

## THE ROLE OF FINAL STATE INTERACTIONS IN $\varepsilon'/\varepsilon$

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The Standard Model prediction for  $\varepsilon'/\varepsilon$  is updated, taking into account the chiral loop corrections induced by final state interactions. The resulting value,  $\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}$ , is in good agreement with present measurements.

### 1. Introduction

The CP-violating ratio  $\varepsilon'/\varepsilon$  constitutes a fundamental test for our understanding of flavour-changing phenomena. The present experimental world average,<sup>1</sup>  $\text{Re}(\varepsilon'/\varepsilon) = (19.3 \pm 2.4) \cdot 10^{-4}$ , provides clear evidence for a non-zero value and, therefore, the existence of direct CP violation.

The theoretical prediction has been rather controversial since different groups, using different models or approximations, have obtained different results.<sup>2,3,4,5,6,7,8,9</sup> In terms of the  $K \rightarrow \pi\pi$  isospin amplitudes,  $\mathcal{A}_I = A_I e^{i\delta_I}$  ( $I = 0, 2$ ),

$$\frac{\varepsilon'}{\varepsilon} = e^{i\Phi} \frac{\omega}{\sqrt{2}|\varepsilon|} \left[ \frac{\text{Im}A_2}{\text{Re}A_2} - \frac{\text{Im}A_0}{\text{Re}A_0} \right], \quad \Phi \approx \delta_2 - \delta_0 + \frac{\pi}{4} \approx 0, \quad (1)$$

where  $\omega = \text{Re}A_2/\text{Re}A_0 \approx 1/22$ . The CP-conserving amplitudes  $\text{Re}A_I$ , their ratio  $\omega$  and  $\varepsilon$  are usually set to their experimentally determined values. A theoretical calculation is then only needed for the quantities  $\text{Im}A_I$ .

Since  $M_W \gg M_K$ , there are large short-distance logarithmic contributions which can be summed up using the Operator Product Expansion and the renormalization group.<sup>10,11</sup> To predict the physical amplitudes one also needs to compute long-distance hadronic matrix elements of light four-quark operators  $Q_i$ . They are usually parameterized in terms of the so-called bag parameters  $B_i$ , which measure them in units of their vacuum insertion approximation values.

To a very good approximation, the Standard Model prediction for  $\varepsilon'/\varepsilon$  can be written (up to global factors) as<sup>5</sup>

$$\frac{\varepsilon'}{\varepsilon} \sim \left[ B_6^{(1/2)}(1 - \Omega_{IB}) - 0.4 B_8^{(3/2)} \right], \quad \Omega_{IB} = \frac{1}{\omega} \frac{(\text{Im}A_2)_{IB}}{\text{Im}A_0}. \quad (2)$$

Thus, only two operators are numerically relevant: the QCD penguin operator  $Q_6$  governs  $\text{Im}A_0$  ( $\Delta I = 1/2$ ), while  $\text{Im}A_2$  ( $\Delta I = 3/2$ ) is dominated by the electroweak

penguin operator  $Q_8$ . The parameter  $\Omega_{IB}$  takes into account isospin breaking corrections; the value  $\Omega_{IB} = 0.25$  was usually adopted in all calculations.<sup>12</sup> Together with  $B_i \sim 1$ , this produces a numerical cancellation leading to values of  $\varepsilon'/\varepsilon \sim 7 \times 10^{-4}$ . This number has been slightly increased by a recent Chiral Perturbation Theory ( $\chi$ PT) calculation at  $O(p^4)$  which finds  $\Omega_{IB} = 0.16 \pm 0.03$ .<sup>13</sup>

## 2. Chiral Loop Corrections

Chiral symmetry determines the low-energy hadronic realization of the operators  $Q_i$ , through a perturbative expansion in powers of momenta and quark masses. The corresponding chiral couplings can be calculated in the large- $N_C$  limit of QCD. The usual input values  $B_8^{(3/2)} \approx B_6^{(1/2)} = 1$  correspond to the lowest-order approximation in both the  $1/N_C$  and  $\chi$ PT expansions.

The lowest-order calculation does not provide any strong phases  $\delta_I$ . Those phases originate in the final rescattering of the two pions and, therefore, are generated by higher-order chiral loops. Analyticity and unitarity require the presence of a corresponding dispersive effect in the moduli of the isospin amplitudes. Since the S-wave strong phases are quite large, specially in the isospin-zero case, one should expect large unitarity corrections.

The one-loop analyses of  $K \rightarrow 2\pi$  show in fact that pion loop diagrams provide an important enhancement of the  $\mathcal{A}_0$  amplitude.<sup>14</sup> This chiral loop correction destroys the accidental numerical cancellation in eq. (2), generating a sizeable enhancement of the  $\varepsilon'/\varepsilon$  prediction.<sup>2</sup> The large one-loop correction to  $\mathcal{A}_0$  has its origin in the strong final state interaction (FSI) of the two pions in S-wave, which generates large infrared logarithms involving the light pion mass.<sup>3</sup> Using analyticity and unitarity constraints, these logarithms can be exponentiated to all orders in the chiral expansion.<sup>2,3</sup> For the CP-conserving amplitudes, the result can be written as

$$\mathcal{A}_I = (M_K^2 - M_\pi^2) a_I(M_K^2) = (M_K^2 - M_\pi^2) \Omega_I(M_K^2, s_0) a_I(s_0), \quad (3)$$

where  $a_I(s)$  denote reduced off-shell amplitudes with  $s \equiv (p_{\pi_1} + p_{\pi_2})^2$  and

$$\Omega_I(s, s_0) \equiv e^{i\delta_I(s)} \Re_I(s, s_0) = \exp \left\{ \frac{(s - s_0)}{\pi} \int \frac{dz}{(z - s_0)} \frac{\delta_I(z)}{(z - s - i\epsilon)} \right\} \quad (4)$$

provides an evolution of  $a_I(s)$  from an arbitrary low-energy point  $s_0$  to  $s = M_K^2$ . The physical amplitude  $a_I(M_K^2)$  is of course independent of  $s_0$ .

Taking the chiral prediction for  $\delta_I(z)$  and expanding the exponential to first order, one just reproduces the one-loop  $\chi$ PT result. Eq. (4) allows us to get a much more accurate prediction, by taking  $s_0$  low enough that the  $\chi$ PT corrections to  $a_I(s_0)$  are small and exponentiating the large logarithms with the Omnès factor  $\Omega_I(M_K^2, s_0)$ . Moreover, using the experimental phase-shifts in the dispersive integral one achieves an all-order resummation of FSI effects. The numerical accuracy of this exponentiation has been successfully tested through an analysis of the scalar pion form factor,<sup>3</sup> which has identical FSI than  $\mathcal{A}_0$ .

### 3. Numerical Predictions

At  $s_0 = 0$ , the chiral corrections are rather small. To a very good approximation,<sup>4</sup> we can just multiply the tree-level  $\chi$ PT result for  $a_I(0)$  with the experimentally determined Omnès exponentials:<sup>3</sup>

$$\Re_0 \equiv \Re_0(M_K^2, 0) = 1.55 \pm 0.10, \quad \Re_2 \equiv \Re_2(M_K^2, 0) = 0.92 \pm 0.03. \quad (5)$$

Thus,  $B_6^{(1/2)} \approx \Re_0 \times B_6^{(1/2)} \Big|_{N_C \rightarrow \infty} = 1.55$ ,  $B_8^{(3/2)} \approx \Re_2 \times B_8^{(3/2)} \Big|_{N_C \rightarrow \infty} \approx 0.92$  and  $\Omega_{IB} \approx 0.16 \times \Re_2/\Re_0 = 0.09$ . This agrees with the result  $\Omega_{IB} = 0.08 \pm 0.05$ , obtained recently with an explicit chiral loop calculation.<sup>15</sup>

The large FSI correction to the  $I = 0$  amplitude gets reinforced by the mild suppression of the  $I = 2$  contributions. The net effect is a large enhancement of  $\varepsilon'/\varepsilon$  by a factor 2.4, pushing the predicted central value from<sup>5,6</sup>  $7 \times 10^{-4}$  to<sup>3</sup>  $17 \times 10^{-4}$ . A more careful analysis, taking into account all hadronic and quark-mixing inputs gives the Standard Model prediction:<sup>4</sup>

$$\varepsilon'/\varepsilon = (17 \pm 6) \times 10^{-4}, \quad (6)$$

which compares well with the present experimental world average.

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