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Electromagnetism in Nonleptonic Weak Interactions*

G. Ecker¹, G. Isidori², G. Müller^{1,#}, H. Neufeld^{1,3} and A. Pich³

¹) Institut für Theoretische Physik, Universität Wien
Boltzmannngasse 5, A-1090 Wien, Austria

²) INFN, Laboratori Nazionali di Frascati, P.O. Box 13, I-00044 Frascati, Italy

³) Departament de Física Teòrica, IFIC, Universitat de València - CSIC
Apt. Correus 2085, E-46071 València, Spain

Abstract

We construct a low-energy effective field theory that permits the complete treatment of isospin-breaking effects in nonleptonic weak interactions to next-to-leading order. To this end, we enlarge the chiral Lagrangian describing strong and $\Delta S = 1$ weak interactions by including electromagnetic terms with the photon as additional dynamical degree of freedom. The complete and minimal list of local terms at next-to-leading order is given. We perform the one-loop renormalization at the level of the generating functional and specialize to $K \rightarrow \pi\pi$ decays.

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Present address: Jagdweg 23, D-53115 Bonn, Germany

1 Introduction

Although isospin violation in nonleptonic weak interactions has been investigated many times in the past systematic treatments have appeared only rather recently [1, 2, 3, 4, 5]. The topic is both of general interest and of considerable phenomenological relevance. Precise determinations of weak decay amplitudes are needed for many purposes, in particular for a reliable calculation of CP violation in the $K^0 - \bar{K}^0$ system. In the standard model, isospin violation arises from the quark mass difference $m_u - m_d$ and from electromagnetic corrections. Although these effects are expected to be small in general they are amplified in nonleptonic weak transitions. Because of the suppression of amplitudes with $\Delta I > 1/2$, isospin violation in the dominant $\Delta I = 1/2$ amplitudes leads to significantly enhanced corrections for the sub-dominant amplitudes. In fact, a quantitative analysis of the $\Delta I = 1/2$ rule is only possible with the inclusion of isospin-violating effects.

At first order in a systematic low-energy expansion, isospin breaking in the leading octet amplitudes of nonleptonic kaon decays is of order $G_8(m_u - m_d)$ and $G_8 e^2$ where G_8 denotes the strength of the effective octet coupling. The corrections appear in the mass differences of charged and neutral mesons, via $\pi^0 - \eta$ mixing and through electromagnetic penguins [6] in the effective nonleptonic weak Hamiltonian. However, there are good reasons to believe that the problem cannot be understood at lowest order only. For instance, the resulting (tree-level) corrections do not produce a $\Delta I = 5/2$ component for which there is some phenomenological evidence [1].

The chiral realization of isospin violation due to the light quark mass difference is available also at next-to-leading order. The purpose of this paper is to close the gap in the electromagnetic sector by

- completing the construction of the effective chiral Lagrangian of $\mathcal{O}(G_8 e^2 p^2)$ and
- performing the complete renormalization at the one-loop level for nonleptonic weak transitions including electromagnetic corrections.

As our notation indicates, we only consider corrections to the leading octet part of the nonleptonic weak Hamiltonian. The results are applicable to the analysis of both $K \rightarrow 2\pi$ [2, 3, 4, 7] and $K \rightarrow 3\pi$ decays.

We start in Sec. 2 by recalling the ingredients for the construction of effective theories of strong, electromagnetic and nonleptonic weak interactions. In Sec. 3 we review the effective Lagrangian of lowest order. For this Lagrangian, we evaluate the one-loop divergence functional by standard heat-kernel techniques in Sec. 4. The new parts are terms of $\mathcal{O}(G_8 e^2 p^2)$ which arise also from using the equations of motion to transform to the standard bases for the nonleptonic weak Lagrangian of $\mathcal{O}(G_8 p^4)$ [8] and for the electromagnetic Lagrangian of $\mathcal{O}(e^2 p^2)$ [9]. In the following section we construct the complete and minimal Lagrangian of $\mathcal{O}(G_8 e^2 p^2)$ making use of CPS symmetry [10], Cayley-Hamilton relations, partial integration in the action and of the equations of motion. We order the terms in the effective Lagrangian according to their physical relevance: $K \rightarrow \pi\pi$ amplitudes receive contributions from the first 12 operators, the next two appear in $K \rightarrow 3\pi$ and the rest turns out not to be relevant phenomenologically. In Sec. 6, we present the divergences

for the three $K \rightarrow 2\pi$ amplitudes and compare with the results of direct one-loop calculations [3, 7]. We summarize our findings in Sec. 7. Various quantities appearing in the heat-kernel expansion of the generating functional are collected in the Appendix.

2 Symmetries

For a complete treatment of isospin-breaking effects in nonleptonic kaon decays, an appropriate effective Lagrangian with the pseudoscalar octet and the photon as dynamical degrees of freedom has to be used. The symmetries of the standard model are serving as the basic guiding principles for its construction. Starting with QCD in the chiral limit $m_u = m_d = m_s = 0$, the resulting symmetry under the chiral group $G = SU(3)_L \times SU(3)_R$ is spontaneously broken to $SU(3)_V$. The pseudoscalar mesons (π, K, η) are interpreted as the corresponding Goldstone fields φ_i ($i = 1, \dots, 8$) acting as coordinates of the coset space $SU(3)_L \times SU(3)_R / SU(3)_V$. The coset variables $u_{L,R}(\varphi)$ are transforming as

$$\begin{aligned} u_L(\varphi) &\xrightarrow{G} g_L u_L(\varphi) h(g, \varphi)^{-1} , \\ u_R(\varphi) &\xrightarrow{G} g_R u_R(\varphi) h(g, \varphi)^{-1} , \\ g &= (g_L, g_R) \in SU(3)_L \times SU(3)_R , \end{aligned} \quad (2.1)$$

where $h(g, \varphi)$ is the nonlinear realization of G [11].

The photon field A_μ is introduced in

$$u_\mu = i[u_R^\dagger(\partial_\mu - ir_\mu)u_R - u_L^\dagger(\partial_\mu - il_\mu)u_L] \quad (2.2)$$

by adding appropriate terms to the usual external vector and axial-vector sources v_μ, a_μ :

$$\begin{aligned} l_\mu &= v_\mu - a_\mu - eQ_L A_\mu , \\ r_\mu &= v_\mu + a_\mu - eQ_R A_\mu . \end{aligned} \quad (2.3)$$

The 3×3 matrices $Q_{L,R}$ are spurion fields with the transformation properties

$$Q_L \xrightarrow{G} g_L Q_L g_L^\dagger , \quad Q_R \xrightarrow{G} g_R Q_R g_R^\dagger \quad (2.4)$$

under the chiral group. We also define

$$\mathcal{Q}_L := u_L^\dagger Q_L u_L , \quad \mathcal{Q}_R := u_R^\dagger Q_R u_R \quad (2.5)$$

transforming as

$$\begin{aligned} \mathcal{Q}_L &\xrightarrow{G} h(g, \varphi) \mathcal{Q}_L h(g, \varphi)^{-1} , \\ \mathcal{Q}_R &\xrightarrow{G} h(g, \varphi) \mathcal{Q}_R h(g, \varphi)^{-1} . \end{aligned} \quad (2.6)$$

At the end, one identifies $Q_{L,R}$ with the quark charge matrix

$$Q = \begin{bmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{bmatrix} . \quad (2.7)$$

External scalar and pseudoscalar sources are combined in

$$\chi = s + ip . \quad (2.8)$$

For the construction of the effective Lagrangian, it is convenient to introduce the quantities

$$\chi_{\pm} = u_R^{\dagger} \chi u_L \pm u_L^{\dagger} \chi^{\dagger} u_R \quad (2.9)$$

with the same chiral transformation properties as $\mathcal{Q}_L, \mathcal{Q}_R$ in (2.6).

After integrating out the heavy degrees of freedom, the $\Delta S = 1$ weak interactions can be described in terms of an effective four-fermion Hamiltonian [12]. With respect to the chiral group G , this effective Hamiltonian transforms as the direct sum

$$(8_L, 1_R) + (27_L, 1_R) + (8_L, 8_R) , \quad (2.10)$$

where the first piece, contributing only to $\Delta I = \frac{1}{2}$ transitions, is largely dominant. In this work we shall consider only the electromagnetic corrections induced by the dominant octet part of the effective Hamiltonian. To this end we introduce a weak spurion λ that is finally taken at

$$\lambda = \frac{\lambda_6 - i\lambda_7}{2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} , \quad (2.11)$$

where $\lambda_{6,7}$ are Gell-Mann matrices. In analogy to (2.5) we also define

$$\Delta := u_L^{\dagger} \lambda u_L , \quad (2.12)$$

transforming again as in (2.6) under chiral transformations.

Although CP is broken by the weak interactions, the $\Delta S = 1$ transitions are still invariant under the so-called CPS symmetry [10]: a CP transformation followed by a subsequent interchange of d and s quarks. This symmetry is also obeyed by strong and electromagnetic interactions, provided the 2-3 indices of the external fields are also exchanged appropriately (this implies, in particular, the exchange $m_s \leftrightarrow m_d$ in the mass terms). The explicit CPS transformation properties of the several building blocks introduced so far are given by

$$\begin{aligned} u_{\mu}(x) &\xrightarrow{CPS} -\epsilon(\mu) S u_{\mu}^T(\tilde{x}) S , \\ \chi_{\pm}(x) &\xrightarrow{CPS} \pm S \chi_{\pm}^T(\tilde{x}) S , \\ \mathcal{Q}_{L,R}(x) &\xrightarrow{CPS} S \mathcal{Q}_{L,R}^T(\tilde{x}) S , \\ \Delta(x) &\xrightarrow{CPS} S \Delta^T(\tilde{x}) S , \end{aligned} \quad (2.13)$$

with

$$\tilde{x} = (x^0, -\vec{x}) , \quad \epsilon(0) = 1 , \quad \epsilon(1) = \epsilon(2) = \epsilon(3) = -1 , \quad (2.14)$$

and

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} . \quad (2.15)$$

3 The effective Lagrangian at lowest order

With the building blocks introduced in the previous section we may now assemble our effective Lagrangian. We adopt an expansion scheme where the n -th order is related to terms of order p^n in the strong and weak sector and to terms of order $e^2 p^{n-2}$ in the electromagnetic sector where p denotes a typical meson momentum. Terms of $\mathcal{O}(e^4)$ will be neglected throughout.

To lowest order ($n = 2$), our effective theory consists of the following parts: the strong sector is represented by the nonlinear sigma model in the presence of the external sources v_μ, a_μ, χ [13] and the photon coupling introduced in (2.3):

$$\frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle . \quad (3.1)$$

The symbol $\langle \rangle$ denotes the trace in three-dimensional flavour space and F is the pion decay constant in the chiral limit. Explicit chiral symmetry breaking by the non-vanishing masses of the light quarks is achieved by evaluating the generating functional at

$$\chi = 2B\mathcal{M}_{\text{quark}} = 2B \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix} . \quad (3.2)$$

The quantity B is related to the quark condensate in the chiral limit by $\langle 0|\bar{q}q|0\rangle = -F^2 B$.

The $(8_L, 1_R)$ piece of the nonleptonic weak interactions is represented by the well-known Cronin Lagrangian [14],

$$F^2 \langle \Xi u_\mu u^\mu \rangle , \quad \Xi = G_8 F^2 \Delta + \text{h.c.} . \quad (3.3)$$

At lowest order, the parameter G_8 can be determined [15] from $K \rightarrow 2\pi$ decays to be $|G_8| \simeq 9 \times 10^{-6} \text{GeV}^{-2} \simeq 5(G_F/\sqrt{2})|V_{ud}V_{us}|$.

Now also the electromagnetic interaction has to be included. Apart from the necessary modification in (2.2), we have to add a kinetic term for the photon field,

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (3.4)$$

and a strangeness-conserving term of $\mathcal{O}(e^2 p^0)$ [16],

$$e^2 F^4 Z \langle \mathcal{Q}_L \mathcal{Q}_R \rangle . \quad (3.5)$$

The numerical value of the parameter Z can be determined from the mass difference of charged and neutral pions. The relation $M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2 Z F^2$ implies $Z \simeq 0.8$.

Finally, we have to introduce a weak-electromagnetic term characterized by a coupling constant g_{ewk} ,

$$e^2 F^4 \langle \Upsilon \mathcal{Q}_R \rangle , \quad \Upsilon = g_{\text{ewk}} G_8 F^2 \Delta + \text{h.c.} . \quad (3.6)$$

Note that to lowest order only a single (linear independent) term of this type can be constructed once the relation

$$\mathcal{Q}_L \Delta = \Delta \mathcal{Q}_L = -\frac{1}{3} \Delta \quad (3.7)$$

is taken into account. This term is the lowest-order chiral realization of electromagnetic penguins [6, 17] and transforms as $(8_L, 8_R)$ under G when the Q_R spurion field is “frozen” to the constant value (2.7). By chiral dimensional analysis we expect the coupling constant g_{ewk} to be of $\mathcal{O}(1)$. A recent estimate in Ref. [2] corresponds in fact to $g_{\text{ewk}} = -1.0 \pm 0.3$ (see also Ref. [18]).

Summing up all these contributions, our lowest-order effective Lagrangian assumes the form

$$\begin{aligned} \mathcal{L}_2 = & \frac{F^2}{4} \langle u_\mu u^\mu + \chi_+ \rangle + F^2 \langle \Xi u_\mu u^\mu \rangle \\ & - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + e^2 F^4 Z \langle \mathcal{Q}_L \mathcal{Q}_R \rangle + e^2 F^4 \langle \Upsilon \mathcal{Q}_R \rangle . \end{aligned} \quad (3.8)$$

Using (2.13), one easily verifies that (3.8) is CPS invariant.

4 One-loop divergences

For the construction of the one-loop functional, we first add a gauge-breaking term (we are using the Feynman gauge) and external sources to (3.8):

$$\mathcal{L}_2 \rightarrow \mathcal{L}_2 - \frac{1}{2} (\partial_\mu A^\mu)^2 - J_\mu A^\mu . \quad (4.1)$$

Then we expand the lowest-order action associated with (4.1) around the solutions $\varphi_{\text{cl}}, A_{\text{cl}}^\mu$ of the classical equations of motion. In the standard “gauge” $u_R(\varphi_{\text{cl}}) = u_L(\varphi_{\text{cl}})^\dagger =: u(\varphi_{\text{cl}})$, a convenient choice of the pseudoscalar fluctuation variables ξ_i ($i = 1, \dots, 8$) is given by

$$u_R = u_{\text{cl}} e^{i\xi_i \lambda_i / 2F} , \quad u_L = u_{\text{cl}}^\dagger e^{-i\xi_i \lambda_i / 2F} , \quad \xi_i(\varphi_{\text{cl}}) = 0 , \quad (4.2)$$

with the Gell-Mann matrices λ_i ($i = 1, \dots, 8$). The photon field is decomposed as

$$A^\mu = A_{\text{cl}}^\mu + \varepsilon^\mu \quad (4.3)$$

with a fluctuation field ε^μ . In the following formulas, we shall drop the subscript “cl” for simplicity. The classical equations of motion take the form

$$\begin{aligned} \nabla_\mu u^\mu = & \frac{i}{2} \left(\chi_- - \frac{1}{3} \langle \chi_- \rangle \right) + 2ie^2 F^2 Z[\mathcal{Q}_R, \mathcal{Q}_L] \\ & + i[u_\mu u^\mu, \Xi] - 2(\nabla_\mu \{u^\mu, \Xi\} - \frac{1}{3} \langle \nabla_\mu \{u^\mu, \Xi\} \rangle) \\ & + 2ie^2 F^2 [\mathcal{Q}_R, \Upsilon] , \end{aligned} \quad (4.4)$$

$$\square A_\mu = J_\mu + \frac{eF^2}{2} \langle u_\mu (\mathcal{Q}_R - \mathcal{Q}_L) \rangle + eF^2 \langle \Xi \{ \mathcal{Q}_R - \mathcal{Q}_L, u_\mu \} \rangle , \quad (4.5)$$

where

$$\begin{aligned} \nabla_\mu = & \partial_\mu + [\Gamma_\mu, \] , \\ \Gamma_\mu = & \frac{1}{2} [u^\dagger (\partial_\mu - ir_\mu) u + u (\partial_\mu - il_\mu) u^\dagger] . \end{aligned} \quad (4.6)$$

The solutions of (4.4) and (4.5) are uniquely determined functionals of the external sources v_μ , a_μ , χ , J_μ . (Note that the usual Feynman boundary conditions are always implicitly understood.)

Expanding (4.1) up to terms quadratic in the fields ξ_i , ε_μ , we obtain the second-order fluctuation Lagrangian $\mathcal{L}^{(2)}$. The one-loop functional $W_{L=1}$ is then given by the Gaussian functional integral

$$e^{iW_{L=1}} = \int [d\xi_i d\varepsilon_\mu] e^{i \int d^d x \mathcal{L}^{(2)}} . \quad (4.7)$$

In our case, $\mathcal{L}^{(2)}$ reads

$$\begin{aligned} \mathcal{L}^{(2)} = & \frac{F^2}{4} \langle \nabla_\mu \xi \nabla^\mu \xi + \frac{1}{2} u_\mu \xi u^\mu \xi - \frac{1}{2} (u_\mu u^\mu + \chi_+) \xi^2 \rangle \\ & + e^2 F^4 Z \langle \xi \mathcal{Q}_L \xi \mathcal{Q}_R - \frac{1}{2} \xi^2 \{ \mathcal{Q}_L, \mathcal{Q}_R \} \rangle \\ & + \frac{F^2}{4} \langle 4 \Xi (\nabla_\mu \xi \nabla^\mu \xi - \frac{1}{4} \{ u_\mu, \{ u^\mu, \xi^2 \} \}) + \frac{1}{4} \{ u_\mu, \xi \} \{ u^\mu, \xi \} \rangle - 2i \langle [\xi, \Xi] \{ \nabla_\mu \xi, u^\mu \} \rangle \\ & + e^2 F^4 \langle \Upsilon (\xi \mathcal{Q}_R \xi - \frac{1}{2} \{ \xi^2, \mathcal{Q}_R \}) \rangle \\ & + \frac{1}{2} \varepsilon_\mu \square \varepsilon^\mu + \frac{e^2 F^2}{4} \langle (\mathcal{Q}_R - \mathcal{Q}_L)^2 \rangle \varepsilon_\mu \varepsilon^\mu + e^2 F^2 \langle \Xi (\mathcal{Q}_R - \mathcal{Q}_L)^2 \rangle \varepsilon_\mu \varepsilon^\mu \\ & - \frac{ieF^2}{4} \langle [u_\mu, \mathcal{Q}_R + \mathcal{Q}_L] \xi \rangle \varepsilon^\mu + \frac{eF^2}{2} \langle (\mathcal{Q}_R - \mathcal{Q}_L) \nabla_\mu \xi \rangle \varepsilon^\mu \\ & - \frac{ieF^2}{2} \langle \Xi \{ [\mathcal{Q}_R + \mathcal{Q}_L, \xi], u_\mu \} \rangle \varepsilon^\mu - \frac{ieF^2}{2} \langle [\xi, \Xi] \{ \mathcal{Q}_R - \mathcal{Q}_L, u_\mu \} \rangle \varepsilon^\mu \\ & + eF^2 \langle \Xi \{ \mathcal{Q}_R - \mathcal{Q}_L, \nabla_\mu \xi \} \rangle \varepsilon^\mu , \end{aligned} \quad (4.8)$$

where

$$\xi = \xi_i \lambda_i / F . \quad (4.9)$$

In the next step, we perform the field transformation

$$\xi \rightarrow \xi - \{ \Xi, \xi \} + \frac{2}{3} \langle \Xi \xi \rangle \mathbf{1} . \quad (4.10)$$

Because of $\langle \Delta \rangle = 0$, we do not pick up an additional contribution from the Jacobi determinant and the fluctuation Lagrangian (4.8) assumes the form

$$\mathcal{L}^{(2)} = -\frac{1}{2} \xi_i (d_\mu d^\mu + \sigma)_{ij} \xi_j + \frac{1}{2} \varepsilon_\mu (\square + \kappa) \varepsilon^\mu + \varepsilon_\mu a_i^\mu \xi_i + \varepsilon_\mu b_i d_{ij}^\mu \xi_j , \quad (4.11)$$

where

$$d_{ij}^\mu = \delta_{ij} \partial^\mu + \gamma_{ij}^\mu . \quad (4.12)$$

The explicit expressions for γ_{ij}^μ , σ_{ij} , κ , a_i^μ , b_i are given in the Appendix.

The divergent part of the one-loop functional,

$$W_{L=1}^{\text{div}} = \int d^d x \mathcal{L}_{L=1}^{\text{div}} , \quad (4.13)$$

is determined by

$$\begin{aligned} \mathcal{L}_{L=1}^{\text{div}} = & -\frac{1}{(4\pi)^2(d-4)} \left[\text{tr} \left(\frac{1}{12} \gamma_{\mu\nu} \gamma^{\mu\nu} + \frac{1}{2} \sigma^2 \right) - a_i^\mu a_{\mu i} + a_i^\mu (d_\mu b)_i \right. \\ & \left. + \frac{1}{2} (b_i b_i)^2 - b_i \sigma_{ij} b_j - \kappa b_i b_i + 2\kappa^2 \right], \end{aligned} \quad (4.14)$$

where

$$\gamma_{\mu\nu} = \partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu + [\gamma_\mu, \gamma_\nu]. \quad (4.15)$$

This formula can easily be derived from the well-known second Seeley-deWitt coefficient for bosonic systems [19].

5 The chiral Lagrangian at next-to-leading order

We are now in the position to construct the most general local action at next-to-leading order which will also renormalize the one-loop divergences discussed in the previous section.

The strong part of the local action of $\mathcal{O}(p^4)$ is, of course, generated by the well-known Gasser-Leutwyler Lagrangian [13] associated with the low-energy constants L_1, \dots, L_{12} . In the presence of virtual photons, the structure of the operators given in [13] remains unchanged. The only necessary modification is the inclusion of the dynamical photon field in the generalized “sources” ℓ_μ and r_μ (see (2.3)). The divergences corresponding to the strong sector of (4.14) are absorbed by the divergent parts of the L_i [13]. In the relevant case of chiral $SU(3)$, the strong terms generated by (4.14) can be written immediately as a linear combination of the $\mathcal{O}(p^4)$ operators of the Gasser-Leutwyler basis without using the equations of motion (4.4) or (4.5). Consequently, no additional (weak-)electromagnetic terms are induced at this point.

The strangeness-conserving terms of $\mathcal{O}(e^2 p^2)$ have been constructed by Urech [9]. His list of electromagnetic counterterms is associated with the coupling constants K_1, \dots, K_{14} . In this case, (4.14) leads to that canonical basis only after the use of the equation of motion (4.4). In this way, also some divergent weak-electromagnetic contributions of $\mathcal{O}(G_8 e^2 p^2)$ are generated.

For the octet part of the nonleptonic weak Lagrangian of $\mathcal{O}(G_F p^4)$ [20] we refer to the standard form of Ecker, Kambor and Wyler [8] with couplings N_1, \dots, N_{37} . Again, because of the mismatch between (4.14) and the standard basis, the equation of motion has to be used and the (purely) electromagnetic piece in (4.4) induces divergent terms of $\mathcal{O}(G_8 e^2 p^2)$.

Finally, we have to construct the most general weak-electromagnetic Lagrangian of $\mathcal{O}(G_8 e^2 p^2)$. Some parts of this Lagrangian have appeared before in the literature [1, 3, 21]. The complete minimal Lagrangian of $\mathcal{O}(G_8 e^2 p^2)$ takes the form

$$\mathcal{L}_{G_8 e^2 p^2} = G_8 e^2 F^4 \sum_{i=1}^{32} Z_i Q_i + \text{h.c.}, \quad (5.1)$$

with operators Q_i of $\mathcal{O}(p^2)$ and dimensionless coupling constants Z_i . A linear independent set of operators is given by

$$\begin{aligned}
Q_1 &= \langle \Delta \{ \mathcal{Q}_R, \chi_+ \} \rangle , \\
Q_2 &= \langle \Delta \mathcal{Q}_R \rangle \langle \chi_+ \rangle , \\
Q_3 &= \langle \Delta \mathcal{Q}_R \rangle \langle \chi_+ \mathcal{Q}_R \rangle , \\
Q_4 &= \langle \Delta \chi_+ \rangle \langle \mathcal{Q}_L \mathcal{Q}_R \rangle , \\
Q_5 &= \langle \Delta u_\mu u^\mu \rangle , \\
Q_6 &= \langle \Delta \{ \mathcal{Q}_R, u_\mu u^\mu \} \rangle , \\
Q_7 &= \langle \Delta u_\mu u^\mu \rangle \langle \mathcal{Q}_L \mathcal{Q}_R \rangle , \\
Q_8 &= \langle \Delta u_\mu \rangle \langle \mathcal{Q}_L u^\mu \rangle , \\
Q_9 &= \langle \Delta u_\mu \rangle \langle \mathcal{Q}_R u^\mu \rangle , \\
Q_{10} &= \langle \Delta u_\mu \rangle \langle \{ \mathcal{Q}_L, \mathcal{Q}_R \} u^\mu \rangle , \\
Q_{11} &= \langle \Delta \{ \mathcal{Q}_R, u_\mu \} \rangle \langle \mathcal{Q}_L u^\mu \rangle , \\
Q_{12} &= \langle \Delta \{ \mathcal{Q}_R, u_\mu \} \rangle \langle \mathcal{Q}_R u^\mu \rangle , \\
Q_{13} &= \langle \Delta \mathcal{Q}_R \rangle \langle u_\mu u^\mu \rangle , \\
Q_{14} &= \langle \Delta \mathcal{Q}_R \rangle \langle u_\mu u^\mu \mathcal{Q}_R \rangle , \\
Q_{15} &= \langle \Delta \mathcal{Q}_R \rangle \langle u_\mu u^\mu (\mathcal{Q}_L - \mathcal{Q}_R) \rangle , \\
Q_{16} &= \langle \Delta \chi_+ \rangle , \\
Q_{17} &= \frac{2}{3} \langle \Delta \chi_+ \rangle + \langle \Delta \{ \mathcal{Q}_R, \chi_+ \} \rangle + \langle \Delta [\mathcal{Q}_R, \chi_-] \rangle , \\
Q_{18} &= \langle \Delta \{ \mathcal{Q}_R, \chi_+ \} \rangle - \frac{1}{3} \langle \Delta [\mathcal{Q}_R, \chi_-] \rangle + \langle \Delta (\chi_- \mathcal{Q}_L \mathcal{Q}_R - \mathcal{Q}_R \mathcal{Q}_L \chi_-) \rangle \\
&\quad - \frac{4}{3} \langle \Delta \mathcal{Q}_R \rangle \langle \chi_+ \rangle - \langle \Delta \mathcal{Q}_R \rangle \langle \chi_+ \mathcal{Q}_R \rangle + \langle \Delta \chi_+ \rangle \langle \mathcal{Q}_L \mathcal{Q}_R \rangle , \\
Q_{19} &= \langle \Delta \mathcal{Q}_R \rangle \langle \chi_+ (\mathcal{Q}_L - \mathcal{Q}_R) \rangle , \\
Q_{20} &= i \langle (\widehat{\nabla}_\mu \Delta) [\mathcal{Q}_L, u^\mu] \rangle , \\
Q_{21} &= i \langle (\widehat{\nabla}_\mu \Delta) [\mathcal{Q}_R, u^\mu] \rangle , \\
Q_{22} &= i \langle (\widehat{\nabla}_\mu \Delta) (\mathcal{Q}_L u^\mu \mathcal{Q}_R - \mathcal{Q}_R u^\mu \mathcal{Q}_L) \rangle , \\
Q_{23} &= i \langle (\widehat{\nabla}_\mu \Delta) (u^\mu \mathcal{Q}_L \mathcal{Q}_R - \mathcal{Q}_R \mathcal{Q}_L u^\mu) \rangle , \\
Q_{24} &= i \langle \Delta (u_\mu (\widehat{\nabla}^\mu \mathcal{Q}_L) \mathcal{Q}_R - \mathcal{Q}_R (\widehat{\nabla}^\mu \mathcal{Q}_L) u_\mu) \rangle , \\
Q_{25} &= i \langle (\widehat{\nabla}_\mu \Delta) (u^\mu \mathcal{Q}_R \mathcal{Q}_L - \mathcal{Q}_L \mathcal{Q}_R u^\mu) \rangle , \\
Q_{26} &= i \langle \Delta (u_\mu \mathcal{Q}_R (\widehat{\nabla}^\mu \mathcal{Q}_R) - (\widehat{\nabla}^\mu \mathcal{Q}_R) \mathcal{Q}_R u_\mu) \rangle , \\
Q_{27} &= i \langle \Delta (\mathcal{Q}_R u_\mu (\widehat{\nabla}^\mu \mathcal{Q}_R) - (\widehat{\nabla}^\mu \mathcal{Q}_R) u_\mu \mathcal{Q}_R) \rangle , \\
Q_{28} &= \langle (\widehat{\nabla}_\mu \Delta) (\widehat{\nabla}^\mu \mathcal{Q}_L) \rangle , \\
Q_{29} &= \langle (\widehat{\nabla}_\mu \Delta) (\widehat{\nabla}^\mu \mathcal{Q}_R) \rangle , \\
Q_{30} &= \langle \Delta (\widehat{\nabla}_\mu \mathcal{Q}_R) (\widehat{\nabla}^\mu \mathcal{Q}_R) \rangle , \\
Q_{31} &= \langle (\widehat{\nabla}^\mu \Delta) \{ \widehat{\nabla}_\mu \mathcal{Q}_L, \mathcal{Q}_R \} \rangle ,
\end{aligned}$$

$$Q_{32} = \langle (\widehat{\nabla}^\mu \Delta) \{ \widehat{\nabla}_\mu \mathcal{Q}_R, \mathcal{Q}_L \} \rangle , \quad (5.2)$$

where

$$\begin{aligned} \widehat{\nabla}_\mu \Delta &= \nabla_\mu \Delta + \frac{i}{2} [u_\mu, \Delta] = u(D_\mu \lambda) u^\dagger , \\ \widehat{\nabla}_\mu \mathcal{Q}_L &= \nabla_\mu \mathcal{Q}_L + \frac{i}{2} [u_\mu, \mathcal{Q}_L] = u(D_\mu \mathcal{Q}_L) u^\dagger , \\ \widehat{\nabla}_\mu \mathcal{Q}_R &= \nabla_\mu \mathcal{Q}_R - \frac{i}{2} [u_\mu, \mathcal{Q}_R] = u^\dagger (D_\mu \mathcal{Q}_R) u , \end{aligned} \quad (5.3)$$

with

$$\begin{aligned} D_\mu \lambda &= \partial_\mu \lambda - i[l_\mu, \lambda] , \\ D_\mu \mathcal{Q}_L &= \partial_\mu \mathcal{Q}_L - i[l_\mu, \mathcal{Q}_L] , \\ D_\mu \mathcal{Q}_R &= \partial_\mu \mathcal{Q}_R - i[r_\mu, \mathcal{Q}_R] . \end{aligned} \quad (5.4)$$

For the construction of the list of local terms (5.2) we have used CPS invariance, the relations (3.7) and

$$\mathcal{Q}_{L,R}^2 = \frac{2}{9} \mathbf{1} + \frac{1}{3} \mathcal{Q}_{L,R} , \quad (5.5)$$

the Cayley-Hamilton formula, partial integration and the equations of motion (4.4).

If the spurion fields $\mathcal{Q}_{L,R}$ and λ are fixed to the constant values in (2.7) and (2.11), respectively, then $\mathcal{L}_{G_8 e^2 p^2}$ transforms under G as

$$(8_L, 1_R) + (8_L, 8_R) + (27_L, 1_R) + (27_L, 8_R) + (8_L, 27_R) . \quad (5.6)$$

This structure is richer than the one of the $\mathcal{O}(G_8)$ terms in \mathcal{L}_2 and also of the weak four-fermion effective Hamiltonian [12]. The last two pieces, in particular, which are responsible for $\Delta I = 5/2$ transitions, have no analog in the effective Hamiltonian of dimension six.

The operator Q_{16} does not contribute to on-shell matrix elements [10, 22, 23, 20]. The terms Q_{17} , Q_{18} , Q_{19} vanish for electrically neutral (pseudo)scalar sources,

$$[\chi, Q] = 0 , \quad (5.7)$$

which is, of course, the case for all realistic physical processes. Also the operators Q_{20}, \dots, Q_{32} are irrelevant for practical purposes. Because of (5.3) and (5.4), they contribute only in the presence of non-vanishing external (axial-)vector sources.

The coupling constants Z_1, \dots, Z_{12} appear in the amplitudes of $K \rightarrow 2\pi$ decays. The operators Q_{13} and Q_{14} do not contribute to $K \rightarrow 2\pi$ but they enter for $K \rightarrow 3\pi$. Q_{15} involves at least five pseudoscalar fields and is therefore irrelevant for K decays. A few linear combinations of the operators in (5.2) were already given some time ago by de Rafael [1]. His list was restricted to terms contributing to $K \rightarrow 2\pi$, neglecting contributions $\sim M_\pi^2$ and those renormalizing G_8 . A more recent extension of de Rafael's list can be found in Ref. [3]. However, their Lagrangian is still incomplete even for the

$K \rightarrow 2\pi$ amplitudes, as we shall discuss in the following section. There is in addition an obvious misprint in the operator multiplied by s_6 in [3], which would be in conflict with chiral symmetry. Some of the operators in (5.2) have also appeared in attempts [21] to bosonize the $\Delta S = 1$ four-fermion effective Hamiltonian.

The low-energy couplings Z_i are in general divergent. They absorb the divergences of the one-loop graphs via the renormalization

$$\begin{aligned} Z_i &= Z_i^r(\mu) + z_i \Lambda(\mu), \quad i = 1, \dots, 32, \\ \Lambda(\mu) &= \frac{\mu^{d-4}}{(4\pi)^2} \left\{ \frac{1}{d-4} - \frac{1}{2} [\ln(4\pi) + \Gamma'(1) + 1] \right\}, \end{aligned} \quad (5.8)$$

in the dimensional regularization scheme. The coefficients z_1, \dots, z_{32} are determined in such a way that the divergences generated by (4.14) are cancelled:

$$\begin{aligned} z_1 &= -\frac{17}{12} - 3Z + \frac{3}{2}g_{\text{ewk}}, & z_2 &= 1 + \frac{16}{3}Z + g_{\text{ewk}}, & z_3 &= \frac{3}{4} + 7Z, & z_4 &= -\frac{3}{4} - 7Z, \\ z_5 &= -2, & z_6 &= \frac{7}{2} + 5Z + \frac{3}{2}g_{\text{ewk}}, & z_7 &= \frac{3}{2} + 5Z, & z_8 &= -\frac{1}{2}, \\ z_9 &= -\frac{11}{6} + \frac{4}{3}Z + 2g_{\text{ewk}}, & z_{10} &= -\frac{3}{2} - Z, & z_{11} &= -\frac{3}{2} - 2Z, & z_{12} &= \frac{3}{2}, \\ z_{13} &= -\frac{35}{12} - 3Z + g_{\text{ewk}}, & z_{14} &= 3 + 15Z, & z_{15} &= \frac{3}{2} + 15Z, & z_{16} &= -\frac{4}{9} - \frac{4}{3}Z, \\ z_{17} &= \frac{2}{3} + 2Z, & z_{18} &= \frac{3}{4} + 3Z, & z_{19} &= 4Z, & z_{20} &= -\frac{1}{2}, \\ z_{21} &= \frac{1}{6}, & z_{22} &= 3 + 6Z, & z_{23} &= -3 - 9Z, & z_{24} &= 0, \\ z_{25} &= -3Z, & z_{26} &= -1, & z_{27} &= 0, & z_{28} &= -\frac{1}{2}, \\ z_{29} &= -\frac{1}{2}, & z_{30} &= 0, & z_{31} &= \frac{3}{2} + 6Z, & z_{32} &= \frac{3}{2} + 6Z. \end{aligned} \quad (5.9)$$

As already discussed above, the values in this list depend on our conventions for the basis systems in the strong, electromagnetic and weak parts of the next-to-leading order Lagrangian. The z_i given in (5.9) have to be used together with the divergent parts of the coupling constants L_i [13], K_i [9] and N_i [8], respectively. The divergences involving the electroweak penguin coupling g_{ewk} are independent of this choice of basis and they agree with a recent calculation of Cirigliano and Golowich [24]. Note that g_{ewk} appears only in the couplings of $(8_L, 8_R)$ operators. This is because the lowest-order term proportional to g_{ewk} is already of $\mathcal{O}(G_8 e^2)$. Therefore, the $\mathcal{O}(G_8 e^2 p^2)$ terms proportional to g_{ewk} arise from the product of the lowest-order $(8_L, 8_R)$ weak operator times the $\mathcal{O}(p^2)$ invariant part of the strong Lagrangian.

The renormalized low-energy constants $Z_i^r(\mu)$ are in general scale dependent. The coefficients z_i govern this scale dependence through the renormalization group equations

$$\mu \frac{dZ_i^r(\mu)}{d\mu} = -\frac{z_i}{(4\pi)^2}. \quad (5.10)$$

By construction, the complete generating functional at next-to-leading order is then scale independent.

6 $K \rightarrow \pi\pi$

In the modern framework of chiral perturbation theory, electromagnetic corrections for $K \rightarrow \pi\pi$ decays to $\mathcal{O}(G_8 e^2 p^2)$ were discussed by de Rafael [1] and have been treated in more detail by Cirigliano, Donoghue and Golowich [2, 3, 4]. Together with corrections of $\mathcal{O}(G_8(m_u - m_d)p^2)$ [5], the complete isospin-breaking effects of next-to-leading order have obvious phenomenological implications, from the $\Delta I = 1/2$ rule to CP violation [25].

In this section, we present the tree-level contributions to the $K \rightarrow \pi\pi$ amplitudes from the Lagrangian (5.1). We compare those amplitudes and in particular their divergent parts with the results of Ref. [3]. Using our own one-loop calculation of isospin-breaking corrections [7] and the heat-kernel results (5.9), we find that the complete amplitudes of $\mathcal{O}(G_8 e^2 p^2)$ are indeed finite. We demonstrate the cancellation of divergences explicitly for the subset of amplitudes proportional to the electromagnetic penguin coupling g_{ewk} defined in (3.6).

From the Lagrangian (5.1) of $\mathcal{O}(G_8 e^2 p^2)$, we obtain the following amplitudes in units of $C_{\text{ewk}} := iG_8 e^2 F$:

$$\begin{aligned}
A(K^0 \rightarrow \pi^+ \pi^-) &= C_{\text{ewk}} \sqrt{2} \left[(M_K^2 - M_\pi^2)(2Z_1 + 4Z_2 - 4/3Z_3 + 4Z_4 - Z_5 - 1/3Z_6 \right. \\
&\quad \left. - 2/3Z_7) + M_\pi^2(6Z_1 + 6Z_2 - Z_6) \right] , \\
A(K^0 \rightarrow \pi^0 \pi^0) &= C_{\text{ewk}} \sqrt{2} (M_K^2 - M_\pi^2) (-Z_5 + 2/3Z_6 - 2/3Z_7 + Z_8 + Z_9 + 2/3Z_{10} \\
&\quad - 2/3Z_{11} - 2/3Z_{12}) , \\
A(K^+ \rightarrow \pi^+ \pi^0) &= C_{\text{ewk}} \left[(M_K^2 - M_\pi^2)(2Z_1 + 4Z_2 - 4/3Z_3 - Z_6 - Z_8 - Z_9 - 2/3Z_{10} \right. \\
&\quad \left. - 4/3Z_{11} - 4/3Z_{12}) + M_\pi^2(6Z_1 + 6Z_2 - Z_6) \right] . \tag{6.1}
\end{aligned}$$

These amplitudes agree with Ref. [3] for $Z_3 = 3Z_2, Z_{10} = Z_{11} = 0$. In addition, the coefficients s_8, s_9 in Eq. (35) of [3] should be multiplied by 2/3.

In the $SU(3)$ limit for the mass matrix (3.2), the amplitudes (6.1) satisfy the relations

$$\begin{aligned}
A(K^0 \rightarrow \pi^+ \pi^-)_{SU(3)} &= \sqrt{2} A(K^+ \rightarrow \pi^+ \pi^0)_{SU(3)} , \\
A(K^0 \rightarrow \pi^0 \pi^0)_{SU(3)} &= 0 , \tag{6.2}
\end{aligned}$$

in accordance with a general theorem on $K \rightarrow \pi\pi$ transitions in the presence of electromagnetism [7].

The divergent parts of the Z_i in (5.9) give rise to the following divergent tree-level amplitudes, with $\Lambda(\mu)$ and Z defined in (5.8) and (3.5), respectively:

$$\begin{aligned}
A(K^0 \rightarrow \pi^+ \pi^-)_{\text{div}} &= C_{\text{ewk}} \sqrt{2} \Lambda(\mu) \left[M_K^2 (-3 - 27Z + 13/2g_{\text{ewk}}) \right. \\
&\quad \left. + M_\pi^2 (-3 + 36Z + 7g_{\text{ewk}}) \right] , \\
A(K^0 \rightarrow \pi^0 \pi^0)_{\text{div}} &= C_{\text{ewk}} \sqrt{2} \Lambda(\mu) (M_K^2 - M_\pi^2) (2Z + 3g_{\text{ewk}}) , \tag{6.3} \\
A(K^+ \rightarrow \pi^+ \pi^0)_{\text{div}} &= C_{\text{ewk}} \Lambda(\mu) \left[M_K^2 (3Z + 7/2g_{\text{ewk}}) + M_\pi^2 (-6 + 6Z + 10g_{\text{ewk}}) \right] .
\end{aligned}$$

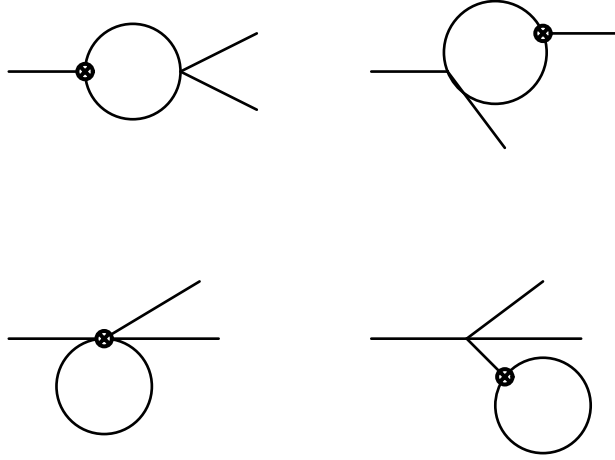


Figure 1: Loop diagrams for $K \rightarrow \pi\pi$ involving the electromagnetic penguin coupling g_{ewk} . The associated vertex from the Lagrangian (3.6) is denoted by a crossed circle. Normal vertices are from the lowest-order strong Lagrangian (3.1).

The (ultraviolet) divergences in (6.3) arise from three different sources:

- Photon loops proportional to $G_8 e^2$;
- Loops involving the electromagnetic coupling (3.5) proportional to $G_8 e^2 Z$;
- Loops involving the coupling (3.6) proportional to $G_8 e^2 g_{\text{ewk}}$.

Strong and electromagnetic wave function renormalization [26] is included in all three categories.

We have performed a complete calculation of $K \rightarrow \pi\pi$ amplitudes to $\mathcal{O}(G_8 e^2 p^2)$ and $\mathcal{O}(G_8 (m_u - m_d) p^2)$ [7]. For $m_u = m_d$, we find that the explicit loop divergences are exactly cancelled by the divergent tree-level amplitudes (6.3). We exhibit those cancellations in detail for the divergences proportional to g_{ewk} . Divergences arise both in loops with an electromagnetic penguin vertex shown in Fig. 1 and from (strong) wave function renormalization of tree diagrams from the Lagrangian (3.6).

In the exponential parametrization, the divergences due to the diagrams of Fig. 1 take the form

$$\begin{aligned}
 A(K^0 \rightarrow \pi^+ \pi^-)_{\text{loops}} &= -\frac{\sqrt{2}}{2} C_{\text{ewk}} g_{\text{ewk}} \Lambda(\mu) (7M_K^2 + 8M_\pi^2) , \\
 A(K^0 \rightarrow \pi^0 \pi^0)_{\text{loops}} &= -3\sqrt{2} C_{\text{ewk}} g_{\text{ewk}} \Lambda(\mu) (M_K^2 - M_\pi^2) , \\
 A(K^+ \rightarrow \pi^+ \pi^0)_{\text{loops}} &= -C_{\text{ewk}} g_{\text{ewk}} \Lambda(\mu) (M_K^2/2 + 7M_\pi^2) .
 \end{aligned} \tag{6.4}$$

Wave function renormalization (again in exponential parametrization) leads to

$$\begin{aligned}
 A(K^0 \rightarrow \pi^+ \pi^-)_{\text{wfr}} &= -3\sqrt{2} C_{\text{ewk}} g_{\text{ewk}} \Lambda(\mu) (M_K^2 + M_\pi^2) , \\
 A(K^0 \rightarrow \pi^0 \pi^0)_{\text{wfr}} &= 0 , \\
 A(K^+ \rightarrow \pi^+ \pi^0)_{\text{wfr}} &= -3C_{\text{ewk}} g_{\text{ewk}} \Lambda(\mu) (M_K^2 + M_\pi^2) .
 \end{aligned} \tag{6.5}$$

The sum of (6.4) and (6.5) is parametrization independent and it is exactly cancelled by the terms in (6.3) proportional to g_{ewk} .

We have exhibited (part of) the loop divergences explicitly also because we do not completely agree with the results of Ref. [3]. Although the divergences due to photon loops are identical, we obtain different results for some of the other divergences¹. Only for the channel $K^0 \rightarrow \pi^+\pi^-$, there is complete agreement for all three types of divergences.

The complete amplitudes of $\mathcal{O}(G_8 e^2 p^2)$ and $\mathcal{O}(G_8(m_u - m_d)p^2)$ together with a phenomenological analysis will be presented elsewhere [7].

7 Conclusions

We have supplied the missing ingredients for a complete analysis at next-to-leading order of the combined strong, nonleptonic weak and electromagnetic interactions of mesons. The main results are:

- i. The complete and minimal Lagrangian (5.1) of $\mathcal{O}(G_8 e^2 p^2)$ contains 32 operators Q_i and associated dimensionless coupling constants Z_i . Of these 32 operators, only 14 are of immediate phenomenological relevance. We have ordered the terms in a way most suitable for applications: the first 12 operators contribute to $K \rightarrow 2\pi$ decays whereas the remaining two enter in $K \rightarrow 3\pi$ amplitudes.
- ii. The one-loop divergence functional (4.13) determines the renormalization of the effective theory. Together with the previously known divergences, the new terms (5.9) in the coupling constants Z_i ensure that the complete amplitudes for strong, nonleptonic weak and electromagnetic interactions of mesons at next-to-leading order are finite.

As a first application, we have presented the tree-level amplitudes of $\mathcal{O}(G_8 e^2 p^2)$ for $K \rightarrow \pi\pi$ decays. The associated divergent parts cancel with the explicit one-loop divergences [7] to yield finite and scale independent decay amplitudes.

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¹V. Cirigliano has informed us that they now agree with the divergences (6.3); see forthcoming erratum for Ref. [3].

Appendix

The quantities occurring in (4.14) can be decomposed with respect to (explicit²) powers of e and G_8 in the following way:

$$\begin{aligned}
\sigma_{ij} &= \sigma_{ij}|_{e^0 G_8^0} + \sigma_{ij}|_{e^2 G_8^0} + \sigma_{ij}|_{e^0 G_8} + \sigma_{ij}|_{e^2 G_8} , \\
\gamma_\mu &= \gamma_\mu|_{e^0 G_8^0} + \gamma_\mu|_{e^0 G_8} , \\
a_i^\mu &= a_i^\mu|_{e G_8^0} + a_i^\mu|_{e G_8} , \\
b_i &= b_i|_{e G_8^0} + b_i|_{e G_8} , \\
\kappa &= \kappa|_{e^2 G_8^0} + \kappa|_{e^2 G_8} .
\end{aligned} \tag{A.1}$$

The explicit expressions for the various terms are given by

$$\sigma_{ij}|_{e^0 G_8^0} = \frac{1}{8} \langle (u_\mu u^\mu + \chi_+) \{ \lambda_i, \lambda_j \} \rangle - \frac{1}{4} \langle u_\mu \lambda_i u^\mu \lambda_j \rangle , \tag{A.2}$$

$$\sigma_{ij}|_{e^2 G_8^0} = e^2 F^2 Z \langle \frac{1}{2} \{ \mathcal{Q}_R, \mathcal{Q}_L \} \{ \lambda_i, \lambda_j \} - \lambda_i \mathcal{Q}_R \lambda_j \mathcal{Q}_L - \lambda_j \mathcal{Q}_R \lambda_i \mathcal{Q}_L \rangle , \tag{A.3}$$

$$\begin{aligned}
\sigma_{ij}|_{e^0 G_8} &= \frac{1}{4} \langle \{ \Xi, \lambda_i \} u_\mu \lambda_j u^\mu \rangle + \frac{1}{4} \langle \{ \Xi, \lambda_j \} u_\mu \lambda_i u^\mu \rangle \\
&\quad - \frac{1}{8} \langle (u_\mu u^\mu + \chi_+) (\{ \lambda_i, \{ \Xi, \lambda_j \} \} + \{ \lambda_j, \{ \Xi, \lambda_i \} \}) \rangle \\
&\quad + \frac{1}{6} \langle \Xi \lambda_i \rangle \langle \chi_+ \lambda_j \rangle + \frac{1}{6} \langle \Xi \lambda_j \rangle \langle \chi_+ \lambda_i \rangle \\
&\quad + \frac{1}{4} \langle \Xi \{ u_\mu, \{ u^\mu, \{ \lambda_i, \lambda_j \} \} \} \rangle \\
&\quad - \frac{1}{4} \langle \Xi (\{ u_\mu, \lambda_i \} \{ u^\mu, \lambda_j \} + \{ u_\mu, \lambda_j \} \{ u^\mu, \lambda_i \}) \rangle \\
&\quad + \frac{i}{4} \langle [u_\mu, \nabla^\mu \Xi] \{ \lambda_i, \lambda_j \} \rangle + \frac{i}{4} \langle [\nabla^\mu u_\mu, \Xi] \{ \lambda_i, \lambda_j \} \rangle \\
&\quad - \frac{1}{2} \langle \{ \lambda_i, \lambda_j \} \nabla_\mu \nabla^\mu \Xi \rangle ,
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
\sigma_{ij}|_{e^2 G_8} &= e^2 F^2 Z \langle \Xi (\lambda_i \mathcal{Q}_R \lambda_j \mathcal{Q}_L + \lambda_j \mathcal{Q}_R \lambda_i \mathcal{Q}_L + \mathcal{Q}_R \lambda_i \mathcal{Q}_L \lambda_j + \mathcal{Q}_R \lambda_j \mathcal{Q}_L \lambda_i \\
&\quad + \lambda_i \mathcal{Q}_L \lambda_j \mathcal{Q}_R + \lambda_j \mathcal{Q}_L \lambda_i \mathcal{Q}_R + \mathcal{Q}_L \lambda_i \mathcal{Q}_R \lambda_j + \mathcal{Q}_L \lambda_j \mathcal{Q}_R \lambda_i \\
&\quad - \lambda_i \{ \mathcal{Q}_L, \mathcal{Q}_R \} \lambda_j - \lambda_j \{ \mathcal{Q}_L, \mathcal{Q}_R \} \lambda_i - \frac{1}{2} \{ \{ \mathcal{Q}_L, \mathcal{Q}_R \}, \{ \lambda_i, \lambda_j \} \} \rangle \\
&\quad + e^2 F^2 \langle \Upsilon (\frac{1}{2} \{ \{ \lambda_i, \lambda_j \}, \mathcal{Q}_R \} - \lambda_i \mathcal{Q}_R \lambda_j - \lambda_j \mathcal{Q}_R \lambda_i) \rangle ,
\end{aligned} \tag{A.5}$$

$$\gamma_{ij}^\mu|_{e^0 G_8^0} = -\frac{1}{2} \langle \Gamma^\mu [\lambda_i, \lambda_j] \rangle , \tag{A.6}$$

$$\gamma_{ij}^\mu|_{e^0 G_8} = \frac{i}{4} \langle [\lambda_i, \Xi] \{ u^\mu, \lambda_j \} \rangle - \frac{i}{4} \langle [\lambda_j, \Xi] \{ u^\mu, \lambda_i \} \rangle , \tag{A.7}$$

$$a_i^\mu|_{e G_8^0} = -\frac{ieF}{4} \langle u^\mu [\mathcal{Q}_R + \mathcal{Q}_L, \lambda_i] \rangle , \tag{A.8}$$

²Note that e also appears in the vielbein u_μ (2.2) and in the connection Γ_μ (4.6) via (2.3).

$$\begin{aligned}
a_i^\mu|_{eG_8} &= \frac{ieF}{4} \langle \Xi(u^\mu \lambda_i \mathcal{Q}_R - \mathcal{Q}_R \lambda_i u^\mu) \rangle + \frac{3ieF}{4} \langle \Xi(u^\mu \lambda_i \mathcal{Q}_L - \mathcal{Q}_L \lambda_i u^\mu) \rangle \\
&\quad - \frac{ieF}{2} \langle \Xi(u^\mu \mathcal{Q}_R \lambda_i - \lambda_i \mathcal{Q}_R u^\mu) \rangle + \frac{ieF}{4} \langle \Xi(\lambda_i u^\mu \mathcal{Q}_R - \mathcal{Q}_R u^\mu \lambda_i) \rangle \\
&\quad + \frac{ieF}{4} \langle \Xi(\lambda_i u^\mu \mathcal{Q}_L - \mathcal{Q}_L u^\mu \lambda_i) \rangle - \frac{eF}{2} \langle (\mathcal{Q}_R - \mathcal{Q}_L) \{ \nabla^\mu \Xi, \lambda_i \} \rangle, \quad (\text{A.9})
\end{aligned}$$

$$b_i|_{eG_8^0} = \frac{eF}{2} \langle (\mathcal{Q}_R - \mathcal{Q}_L) \lambda_i \rangle, \quad (\text{A.10})$$

$$b_i|_{eG_8} = \frac{eF}{2} \langle \Xi \{ \mathcal{Q}_R - \mathcal{Q}_L, \lambda_i \} \rangle, \quad (\text{A.11})$$

$$\kappa|_{e^2 G_8^0} = \frac{e^2 F^2}{2} \langle (\mathcal{Q}_R - \mathcal{Q}_L)^2 \rangle, \quad (\text{A.12})$$

$$\kappa|_{e^2 G_8} = 2e^2 F^2 \langle \Xi (\mathcal{Q}_R - \mathcal{Q}_L)^2 \rangle. \quad (\text{A.13})$$

The expressions (A.12) and (A.13) are included for completeness only; κ does not contribute to the order we are concerned with.

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