PRESENT STATUS OF CHIRAL PERTURBATION THEORY[*]

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The basic ideas and methods of chiral perturbation theory are briefly reviewed. I discuss the recent attempts to build an effective Lagrangian in the resonance region and summarize the known large–N_C constraints on the low-energy chiral couplings.

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1. Chiral Symmetry

With n_f massless quark flavours, the QCD Lagrangian is invariant under global $SU(n_f)_L \otimes SU(n_f)_R$ transformations of the left- and right-handed quarks in flavour space. The symmetry group spontaneously breaks down to the diagonal subgroup $SU(n_f)_{L+R}$ and $n_f^2 - 1$ pseudoscalar massless Goldstone bosons appear in the theory, which for $n_f = 3$ can be identified with the eight lightest hadronic states $\phi^a = \{\pi, K, \eta\}$. These pseudoscalar fields are usually parameterized through the 3×3 unitary matrix $U(\phi) = u(\phi)^2 = \exp\{i\lambda^a \phi^a / f\}$.

The Goldstone nature of the pseudoscalar mesons implies strong constraints on their interactions, which can be most easily analyzed on the basis of an effective Lagrangian.[1] Since there is a mass gap separating the pseudoscalar octet from the rest of the hadronic spectrum, we can build an effective field theory containing only the Goldstone modes.[2,3] The low-energy effective Lagrangian can be organized in terms of increasing powers of momenta (derivatives): $\mathcal{L} =? \sum_n \mathcal{L}_{2n}$.

It is convenient to consider an extended QCD Lagrangian, with quark currents coupled to external Hermitian matrix-valued sources l_μ, r_μ, s, p. In addition to generate the QCD Green functions, the external fields can be used to incorporate the electromagnetic and semileptonic weak interactions, and the explicit breaking of chiral symmetry through the quark masses: $s = \mathcal{M} + \dots$, $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$.

At lowest order in derivatives and quark masses, the most general effective Lagrangian consistent with chiral symmetry has the form:[4]

$$\mathcal{L}_2 = \frac{f^2}{4} \left\langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \right\rangle, \qquad \chi \equiv 2B_0\,(s + ip), \tag{1}$$

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Table 1. Phenomenological values of the renormalized couplings $L_i^r(M_\rho)$ in units of 10^{-3}. The large–N_C predictions obtained within the single-resonance approximation are given in the last column.[2,3]

i	$L_i^r(M_\rho)$	$O(N_C)$	Source	$L_i^{N_C \to \infty}$
$2L_1 - L_2$	-0.6 ± 0.6	$O(1)$	K_{e4}, $\pi\pi \to \pi\pi$	0
L_2	1.4 ± 0.3	$O(N_C)$	K_{e4}, $\pi\pi \to \pi\pi$	1.8
L_3	-3.5 ± 1.1	$O(N_C)$	K_{e4}, $\pi\pi \to \pi\pi$	-4.3
L_4	-0.3 ± 0.5	$O(1)$	Zweig rule	0
L_5	1.4 ± 0.5	$O(N_C)$	$F_K : F_\pi$	2.1
L_6	-0.2 ± 0.3	$O(1)$	Zweig rule	0
L_7	-0.4 ± 0.2	$O(1)$	GMO, L_5, L_8	-0.3
L_8	0.9 ± 0.3	$O(N_C)$	M_ϕ, L_5	0.8
L_9	6.9 ± 0.7	$O(N_C)$	$\langle r^2 \rangle_V^\pi$	7.1
L_{10}	-5.5 ± 0.7	$O(N_C)$	$\pi \to e\nu\gamma$	-5.4

where $D_\mu U = \partial_\mu U - i r_\mu U + i U l_\mu$, $\langle A \rangle$ denotes the flavour trace of the matrix A and B_0 is a constant, which, like f, is not fixed by symmetry requirements alone. One finds that f equals the pion decay constant (at lowest order) $f = f_\pi = 92.4$ MeV, while B_0 is related to the quark condensate:

$$B_0 = -\frac{\langle \bar{q}q \rangle}{f^2} = \frac{M_\pi^2}{m_u + m_d} = \frac{M_{K^0}^2}{m_s + m_d} = \frac{M_{K^\pm}^2}{m_s + m_u} \, . \tag{2}$$

With only two low-energy constants, the lowest-order chiral Lagrangian $\mathcal{L}_2$ encodes in a very compact way all the Current Algebra results obtained in the sixties.

The symmetry constraints become less powerful at higher orders. At $O(p^4)$ we need ten additional coupling constants L_i to determine the low-energy behaviour of the Green functions:[4]

$$\mathcal{L}_4 = L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle + \dots \tag{3}$$

One-loop graphs with the lowest-order Lagrangian $\mathcal{L}_2$ contribute also at $O(p^4)$. Their divergent parts are renormalized by the $\mathcal{L}_4$ couplings, which introduces a renormalization scale dependence. The chiral loops generate non-polynomial contributions, with logarithms and threshold factors as required by unitarity, which are completely determined as functions of f and the Goldstone masses.

The precision required in present phenomenological applications makes necessary to include corrections of $O(p^6)$. This involves contributions from $\mathcal{L}_4$ at one-loop and $\mathcal{L}_2$ at two-loops, which can be fully predicted.[5] However, the $O(p^6)$ chiral Lagrangian $\mathcal{L}_6$ contains 90 (23) independent local terms of even (odd) intrinsic parity.[5,6] The huge number of unknown couplings limits the achievable accuracy. Clearly, further progress will depend on our ability to estimate these chiral couplings, which encode the underlying QCD dynamics.

Several two-loop calculations have been already performed.[7–12] Thus, the non-local $O(p^6)$ contributions (chiral logarithms) to many observables are known and, in some cases, the local ambiguities can be reduced to a few subtraction constants using dispersion relation techniques.

2. Resonance Chiral Theory

The limit of an infinite number of quark colours is a very useful starting point to understand many features of QCD.[13, 14] Assuming confinement, the strong dynamics at $N_C \to \infty$ is given by tree diagrams with infinite sums of hadron exchanges, which correspond to the tree approximation to some local effective Lagrangian. Hadronic loops generate corrections suppressed by factors of $1/N_C$. At $N_C \to \infty$, QCD has a larger symmetry $U(3)_L \otimes U(3)_R \to U(3)_{L+R}$, and one needs to include in the matrix $U(\phi)$ a ninth Goldstone boson field, the η_1. Resonance chiral theory provides the correct framework to incorporate the massive mesonic states.[15]

Let us consider a chiral-invariant Lagrangian $\mathcal{L}_R$, describing the couplings of resonance nonet multiplets $V_i^{\mu\nu}(1^{--})$, $A_i^{\mu\nu}(1^{++})$, $S_i(0^{++})$ and $P_i(0^{-+})$ to the Goldstone bosons:[16]

$$\mathcal{L}_R = \sum_i \left\{ \frac{F_{V_i}}{2\sqrt{2}} \langle V_i^{\mu\nu} f_{+\,\mu\nu}\rangle + \frac{i\,G_{V_i}}{\sqrt{2}} \langle V_i^{\mu\nu} u_\mu u_\nu\rangle + \frac{F_{A_i}}{2\sqrt{2}} \langle A_i^{\mu\nu} f_{-\,\mu\nu}\rangle \right.$$

$$\left. + c_{d_i} \langle S_i\, u^\mu u_\mu\rangle + c_{m_i} \langle S_i\, \chi_+\rangle + i\,d_{m_i} \langle P_i\, \chi_-\rangle \right\}, \tag{4}$$

where $u_\mu \equiv i\,u^\dagger D_\mu U u^\dagger$, $f_\pm^{\mu\nu} \equiv u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u$ with $F_{L,R}^{\mu\nu}$ the field-strength tensors of the l^μ and r^μ flavour fields and $\chi_\pm \equiv u^\dagger \chi u^\dagger \pm u \chi^\dagger u$. The resonance couplings F_{V_i}, G_{V_i}, F_{A_i}, c_{d_i}, c_{m_i} and d_{m_i} are of $O\left(\sqrt{N_C}\right)$.

The lightest resonances have an important impact on the low-energy dynamics of the pseudoscalar bosons. Below the resonance mass scale, the singularity associated with the pole of a resonance propagator is replaced by the corresponding momentum expansion; therefore, the exchange of virtual resonances generates derivative Goldstone couplings proportional to powers of $1/M_R^2$. At lowest order in derivatives, this gives the large–N_C predictions for the $O(p^4)$ couplings of chiral perturbation theory:[16]

$$2\,L_1 = L_2 = \sum_i \frac{G_{V_i}^2}{4\,M_{V_i}^2}, \qquad L_3 = \sum_i \left\{ -\frac{3\,G_{V_i}^2}{4\,M_{V_i}^2} + \frac{c_{d_i}^2}{2\,M_{S_i}^2} \right\},$$

$$L_5 = \sum_i \frac{c_{d_i}\,c_{m_i}}{M_{S_i}^2}, \qquad L_8 = \sum_i \left\{ \frac{c_{m_i}^2}{2\,M_{S_i}^2} - \frac{d_{m_i}^2}{2\,M_{P_i}^2} \right\}, \tag{5}$$

$$L_9 = \sum_i \frac{F_{V_i}\,G_{V_i}}{2\,M_{V_i}^2}, \qquad L_{10} = \sum_i \left\{ \frac{F_{A_i}^2}{4\,M_{A_i}^2} - \frac{F_{V_i}^2}{4\,M_{V_i}^2} \right\}.$$

All these couplings are of $O(N_C)$, in agreement with the counting indicated in Table 1, while for the couplings of $O(1)$ we get $2\,L_1 - L_2 = L_4 = L_6 = L_7 = 0$.

Owing to the $U(1)_A$ anomaly, the η_1 field is massive and it is often integrated out from the low-energy chiral theory. In that case, the $SU(3)_L \otimes SU(3)_R$ chiral coupling L_7 gets a contribution from η_1 exchange:[4, 16]

$$L_7 = -\frac{\tilde{d}_m^2}{2\,M_{\eta_1}^2}, \qquad\qquad \tilde{d}_m = -\frac{f}{\sqrt{24}}. \tag{6}$$

3. Short-Distance Constraints

The short-distance properties of the underlying QCD dynamics impose some constraints on the low-energy parameters.[15, 17] At leading order in $1/N_C$, the two-Goldstone matrix element of the vector current, is characterized by

$$F_V(t) = 1 + \sum_i \frac{F_{V_i} G_{V_i}}{f^2} \frac{t}{M_{V_i}^2 - t}.$$

(7)

Since the vector form factor $F_V(t)$ should vanish at infinite momentum transfer t, the resonance couplings should satisfy

$$\sum_i F_{V_i} G_{V_i} = f^2.$$

(8)

Similarly, the matrix element of the axial current between one Goldstone and one photon is parameterized by the so-called axial form factor $G_A(t)$, which vanishes at $t \to \infty$ provided that

$$\sum_i \left(2 F_{V_i} G_{V_i} - F_{V_i}^2 \right) / M_{V_i}^2 = 0.$$

(9)

Requiring the scalar form factor $F^S(t)$, which governs the two-pseudoscalar matrix element of the scalar quark current, to vanish at $t \to \infty$, one gets the constraints:[18]

$$4 \sum_i c_{d_i} c_{m_i} = f^2, \qquad \sum_i c_{m_i} \left(c_{m_i} - c_{d_i} \right) / M_{S_i}^2 = 0.$$

(10)

Since gluonic interactions preserve chirality, the two-point function built from a left-handed and a right-handed vector quark currents $\Pi_{LR}(t)$ satisfies an unsubtracted dispersion relation. In the chiral limit, it vanishes faster than $1/t^2$ when $t \to \infty$; this implies the well-known Weinberg conditions:[19]

$$\sum_i \left(F_{V_i}^2 - F_{A_i}^2 \right) = f^2, \qquad \sum_i \left(M_{V_i}^2 F_{V_i}^2 - M_{A_i}^2 F_{A_i}^2 \right) = 0.$$

(11)

The two-point correlators of two scalar or two pseudoscalar currents would be equal if chirality was preserved. For massless quarks, $\Pi_{SS-PP}(t)$ vanishes as $1/t^2$ when $t \to \infty$, with a coefficient proportional to $\alpha_s \langle \bar{q}\Gamma q\, \bar{q}\Gamma q \rangle \sim \alpha_s \langle \bar{q}q \rangle^2 \sim \alpha_s B_0^2$. Imposing this behaviour, one gets:[20]

$$8 \sum_i \left(c_{m_i}^2 - d_{m_i}^2 \right) = f^2, \qquad \sum_i \left(c_{m_i}^2 M_{S_i}^2 - d_{m_i}^2 M_{P_i}^2 \right) = 3 \pi \alpha_s f^4/4.$$

(12)

4. Single-Resonance Approximation

Let us approximate each infinite resonance sum with the first meson nonet contribution. This is meaningful at low energies where the contributions from higher-mass states are suppressed by their corresponding propagators. The resulting short-distance constraints are matching conditions between an effective theory below the scale of the second resonance multiplets and the underlying QCD dynamics.

With this approximation, Eqs. (8), (9) and (11) determine the vector and axial-vector couplings in terms of M_V and f:[17]

$$F_V = 2\,G_V = \sqrt{2}\,F_A = \sqrt{2}\,f\,, \qquad\qquad M_A = \sqrt{2}\,M_V\,. \qquad (13)$$

The scalar[18] and pseudoscalar parameters are obtained from (10) and (12):[15]

$$c_m = c_d = \sqrt{2}\,d_m = f/2\,, \qquad\qquad M_P = \sqrt{2}\,M_S\,(1-\delta)^{1/2}\,. \qquad (14)$$

The last relation involves a small correction $\delta \approx 3\,\pi\alpha_s f^2/M_S^2 \sim 0.08\,\alpha_s$, which we can neglect together with the tiny effects from light quark masses.

Inserting these predictions into Eqs. (5), one finally gets all $O(N_C\,p^4)$ chiral perturbation theory couplings, in terms of M_V, M_S and f:

$$2\,L_1 = L_2 = \frac{1}{4}\,L_9 = -\frac{1}{3}\,L_{10} = \frac{f^2}{8\,M_V^2}\,, \qquad (15)$$

$$L_3 = -\frac{3\,f^2}{8\,M_V^2} + \frac{f^2}{8\,M_S^2}\,, \qquad L_5 = \frac{f^2}{4\,M_S^2}\,, \qquad L_8 = \frac{3\,f^2}{32\,M_S^2}\,. \qquad (16)$$

The last column in Table 1 shows the results obtained with $M_V = 0.77$ GeV, $M_S = 1.0$ GeV and $f = 92$ MeV. Also shown is the L_7 prediction in (6), taking $M_{\eta_1} = 0.80$ GeV. The agreement with the measured values is a clear success of the large–N_C approximation. It demonstrates that the lightest resonance multiplets give indeed the dominant effects at low energies.

The study of other Green functions provides further matching conditions between the hadronic and fundamental QCD descriptions. Clearly, it is not possible to satisfy all of them within the single-resonance approximation. A useful generalization is the so-called *Minimal Hadronic Ansatz*, which keeps the minimum number of resonances compatible with all known short-distance constraints for the problem at hand.[21] Some $O(p^6)$ chiral couplings have been already analyzed in this way, by studying an appropriate set of three-point functions.[22–25]

5. Subleading $1/N_C$ Corrections

The large–N_C limit provides a very successful description of the low-energy dynamics.[15] However, we are still lacking a systematic procedure to incorporate next-to-leading contributions in the $1/N_C$ counting. Up to now, the effort has concentrated in pinning down the most relevant subleading effects, such as the resonance widths which regulate the corresponding poles in the meson propagators,[26] or the role of final state interactions in the physical amplitudes.[18, 26–28]

Quantum loops including virtual resonance propagators constitute a major technical challenge.[29, 30] Their ultraviolet divergences require higher dimensional counterterms, which could generate a problematic behaviour at large momenta. Thus, it is necessary to investigate the short-distance QCD constraints at the next-to-leading order in $1/N_C$. A first step in this direction is the recent one-loop calculation of the vector form factor in the resonance chiral theory.[30] Further work towards a more formal renormalization procedure is in progress.

Acknowledgements

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