Caltech MAP-306
UWThPh-2005-2
IFIC/05-12
FTUV/05-0309

The $\langle SPP \rangle$ Green Function and SU(3) Breaking in $K_{\ell 3}$ Decays*

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Abstract

Using the $1/N_C$ expansion scheme and truncating the hadronic spectrum to the lowest-lying resonances, we match a meromorphic approximation to the $\langle SPP \rangle$ Green function onto QCD by imposing the correct large-momentum falloff, both off-shell and on the relevant hadron mass shells. In this way we determine a number of chiral low-energy constants of $O(p^6)$, in particular the ones governing $SU(3)$ breaking in the $K_{\ell 3}$ vector form factor at zero momentum transfer. The main result of our matching procedure is that the known loop contributions largely dominate the corrections of $O(p^6)$ to $f_+(0)$. We discuss the implications of our final value $f_+^{K^0\pi^-}(0) = 0.984 \pm 0.012$ for the extraction of V_{us} from $K_{\ell 3}$ decays.

* Work supported in part by HPRN-CT2002-00311 (EURIDICE) and by Acciones Integradas, HU2002-0044 (MCYT, Spain), Project No. 19/2003 (Austria).

1 Introduction

Chiral perturbation theory [1–3] (CHPT) is the effective theory describing the low-energy expansion of QCD Green functions. It is a fundamental tool in low-energy hadron phenomenology. State-of-the-art calculations involve expansions to NNLO (p^6) in external momenta and quark masses [4]. Given the large number of low-energy constants (LECs) appearing in the CHPT effective Lagrangian to order p^6 [5–7], in order to retain predictive power it is highly desirable to develop a non-perturbative framework to match the effective low-energy description to QCD and to estimate the LECs. In this work we use the $1/N_C$ expansion framework together with the Minimal Hadronic Ansatz [8–10] to study the matching between low and high energies for the $\langle SPP \rangle$ three-point function of one scalar and two pseudoscalar densities in the chiral limit.

The purpose of our investigation is twofold. First, we wish to extend the $1/N_C$-motivated matching scheme that has proved successful in other cases [8–16] to the $\langle SPP \rangle$ three-point function. The scheme entails the construction of a hadronic interpolation between the known low- and high-momentum regimes, dictated respectively by chiral symmetry and by the QCD asymptotic behaviour. The approximations involve the choice of the hadronic content and the corresponding set of short-distance constraints to be satisfied. We truncate the spectrum to the lowest-lying resonance multiplet per channel, based on the observation that the low-lying hadronic spectrum has the largest impact on the LECs. This choice defines our hadronic ansatz as the most general meromorphic function with poles corresponding to Goldstone bosons and $\mathcal{S}$, $\mathcal{P}$ resonances. Concerning the short-distance behaviour, the leading power in the OPE for the $\langle SPP \rangle$ Green function displays anomalous scaling, making the explicit matching with a meromorphic function problematic. In our analysis we therefore impose that the behaviour of our ansatz at large momenta is not worse than the one required by QCD. These constraints are then fully compatible with the asymptotic vanishing of those hadronic form factors that we need to consider for fixing the relevant LECs.

Beyond the general aspects mentioned above, the $\langle SPP \rangle$ Green function is of special phenomenological interest, as its low-energy behaviour allows to determine the LECs governing SU(3) breaking in $K_{\ell 3}$ decays to $O(p^6)$. In turn, $K_{\ell 3}$ decays offer the possibility of a precise determination of the CKM mixing parameter V_{us}, and thus an accurate test of CKM unitarity when combined with knowledge of V_{ud} [17]. Our aim is to explore the quantitative implications of our matching framework for $K_{\ell 3}$ decays and to assess the attendant uncertainty.

The material in this paper is organized as follows. In Sec. 2 we describe in detail the matching procedure for the $\langle SPP \rangle$ correlator. We give its low- and high-momentum limits, the interpolating form and we discuss the implications for the LECs. In Sec. 3 we then review the status of CHPT calculations of $K_{\ell 3}$ form factors and the impact of our findings on the local contribution of $O(p^6)$. Finally, in Sec. 4 we summarize our main results and conclusions.

2 The $\langle SPP \rangle$ Green function

Our starting point is the Fourier transform of the octet $\langle SPP \rangle$ Green function in massless QCD. On account of SU(3) and C invariance it is given by a single scalar function[1],

$$i^2 \int dx\, dy\, e^{ipx+iqy+irz} \langle 0|T S^a(x) P^b(y) P^c(z)|0\rangle = d^{abc}\, \Pi_{SPP}(p^2, q^2, r^2) \; , \tag{1}$$

with $p + q + r = 0$. Bose symmetry implies that the function is symmetric in its second and third arguments

$$\Pi_{SPP}(s, u, t) = \Pi_{SPP}(s, t, u) \; . \tag{2}$$

2.1 Chiral symmetry

The low-energy expansion of the function Π_{SPP} may be worked out in CHPT:

$$\begin{aligned} \Pi_{SPP}(s,t,u) \;=\; & \frac{(2B_0)^3}{tu} \Big\{ F_0^2 + 4L_5\, s + 4(4L_8 - L_5)\,(t+u) \\ & -8C_{12}\, s^2 + 8(2C_{12} + C_{34} + C_{38})\, s(t+u) - 8(C_{12} + C_{34} - C_{38})\,(t^2+u^2) \Big\} \\ & -(4B_0)^3 (2C_{12} - 4C_{31} - 2C_{34} + 2C_{38} - C_{94}) + O(p^2) \; , \end{aligned} \tag{3}$$

where the loop contributions have been discarded. The leading-order contribution to Π_{SPP} depends exclusively on the pion decay constant F_0 and on the quark condensate, $B_0 = -\langle 0|\bar{u}u|0\rangle_0 / F_0^2$ in the chiral limit $m_u = m_d = m_s = 0$. The coefficients of the higher-order contributions are written in terms of the coupling constants L_i and C_i [3, 6][2].

2.2 Asymptotic behaviour

The Operator Product Expansion (OPE) implies the following short-distance behaviour of the $\langle SPP \rangle$ Green function when $s, t, u \to \infty$:

$$\Pi_{SPP}(s,t,u) \to 2\langle 0|\bar{u}u|0\rangle_0\, \frac{s^2 - (t-u)^2}{stu} \; , \tag{4}$$

to leading order in inverse powers of momenta, but to zeroth order in the strong coupling constant. In fact, the anomalous dimensions of the (pseudo)scalar currents and of the quark

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[1] The quark currents are normalized according to

$$P^a(x) = \bar{q}(x) i\gamma_5 \lambda^a q(x) \;\; , \;\; S^a(x) = \bar{q}(x)\lambda^a q(x) \; ,$$

with $\langle \lambda^a \lambda^b \rangle = 2\delta^{ab}$; furthermore, $d^{abc} = \frac{1}{4}\langle \lambda^a \{\lambda^b, \lambda^c\}\rangle$.

[2] In the literature, there exist different conventions as to the normalization of the C_i [6, 7]. We prefer to work with the C_i of mass dimension -2 since they have the canonical large-N_c behaviour.

condensate imply nontrivial Wilson coefficients in (4) that ensure the correct scale dependence and modify the asymptotic behaviour by logarithms. We come back to this important point in subsection 2.4.

In the case where only two of the three coordinates x, y, z approach each other, the analysis reduces to the OPE of pairs of quark currents and yields in momentum space[3]

$$\Pi_{SPP}(p^2,q^2,(p+q)^2) = O(q^{-2}) \ , \qquad p \text{ fixed} \ , \ q \to \infty \ , \tag{5}$$

$$\Pi_{SPP}(p^2,q^2,(p+q)^2) = -8\langle 0|\bar{u}u|0\rangle_0 \frac{pq}{p^2q^2} + O(p^{-2}) \ , \qquad q \text{ fixed} \ , \ p \to \infty \ . \tag{6}$$

There is one additional constraint that follows from chiral symmetry: chiral Ward identities and pion pole dominance lead to the relation

$$\lim_{t\to 0} \frac{t}{2B_0} \Pi_{SPP}(s,t,s) = \Pi_{SS}(s) - \Pi_{PP}(s) \, , \tag{7}$$

where we have introduced the scalar two-point functions Π_{SS} and Π_{PP} according to

$$\begin{aligned} i\int dx\, e^{ip(x-y)} \langle 0|TS^a(x)S^b(y)|0\rangle &= \delta^{ab}\, \Pi_{SS}(p^2) \ , \\ i\int dx\, e^{ip(x-y)} \langle 0|TP^a(x)P^b(y)|0\rangle &= \delta^{ab}\, \Pi_{PP}(p^2) \ . \end{aligned} \tag{8}$$

It is well known that the OPE for $TS^a(x)S^b(y)$ coincides with the one for $TP^a(x)P^b(y)$ in the chiral limit up to terms of order $(x-y)^{-2}$. We thus deduce

$$\lim_{t\to 0} \frac{t}{2B_0} \Pi_{SPP}(s,t,s) = O(s^{-2}) \ , \tag{9}$$

when $s \to \infty$, not conflicting with (6).

A different class of asymptotic constraints comes from considering the behaviour of various hadronic form factors at high momentum transfer. Form factors may be obtained from Π_{SPP} by extracting the residues of the appropriate double poles. As an explicit example, the scalar form factor of the pion $F_S^{\pi\pi}(s)$ is given by the residue of the double pole at $t = u = 0$,

$$F_S^{\pi\pi}(s) = \lim_{t,u\to 0} \frac{tu}{(2B_0F_0)^2} \Pi_{SPP}(s,t,u) \ . \tag{10}$$

The asymptotic condition [18] in this case reads

$$\lim_{s\to\infty} F_S^{\pi\pi}(s) = 0 \ . \tag{11}$$

Similar constraints exist for other (transition) form factors.

[3] The term given explicitly does in fact involve the two-point function $\langle 0|TA_\mu^a(x)P^b(y)|0\rangle$. However, for vanishing quark masses, that correlation function coincides with its short-distance limit,

$$\langle 0|TA_\mu^a(x)P^b(y)|0\rangle = -2i\delta^{ab}\langle 0|\bar{u}u|0\rangle_0 \partial_\mu \Delta_0(x-y) + O(m_q) \ .$$

2.3 Model approximation

In the large-N_C limit the only singularities in Π_{SPP} are single-meson poles. In this spirit, and truncating the spectrum to one resonance per channel, we construct a meromorphic approximation to the function Π_{SPP} with explicit scalar ($\mathcal{S}$) and pseudoscalar ($\mathcal{P}$) poles (besides the pion),

$$\Pi^{\mathcal{SP}}_{SPP}(s,t,u) = 8B_0^3F_0^2M_S^2M_P^4\frac{P_0+P_1+P_2+P_3+P_4}{[M_S^2-s][-t][-u][M_P^2-t][M_P^2-u]} \ , \tag{12}$$

where the P_n are polynomials of degree n in s, t, u:

$$P_n = \sum_{k=0}^{n}\sum_{l=0}^{k} c_{n-k,k-l,l}\, s^{n-k}t^{k-l}u^l \ . \tag{13}$$

The ansatz in Eq. (12) represents the most general expression for the given particle content (fixing the denominator) that does not violate the short-distance behaviour $\Pi_{SPP} = O(p^{-2})$ required by the OPE (4). The normalization is chosen such that $P_0 \equiv c_{000} = 1$ yields the correct low-energy limit. Since the function Π_{SPP} is symmetric under the interchange of t and u we have in addition

$$c_{kml} = c_{klm} \ . \tag{14}$$

We are thus left with 21 parameters $c_{100}, \ldots, c_{022}$ to be determined.

It is straightforward to work out the implications of our model for the chiral coupling constants that enter in Eq. (3). Aside from $c_{000} = 1$, the expressions for the LECs involve the coefficients in P_1 and, in case of the C_i, also those from P_2:

$$\begin{aligned}
L_5^{\mathcal{SP}} &= \frac{F_0^2}{4}\left[\frac{1}{M_S^2}+c_{100}\right] \ , \quad L_8^{\mathcal{SP}} = \frac{F_0^2}{16}\left[\frac{1}{M_S^2}+\frac{1}{M_P^2}+c_{100}+c_{010}\right] \ , \\
C_{12}^{\mathcal{SP}} &= -\frac{1}{2M_S^2}L_5^{\mathcal{SP}} - \frac{F_0^2}{8}c_{200} \ , \\
C_{34}^{\mathcal{SP}} &= \frac{1}{2}\left[\frac{1}{M_S^2}+\frac{1}{M_P^2}\right]L_5^{\mathcal{SP}} + \left[\frac{1}{M_S^2}-\frac{1}{M_P^2}\right]L_8^{\mathcal{SP}} - \frac{F_0^2}{16}\left[\frac{1}{M_S^2M_P^2}-3c_{200}-c_{110}+c_{020}\right] \ , \\
C_{38}^{\mathcal{SP}} &= \left[\frac{1}{M_S^2}+\frac{1}{M_P^2}\right]L_8^{\mathcal{SP}} - \frac{F_0^2}{16}\left[\frac{1}{M_S^2M_P^2}-c_{200}-c_{110}-c_{020}\right] \ .
\end{aligned} \tag{15}$$

The coupling constants C_{31} and C_{94} are not fixed individually. For the combination $C_{31}+\frac{1}{4}C_{94}$ one finds

$$C_{31}^{\mathcal{SP}} + \frac{1}{4}C_{94}^{\mathcal{SP}} = -C_{34}^{\mathcal{SP}} + C_{38}^{\mathcal{SP}} - \frac{F_0^2}{32}\left[\frac{1}{M_P^4}+2c_{020}-c_{011}\right] \ . \tag{16}$$

Before moving on to the asymptotic constraints, let us observe that the relation between Π_{SPP} and $\Pi_{SS}-\Pi_{PP}$ given in Eq. (7) implies that the combination $c_{100}+c_{010}$ relevant for L_8

is given in terms of quantities that specify the two-point functions. In fact, defining c_m and d_m in terms of the one-particle matrix elements of the scalar and pseudoscalar currents [19],

$$|\langle 0|S^a|\mathcal{S}^b\rangle| = \delta^{ab}\, 4\sqrt{2}B_0\, c_m \ \ , \ \ |\langle 0|P^a|\mathcal{P}^b\rangle| = \delta^{ab}\, 4\sqrt{2}B_0\, d_m \ , \tag{17}$$

one derives from Eq. (7)

$$\frac{1}{M_S^2} + \frac{1}{M_P^2} + c_{100} + c_{010} = \frac{8}{F_0^2}\left[\frac{c_m^2}{M_S^2} - \frac{d_m^2}{M_P^2}\right] \ , \tag{18}$$

which implies the well-known result [19] $L_8^{\mathcal{SP}} = 1/2\,(c_m^2/M_S^2 - d_m^2/M_P^2)$.

The $\langle SPP\rangle$ Green function has also been studied in Ref. [14] within a ladder resummation inspired hadronic model. Phenomenological consequences such as the LECs of $O(p^6)$ were not considered in that approach.

2.4 Implementing asymptotic constraints

The aim of the present investigation is to obtain information on the chiral coupling constants by exposing the ansatz in Eq. (12) to suitable constraints implied by QCD asymptotic behaviour. In particular, it has been proven successful [8–16] to match a meromorphic representation to the QCD short-distance behaviour, given in this case by Eqs. (4), (5), and (6). Here we encounter a problem related to the missing Wilson coefficients in those relations. For instance, matching to Eq. (4) would imply that the nine coefficients in P_4 are fixed completely,

$$2c_{211} = c_{022} = -2c_{031} = \frac{1}{2B_0^2 M_S^2 M_P^4} \ , \tag{19}$$

with all other coefficients in P_4 being zero.

The problem alluded to is manifest in Eq. (19), which would imply that non-vanishing coefficients c_{klm} depend on the running scale of QCD in the same manner as the (quark condensate)$^{-2}$. Including the appropriate Wilson coefficients in Eqs. (4) and (6) would repair the scale dependence but it would make the c_{klm} momentum dependent at the same time invalidating our ansatz. Moreover, that momentum dependence would involve the strong coupling in the non-perturbative regime. It is also obvious that the logarithmic dependence induced by nontrivial Wilson coefficients can never be matched by a meromorphic approximation with a finite number of resonances (see, e.g., Ref. [20]).

Therefore, we have to find an alternative set of criteria to determine the parameters of our model ansatz and the relevant LECs. Of course, this discussion is not limited to $\langle SPP\rangle$ but applies to all Green functions with anomalous scaling to leading power in inverse momenta.

1. While matching onto the short-distance result leads to the problems discussed above, we certainly wish to ensure that the short-distance behaviour of our ansatz is not worse than what is predicted in QCD. The behaviour $\Pi_{SPP} = O(p^{-2})$ (up to logarithms) was built into our

ansatz from the start. There are, however, nontrivial restrictions that follow from the limits in which the momenta are treated asymmetrically:

$$\Pi_{SPP}(p^2, q^2, (p+q)^2) = O(q^{-2}) \,, \qquad p \text{ fixed} \,,\ q \to \infty \,, \tag{5'}$$
$$\Pi_{SPP}(p^2, q^2, (p+q)^2) = O(p^{-1}) \,, \qquad q \text{ fixed} \,,\ p \to \infty \,. \tag{6'}$$

Our model (12) reproduces this behaviour, provided the following relations hold:

$$\begin{aligned} c_{400} + c_{310} + c_{220} + c_{130} + c_{040} &= 0 \,, \\ 8c_{400} + 3c_{310} - c_{130} &= 0 \,, \\ 6c_{400} + 3c_{310} + c_{220} &= 0 \,, \\ c_{310} + c_{211} + c_{121} + c_{031} &= 0 \,, \\ 2c_{040} + 2c_{031} + c_{022} &= 0 \,, \\ c_{300} + c_{210} + c_{120} + c_{030} &= 0 \,. \end{aligned} \tag{20}$$

2. More conditions arise from the short-distance behaviour of $\Pi_{SS}(s) - \Pi_{PP}(s)$ in (9). First, the parameters c_m and d_m must satisfy [11, 21]

$$c_m^2 - d_m^2 = \frac{F_0^2}{8} \,, \tag{21}$$

a relation analogous to the first Weinberg sum rule [22]. In addition, Eq. (9) requires that the following combinations of coefficients vanish:

$$\begin{aligned} c_{200} + c_{110} + c_{020} &= 0 \,, \\ c_{300} + c_{210} + c_{120} + c_{030} &= 0 \,, \\ c_{400} + c_{310} + c_{220} + c_{130} + c_{040} &= 0 \,. \end{aligned} \tag{22}$$

3. The asymptotic vanishing of the scalar form factor of the pion requires

$$c_{100} = c_{200} = c_{300} = c_{400} = 0 \,. \tag{23}$$

When combining the restrictions from the pion scalar form factor in Eq. (23), the scalar/ pseudoscalar Weinberg sum rule in Eq. (21), the OPE conditions in Eq. (20), and Eq. (18), we are left with a total of 9 undetermined parameters:

$$\begin{aligned} P_2 &= c_{110}[s(t+u) - t^2 - u^2] + c_{011} tu \,, \\ P_3 &= c_{210}[s^2(t+u) - t^3 - u^3] + c_{111} stu + c_{021}[t+u]tu + c_{120}[s(t^2+u^2) - t^3 - u^3] \,, \\ P_4 &= c_{310}[s^3(t+u) - 3s^2(t^2+u^2) + 3s(t^3+u^3) - (t^2 + 3tu + u^2)(t-u)^2] \\ &\quad + c_{211}[s^2 - (t-u)^2]tu + c_{121}[s(t+u) - (t-u)^2]tu \,. \end{aligned} \tag{24}$$

4. Once the conditions from one-pion transition form factors $\langle \pi | S | \mathcal{P} \rangle$ ($c_{310} = c_{210} = 0$, $c_{110} = -M_P^2 c_{120}$) and $\langle \pi | P | \mathcal{S} \rangle$ ($c_{120} = 0$) are included, the ansatz reduces further to

$$\begin{aligned} P_2 &= c_{011} tu \ , \\ P_3 &= [c_{111} s + c_{021}(t+u)] tu \ , \\ P_4 &= [c_{211}(s^2 - (t-u)^2) + c_{121}(s(t+u) - (t-u)^2)] tu \ . \end{aligned} \tag{25}$$

The constraints discussed so far are sufficient to fix the LECs of order p^4 and p^6 within our scheme.

We could fix additional parameters in the polynomials (25) by also considering the various form factors of the $\mathcal{S}$ and $\mathcal{P}$ resonance states. As pointed out in Ref. [14], this procedure in general leads to a representation in conflict with the OPE constraints. We do not dwell on this issue further since it is of no concern for our purpose of determining the LECs occurring up to $O(p^6)$.

2.5 Low-energy constants

The constraints enumerated above determine the LECs L_5, L_8, C_{12}, C_{34} and C_{38} in terms of resonance masses and couplings.

- The asymptotic vanishing of the pion scalar form factor fixes L_5 and C_{12} through $c_{100} = c_{200} = 0$. The scalar form factor defined in Eq. (10) takes the simple form

$$F_S^{\pi\pi}(s) = F_S^{\pi\pi}(0) \frac{M_S^2}{M_S^2 - s} \ . \tag{26}$$

  The coupling constants describing the dependence on the variable s in the low-energy expansion of Π_{SPP}, viz. L_5 and C_{12} in Eq. (3), are determined by the momentum dependence of the scalar form factor alone. Within the single-scalar resonance approximation that implies (see also [23])

$$L_5 = \frac{F_0^2}{4M_S^2} \ , \quad C_{12} = -\frac{F_0^2}{8M_S^4} \ . \tag{27}$$

  In the vector sector the analogous consideration leads to the well-known predictions [12, 13, 16, 24]

$$L_9 = \frac{F_0^2}{2M_V^2} \ , \quad C_{88} - C_{90} = -\frac{F_0^2}{4M_V^4} \ . \tag{28}$$

- Enforcing the correct short-distance behaviour of $\Pi_{SS}(s) - \Pi_{PP}(s)$ by virtue of the first of Eqs. (22) determines the value of C_{38}.

- The asymptotic vanishing of one-pion transition form factors $\langle \pi | S | \mathcal{P} \rangle$ and $\langle \pi | P | \mathcal{S} \rangle$ implies $c_{110} = c_{020} = 0$ and thus determines C_{34}.

The combination $C_{31} + 1/4\, C_{94}$ remains undetermined, as the coefficient c_{011} is not fixed by the constraints we have considered.

Within our framework the relevant LECs of $O(p^6)$ are determined in terms of the scalar and pseudoscalar octet masses (M_S and M_P), the pion decay constant F_0 and the couplings c_m and d_m defined in Eq. (17). Using also the Weinberg-like sum rule (21), we obtain

$$
\begin{aligned}
L_5^{\mathcal{SP}} &= \frac{F_0^2}{4\,M_S^2} \;, \quad L_8^{\mathcal{SP}} = \frac{1}{2}\left(\frac{c_m^2}{M_S^2} - \frac{d_m^2}{M_P^2}\right) \;, \\
C_{12}^{\mathcal{SP}} &= -\frac{F_0^2}{8M_S^4} \;, \\
C_{34}^{\mathcal{SP}} &= \frac{3\,F_0^2}{16M_S^4} + \frac{d_m^2}{2}\left(\frac{1}{M_S^2} - \frac{1}{M_P^2}\right)^2 \;, \\
C_{38}^{\mathcal{SP}} &= \frac{F_0^2}{16M_S^4} + \frac{d_m^2}{2}\left(\frac{1}{M_S^4} - \frac{1}{M_P^4}\right) \;.
\end{aligned}
\qquad (29)
$$

To estimate these couplings, we need numerical values for the input parameters. For the pion decay constant we use the physical value $F_0 = F_\pi = 92.4$ MeV. The coupling d_m can be fixed by studying the pion scalar form factor away from the chiral limit [9], resulting in $d_m = F_0/(2\sqrt{2})$ (or $c_m = F_0/2$), which we take as central value. As for the mass parameters, spectroscopy and chiral symmetry [25,26] suggest a central value $M_P = 1.3$ GeV, while M_S is more controversial. The analysis of Ref. [26] would suggest $M_S = 1.48$ GeV for the lightest scalar nonet that survives in the large-N_C limit. This result is supported by recent lattice calculations (see for example Ref. [27] and references therein). It implies a value of $L_5^{\mathcal{SP}} \simeq 10^{-3}$, in good agreement with most recent fits to the $O(p^4)$ LECs [28].

	$C_i \cdot 10^4$ GeV^2	$C_i \cdot (4\pi)^4 F_\pi^2$	$\delta C_i \cdot 10^4$ GeV^2
$C_{12}^{\mathcal{SP}}$	-4.4	-0.09	1.6
$C_{34}^{\mathcal{SP}}$	6.6	0.14	4.7
$C_{38}^{\mathcal{SP}}$	2.5	0.05	2.2

Table 1: Numerical values for the LECs of $O(p^6)$ in GeV^{-2} and in natural units $(4\pi)^{-4}F_\pi^{-2}$ for $M_S = 1.25$ GeV. The last column contains the variations of the $C_i(\mu)$ for $M_\eta \leq \mu \leq 1$ GeV. More precisely, we display the quantities $\delta C_i = \max\{|C_i(M_\rho) - C_i(M_\eta)|, |C_i(M_\rho) - C_i(1\,\mathrm{GeV})|\}$, using the $L_i^r(\mu)$ from fit 10 in Ref. [28].

In the estimates below, we use $M_P = 1.3$ GeV and we vary M_S between 1 and 1.5 GeV. The numerical values for $M_S = 1.25$ GeV are collected in Table 1. The above results represent the leading term in the large-N_C expansion of the couplings (within our simplified scheme of truncating the spectrum to the lowest-lying resonances). One way to estimate the size of subleading corrections in $1/N_C$ is to look at the renormalization scale dependence of the couplings. This effect is formally higher order in $1/N_C$ and a leading-order estimate is unable to provide the scale at which the expressions (29) apply. At $O(p^4)$, resonance saturation works well for $\mu = M_\rho$. On this basis, a crude estimate of the uncertainty is given by the variation of $C_i(\mu)$ for μ between M_η and 1 GeV.

In this way, we obtain the uncertainties shown in Table 1. Two comments are in order here. First of all, it is not the uncertainty of any given LEC that matters[4] but the overall scale dependence of a measurable quantity, as will be discussed below for the $K_{\ell 3}$ vector form factor at $t = 0$. Secondly, the values in Table 1 are of course very sensitive to the scalar resonance mass, while the scale dependence stays the same. This strong dependence on M_S seems rather discomforting at first sight but, as above, physical observables may exhibit a smoother dependence as we will demonstrate in the following section.

3 SU(3) breaking in $K_{\ell 3}$ decays and V_{us}

We now investigate the consequences of our results for the estimate of SU(3) breaking in $K_{\ell 3}$ form factors. As is well known, $K_{\ell 3}$ decays offer one of the most accurate determinations of the CKM element V_{us}. After the recent re-evaluation of radiative corrections [29, 30] and the new experimental results [31–34] (see Ref. [35] for a review of the present experimental and theoretical status), the main uncertainty in extracting V_{us} comes from theoretical calculations of the vector form factor $f_+^{K^0\pi^-}(0)$ at zero momentum transfer defined by

$$\langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = f_+^{K^0\pi^-}(t)\,(p_K+p_\pi)_\mu + f_-^{K^0\pi^-}(t)\,(p_K-p_\pi)_\mu \ , \tag{30}$$

where $t = (p_K - p_\pi)^2$. Here we are interested in the $SU(3)$ breaking corrections to $f_+^{K^0\pi^-}(0)$. We break up the form factor in terms of its expansion in quark masses:

$$f_+^{K^0\pi^-}(0) = 1 + f_{p^4} + f_{p^6} + \ldots \ . \tag{31}$$

Deviations from unity (the octet symmetry limit) are of second order in $SU(3)$ breaking [36]. The first correction arises to $O(p^4)$ in CHPT: a finite one-loop contribution [37,38] determines $f_{p^4} = -0.0227$ in terms of F_π, M_K and M_π, with essentially no uncertainty. The p^6 term receives contributions from pure two-loop diagrams, one-loop diagrams with insertion of one vertex from the p^4 effective Lagrangian, and pure tree-level diagrams with two insertions from the p^4 Lagrangian or one insertion from the p^6 Lagrangian [39, 40]:

$$f_{p^6} = f_{p^6}^{2-\mathrm{loops}}(\mu) + f_{p^6}^{L_i\times \mathrm{loop}}(\mu) + f_{p^6}^{\mathrm{tree}}(\mu) \ . \tag{32}$$

Individual components depend on the chiral renormalization scale μ, their sum being scale independent. Formally speaking, the three contributions scale as $O(1/N_C^2)$, $O(1/N_C)$, and

[4] After all, the choice of LECs is basis dependent.

$O(1)$, respectively. Although our main concern here is with $f_{p^6}^{\rm tree}$, the other terms have to be accounted for in a consistent phenomenological analysis, as infrared logs tend to upset the $1/N_C$ counting. Using as reference scale $\mu = M_\rho = 0.77$ GeV and the L_i from fit 10 in Ref. [28], one has [40]:

$$f_{p^6}^{2-\rm loops}(M_\rho) = 0.0113 \, , \tag{33}$$

$$f_{p^6}^{L_i \times \rm loop}(M_\rho) = -0.0020 \pm 0.0005 \, . \tag{34}$$

Note that we have subtracted the tree-level piece proportional to $L_5 \times L_5$ from the corresponding quantity $\Delta(0)$ in Ref. [40].

The explicit form for the tree-level contribution is then [40, 41]

$$f_{p^6}^{\rm tree}(M_\rho) = 8 \frac{(M_K^2 - M_\pi^2)^2}{F_\pi^2} \left[\frac{(L_5^r(M_\rho))^2}{F_\pi^2} - C_{12}^r(M_\rho) - C_{34}^r(M_\rho) \right] . \tag{35}$$

Upon substituting $L_5^r(M_\rho) \to L_5^{\mathcal{SP}}$ and $C_{12,34}^r(M_\rho) \to C_{12,34}^{\mathcal{SP}}$ one gets

$$f_{p^6}^{\rm tree}(M_\rho) = -\frac{(M_K^2 - M_\pi^2)^2}{2\, M_S^4} \left(1 - \frac{M_S^2}{M_P^2} \right)^2 \, , \tag{36}$$

where we used $d_m = F_\pi/(2\sqrt{2})$. In Fig. 1 we plot as a function of M_S our full result for $f_{p^6}^{\rm tree}(M_\rho)$ (solid line) as well as its two components: the $L_5 \times L_5$ term (dashed line) and the $C_{12} + C_{34}$ piece (dotted line). The two contributions tend to largely cancel each other, reducing the full result to $\sim 10\%$ of each individual term. As a consequence, the ambiguity related to M_S is strongly reduced, given the size of the resulting effect. This is simply a consequence of treating all tree-level contributions to f_{p^6} on the same footing, as suggested by the $1/N_C$ counting.

In addition to varying M_S in the range 1 GeV $\leq M_S \leq 1.5$ GeV, we need to estimate the intrinsic uncertainty due to our choice of $\mu = M_\rho$ as matching point for $f_{p^6}^{\rm tree}(M_\rho)$. We do this by performing the matching for any $\mu \in [M_\eta, 1\,\rm GeV]$ and then running the result back to $\mu = M_\rho$ via the renormalization group [7]. This way we find $\delta f_{p^6}^{\rm tree}(1/N_C) = \pm 0.008$. Accounting also for the uncertainty in M_S, we get

$$f_{p^6}^{\rm tree}(M_\rho) = -0.002 \pm 0.008_{\,1/N_C} \pm 0.002_{\,M_S} \, . \tag{37}$$

We now discuss the two main features of our result:

i. The smallness of the local contribution $f_{p^6}^{\rm tree}(M_\rho)$.

ii. The size of formally subleading terms in the $1/N_C$ expansion: the scale dependence of the LECs and the loop contribution to f_{p^6}.

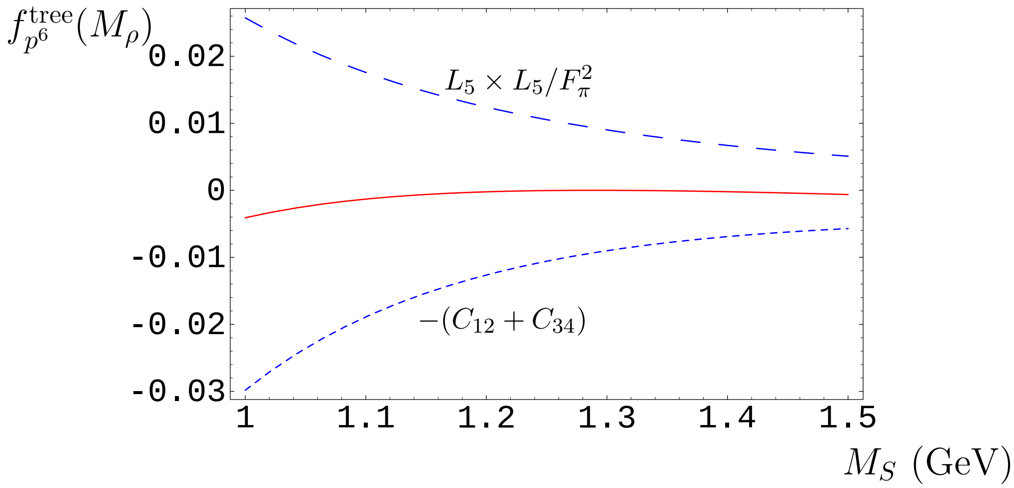

Figure 1: We display $f_{p^6}^{\rm tree}(M_\rho)$ according to Eq. (36) as a function of M_S for $M_P = 1.3$ GeV (solid line). We also plot the two components according to Eq. (35): the dashed line represents the term proportional to $L_5 \times L_5$, while the dotted line represents the term proportional to $-(C_{12}+C_{34})$.

The naive expectation for the size of the local contribution to f_{p^6} is $(M_K^2 - M_\pi^2)^2/M_S^4 \sim 5\cdot 10^{-2}\,\text{GeV}^4/M_S^4$. An order of magnitude is lost through the factor $1/2(1-M_S^2/M_P^2)^2 < 0.1$ (for resonance masses in the range considered) in Eq. (36). This extra suppression is a consequence of imposing, within the particle content of our ansatz, the correct asymptotic behaviour for the two- and one-pion form factors $\langle\pi|S|\pi\rangle$, $\langle\pi|S|\mathcal{P}\rangle$ and $\langle\pi|P|\mathcal{S}\rangle$.

We do not have a complete answer to the question whether this suppression persists when using a more sophisticated ansatz. However, we have examined the stability of our result in two different directions. Omitting the pseudoscalar resonances altogether in our ansatz (12) gives rise to a solution that is equivalent to setting $d_m = 0$ in Eqs. (29). There is a complete destructive interference in this case: the scalar contributions cancel in Eq. (35) implying $f_{p^6}^{\rm tree}(M_\rho) = 0$ instead of Eq. (36). On the other hand, adding an additional pseudoscalar nonet leads to a straightforward generalization of Eq. (36). Instead of

$$d_m^2\left(1-\frac{M_S^2}{M_P^2}\right)^2 \qquad \text{with} \qquad d_m^2 = c_m^2 - \frac{F_\pi^2}{8} = \frac{F_\pi^2}{8}\ , \tag{38}$$

one gets by including a second pseudoscalar multiplet with mass $M_{P'}$ and coupling d'_m

$$d_m^2\left(1-\frac{M_S^2}{M_P^2}\right)^2 + d_m'^2\left(1-\frac{M_S^2}{M_{P'}^2}\right)^2 \qquad \text{with} \qquad d_m^2 + d_m'^2 = \frac{F_\pi^2}{8}\ . \tag{39}$$

Assuming $d_m'^2 \le d_m^2$, $M_S = 1.25$ GeV and taking $M_{P'}$ in the range $1.5 \to 2$ GeV, $|f_{p^6}^{\rm tree}(M_\rho)|$ may increase by up to 0.002. Thus, additional pseudoscalar multiplets do not modify the general result of a small tree-level contribution $f_{p^6}^{\rm tree}(M_\rho)$.

The second issue concerns higher-order corrections in the $1/N_C$ expansion. We have taken the variation of the LECs with the renormalization scale as a measure of those corrections. The relatively big variation for μ in the range $M_\eta \leq \mu \leq 1$ GeV is due to large infrared logs that typically occur in the scalar sector. This also explains why loop contributions cannot be neglected in this case.

Adding all uncertainties linearly, but omitting the small error of the loop contribution (34), we get finally[5]

$$
\begin{aligned}
f_{p^6}^{\text{tree}}(M_\rho) &= -0.002 \pm 0.008_{\,1/N_C} \pm 0.002_{\,M_S} {}^{+0.000}_{-0.002}{}_{\,P'} \,, &(40)\\
f_{p^6} &= 0.007 \pm 0.012 \,, &(41)\\
f_+^{K^0\pi^-}(0) &= 0.984 \pm 0.012 \,. &(42)
\end{aligned}
$$

Our final result (42) differs from other determinations [38, 42, 43]. This difference is due to the small central value for $f_{p^6}^{\text{tree}}$, which appears to be a generic consequence of a few-resonance approximation. When combined with the size and sign of the loop contribution of $O(p^6)$ [40], our central value for f_{p^6} in (41) is positive in contrast to most other estimates, e.g., $f_{p^6}^{\text{LR}} = -0.016 \pm 0.008$ of Leutwyler and Roos [38].

For the purpose of illustration, we use the recent K_L branching ratio measurements of KTeV [31], together with their results on K_{e3} form factors where the curvature in $f_+^{K^0\pi^-}(t)$ has been included in the analysis [32]. Using also the recent precise determination of the K_L lifetime [33], one finds [44] $f_+^{K^0\pi^-}(0) \cdot |V_{us}| = 0.2166 \pm 0.0010$. Our result for $f_+^{K^0\pi^-}(0)$ in (42) then implies

$$|V_{us}| = 0.2201 \pm 0.0027_{f_+(0)} \pm 0.0010_{\text{exp}} \,. \tag{43}$$

The central value is smaller than the one from CKM unitarity, using the most recent value of V_{ud} [45]. It is also smaller than what one would obtain using the same experimental input and $f_+^{K^0\pi^-}(0)$ from Refs. [38, 42, 43]. On the other hand, our result is in better agreement with alternative extractions of V_{us} from τ decays [46] and $K_{\ell 2}/\pi_{\ell 2}$ [47].

The LECs obtained in this article determine also the deviation from the original Callan-Treiman relation [48]

$$\Delta_{CT} \,=\, f_0^{K^0\pi^-}(M_K^2 - M_\pi^2) \,-\, \frac{F_K}{F_\pi} \,, \tag{44}$$

involving the scalar form factor $f_0(t) = f_+(t) + t f_-(t)/(M_K^2 - M_\pi^2)$. The tree-level contribution of $O(p^6)$ is given by [42]

$$\Delta_{CT}^{\text{tree,p}^6} \;=\; 16\,\frac{M_\pi^2}{F_\pi^2}\,(M_K^2 - M_\pi^2)\,(2\,C_{12}^r \,+\, C_{34}^r) \,=\, \frac{M_\pi^2(M_K^2 - M_\pi^2)}{M_P^4}\,\left(1 \,-\, 2\,\frac{M_P^2}{M_S^2}\right)\,. \tag{45}$$

[5]Estimates of the uncertainty due to higher-order corrections beyond $O(p^6)$ [40] essentially do not modify the final result (42) for $f_+^{K^0\pi^-}(0)$ when adding the errors in quadrature.

With the values of M_S and M_P considered before, $\Delta_{CT}^{\rm tree,p^6}$ is negative and small (a few times 10^{-3} in magnitude). Finally, these LECs also provide a prediction for the slope[6] of the scalar form factor. In the representation for λ_0 given in Ref. [40], the LECs appear again in the combination $2\,C_{12} + C_{34}$. Including the loop contributions [40], we obtain

$$\lambda_0 = 0.013 \pm 0.002\,_{1/N_C} \pm 0.001\,_{M_S} = (13 \pm 3) \cdot 10^{-3} \ , \tag{46}$$

in agreement with $\lambda_0 = (17 \pm 4) \cdot 10^{-3}$ from Ref. [37], $\lambda_0 = (15.7 \pm 1.0) \cdot 10^{-3}$ from Ref. [42] and with the value measured by KTeV [32] in $K^L_{\mu 3}$ decays, $\lambda_0 = (13.72 \pm 1.31) \cdot 10^{-3}$.

4 Conclusions

We have constructed a meromorphic approximation to the $\langle SPP \rangle$ Green function with pole singularities corresponding to Goldstone modes and lowest-lying scalar ($\mathcal{S}$) and pseudoscalar ($\mathcal{P}$) resonances. The highlights of our analysis are:

- We have shown how the polynomial terms in our ansatz can be fixed by imposing the correct large-momentum behaviour implied by QCD both off-shell (OPE constraints) and on the relevant hadron mass shells (form factor constraints). Because of nontrivial Wilson coefficients, one cannot match the OPE constraints exactly with a finite number of resonance poles. We have instead required a large-momentum behaviour for our ansatz that is not worse than predicted by QCD. For the particle content used, no inconsistencies arise with this set of constraints.

- This matching procedure has allowed us to determine three of the LECs of $O(p^6)$ (C_{12}, C_{34}, C_{38}) appearing in the low-energy expansion of $\langle SPP \rangle$ in terms of resonance masses $M_{S,P}$ and F_π. In particular, C_{12} is uniquely determined by requiring the correct behaviour of the pion scalar form factor $\langle \pi | S | \pi \rangle$, while C_{34} is fixed by the correct scaling of the one-pion form factors $\langle \pi | S | \mathcal{P} \rangle$ and $\langle \pi | P | \mathcal{S} \rangle$.

- We have estimated the uncertainty of the large-N_C matching procedure by varying the chiral renormalization scale at which the matching is performed. While being a sub-leading effect in the $1/N_C$ counting, the scale ambiguity $M_\eta \leq \mu \leq 1$ GeV gives rise to sizable uncertainties. For the same reason, loop contributions must be included in phenomenological applications.

- We have explored the impact of our results on the estimate of the local p^6 contribution to $SU(3)$ breaking in $K_{\ell 3}$ decays. We find that the resulting effect is much smaller than

[6] The slope λ_0 is defined by

$$f_0(t) = f_+(0) \left[1 \, + \, \lambda_0 \, \frac{t}{M^2_{\pi^+}} + \ldots \right] \ .$$

the ratio of mass scales $(M_K^2 - M_\pi^2)^2/M_S^4$ would suggest, due to interfering contributions. When combined with known loop corrections [40], our estimate leads to $f_{p^6} = 0.007 \pm 0.012$, the mean value being opposite in sign to most other existing calculations. Using this input and the most recent experimental results for the K_{e3} branching ratio [31] and lifetime for K_L [33], we find $|V_{us}| = 0.2201 \pm 0.0027_{f_+(0)} \pm 0.0010_{\rm exp}$. The mean value of our result is smaller than the value inferred from CKM unitarity $|V_{us}|^{\rm unit.} = 0.2265 \pm 0.0022$, using the most recent determination of V_{ud} [45]. If the recent precision measurement of the neutron lifetime [49] were confirmed our preferred value for V_{us} would however be in perfect agreement with CKM unitarity.

- We have considered variations of the hadronic ansatz to investigate the stability of our results. The smallness of the tree-level part compared to the loop contribution of $O(p^6)$ for $f_+^{K^0\pi^-}(0)$ appears as generic feature of a few-resonance approximation for the set of large-momentum constraints considered. In view of the significant implications for the determination of V_{us}, the validity of our approach will be further investigated also for other Green functions.

- Finally, we have also used our results to estimate the deviation from the Callan–Treiman relation and to calculate the slope of the scalar $K_{\ell 3}$ form factor.

Acknowledgements

V.C., A.P. and J.P. thank Maarten Golterman and Santi Peris for hospitality at the Benasque Workshop *"Matching Light Quarks to Hadrons"*, where an early part of this work was carried out. We also thank Hans Bijnens, Jürg Gasser and Helmut Neufeld for useful information and discussions. V.C. is supported by a Sherman Fairchild Fellowship from Caltech. M.E. thanks the FAPESP (Brazil) for financial support. This work has been supported in part by MCYT (Spain) under Grant FPA2004-00996, by Generalitat Valenciana (Grants GRUPOS03/013 and GV04B-594) and by ERDF funds from the European Commission.

References

[1] S. Weinberg, Physica A **96** (1979) 327.

[2] J. Gasser and H. Leutwyler, Ann. Phys. **158** (1984) 142.

[3] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 465.

[4] For a recent review see J. Bijnens, *Chiral meson physics at two loops*, arXiv:hep-ph/0409068.

[5] H. W. Fearing and S. Scherer, Phys. Rev. D **53** (1996) 315 [arXiv:hep-ph/9408346].

[6] J. Bijnens, G. Colangelo and G. Ecker, JHEP **9902** (1999) 020 [arXiv:hep-ph/9902437].

[7] J. Bijnens, G. Colangelo and G. Ecker, Ann. Phys. **280** (2000) 100 [arXiv:hep-ph/9907333].

[8] S. Peris, M. Perrottet and E. de Rafael, JHEP **9805** (1998) 011 [arXiv:hep-ph/9805442];
M. Knecht, S. Peris, M. Perrottet and E. de Rafael, Phys. Rev. Lett. **83** (1999) 5230 [arXiv:hep-ph/9908283];
S. Peris, B. Phily and E. de Rafael, Phys. Rev. Lett. **86** (2001) 14 [arXiv:hep-ph/0007338].

[9] A. Pich, in *Phenomenology of large-N_C QCD*, ed. R.F. Lebed, World Scientific, Singapore 2002, p. 239, arXiv:hep-ph/0205030.

[10] E. de Rafael, Nucl. Phys. Proc. Suppl. **119** (2003) 71 [arXiv:hep-ph/0210317].

[11] B. Moussallam and J. Stern, in *Two-photon physics: From DAΦNE to LEP 200 and beyond*, F. Kapusta and J. Parisi eds., World Scientific, Singapore 1994, arXiv:hep-ph/9404353.

[12] B. Moussallam, Nucl. Phys. B **504** (1997) 381 [arXiv:hep-ph/9701400].

[13] M. Knecht and A. Nyffeler, Eur. Phys. J. C **21** (2001) 659 [arXiv:hep-ph/0106034].

[14] J. Bijnens, E. Gamiz, E. Lipartia and J. Prades, JHEP **0304** (2003) 055 [arXiv:hep-ph/0304222].

[15] P. D. Ruiz-Femenia, A. Pich and J. Portolés, JHEP **0307** (2003) 003 [arXiv:hep-ph/0306157].

[16] V. Cirigliano, G. Ecker, M. Eidemüller, A. Pich and J. Portolés, Phys. Lett. B **596** (2004) 96 [arXiv:hep-ph/0404004].

[17] M. Battaglia et al., *The CKM matrix and the unitarity triangle*, arXiv:hep-ph/0304132.

[18] G.P. Lepage and S.J. Brodsky, Phys. Lett. B **87** (1979) 359, Phys. Rev. D **22** (1980) 2157;
S.J. Brodsky and G.P. Lepage, Phys. Rev. D **24** (1981) 1808.

[19] G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. B **321** (1989) 311.

[20] M.A. Shifman, in *At the frontier of particle physics: Handbook of QCD. Boris Ioffe festschrift*, M. A. Shifman ed., World Scientific, Singapore 2000, arXiv:hep-ph/0009131;
M. Golterman, S. Peris, B. Phily and E. de Rafael, JHEP **01** (2002) 024 [arXiv:hep-ph/0112042].

[21] M. F. L. Golterman and S. Peris, Phys. Rev. D **61** (2000) 034018 [arXiv:hep-ph/9908252].

[22] S. Weinberg, Phys. Rev. Lett. **18** (1967) 507.

[23] H. Leutwyler, Nucl. Phys. B **337** (1990) 108.

[24] G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B **223** (1989) 425.

[25] S. Eidelman et al. [Particle Data Group Collaboration], Phys. Lett. B **592** (2004) 1.

[26] V. Cirigliano, G. Ecker, H. Neufeld and A. Pich, JHEP **0306** (2003) 012 [arXiv:hep-ph/0305311].

[27] S. Prelovsek, C. Dawson, T. Izubuchi, K. Orginos and A. Soni, Phys. Rev. D **70** (2004) 094503 [arXiv:hep-lat/0407037].

[28] G. Amorós, J. Bijnens and P. Talavera, Nucl. Phys. B **602** (2001) 87 [arXiv:hep-ph/0101127].

[29] V. Cirigliano, H. Neufeld and H. Pichl, Eur. Phys. J. C **35** (2004) 53 [arXiv:hep-ph/0401173];
V. Cirigliano, M. Knecht, H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C **23** (2002) 121 [arXiv:hep-ph/0110153].

[30] T. C. Andre, *Radiative corrections to K^0_{l3} decays*, arXiv:hep-ph/0406006 ;
V. Bytev, E. Kuraev, A. Baratt and J. Thompson, Eur. Phys. J. C **27** (2003) 57 [Erratum-ibid. C **34** (2004) 523] [arXiv:hep-ph/0210049].

[31] T. Alexopoulos et al. [KTeV Collaboration], Phys. Rev. Lett. **93** (2004) 181802 [arXiv:hep-ex/0406001].

[32] T. Alexopoulos *et al.* [KTeV Collaboration], Phys. Rev. D **70** (2004) 092007 [arXiv:hep-ex/0406003].

[33] P. Franzini, *Kaon decays and V_{us}*, arXiv:hep-ex/0408150.

[34] A. Lai *et al.* [NA48 Collaboration], Phys. Lett. B **602** (2004) 41 [arXiv:hep-ex/0410059].

[35] F. Mescia, *V_{us} from K_{l3} decays*, arXiv:hep-ph/0411097.

[36] M. Ademollo and R. Gatto, Phys. Rev. Lett. **13** (1964) 264;
R. E. Behrends and A. Sirlin, Phys. Rev. Lett. **4** (1960) 186.

[37] J. Gasser and H. Leutwyler, Nucl. Phys. B **250** (1985) 517.

[38] H. Leutwyler and M. Roos, Z. Phys. C **25** (1984) 91.

[39] P. Post and K. Schilcher, Eur. Phys. J. C **25** (2002) 427 [arXiv:hep-ph/0112352].

[40] J. Bijnens and P. Talavera, Nucl. Phys. B **669** (2003) 341 [arXiv:hep-ph/0303103]; see also http://www.thep.lu.se/∼bijnens/chpt.html.

[41] J. Bijnens, G. Colangelo and G. Ecker, Phys. Lett. B **441** (1998) 437 [arXiv:hep-ph/9808421].

[42] M. Jamin, J. A. Oller and A. Pich, JHEP **0402** (2004) 047 [arXiv:hep-ph/0401080].

[43] D. Becirevic et al., Nucl. Phys. B **705** (2005) 339 [arXiv:hep-ph/0403217];
M. Okamoto [Fermilab Lattice Collaboration], *Full CKM matrix with lattice QCD*, arXiv:hep-lat/0412044.

[44] H. Neufeld, *Isospin violation in semileptonic decays*, Talk presented at the Workshop on Effective Field Theories in Nuclear, Particle and Atomic Physics, Bad Honnef, Germany, Dec. 2004 and private communication.

[45] A. Czarnecki, W. J. Marciano and A. Sirlin, Phys. Rev. D **70** (2004) 093006 [arXiv:hep-ph/0406324].

[46] E. Gamiz, M. Jamin, A. Pich, J. Prades and F. Schwab, Phys. Rev. Lett. **94** (2005) 011803 [arXiv:hep-ph/0408044].

[47] W. J. Marciano, Phys. Rev. Lett. **93** (2004) 231803 [arXiv:hep-ph/0402299].

[48] C. G. Callan and S. B. Treiman, Phys. Rev. Lett. **16** (1966) 153;
R. F. Dashen and M. Weinstein, Phys. Rev. Lett. **22** (1969) 1337.

[49] A. Serebrov et al., Phys. Lett. B **605** (2005) 72 [arXiv:nucl-ex/0408009].