryons. All constants can be eliminated in the relations

$$
\begin{aligned}
A_{M1}(\Sigma_c^{+*}) - A_{M1}(\Xi_c^{+'*}) &= -3\,A_{M1}(\Xi_c^{0'*})\,, \\
A_{M1}(\Sigma_b^{0*}) - A_{M1}(\Xi_b^{0'*}) &= -3\,A_{M1}(\Xi_b^{-'*})\,.
\end{aligned}
\tag{38}
$$

It is interesting to notice that $A_{M1}(\Xi_c^{0'*})$ does not depend on c_{ST}. Since at $\mathcal{O}(1/\Lambda_\chi^2)$ this decay does not get any contribution from local terms, its M1 amplitude results from a *finite* chiral loop calculation (it cannot be divergent because there is no possible counter-term to renormalize it), so that we have an absolute prediction for its value in terms of g_2 and g_3. Using for g_2 and g_3 the same values as in Eq. (29), we find

$$
\Gamma_{M1}(\Xi_c^{0'*}) = 5.1 \pm 2.7 \text{ KeV} \,.
\tag{39}
$$

Lower estimates of this decay width are reported in ref. [11], $\Gamma_{M1}(\Xi_c^{0'*}) = 0.68 \pm 0.04$ KeV, and in ref. [24], $\Gamma_{M1}(\Xi_c^{0'*}) = 1.1$ KeV. These authors do not consider chiral corrections to their result which cannot be neglected. In particular the result of ref. [24] is worth a further comment. The effective coupling to the M1 operator in this decay is estimated using the non relativistic quark model. This coupling is found to be proportional to $1/M_d - 1/M_s$, where $M_{d,s}$ are the constituent quark mass of the down and strange quarks respectively. However

$$
\frac{1}{M_d} - \frac{1}{M_s} = \frac{M_s - M_d}{M_s M_d} \sim \mathcal{O}\left(\frac{m_K^2 - m_\pi^2}{\Lambda_\chi^3}\right) \,.
\tag{40}
$$

c quark	b quark	b_χ	$b_{g'}$	b_g
$\Sigma_c^{+*} \to \Lambda_c^+ \gamma$	$\Sigma_b^{0*} \to \Lambda_b^0 \gamma$	1	$2J_\pi + J_K/2$	$2G_\pi + G_K/2$
$\Xi_c^{+*} \to \Xi_c^+ \gamma$	$\Xi_b^{0*} \to \Xi_b^0 \gamma$	1	$J_\pi/2 + 2J_K$	$G_\pi/2 + 2G_K$
$\Xi_c^{0*} \to \Xi_c^0 \gamma$	$\Xi_b^{-*} \to \Xi_b^- \gamma$	0	$-J_\pi/2 + J_K/2$	$-G_\pi/2 + G_K/2$

Table 7: Contributions to E2 amplitudes for $S^* \to T\gamma$.

Thus, the effect they calculate represents a higher order correction to our result.

The corresponding decay for b baryons, $\Xi_b^{-'*} \to \Xi_b^- \gamma$ can be also predicted, using the existing estimates for the masses of these baryons (see Table 5),

$$\Gamma_{M1}(\Xi_b^{-'*}) = 4.2 \pm 2.4 \text{ KeV} . \tag{41}$$

The dominant error of Eq. (39) and Eq. (41) comes from the determination of the couplings $g_{2,3}$.

The E2 amplitude in $S^* \to T\gamma$ is suppressed by an extra power of $1/m_Q$. The first non-zero contributions comes at $\mathcal{O}(1/m_Q\Lambda_\chi^2)$. At this order we find:

- a divergent contribution [13] arising from the lowest–order Lagrangian (7), through the loop in fig. 3, which is proportional to the mass splitting between S and S^* baryons [27],

$$\Delta M_Q = 3\frac{\lambda_{2S}}{m_Q} ; \tag{42}$$

- a spin symmetry breaking operator of $\mathcal{O}(1/m_Q)$,

$$\mathcal{L}' = i\frac{g'}{m_Q} \left[\epsilon_{ijk} \bar{T}^i \sigma^{\mu\nu} (\xi_\mu)_l^j S_\nu^{kl} + \epsilon^{ijk} \bar{S}_{kl}^\mu \sigma_{\mu\nu} (\xi^\nu)_j^l T_i \right] , \tag{43}$$

which gives rise to divergent loop diagrams, as the one in fig. 3, where one of the vertices is proportional to g';

- further, there are finite contributions of the same order coming from

$$-i\frac{c_T^{E2}}{m_Q\Lambda_\chi^2} \epsilon_{ijk} \bar{T}^i \sigma_{\mu\nu} Q_l^j S_\alpha^{kl} \partial^\alpha \tilde{F}^{\mu\nu} . \tag{44}$$

We could also include the operator

$$i\epsilon_{ijk} \bar{T}^i \sigma_{\mu\nu} Q_l^j S_\alpha^{kl} \partial^\nu \tilde{F^{\mu\alpha}} , \tag{45}$$

but its contribution is proportional to that in Eq. (44) up to higher order corrections.

13

Finally, the E2 amplitude can be written as

$$A_{E2}(B^*) = -\frac{1}{\sqrt{2}} \frac{c_T^{E2}}{m_Q \Lambda_\chi^2} b_\chi(B^*) - \frac{1}{24\sqrt{2}} \frac{g'g_2}{m_Q(4\pi f_\pi)^2} b_{g'}(B^*) + \frac{\lambda_{2S}}{24\sqrt{2}} \frac{g_2 g_3}{m_Q(4\pi f_\pi)^2} b_g(B^*) \; . \quad (46)$$

The values of the different contributions are collected in Table 7, where

$$G_i = \left. \frac{\partial\, J_i}{\partial \Delta_{ST}} \right|_{\Delta_{ST}=0} = \frac{2\pi}{m_i} \; . \quad (47)$$

We underline the infrared divergent behavior of this term. Neither the interaction in Eq. (43) nor the local term Eq. (44) have been taken into account in the literature. An estimate of E2 for $\Sigma_c^{+*} \to \Lambda_c^+ \gamma$ is provided in ref. [13], considering only the b_g contribution.

By eliminating the unknown coupling constants, one can deduce the relation

$$A_{E2}(\Sigma_c^{+*}) - A_{E2}(\Xi_c^{+'*}) = -3\, A_{E2}(\Xi_c^{0'*}) \; . \quad (48)$$

The same relation holds for the corresponding b baryons, since

$$A_{E2}(B_b^*) = \frac{m_c}{m_b} A_{E2}(B_c^*) \; . \quad (49)$$

The decays $\Xi_c^{0*} \to \Xi_c^0 \gamma$ and $\Xi_b^{-*} \to \Xi_b^- \gamma$ do not get any contribution from the local term proportional to c_T^{E2}; their $O(1/m_Q\Lambda_\chi^2)$ E2 amplitude is also given by a finite loop calculation. Unfortunately, since the coupling g' is not known, there is no absolute prediction in this case. An experimental measurement of these E2 amplitudes would provide a direct estimate of g'.

5   Results for $S \to T\gamma$

The calculation of the M1 amplitude for $S \to T\gamma$ decays is analogous to that of the previous section. Now the M1 operator is defined as

$$\mathcal{O}_{M1} = ie\, \bar{B}_T \sigma_{\mu\nu} B_S\, F^{\mu\nu} \quad (50)$$

and the corresponding amplitude can be written in the form

$$A_{M1}(B) = \frac{1}{\sqrt{6}} \frac{c_{ST}}{\Lambda_\chi} a_\chi(B) - g_2 g_3 \frac{\Delta_{ST}}{4\sqrt{6}(4\pi f_\pi)^2} a_g(B) \; , \quad (51)$$

where the coefficients satisfy

$$a_\chi(B) = a_\chi(B^*), \qquad a_g(B) = a_g(B^*) \; . \quad (52)$$

Therefore, the relation (38) is also valid in this case. The widths of the decays $\Xi_c^{0'} \to \Xi_c^0 \gamma$ and $\Xi_b^{-'} \to \Xi_b^- \gamma$ can be predicted through a finite loop calculation. From

$$\Gamma(S \to T\gamma) = 16\alpha_{em} \frac{E_\gamma^3 M_T}{M_S} |A_{M1}|^2 \; , \quad (53)$$

14

we find

$$\begin{aligned}
\Gamma(\Xi_c^{0\prime}) &= (1.2 \pm 0.7) \text{ KeV} , \\
\Gamma(\Xi_b^{-\prime}) &= (3.1 \pm 1.8) \text{ KeV} .
\end{aligned} \tag{54}$$

Again the dominant error in Eq. (54) is given by the uncertainty of $g_{2,3}$.

As in section 3 in order to estimate the importance of chiral corrections we use the ratios (see Eq. (36) and (51))

$$R(B^{(*)}) = \frac{g_2 g_3 \Delta_{ST} a_g(B^{(*)})}{16\pi f_\pi a_\chi(B^{(*)}) |c_{ST}(\mu)|} . \tag{55}$$

We find (we consider $\Lambda_\chi/2 < \mu < \Lambda_\chi$)

$$\begin{aligned}
R(\Sigma_c^{+(*)}) = R(\Sigma_b^{0(*)}) &= -(1.6 \pm 0.6)/|c_{ST}(\mu)|, \\
R(\Xi_c^{+\prime(*)}) = R(\Xi_b^{0\prime(*)}) &= -(2.4 \pm 0.8)/|c_{ST}(\mu)|
\end{aligned} \tag{56}$$

Therefore the one loop chiral contribution cannot be neglected for $|c_{ST}(\mu)| \sim \mathcal{O}(1)$. In refs. [11] and [9, 24], numerical values for all $S_c^{(*)} \to T\gamma$ at $\mathcal{O}(1/\Lambda_\chi)$ are given using respectively the relativistic three quark model and the constituent quark-model. As in section 3 we can define

$$c_{ST}(\mu)_{\overline{MS}} = c_{ST}^0 + \frac{(c_{ST}^1(\mu))_{\overline{MS}}}{\Lambda_\chi} . \tag{57}$$

From ref. [9, 11, 24] we find

$$0.83 < |c_{ST}^0| < 1.6 . \tag{58}$$

Our results in Eq. (54) can be compared with other estimates existing in the literature. Ref. [11] reports $\Gamma(\Xi_c^{0\prime}) = 0.17 \pm 0.02$ KeV, while ref. [9] quotes $\Gamma(\Xi_c^{0\prime}) = 0.3$ KeV. Further, the same argument as in section 4 can be used also now to understand these low values obtained in the constituent quark model [9].

For these decays the E2 amplitude is further suppressed than in the previous cases. The lowest–order contribution appears at $\mathcal{O}(1/m_Q^3 \Lambda_\chi^2)$ and, therefore, can be neglected.

6 Conclusions

We have calculated the electromagnetic one photon decays $S^* \to S\gamma$ and $S^{(*)} \to T\gamma$ using Heavy Hadron Chiral Perturbation Theory. For each of these decays we have provided an estimate of both the M1 and E2 amplitudes. The computation of the M1 amplitudes up to $\mathcal{O}(1/\Lambda_\chi^2)$ involves the introduction of the unknown constants c_S for $S^* \to S\gamma$ and c_{ST} for $S^{(*)} \to T\gamma$. Eliminating these couplings we derive relations among different amplitudes. Moreover, since charm and bottom baryons are described by the same arbitrary constants, we can connect the amplitudes of the two kinds of hadrons.

The E2 contributions appear at different higher orders for the three kinds of decays: $\mathcal{O}(1/\Lambda_\chi^2)$ for $S^* \to S\gamma$, $\mathcal{O}(1/m_Q\Lambda_\chi^2)$ for $S^* \to T\gamma$ and $\mathcal{O}(1/m_Q^3\Lambda_\chi^2)$ for $S \to T\gamma$. They

introduce additional unknown constants: c_S^{E2} for $S^* \to S\gamma$; c_T^{E2} and g' for $S^* \to T\gamma$ (the E2 amplitude for $S \to T\gamma$ is completely negligible). The E2 effects can be strongly enhanced by a term which is infrared divergent in the exact chiral limit. The possibility of measuring the ratio A_{E2}/A_{M1}, using polarized initial baryons, has been suggested in ref. [13] and could be performed with an analysis of the photon distribution.

Furthermore, we obtain an absolute prediction for $\Gamma(\Xi_c^{0'(*)} \to \Xi_c^0 \gamma)$ and $\Gamma(\Xi_b^{-'(*)} \to \Xi_b^- \gamma)$. At $\mathcal{O}(1/\Lambda_\chi^2)$, these decay widths do not get any contribution from local terms in the Lagrangian and, therefore, their values are fixed by a finite chiral loop calculation.

Finally, we have shown that chiral loops involving photon–meson coupling cannot be neglected in the computation of the amplitudes of these decays. These interactions generate the dominant contribution to the electromagnetic decays of heavy baryons.

Acknowledgements

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