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Meson resonances, large N_c and chiral symmetry

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Abstract

We investigate the implications of large N_c and chiral symmetry for the mass spectra of meson resonances. Unlike for most other mesons, the mass matrix of the light scalars deviates strongly from its large-N_c limit. We discuss the possible assignments for the lightest scalar nonet that survives in the large-N_c limit.

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1. The interpretation of scalar meson resonances has been controversial for a long time. The problems are of both experimental and theoretical nature [1, 2]. As a distinctive feature of scalar mesons, the $SU(3)$ singlet has vacuum quantum numbers. Scalar mesons may therefore be especially susceptible to the non-trivial structure of the QCD vacuum.

To explore the peculiar properties of 0^{++} mesons, we propose a general analysis of the mass spectra of all light meson resonances that is only based on established consequences of QCD for light hadrons. In particular, we make no reference to the internal structure of meson resonances ($\overline{q}q$, multi-quark states, meson-meson bound states, glueballs, ...).

Our main assumptions are two-fold.

i. We assume that the mass splittings of light meson multiplets and their couplings to pseudoscalar mesons can be understood in the framework of a chiral resonance Lagrangian [3, 4]. Only leading terms in the chiral expansion will be considered.

ii. In first approximation, we assume a nonet structure for the mesons as predicted by QCD in the limit of large N_c [5]. In order to parametrize the deviations from the nonet limit, we include in a second step all possible sub-leading terms in $1/N_c$ of relevance for the mass spectrum as long as they are of leading order in the chiral expansion.

2. We first recall the main features of chiral resonance theory [3]. The resonance fields come in $SU(3)$ octets and singlets and they transform in the usual way under a non-linear realization of chiral $SU(3)$. The octet (R_i) and singlet (R_0) fields are grouped together in a nonet field R:

$$R = \lambda_i R_i/\sqrt{2} + R_0/\sqrt{3}\ \mathbf{1}\ . \tag{1}$$

In the limit of large N_c, these nine fields are degenerate in the chiral limit with a common mass M_R. To understand the phenomenological values of the low-energy constants (LECs) L_i in the chiral Lagrangian of $O(p^4)$ [6], a chiral resonance Lagrangian of the following generic form is employed:

$$\mathcal{L}_R \;=\; \frac{1}{2}\langle \nabla R \cdot \nabla R - M_R^2 R^2 \rangle + \langle R g_2^R \rangle\ . \tag{2}$$

Following the notation of Ref. [3], ∇R denotes a chiral- and gauge-covariant derivative. All space-time indices are omitted. g_2^R is a chiral field of $O(p^2)$ that couples to the respective resonance multiplet of given spin-parity; $\langle \ldots \rangle$ stands for the three-dimensional flavour trace. The large-N_c relations for the scalar couplings discussed in Ref. [3] are automatically reproduced by the Lagrangian (2).

In order to calculate the contributions of meson resonance exchange to the LECs of $O(p^6)$ [7] , the Lagrangian (2) must be extended:

$$\mathcal{L}_R \;=\; \frac{1}{2}\langle \nabla R \cdot \nabla R - M_R^2 R^2 \rangle + \langle R(g_2^R + g_4^R) \rangle + \langle R^2 h_2^R + \ \ldots\ \rangle\ . \tag{3}$$

Only single flavour traces appear in (3) because we assume large N_c at this point. For our purposes, we only need to consider bilinear interaction terms of the type shown in (3) where R^2 (and therefore h_2^R) is a Lorentz scalar. There are other bilinear terms that will also contribute at $O(p^6)$, e.g., mixed terms with different resonance fields. On the other hand, cubic and higher couplings in the resonance fields do not contribute to the effective low-energy Lagrangian of $O(p^6)$. The order of the chiral fields g_i^R, h_i^R is indicated by the subscript. The resonance Lagrangian (3) induces the following contribution to the effective Lagrangian to $O(p^6)$:

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{1}{2M_R^2}\langle g_2^R g_2^R\rangle \\ &+ \frac{1}{2M_R^4}\langle \nabla g_2^R \cdot \nabla g_2^R\rangle + \frac{1}{M_R^4}\langle g_2^R g_2^R h_2^R\rangle + \frac{1}{M_R^2}\langle g_2^R g_4^R\rangle + \ \ldots \end{aligned} \tag{4}$$

The first line reproduces the result of Ref. [3]. The second line contains the contributions of $O(p^6)$ from the exchange of a specific resonance multiplet with the Lagrangian (3).

Here we are interested in the mass splittings of the mesons. The resonance masses are derived from the non-derivative bilinear part of the Lagrangian (3):

$$\begin{aligned}\mathcal{L}_R^{\text{mass}} &= -\frac{M_R^2}{2}\langle R^2\rangle + e_m^R\langle R^2\chi_+\rangle \\ h_2^R &= e_m^R\chi_+ + \ldots\ ,\end{aligned} \tag{5}$$

with coupling constant e_m^R. The chiral field χ_+ contains the quark mass matrix $\mathcal{M}_q$:

$$\chi_+ = 4B\mathcal{M}_q + \ldots \tag{6}$$

where B is related to the scalar condensate [6]. We always stay in the $SU(2)$ limit with $M_\pi^2 = B(m_u + m_d) = 2B\hat{m}$, $M_K^2 = B(m_s + \hat{m})$.

The structure of the mass Lagrangian (5) is the same for all meson resonances with R^2 the appropriate bilinear field combination [3]. To leading order both in large N_c and in the chiral expansion, the mass splittings in a nonet are governed by a single coupling constant e_m^R. Of course, this constant will in general be different for different resonance nonets.

We use the following notation for the various resonance fields and for the corresponding masses:

$R_{I=1}$	isotriplet fields,
$R_{I=1/2}$	isodoublet fields,
R_0, R_8	singlet and isosinglet octet fields,
R_H, R_L	isosinglet mass eigenfields.

The masses of the non-singlet fields can immediately be extracted from the Lagrangian (5):

$$\begin{aligned}M_{I=1}^2 &= M_R^2 - 4e_m^R M_\pi^2 \\ M_{I=1/2}^2 &= M_R^2 - 4e_m^R M_K^2\ ,\end{aligned} \tag{7}$$

implying

$$
\begin{aligned}
e_m^R &= \frac{M_{I=1}^2 - M_{I=1/2}^2}{4(M_K^2 - M_\pi^2)} \\
M_R^2 &= M_{I=1}^2 + \frac{M_\pi^2 (M_{I=1}^2 - M_{I=1/2}^2)}{M_K^2 - M_\pi^2} \ .
\end{aligned} \tag{8}
$$

3. The mass matrix M_0^2 for the isosinglet fields R_8, R_0 is obtained from (5) as

$$
M_0^2 = \begin{pmatrix} M_R^2 - \frac{4}{3} e_m^R (4M_K^2 - M_\pi^2) & \frac{8\sqrt{2}}{3} e_m^R (M_K^2 - M_\pi^2) \\ \frac{8\sqrt{2}}{3} e_m^R (M_K^2 - M_\pi^2) & M_R^2 - \frac{4}{3} e_m^R (2M_K^2 + M_\pi^2) \end{pmatrix} , \tag{9}
$$

with eigenvalues

$$
M_R^2 - 4 e_m^R M_\pi^2 \qquad \text{and} \qquad M_R^2 - 4 e_m^R (2M_K^2 - M_\pi^2) \ . \tag{10}
$$

In terms of the non-singlet masses (7), the isosinglet masses are given by

$$
\begin{aligned}
M_L^2 &= M_{I=1/2}^2 - |M_{I=1/2}^2 - M_{I=1}^2| \\
M_H^2 &= M_{I=1/2}^2 + |M_{I=1/2}^2 - M_{I=1}^2| \ .
\end{aligned} \tag{11}
$$

The mass matrix (9) can be diagonalized with an orthogonal matrix O:

$$
M_0^2 = O^T M_D^2 O \ , \qquad M_D^2 = \mathrm{diag}(M_H^2, M_L^2) \tag{12}
$$

$$
O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} ,
$$

where the mixing angle θ is defined mod π. It will be convenient to consider the interval $-\pi/2 \leq \theta \leq \pi/2$. The mass eigenfields R_H, R_L are then given by

$$
\begin{aligned}
R_H &= \cos\theta \ R_8 - \sin\theta \ R_0 \\
R_L &= \sin\theta \ R_8 + \cos\theta \ R_0 \ .
\end{aligned} \tag{13}
$$

The fields R_8, R_0 can be expressed in terms of fields with specific flavour content in the $\overline{q}q$ picture:

$$
\begin{aligned}
R_8 &= \frac{1}{\sqrt{3}} R_{\mathrm{non-strange}} - \sqrt{\frac{2}{3}} R_{\mathrm{strange}} \\
R_0 &= \sqrt{\frac{2}{3}} R_{\mathrm{non-strange}} + \frac{1}{\sqrt{3}} R_{\mathrm{strange}} \ .
\end{aligned} \tag{14}
$$

The mass eigenfields can then also be written as

$$\begin{aligned} R_H &= \frac{1}{\sqrt{3}}(\cos\theta - \sqrt{2}\sin\theta)R_{\rm non-strange} - \frac{1}{\sqrt{3}}(\sqrt{2}\cos\theta + \sin\theta)R_{\rm strange} \\ R_L &= \frac{1}{\sqrt{3}}(\sqrt{2}\cos\theta + \sin\theta)R_{\rm non-strange} + \frac{1}{\sqrt{3}}(\cos\theta - \sqrt{2}\sin\theta)R_{\rm strange} \ . \end{aligned} \tag{15}$$

Ideal mixing with $R_H = -R_{\rm strange}$, $R_L = R_{\rm non-strange}$ corresponds to

$$\tan\theta_{\rm ideal} = 1/\sqrt{2} \quad \rightarrow \quad \theta_{\rm ideal} = 35.3^\circ \ . \tag{16}$$

The mass matrix (9) has a special property as already noted in Ref. [8]: the mixing angle θ depends only on the sign but not on the magnitude of e_m^R. We now discuss the two possibilities in turn.

i. $\mathbf{e_m^R > 0}$

The resonance masses are ordered as

$$M_L < M_{I=1/2} < M_H = M_{I=1} \ . \tag{17}$$

The mixing angle is found to be

$$\theta_{e_m^R>0} = -\arctan\sqrt{2} \quad \rightarrow \quad \theta_{e_m^R>0} = -54.7^\circ \ . \tag{18}$$

The ordering of masses is unusual because the strange member of the octet has a smaller mass than the isotriplet state. Most resonance nonets do not display such an inverted hierarchy. Also the mixing pattern is unusual (dual ideal mixing [8]): the light neutral field R_L is identical to $R_{\rm strange}$ and $R_H = R_{\rm non-strange}$.

ii. $\mathbf{e_m^R < 0}$

The isotriplet now changes position compared to (17):

$$M_{I=1} = M_L < M_{I=1/2} < M_H \ . \tag{19}$$

The mixing angle is now

$$\theta_{e_m^R<0} = \arctan\frac{1}{\sqrt{2}} \quad \rightarrow \quad \theta_{e_m^R<0} = \theta_{\rm ideal} = 35.3^\circ \ , \tag{20}$$

and therefore

$$R_L = R_{\rm non-strange} \ , \qquad\qquad R_H = -R_{\rm strange} \ . \tag{21}$$

This pattern is very well satisfied by the vector mesons and, as we shall review below, at least approximately also by other nonets:

$$\begin{gathered} M_\rho \simeq M_\omega < M_{K^*} < M_\phi \\ 2M_{K^*}^2 \simeq M_\omega^2 + M_\phi^2 \\ \theta \simeq \theta_{\rm ideal} \ . \end{gathered} \tag{22}$$

Therefore, only the case $e_m^R < 0$ corresponds to the usual notion of a nonet with ideal mixing [9].

4. Of course, not even the vector mesons are ideally mixed. We consider here a minimal version of nonet symmetry breaking where only the terms bilinear in the resonance fields are affected.

To lowest order in the chiral expansion, the mass Lagrangian (5) acquires two additional terms that are sub-leading in $1/N_c$:

$$\mathcal{L}_R^{\rm mass} = -\frac{M_R^2}{2}\langle R^2\rangle + e_m^R\langle R^2\chi_+\rangle + k_m^R R_0\langle \widehat{R}\chi_+\rangle - \frac{\gamma_R M_R^2}{2}R_0^2 \ , \tag{23}$$

where $\widehat{R} = \lambda_i R_i/\sqrt{2}$ is the octet field. The Lagrangian (23) is the most general lowest-order chiral Lagrangian bilinear in octet and singlet fields that can contribute to the mass matrix. All other terms can be absorbed by a redefinition of the parameters in (23).

The non-singlet fields $R_{I=1}$, $R_{I=1/2}$ are unaffected and their masses are still given by (7). The type of hierarchy is again determined by the sign of e_m^R. The additional parameters k_m^R and γ_R give rise to the isosinglet mass matrix

$$M_0^2 = \begin{pmatrix} M_R^2 - \frac{4}{3}e_m^R(4M_K^2 - M_\pi^2) & \frac{8\sqrt{2}}{3}(e_m^R + \frac{\sqrt{3}}{2}k_m^R)(M_K^2 - M_\pi^2) \\ \frac{8\sqrt{2}}{3}(e_m^R + \frac{\sqrt{3}}{2}k_m^R)(M_K^2 - M_\pi^2) & M_R^2(1+\gamma_R) - \frac{4}{3}e_m^R(2M_K^2 + M_\pi^2) \end{pmatrix} . \tag{24}$$

The interpretation of this mass matrix is straightforward. The first entry corresponds to the isosinglet octet field R_8 and it satisfies a (quadratic) Gell-Mann-Okubo mass formula with the non-singlet masses (7). This field mixes with the $SU(3)$ singlet R_0 via (24) in terms of two arbitrary constants k_m^R, γ_R that parametrize the deviations from the nonet limit. Although the matrix elements are of chiral order p^2 the matrix (24) is therefore effectively of a very general form. In a different notation, it has been used since the early days of resonance physics (see also Ref. [8]).

The non-singlet masses (7) and the mass matrix (24) imply the inequalities

$$M_L^2 \ \leq \ 4M_{I=1/2}^2/3 - M_{I=1}^2/3 \ \leq \ M_H^2 \ . \tag{25}$$

For given nonet masses, this is a necessary and sufficient condition for the existence of solutions in terms of parameters M_R, e_m^R, k_m^R and γ_R. In fact, there are exactly two solutions for a given set of masses satisfying (25) that differ only in the sign of the mixing angle θ and in the value of k_m^R. The two solutions are physically inequivalent but we discuss in the following only the solutions with $\theta \geq 0$ that are closer to the nonet limit.

Because of the new terms in (23) there are now additional contributions of both $O(p^4)$ and $O(p^6)$ to the effective Lagrangian (4) from resonance exchange:

$$\begin{aligned} \mathcal{L}_{\rm eff}^{\rm add} \ = \ & - \frac{\gamma_R}{6M_R^2(1+\gamma_R)}\langle g_2^R\rangle^2 \\ & + \frac{k_m^R}{\sqrt{3}M_R^4(1+\gamma_R)}\langle g_2^R\rangle \left(\langle g_2^R\chi_+\rangle - \langle g_2^R\rangle\langle\chi_+\rangle/3\right) \end{aligned}$$

$$- \frac{\gamma_R(2+\gamma_R)}{6M_R^4(1+\gamma_R)^2}\langle\nabla g_2^R\rangle^2 - \frac{\gamma_R}{3M_R^2(1+\gamma_R)}\langle g_2^R\rangle\langle g_4^R\rangle$$

$$- \frac{2\gamma_R}{3M_R^4(1+\gamma_R)}\langle g_2^R\rangle\langle g_2^R h_2^R\rangle + \frac{\gamma_R^2}{9M_R^4(1+\gamma_R)^2}\langle g_2^R\rangle^2\langle h_2^R\rangle \ . \qquad (26)$$

Anticipating possible large values of the parameter γ_R, we have written down the full expressions instead of expanding in γ_R. We recall that γ_R is of zeroth order in the chiral expansion, albeit sub-leading in $1/N_c$.

5. Before turning to our main subject of scalar mesons, we briefly review the status of the other low-lying meson resonances on the basis of the general mass Lagrangian (23) with isosinglet mass matrix (24).

1^{--}

The lowest-lying vector meson nonet consists of $\rho(770)$, $\omega(782)$, $K^*(892)$ and $\phi(1020)$. From the masses in Ref. [2] one obtains the parameters and the mixing angle collected in Table 1. Not surprisingly, the vector mesons make up an almost ideal nonet.

2^{++}

The lightest nonet of tensor mesons consists of $f_2(1270)$, $a_2(1320)$, $K_2^*(1430)$ and $f_2'(1525)$. The alternative singlet candidate $f_2(1430)$ (omitted from the summary table of Ref. [2]) does not satisfy the inequalities (25). The corresponding parameters and mixing angle in Table 1 document the well-known fact that also the tensor mesons are close to an ideally mixed nonet.

1^{++}

The unambiguous states in this nonet are $a_1(1260)$, $f_1(1285)$ and $f_1(1420)$. The strange isodoublet partner could be $K_1(1270)$ or $K_1(1400)$ or a mixture of these two states [10]. Without mixing, only the $K_1(1270)$ satisfies the inequalities (25). The ALEPH data for $\tau \to K_1\nu_\tau$ [11] are also consistent with a dominant ${}^3P_1$ nature of $K_1(1270)$. Neglecting a possible isodoublet mixing, the masses for the 1^{++} nonet give rise to the solution in Table 1 implying a substantial deviation from ideal mixing (see also Ref. [12]).

1^{+-}

The unambiguous states of this nonet are $h_1(1170)$ and $b_1(1235)$. Consistent with the assignment of $K_1(1270)$ to the 1^{++} nonet, the strange member of the 1^{+-} nonet must be $K_1(1400)$. As before, the situation could be more involved due to isodoublet mixing. For the final isosinglet member of the nonet, the two candidates are $h_1(1380)$ and $h_1(1595)$, neither of which enjoys the status of being listed in the PDG summary table [2]. With $K_1(1400)$ the strange state in this nonet, there is a clear preference: only $h_1(1595)$ satisfies the inequalities (25). The resulting solution can be found in Table 1. It implies an almost ideal mixing although the sub-leading parameter k_m^R is not very small in this case.

0^{-+}

The well-established states are $\pi(1300)$ and $\eta(1295)$. The $K(1460)$ does not appear in the PDG summary table but it is listed in the full review with a mass of either 1400 or 1460 MeV. Again, the inequalities (25) may serve as a guide. Only the lower mass of 1400 MeV allows for the inclusion of the heavy isoscalar $\eta(1440)$ with a mass of at least 1430 MeV. There is growing evidence that there are actually two different 0^{-+} states in that region [2] and we therefore take $M_{\eta(1440)} = 1470$ MeV. The solution with positive θ is again close to ideal mixing as shown in Table 1.

Summarizing the situation for the 1^{--}, 2^{++}, 1^{++}, 1^{+-} and 0^{-+} nonets, the sub-leading parameter k_m^R is in all cases substantially smaller in magnitude than e_m^R. With the possible exception of 1^{++} that may be subject to isodoublet mixing with 1^{+-}, the singlet-octet mixing is close to ideal. All five multiplets display the standard hierarchy: $e_m^R < 0$ in all cases.

	θ(degrees)	M_R(GeV)	e_m^R	k_m^R	γ_R
1^{--}	39	0.760	- 0.23	- 0.02	0.09
2^{++}	32	1.309	- 0.34	- 0.03	- 0.08
1^{++}	79	1.226	- 0.12	0.04	0.29
1^{+-}	37	1.215	- 0.50	- 0.18	- 0.01
0^{-+}	28	1.292	- 0.30	0.07	- 0.05

Table 1: Singlet-octet mixing angle θ and parameters M_R, e_m^R, k_m^R and γ_R of the mass Lagrangian (23) for all light meson nonets except the scalars. The input masses are taken from Ref. [2].

6. Let us now focus on the scalar mesons. Within the framework set up in the previous sections, we want to identify those states which, in the large-N_c limit, make up the lowest-lying nonet of scalar resonances (0^{++}). This is not a straightforward task because $1/N_c$ corrections are known to significantly affect the dynamics of the scalar sector. In particular, in this sector the spectrum of QCD_∞ seems to differ from the spectrum of QCD in the following sense [13, 14]: the inclusion of sub-leading effects in $1/N_c$ in the theoretical description of physical processes (e.g., via loops and unitarization) generates poles in the S-matrix that have no correspondence to the original mass parameters of the effective Lagrangian. This leads to the notion of “pre-existing” and “dynamically generated” resonances [13, 15].

This general feature can be understood within the analysis of Refs. [13, 14] for pseudoscalar meson meson scattering. In the large-N_c limit, the amplitudes are described by tree-level exchange of Goldstone modes and lowest-lying resonances, as described by CHPT and the chiral invariant effective Lagrangian (2). $1/N_c$ corrections are introduced by chiral loops and a suitable unitarization procedure (like N/D or the inverse amplitude method). As a general result, one finds that the full S-wave amplitudes display not only "pre-existing" poles (associated with the mass parameters appearing in the chiral resonance Lagrangian), but also "dynamically generated" poles appearing as an effect of the strong S-wave interaction. The $\sigma(600)$ (see also Ref. [16] where this state emerges in an analysis of the Roy equations for $\pi\pi$ scattering) and $\kappa(900)$ are examples of such "dynamically generated" poles. According to Ref. [13], the $a_0(980)$ falls in this category as well.

The "dynamically generated" poles decouple in the limit of large N_c. Only the "pre-existing" scalar states survive in this limit. We assume then that the latter can be described with a chiral resonance Lagrangian to understand the mass spectrum and the gross features of the $S \to P_1 P_2$ couplings, with the explicit realization [3]

$$g_2^S = c_d u_\mu u^\mu + c_m \chi_+ \tag{27}$$

in the Lagrangian (2). We report here, for future reference, the two-pseudoscalar meson couplings. For the non-singlet scalar fields one has (only the positively charged scalar fields are displayed for simplicity)

$$\begin{aligned}
\mathcal{L}(S^+_{I=1}, S^+_{I=1/2} \to 2 \text{ mesons}) \ = \ & \\
& S^+_{I=1}\frac{2}{F^2}\cdot\left\{c_d\left(\frac{2}{\sqrt{6}}\partial_\mu\pi^-\partial^\mu\eta_8 + \partial_\mu K^-\partial^\mu K^0\right)\right. \\
& \qquad \left. -c_m\left(\frac{2M_\pi^2}{\sqrt{6}}\pi^-\eta_8 + M_K^2 K^- K^0\right)\right\} \\
& +S^+_{I=1/2}\frac{1}{F^2}\cdot\left\{c_d\left(\sqrt{2}\partial_\mu K^-\partial^\mu\pi^0 + 2\partial_\mu\pi^-\partial^\mu\overline{K^0} - \frac{2}{\sqrt{6}}\partial^\mu K^-\partial_\mu\eta_8\right)\right. \\
& \qquad \left. -c_m\left(\frac{M_K^2+M_\pi^2}{\sqrt{2}}(K^-\pi^0 + \sqrt{2}\pi^-\overline{K^0}) + \frac{3M_\pi^2 - 5M_K^2}{\sqrt{6}}K^-\eta_8\right)\right\} \ ,
\end{aligned} \tag{28}$$

while for the strange and non-strange isosinglet fields of Eq. (14) one finds

$$\begin{aligned}
\mathcal{L}(S_{\text{non-strange}}, S_{\text{strange}} \to 2 \text{ mesons}) \ = \ & \\
S_{\text{non-strange}}\frac{\sqrt{2}}{F^2}\cdot & \left\{c_d\left(\partial_\mu\pi^0\partial^\mu\pi^0 + 2\partial_\mu\pi^+\partial^\mu\pi^- + \partial_\mu K^+\partial^\mu K^- + \partial_\mu K^0\partial^\mu\overline{K^0} + \frac{1}{3}\partial_\mu\eta_8\partial^\mu\eta_8\right)\right. \\
& \left. -c_m\left(M_\pi^2(\pi^0\pi^0 + 2\pi^+\pi^-) + M_K^2(K^+K^- + K^0\overline{K^0}) + \frac{M_\pi^2}{3}\eta_8\eta_8\right)\right\} \\
+S_{\text{strange}}\frac{1}{F^2}\cdot & \left\{c_d\left(2\partial_\mu K^+\partial^\mu K^- + 2\partial_\mu K^0\partial^\mu\overline{K^0} + \frac{4}{3}\partial_\mu\eta_8\partial^\mu\eta_8\right)\right. \\
& \left. -c_m\left(2M_K^2(K^+K^- + K^0\overline{K^0}) + \frac{4}{3}(2M_K^2 - M_\pi^2)\eta_8\eta_8\right)\right\} \ .
\end{aligned} \tag{29}$$

Thus, $S_{\rm strange}$ does not couple to pions as expected. On the other hand, $S_{\rm non-strange}$ couples to both strange and non-strange mesons with full strength. This is a straightforward consequence of (softly broken) $SU(3)$ incorporated in the chiral expansion. At the hadronic level, there is no fundamental difference between two- and four-quark states.

The couplings c_d and c_m were originally fixed [3] by requiring that the phenomenologically determined values for $L_5^r(M_\rho)$ and $L_8^r(M_\rho)$ are saturated by scalar resonance exchange. This led to $c_d \simeq 32$ MeV and $c_m \simeq 42$ MeV. Later on, independent information on these couplings was obtained from the study of QCD short-distance constraints on the SS correlator and on the scalar form factor [14, 17]; in the single-resonance approximation, one finds

$$c_d = c_m = F_\pi/2 = 46 \text{ MeV} . \tag{30}$$

Finally, results consistent with the above have been obtained by fitting experimental meson meson phase shifts within a chiral unitary approach [13, 14], with the best fits pointing to somewhat smaller values ($c_d \sim c_m \sim 20$ MeV).

We can now identify possible candidates for the lightest scalar nonet at large N_c, and try to discriminate between them on a phenomenological basis. The $I = 1/2$ member of the nonet is identified without controversy with $K_0^*(1430)$. For the $I = 0$ states, we only consider $f_0(980)$ and $f_0(1500)$ as candidates, excluding $f_0(1370)$ for the arguments given in Ref. [18]. Two scenarios then arise, depending on the assignment for the $I = 1$ state.

A: If we assume that the $a_0(980)$ is dynamically generated [13] (and makes up an octet together with $\sigma(600)$ and $\kappa(900)$ in the $SU(3)$ limit), the remaining candidates for a nonet are

$$f_0(980), K_0^*(1430), a_0(1450), f_0(1500) .$$

B: On the other hand, assuming that $a_0(980)$ is a pre-existing state in the large-N_c limit, the nonet would be composed by [18]

$$f_0(980), a_0(980), K_0^*(1430), f_0(1500) .$$

Contrary to what we have found for other resonance multiplets, both scenarios A and B are very poorly described by the strict nonet limit ($k_m^S = \gamma_S = 0$). In scenario A, the isoscalar masses turn out to be $M_L = 1.35$ GeV and $M_H = 1.47$ GeV, with a sizable deviation of M_L from $M_{f_0(980)}$. Moreover, the dual ideal mixing angle would imply $S_L = S_{\rm strange}$. Consequently, the $f_0(980)$ would not couple to two pions, which is not phenomenologically acceptable. In scenario B, the isoscalar masses turn out to be $M_L = 0.985$ GeV and $M_H = 1.74$ GeV, the latter being significantly bigger than $M_{f_0(1500)}$.

These observations point towards sizable nonet-breaking effects. By fitting to the relevant mass spectra[1] [2], we obtain the parameters and the mixing angle collected in Table

[1] In scenario B, using the input masses from Ref. [2], the inequality (25) is violated at the upper end so that the spectrum cannot be fitted in terms of M_S, e_m^S, k_m^S, and γ_S. For the numerics in Table 2 we use $M_{I=1/2} = 1.39$ GeV, which is fully consistent with the PDG entry, given the large width of 294 MeV for $K_0^*(1430)$.

2. The non-negligible $1/N_c$ effects translate into relatively large values for the couplings k_m^S and γ_S, as compared to e_m^S. Case A displays an inverted hierarchy ($e_m^S > 0$) and a mixing angle close to ideal. Case B displays the standard hierarchy ($e_m^S < 0$) and a mixing angle close to zero, corresponding to the scenario of Ref. [18].

	θ (degrees)	M_S(GeV)	e_m^S	k_m^S	γ_S
$0^{++}(A)$	30	1.48	0.20	- 1.00	- 0.35
0^{++}(B)	6.8	0.94	- 1.06	1.02	- 0.71

Table 2: Singlet-octet mixing angle θ and the parameters M_S, e_m^S, k_m^S, γ_S for the 0^{++} light scalar nonet corresponding to scenarios A and B. The input masses are taken from Ref. [2], with the exception of scenario B, where we use $M_{I=1/2} = 1.39$ GeV.

7. In order to discriminate between the two options, we start with a qualitative argument. Scenario A is attractive because two full multiplets are identified, one nonet of pre-existing states and another octet of dynamically generated poles, which decouple in the large-N_c limit. Put in another way, the role of the well-established $a_0(1450)$ is not clear in scenario B. Although the near-degeneracy of $a_0(980)$ and $f_0(980)$ can be accommodated in scenario B we are in this case very far from the nonet limit that would naturally explain the degeneracy.

More quantitative arguments can be given if we assume that the chiral resonance Lagrangian reproduces at least the salient features of the phenomenology of $S \to P_1\, P_2$ decays. First of all, we determine the contributions from scalar resonance exchange to the LECs of $O(p^4)$. From the effective Lagrangians (4) and (26) one obtains

$$\begin{aligned} L_1^S &= -\frac{\gamma_S c_d^2}{6M_S^2(1+\gamma_S)}\ , & L_3^S &= \frac{c_d^2}{2M_S^2}\ , & L_4^S &= -\frac{\gamma_S c_d c_m}{3M_S^2(1+\gamma_S)}\ , \\ L_5^S &= \frac{c_d c_m}{M_S^2}\ , & L_6^S &= -\frac{\gamma_S c_m^2}{6M_S^2(1+\gamma_S)}\ , & L_8^S &= \frac{c_m^2}{2M_S^2}\ . \end{aligned} \tag{31}$$

Assuming $c_m = c_d = F_\pi/2$, the numerical values of the LECS (in units of 10^{-3}) for the two scenarios are

	L_1^S	L_3^S	L_4^S	L_5^S	L_6^S	L_8^S
A	0.1	0.5	0.2	1.0	0.1	0.5
B	1.0	1.2	2.0	2.4	1.0	1.2

(32)

Although some of the LECs (especially L_5 and L_8 [19]) may have to be reanalysed on the basis of our work there is a certain preference for scenario A. The main drawback of scenario B is the large value of L_4^S, even for smaller values of c_d, c_m.

One can check that the resonance Lagrangian (28) works reasonably well for the decays of the $I = 1$ and $I = 1/2$ states, especially when considering ratios of decay widths[2]. Turning to the isoscalar sector, both scenarios A and B correctly predict that $f_0(980)$ couples predominantly to the $\pi\pi$ state. On the other hand, there is a marked difference between the two scenarios for $f_0(1500)$ due to very different mixing angles. Within option A, the two-pion mode is severely suppressed because of the nearly ideal mixing angle. In scenario B, on the other hand, $f_0(1500)$ couples strongly to two pions. Although for a state as heavy as the $f_0(1500)$ the relative branching ratios cannot be determined reliably from the tree-level amplitude only, the relatively small total width $\Gamma[f_0(1500)] = 109$ MeV [2] seems to favour again scenario A because the $\pi\pi$ channel is strongly suppressed in this case.

Finally, the contributions of scalar resonances to the LECs of $O(p^6)$ are also quite different in the two cases, mainly because they scale as $1/M_S^4$. Moreover, due to the smaller value of e_m^S, scenario A leads to

- better behaved corrections of $O(p^6)$ to masses and decay constants of pseudoscalar mesons [20];
- more reasonable contributions to isospin breaking effects in $K \to \pi\pi$ decays [21, 22].

8. The scalar mesons are very likely the only light meson resonances where large N_c together with chiral symmetry fails dramatically. We have investigated a scenario where the deviations from nonet symmetry occur only in the bilinear part of the resonance Lagrangian and therefore in the scalar mass matrix.

We have identified two possible scenarios for the lightest scalar nonet that survives in the large-N_c limit, and we have discussed arguments to discriminate between the two. Analysis of the mixing of isoscalars seems to favour an inverted hierarchy for the scalar mesons where the isotriplet states $a_0(1450)$ are heavier than the strange particles $K_0^*(1430)$ (Scenario A). The main features of the decays of scalar resonances to two pseudoscalars can also be understood within this framework although a more detailed dynamical study would be needed for a quantitative description. Altogether, our analysis favours a lightest "pre-existing" scalar nonet consisting of the states $f_0(980), K_0^*(1430), a_0(1450), f_0(1500)$.

Besides providing arguments for the composition of the lightest scalar nonet surviving in the large-N_c limit, our analysis has implications for all observables that are especially sensitive to scalar resonance exchange. Among those are the pseudoscalar meson masses and decay constants [20]. Another important application is isospin violation in the CP-violating parameter ϵ' [21]. Our results imply that the coupling constant e_m^S (first considered in Ref. [20]) and the nonet-breaking coupling k_m^S in the Lagrangian (23) produce isospin-violating contributions of similar size. The implications for ϵ' will be considered elsewhere [22].

[2]In the case of $c_m = c_d$, all rates are proportional to c_d^2, which cancels in the ratios.

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