

# Spectral distribution for the decay $\tau \rightarrow \nu_\tau K \pi$

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**Abstract:** With the newly available data sets on hadronic  $\tau$  decays from the  $B$ -factories BABAR and BELLE, and future data from BESIII, precise information on the decay distributions will soon become available. This calls for an improvement of the decay spectra also on the theoretical side. In this work, the distribution function for the decay  $\tau \rightarrow \nu_\tau K \pi$  will be presented with the relevant  $K\pi$  vector and scalar form factors being calculated in the framework of the resonance chiral theory, also taking into account additional constraints from dispersion relations and short-distances. As a by-product the slope and curvature of the vector form factor  $F_+^{K\pi}(s)$  are determined to be  $\lambda'_+ = 25.6 \cdot 10^{-3}$  and  $\lambda''_+ = 1.31 \cdot 10^{-3}$  respectively. From our approach it appears that it should be possible to obtain information on the low lying scalar  $K_0^*(800)$  as well as the second vector  $K^*(1410)$  resonances from the  $\tau$  decay data. In particular, the exclusive branching fraction of the scalar component is found to be  $B[\tau \rightarrow \nu_\tau (K\pi)_{\text{S-wave}}] = (3.88 \pm 0.19) \cdot 10^{-4}$ .

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# 1 Introduction

Already more than a decade ago it was realised that the hadronic decays of the  $\tau$  lepton could serve as an ideal system to study low-energy QCD under rather clean conditions [1–5]. In the following years, detailed investigations of the  $\tau$  hadronic width as well as invariant mass distributions have allowed to determine many QCD parameters, a most prominent example being the QCD coupling  $\alpha_s$ . Especially the experimental separation of the Cabibbo-allowed decays and Cabibbo-suppressed modes into strange particles [6–8] opened a means to also determine the quark-mixing matrix element  $|V_{us}|$  [9–11] as well as the mass of the strange quark [12–19], additional fundamental parameters within the Standard Model, from the  $\tau$  strange spectral function.

The dominant contribution to the Cabibbo-suppressed  $\tau$  decay rate is due to the decay  $\tau \rightarrow \nu_\tau K\pi$ . The corresponding distribution function has been measured experimentally in the past by ALEPH [8] and OPAL [7]. With the large data sets on hadronic  $\tau$  decays from the B-factories BABAR and BELLE, which are currently under investigation, and good prospects for additional data from BESIII in the future, a refined theoretical understanding of the spectral functions is called for. For the decay in question, the general expression for the differential decay distribution takes the form [20]

$$\frac{d\Gamma_{K\pi}}{d\sqrt{s}} = \frac{G_F^2 |V_{us}|^2 M_\tau^3}{32\pi^3 s} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[ \left(1 + 2 \frac{s}{M_\tau^2}\right) q_{K\pi}^3 |F_+^{K\pi}(s)|^2 + \frac{3\Delta_{K\pi}^2}{4s} q_{K\pi} |F_0^{K\pi}(s)|^2 \right], \quad (1)$$

where we have assumed isospin invariance and have summed over the two possible decays  $\tau^- \rightarrow \nu_\tau \bar{K}^0 \pi^-$  and  $\tau^- \rightarrow \nu_\tau K^- \pi^0$ , with the individual decay channels contributing in the ratio 2 : 1 respectively. In this expression,  $F_+^{K\pi}(s)$  and  $F_0^{K\pi}(s)$  are the vector and scalar  $K\pi$  form factors respectively which will be explicated in more detail below,  $\Delta_{K\pi} \equiv M_K^2 - M_\pi^2$ , and  $q_{K\pi}$  is the Kaon momentum in the rest frame of the hadronic system,

$$q_{K\pi}(s) = \frac{1}{2\sqrt{s}} \sqrt{\left(s - (M_K + M_\pi)^2\right) \left(s - (M_K - M_\pi)^2\right)} \cdot \theta\left(s - (M_K + M_\pi)^2\right). \quad (2)$$

By far the dominant contribution to the decay distribution originates from the  $K^*(892)$  meson. In the next section, an effective description of this contribution to the vector form factor  $F_+^{K\pi}(s)$  will be presented in the framework of chiral perturbation theory with resonances (R $\chi$ PT) [21, 22], quite analogously to a similar description of the pion form factor given in refs. [23–25]. To also include a second vector resonance, the effective chiral description can be straightforwardly augmented by an additional  $K^*(1410)$  meson. Finally, the scalar  $K\pi$  form factor  $F_0^{K\pi}(s)$  has very recently been updated in ref. [26]. In section 3, we shall present our main results for the distribution function  $d\Gamma_{K\pi}/d\sqrt{s}$  and total decay rates and investigate the influence of the different vector and scalar form factor contributions.

## 2 The vector form factor $F_+^{K\pi}(s)$

A theoretical representation of the vector form factor  $F_+^{K\pi}(s)$ , which is based on fundamental principles, can be provided by an effective field theory description, in complete analogy to the description of the pion form factor presented in refs. [23–25]<sup>1</sup>. This approach employs our present knowledge on effective hadronic theories, short-distance QCD, the large- $N_C$  expansion as well as analyticity and unitarity. For the pion form factor the resulting expressions provide a very good description of the experimental data [23–25].

Close to  $s$  equal zero,  $F_+^{K\pi}(s)$  is well described by the  $\chi$ PT result at one loop [29], which takes the form:

$$F_+^{K\pi}(s) = 1 + \frac{2}{F_\pi^2} L_9^r s + \frac{3}{2} \left[ \tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s) \right]. \quad (3)$$

The one-loop function  $\tilde{H}(s)$  is related to the corresponding function  $H(s)$  of [29] by  $\tilde{H}(s) \equiv H(s) - 2L_9^r s / (3F_\pi^2) = [sM^r(s) - L(s)] / F_\pi^2$ . Explicit expressions for  $M^r(s)$  and  $L(s)$  can be found in ref. [30]. The vector form factor is an analytic function in the complex  $s$  plane, except for a cut along the positive real axis, starting at  $s_{K\pi} \equiv (M_K + M_\pi)^2$ , where its imaginary part develops a discontinuity. In the elastic region below roughly 1 GeV,  $F_+^{K\pi}(s)$  admits the well-known Omnès representation [31]

$$F_+^{K\pi}(s) = P(s) \exp \left( \frac{s}{\pi} \int_{s_{K\pi}}^{\infty} ds' \frac{\delta_1^{1/2}(s')}{s'(s' - s - i0)} \right), \quad (4)$$

with  $P(s)$  being a real polynomial to take care of the zeros of  $F_+^{K\pi}(s)$  for finite  $s$  and  $\delta_1^{1/2}(s)$  is the P-wave  $I = 1/2$  elastic  $K\pi$  phase shift.

Precisely following the approach of ref. [23], and matching the Omnès formula (4) with the  $\chi$ PT calculation of  $F_+^{K\pi}(s)$  in the presence of vector resonances [21], one finds the following representation of the form factor  $F_+^{K\pi}(s)$ :

$$F_+^{K\pi}(s) = \frac{M_{K^*}^2 e^{\frac{3}{2} \text{Re}[\tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s)]}}{M_{K^*}^2 - s - iM_{K^*} \Gamma_{K^*}(s)}. \quad (5)$$

The one-loop function  $\tilde{H}(s)$  depends on the chiral scale  $\mu$ , and in eq. (5), this scale should be taken as  $\mu = M_{K^*}$ . In ref. [32], the off-shell width of a vector resonance was defined through the two-point vector current correlator, performing a Dyson-Schwinger resummation within R $\chi$ PT [21, 22]. Following this scheme the energy-dependent width  $\Gamma_{K^*}(s)$  is found to be

$$\Gamma_{K^*}(s) = \frac{G_V^2 M_{K^*} s}{64\pi F_\pi^4} \left[ \sigma_{K\pi}^3(s) + \sigma_{K\eta}^3(s) \right], \quad (6)$$

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<sup>1</sup>For an alternative dispersive approach to the pion form factor see also refs. [27, 28].

where  $G_V$  is the chiral vector coupling which appears in the framework of  $\chi$ PT with resonances [21], the phase space function  $\sigma_{K\pi}(s)$  is given by  $\sigma_{K\pi}(s) = 2q_{K\pi}(s)/\sqrt{s}$ , and  $\sigma_{K\eta}(s)$  follows analogously with the replacement  $M_\pi \rightarrow M_\eta$ . Re-expanding eq. (5) in  $s$  and comparing with eq. (3), one reproduces the short-distance constraint for the vector coupling  $G_V = F_\pi/\sqrt{2}$  [22] which guarantees a vanishing form factor at  $s$  to infinity, as well as the lowest-resonance estimate<sup>2</sup> for the  $\mathcal{O}(p^4)$  chiral coupling [21]

$$L_9^r(M_{K^*}) = \frac{F_\pi^2}{2M_{K^*}^2} = 5.34 \cdot 10^{-3}, \quad (7)$$

where  $F_\pi = 92.4$  MeV and the average mass of the charged and neutral  $K^*(892)$ ,  $M_{K^*} = 893.9$  MeV have been used. This result is in very good agreement to a recent determination of  $L_9^r$  from the pion form factor [34] which found  $L_9^r(M_{K^*}) = (5.69 \pm 0.41) \cdot 10^{-3}$ .

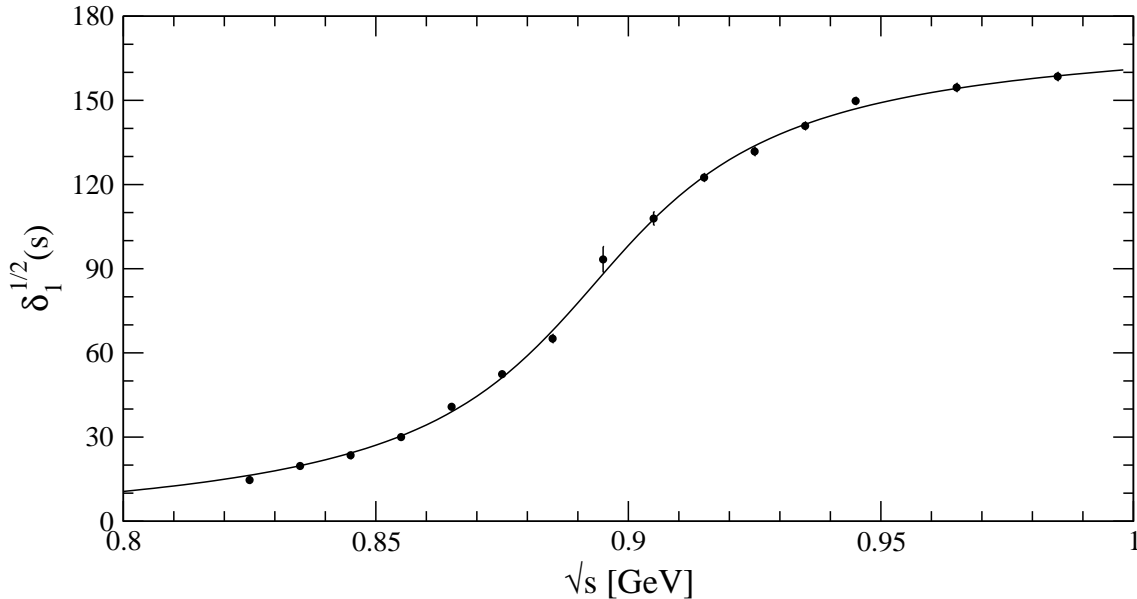


Figure 1:  $K\pi$  scattering P-wave phase-shift data of ref. [35], together with our fit described in the text.

From eq. (5), one obtains a description of the P-wave  $K\pi$  phase shift  $\delta_1^{1/2}(s)$ :

$$\delta_1^{1/2}(s) = \arctan\left(\frac{M_{K^*}\Gamma_{K^*}(s)}{M_{K^*}^2 - s}\right). \quad (8)$$

Improving the  $K^*$ -meson width  $\Gamma_{K^*}(s)$  with the pertinent Blatt-Weisskopf barrier factor  $D_1(rq_{K\pi}(M_{K^*}^2))/D_1(rq_{K\pi}(s))$  [36], where  $D_1(x) = 1 + x^2$  and  $r$  is the interaction radius, we

<sup>2</sup>A recent one-loop calculation of the pion form factor in R $\chi$ PT [33] has found that the NLO corrections to this lowest-resonance approximation are very small.

can perform a fit of eq. (8) to the phase-shift data of Aston et al. [35]. The fit parameters  $M_{K^*}$ ,  $G_V$  and  $r$  are found to be

$$M_{K^*} = 895.9 \pm 0.5 \text{ MeV}, \quad G_V = 64.6 \pm 0.4 \text{ MeV}, \quad r = 3.5 \pm 0.6 \text{ GeV}^{-1}, \quad (9)$$

where the given errors are purely statistical and the corresponding fit is displayed in figure 1.

We observe that the obtained  $K^*$  mass is in complete agreement with the fit result of ref. [35], and also with the PDG average for the mass of the neutral  $K^*$  meson  $M_{K^*} = 896.1 \pm 0.3 \text{ MeV}$  [37], the relevant channel in this case. Our result for  $G_V$  is nicely compatible with the short-distance constraint  $G_V = F_\pi/\sqrt{2} \approx 65.3 \text{ MeV}$  found in ref. [22]. Furthermore, our fit result for  $r$  and our  $\chi^2/\text{NDF} \approx 18.5/12$  are similar to the ones obtained by Aston et al. [35]. However, the result for  $G_V$  found above would correspond to a  $K^*$  width

$$\Gamma_{K^*} \equiv \Gamma_{K^*}(M_{K^*}^2) = 55.8 \pm 0.8 \text{ MeV}, \quad (10)$$

which is about 10% larger than the finding of [35] and the PDG average  $\Gamma_{K^*} = 50.7 \pm 0.6 \text{ MeV}$  [37]. At the origin of this discrepancy may lie the role played by a heavier vector resonance, namely the  $K^*(1410)$ , on which we shall comment further below. As concluded in ref. [25],  $G_V$  goes down a few percent when a second multiplet is included into the analysis of the elastic  $\pi\pi$  scattering amplitude, providing  $G_V = 61.9 \pm 1.5 \text{ MeV}$  [25] which would result in  $\Gamma_{K^*} = 51.2 \pm 0.7 \text{ MeV}$ , in excellent agreement with the PDG average.

Next, we can compare the low-energy expansion of the vector form factor (5) with recent experimental measurements of its slope and curvature. Let us define a general expansion of  $F_+^{K\pi}(s)$  as

$$F_+^{K\pi}(s) \equiv F_+^{K\pi}(0) \left[ 1 + \lambda'_+ \frac{s}{M_\pi^2} + \frac{1}{2} \lambda''_+ \frac{s^2}{M_\pi^4} + \frac{1}{6} \lambda'''_+ \frac{s^3}{M_\pi^6} + \dots \right], \quad (11)$$

where  $\lambda'_+$ ,  $\lambda''_+$  and  $\lambda'''_+$  are the slope, curvature and cubic expansion parameter respectively. Numerically, when employing the following isospin averages for the meson masses  $M_\pi = 138.0 \text{ MeV}$ ,  $M_K = 495.7 \text{ MeV}$  and the mass of the charged  $K^*$  meson,  $M_{K^*} = 891.66 \text{ MeV}$  [37], which is relevant for the leptonic decays of the  $K$  meson, these are found to be:

$$\lambda'_+ = 25.6 \cdot 10^{-3}, \quad \lambda''_+ = 1.31 \cdot 10^{-3}, \quad \lambda'''_+ = 9.74 \cdot 10^{-5}. \quad (12)$$

The parametric uncertainties on these results are rather small. However, it is difficult to estimate the systematic uncertainties and therefore we have chosen not to attach errors to the results of eq. (12). Recent experimental results on the slope  $\lambda'_+$  and the curvature  $\lambda''_+$  have been collected in table 1. One observes that our results of eq. (12) are in nice agreement with the very recent measurement by KLOE [38], and in reasonable agreement with ISTRA [39] as well as NA48 [40], however about  $2.5\sigma$  away from the KTEV result [41].

Collaboration	$\lambda'_+ [10^{-3}]$	$\lambda''_+ [10^{-3}]$
ISTRA 04 [39]	$23.2 \pm 1.6$	$0.84 \pm 0.41$
KTEV 04 [41]	$20.64 \pm 1.75$	$3.20 \pm 0.69$
NA48 04 [40]	$28.0 \pm 2.4$	$0.2 \pm 0.5$
KLOE 06 [38]	$25.5 \pm 1.8$	$1.4 \pm 0.8$

Table 1: Recent experimental results on the slope  $\lambda'_+$  and the curvature  $\lambda''_+$  of the vector form factor  $F_+^{K\pi}(s)$  in units of  $10^{-3}$ .

Since the  $\tau$  lepton can also decay hadronically into the second vector resonance  $K^{*'} \equiv K^*(1410)$ , this particle should be included in our parametrisation of the vector form factor  $F_+^{K\pi}(s)$ . A parametrisation which is motivated by the R $\chi$ PT framework [21, 22] can be written as follows:

$$F_+^{K\pi}(s) = \left[ \frac{M_{K^*}^2 + \gamma s}{M_{K^*}^2 - s - iM_{K^*}\Gamma_{K^*}(s)} - \frac{\gamma s}{M_{K^{*'}}^2 - s - iM_{K^{*'}}\Gamma_{K^{*'}}(s)} \right] e^{\frac{3}{2}\text{Re}[\tilde{H}_{K\pi}(s) + \tilde{H}_{K\eta}(s)]}. \quad (13)$$

The relation of the parameter  $\gamma$  to the R $\chi$ PT couplings takes the form  $\gamma = F_V G_V / F_\pi^2 - 1$ , when one assumes a vanishing form factor at large  $s$  in the  $N_C$  to infinity limit. It is difficult, to asses a precise value for  $\gamma$ , but from the work of ref. [25], we infer that it should be small and positive ( $0 < \gamma \ll 1$ ). We shall come back to the parameter  $\gamma$  below. The width of the second resonance cannot be set unambiguously. Therefore, we have decided to endow the  $K^*(1410)$  contribution with a generic width as expected for a vector resonance. Hence,  $\Gamma_{K^{*'}}(s)$  will be taken to have the form

$$\Gamma_{K^{*'}}(s) = \Gamma_{K^{*'}} \frac{s}{M_{K^{*'}}^2} \left( \frac{\sigma_{K\pi}^3(s)}{\sigma_{K\pi}^3(M_{K^{*'}}^2)} \right). \quad (14)$$

### 3 The differential decay distribution

As a last ingredient for a prediction of the differential decay distribution of the decay  $\tau \rightarrow \nu_\tau K\pi$  according to eq. (1), we require the scalar form factor  $F_0^{K\pi}(s)$ . This form factor was calculated in a series of articles [42–44] in the framework of R $\chi$ PT, again also employing constraints from dispersion theory as well as the short-distance behaviour, which then lead to a determination of the strange quark mass  $m_s$  in [44]. Quite recently, the determination of  $F_0^{K\pi}(s)$  was updated in [26] by employing novel experimental constraints on the form factor at the Callan-Treiman point  $\Delta_{K\pi}$ , and here we shall also make use of this update.

A remaining question is which value to use for the form factors  $F_+^{K\pi}(s)$  and  $F_0^{K\pi}(s)$  at the origin. From our chiral description of the vector form factor (5), one would obtain

$F_+^{K\pi}(0) = 0.978$ . An average over recent determinations from lattice QCD and effective field theory approaches [45–50], however, yields

$$F_+^{K\pi}(0) = F_0^{K\pi}(0) = 0.972 \pm 0.012, \quad (15)$$

somewhat lower and also compatible with the original estimate by Leutwyler and Roos [51]. Furthermore, this is the value to which the scalar form factor had been normalised in ref. [26]. Nevertheless, what is required in the hadronic  $\tau$  decays is only the product  $|V_{us}|F_+^{K\pi}(0)$  which experimentally is known more precisely. Thus, in what follows, to normalise the form factors, we shall employ the most recent average for this number [52]:

$$|V_{us}|F_+^{K\pi}(0) = 0.2173 \pm 0.0008. \quad (16)$$

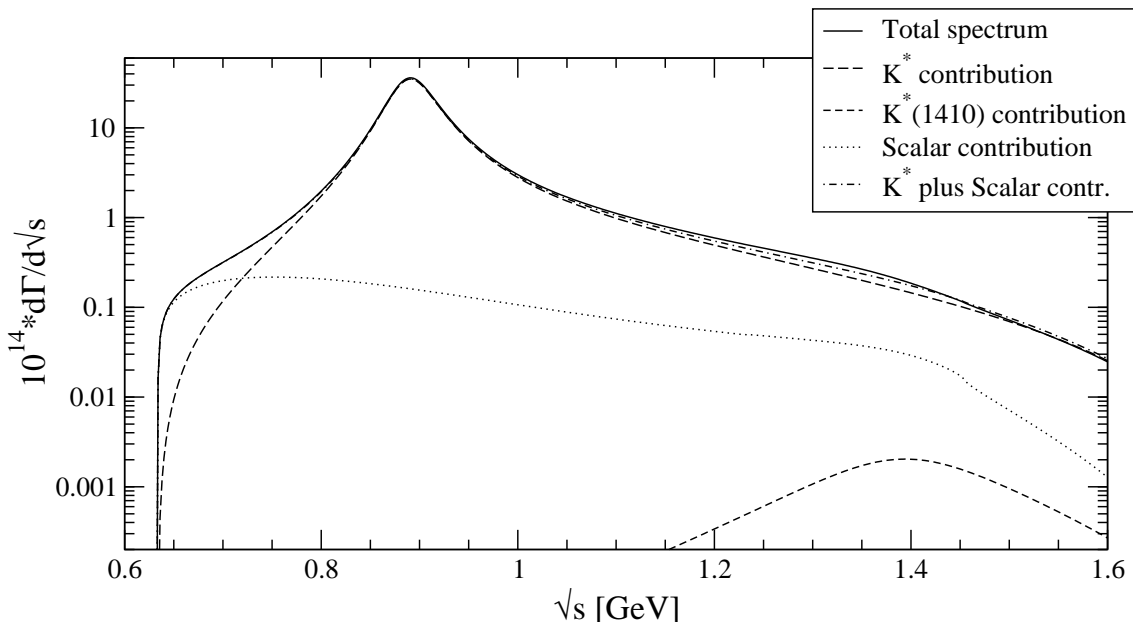


Figure 2: Main result for the differential decay distribution of the decay  $\tau \rightarrow \nu_\tau K\pi$ , together with the individual contributions from the  $K^*$  and  $K^*(1410)$  vector mesons as well as the scalar component residing in the scalar form factor  $F_0^{K\pi}(s)$ .

Our main result for the differential decay distribution is displayed as the solid line in figure 2, together with the individual contributions. Let us discuss our inputs and the individual contributions in more detail. Since the  $K^*$  meson in  $\tau$  decays is the charged one, in contrast to the fit result of eq. (9), for the  $K^*$  mass we have employed the PDG value  $M_{K^{*-}} = 891.66 \pm 0.26$  MeV [37]. However, for  $G_V$  and the barrier factor parameter  $r$ , the fit results of (9) have been used. The resulting contribution of the  $K^*$  meson to the

spectral distribution is shown as the long-dashed line in figure 2. Integrating this part over the phase space and varying the input parameters, one finds

$$B[\tau \rightarrow \nu_\tau K^*(892)] = (1.253_{-0.076}^{+0.062} \pm 0.019)\% = (1.253 \pm 0.078)\%. \quad (17)$$

The first uncertainty represents an estimate of higher order chiral corrections. To this end, in the exponential of eq. (5), we have replaced the factor  $1/F_\pi^2$  by  $1/(F_K F_\pi)$  or by  $1/F_0^2$  with  $F_0 = 87$  MeV being the pion decay constant at the leading order, which should give an idea about unaccounted further chiral corrections. The remaining uncertainty arises from a variation of the fit parameters  $G_V$  and  $r$  of eq. (9) and the value (16) for  $|V_{us}|F_+^{K\pi}(0)$ .

Next, the contribution from the scalar form factor  $F_0^{K\pi}(s)$  in figure 2 is displayed as the dotted line. Its most important contribution arises in the region below the  $K^*$  resonance where the low-lying scalar  $K_0^*(800)$  resonance is active. Integrating over the scalar contribution, we obtain

$$B[\tau \rightarrow \nu_\tau (K\pi)_{\text{S-wave}}] = (3.88 \pm 0.19) \cdot 10^{-4}, \quad (18)$$

where the error dominantly is due to a variation of the form factor shape as discussed in refs. [26, 43, 45]. Since the scalar resonances are not well described by Breit-Wigners and there is also a strong interference between the dynamically generated  $K_0^*(800)$  and the pre-existing (at  $N_C \rightarrow \infty$ )  $K_0^*(1430)$  resonance, we prefer not to resolve the  $K\pi$  S-wave contribution into individual components. (For some remarks on the  $K_0^*(800)$ , also known as the  $\kappa$ , see section 7 of ref. [42].) The sum of the scalar and  $K^*$  contributions in figure 2 is shown as the dashed-dotted line.

The last remaining contribution is the one due to the second vector resonance  $K^*(1410)$ . For its mass and width, we have employed the PDG values  $M_{K^*(1410)} = 1414$  MeV and  $\Gamma_{K^*(1410)} = 232$  MeV [37]. The  $K^*(1410)$  contribution turns out to depend very sensitively on the mixing parameter  $\gamma$  defined in eq. (13), for which an estimate can be obtained on the basis of the total branching fraction  $B[\tau \rightarrow \nu_\tau K\pi]$ . Adding the results of the most recent compilation [6], one finds  $B[\tau \rightarrow \nu_\tau K\pi] = (1.33 \pm 0.05)\%$ . Then adjusting  $\gamma$  such that the experimental total branching fraction, including its uncertainty, is reproduced, we obtain  $\gamma = 0.013 \pm 0.017$ , in agreement with the expectation of the last section, that  $\gamma$  should be small and positive. The contribution of the  $K^*(1410)$  resonance with the central value of  $\gamma$  is shown as the short-dashed line in figure 2. Even though this contribution appears rather small, because of the interference with the leading  $K^*$  resonance, its influence in the energy region above the  $K^*$  resonance is roughly as important as the scalar component. This is also reflected in the corresponding total branching fraction which turns out to be

$$B[\tau \rightarrow \nu_\tau (K^*(892) + K^*(1410))] - B[\tau \rightarrow \nu_\tau K^*(892)] \approx 3.9 \cdot 10^{-4}, \quad (19)$$

however, with rather large uncertainties, but much bigger than the  $K^*(1410)$  branching ratio by itself whose central value reads  $B[\tau \rightarrow \nu_\tau K^*(1410)] = 2.1 \cdot 10^{-6}$ . We notice that



the central result (19) is similar to the scalar branching fraction (18) and compatible to an estimate presented by the ALEPH collaboration [8] which yielded  $B[\tau \rightarrow \nu_\tau K^*(1410)] = (1.5_{-1.0}^{+1.4}) \cdot 10^{-3}$ , but where, however, the scalar component had been neglected.

## 4 Conclusions

Upcoming much improved results on the branching fractions and differential distributions of hadronic  $\tau$  decays from the  $B$ -factories BABAR and BELLE, as well as in the near future from BESIII, necessitate an analogous improvement also on the theoretical side. A step in this direction is taken in the work at hand, where we have presented a description of the decay spectrum of the decay  $\tau \rightarrow \nu_\tau K\pi$  (1) in the framework of R $\chi$ PT [21, 22].

Our approach follows the lines of an analogous description for the pion form factor [23–25], which employs all present knowledge of effective theories, short-distance QCD, the large- $N_C$  expansion as well as analyticity and unitarity, and was successful in describing the experimental data for the pion form factor. Our central result for the  $K\pi$  vector form factor, also including the second vector resonance  $K^*(1410)$  in this channel, has been presented in eq. (13). As a by-product, we have determined the slope and curvature of  $F_+^{K\pi}(s)$ ,

$$\lambda'_+ = 25.6 \cdot 10^{-3}, \quad \lambda''_+ = 1.31 \cdot 10^{-3}, \quad (20)$$

in very good agreement with the recent KLOE results [38]. The required scalar  $K\pi$  form factor  $F_0^{K\pi}(s)$  has been employed from the recent update [26], following the previous analyses [42, 43].

Our central result for the differential decay distribution  $d\Gamma_{K\pi}/d\sqrt{s}$  of eq. (1) is displayed in figure 2, where also the separate contributions originating from the  $K^*$ , the  $K^*(1410)$ , and the scalar component have been shown separately.<sup>3</sup> As can be observed from this figure, the S-wave  $K\pi$  contribution is most prominent in the energy region below the  $K^*$  resonance, and dominated by the dynamically generated  $K_0^*(800)$  resonance (also known as the  $\kappa$ ). Thus, by analysing experimental data in this energy range, valuable information about the still much debated  $K_0^*(800)$  resonance can be obtained. Integrating the scalar component over the phase space, we obtain

$$B[\tau \rightarrow \nu_\tau (K\pi)_{\text{S-wave}}] = (3.88 \pm 0.19) \cdot 10^{-4}, \quad (21)$$

which is already rather precise, so that it will be difficult to improve this accuracy experimentally.

The contribution of the second vector resonance  $K^*(1410)$  dominantly depends on the mixing parameter  $\gamma$ . However, this parameter can be inferred from the total  $\tau \rightarrow \nu_\tau K\pi$

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<sup>3</sup>A Fortran routine to generate this distribution can be obtained on request from the authors.

branching fraction, resulting in the estimate  $\gamma = 0.013 \pm 0.017$ . Even though the  $K^*(1410)$  contribution by itself is small, through interference with the  $K^*$  resonance, in the range around 1.4 GeV it becomes about as important as the scalar resonance in this region, the  $K_0^*(1430)$ . Therefore, with a dedicated experimental analysis in the region above the  $K^*$  peak, it should be possible to also obtain information on the  $K^*(1410)$ .

In summary, in this work we have started a dedicated effort to improve the description of exclusive strangeness-changing hadronic  $\tau$ -decays. We are convinced that our description of the decay spectrum for the decay  $\tau \rightarrow \nu_\tau K\pi$  will prove valuable in the analysis of the high-statistics data on hadronic  $\tau$  decays, acquired at the  $B$ -factories BABAR and BELLE. We plan to return to this subject with other decay channels in the future.

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