

# Soft leptogenesis in the inverse seesaw model

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## J. Garayoa

*Depto. de Física Teórica, and IFIC, Universidad de Valencia-CSIC  
Edificio de Institutos de Paterna, Apt. 22085, 46071 Valencia, Spain  
Email: garayoa@ific.uv.es*

## M. C. Gonzalez-Garcia

*Institució Catalana de Recerca i Estudis Avançats (ICREA),  
Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona,  
Diagonal 647, E-08028 Barcelona, Spain  
and: C.N. Yang Institute for Theoretical Physics  
State University of New York at Stony Brook  
Stony Brook, NY 11794-3840, USA,  
E-mail: concha@insti.physics.sunysb.edu*

## N. Rius

*Depto. de Física Teórica and IFIC, Universidad de Valencia-CSIC  
Edificio de Institutos de Paterna, Apt. 22085, 46071 Valencia, Spain  
Email: nuria@ific.uv.es*

ABSTRACT: We consider leptogenesis induced by soft supersymmetry breaking terms (“soft leptogenesis”), in the context of the inverse seesaw mechanism. In this model there are lepton number ( $L$ ) conserving and  $L$ -violating soft supersymmetry-breaking  $B$ -terms involving the singlet sneutrinos which, together with the – generically small–  $L$ -violating parameter responsible of the neutrino mass, give a small mass splitting between the four singlet sneutrino states of a single generation. In combination with the trilinear soft supersymmetry breaking terms they also provide new CP violating phases needed to generate a lepton asymmetry in the singlet sneutrino decays. We obtain that in this scenario the lepton asymmetry is proportional to the  $L$ -conserving soft supersymmetry-breaking  $B$ -term, and it is not suppressed by the  $L$ -violating parameters. Consequently we find that, as in the standard see-saw case, this mechanism can lead to successful leptogenesis only for relatively small value of the relevant soft bilinear coupling. The right-handed neutrino masses can be sufficiently low to elude the gravitino problem. Also the corresponding Yukawa couplings involving the lightest of the right-handed neutrinos are constrained to be  $\sum |Y_{1k}|^2 \lesssim 10^{-7}$  which generically implies that the neutrino mass spectrum has to be strongly hierarchical.

KEYWORDS: Neutrino Physics, Beyond Standard Model.

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## 1. Introduction

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem [1]. In the standard framework it is usually assumed that the tiny neutrino masses are generated via the (type I) seesaw mechanism [2] and thus the new singlet neutral leptons with heavy (lepton number violating) Majorana masses can produce dynamically a lepton asymmetry through out of equilibrium decay. Eventually, this lepton asymmetry is partially converted into a baryon asymmetry due to fast  $B - L$  violating sphaleron processes.

For a hierarchical spectrum of right-handed neutrinos, successful leptogenesis requires generically quite heavy singlet neutrino masses [3], of order  $M > 2.4(0.4) \times 10^9$  GeV for vanishing (thermal) initial neutrino densities [3, 4], although flavour effects [5] and/or extended scenarios [6] may affect this limit \*. The stability of the hierarchy between this new scale and the electroweak one is natural in low-energy supersymmetry, but in the supersymmetric seesaw scenario there is some conflict between the gravitino bound on the reheat temperature and the thermal production of right-handed neutrinos [9]. This is so because in a high temperature plasma, gravitinos are copiously produced, and their late decay could modify the light nuclei abundances, contrary to observation. This sets an upper bound on the reheat temperature after inflation,  $T_{RH} < 10^{8-10}$  GeV, which may be too low for the right-handed neutrinos to be thermally produced.

Once supersymmetry has been introduced, leptogenesis is induced also in singlet sneutrino decays. If supersymmetry is not broken, the order of magnitude of the asymmetry and

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\*This bound applies when the lepton asymmetry is generated in the decay of the lightest right-handed neutrino. The possibility to evade the bound producing the asymmetry from the second lightest right-handed neutrino has been considered in [7], and flavour effects have been analysed for this case in [8].

the basic mechanism are the same as in the non-supersymmetric case. However, as shown in Refs. [10, 11], supersymmetry-breaking terms can play an important role in the lepton asymmetry generated in sneutrino decays because they induce effects which are essentially different from the neutrino ones. In brief, soft supersymmetry-breaking terms involving the singlet sneutrinos remove the mass degeneracy between the two real sneutrino states of a single neutrino generation, and provide new sources of lepton number and CP violation. As a consequence, the mixing between the two sneutrino states generates a CP asymmetry in the decay, which can be sizable for a certain range of parameters. In particular the asymmetry is large for a right-handed neutrino mass scale relatively low, in the range  $10^5 - 10^8$  GeV, well below the reheat temperature limits, what solves the cosmological gravitino problem. Moreover, contrary to the traditional leptogenesis scenario, where at least two generations of right-handed neutrinos are required to generate a CP asymmetry in neutrino/sneutrino decays, in this new mechanism for leptogenesis the CP asymmetry in sneutrino decays is present even if a single generation is considered. This scenario has been termed “soft leptogenesis”, since the soft terms and not flavour physics provide the necessary mass splitting and CP-violating phase. It has also been studied in the minimal supersymmetric triplet seesaw model [12].

In this paper we want to explore soft leptogenesis in the framework of an alternative mechanism to generate small neutrino masses, namely the inverse seesaw scheme [13]. This scheme is characterized by a *small* lepton number violating Majorana mass term  $\mu$ , while the effective light neutrino mass is  $m_\nu \propto \mu$ . Small values of  $\mu$  are technically natural, given that when  $\mu \rightarrow 0$  a larger symmetry is realized [14]: lepton number is conserved and neutrinos become massless. In the inverse seesaw scheme lepton flavour and CP violation can arise even in the limit where lepton number is strictly conserved and the light neutrinos are massless [15], due to the mixing of the SU(2) doublet neutrinos with new SU(2)  $\times$  U(1) singlet leptons.

As opposite to the standard seesaw case, these singlet leptons do not need to be very heavy [16], and, as a result, lepton flavour and CP violating processes are highly enhanced [15]. In Ref. [17] it was studied the possibility that the baryon asymmetry is generated in this type of models during the electroweak phase transition, in the limit  $\mu = 0$ . A suppression was found due to the experimental constraints on the mixing angles of the neutrinos [18]. Therefore we consider here the supersymmetric version of the model and the soft leptogenesis mechanism, since (i) in this case we expect that a CP asymmetry will be generated in sneutrino decays even with a single-generation and no suppression due to the mixing angles is expected, and (ii) this scheme provides a more natural framework for the relatively low right-handed neutrino mass scale.

The outline of the paper is as follows. Section 2 presents the main features of the inverse seesaw model in the presence of supersymmetry breaking terms. In Sec. 3 we evaluate the lepton asymmetry generated in the decay of the singlet sneutrinos using a field-theoretical approach assuming a hierarchy between the SUSY and  $L$  breaking scales and the mass scale of the singlet sneutrinos. The relevant Boltzmann equations describing the decay, inverse decay and scattering processes involving the singlet sneutrino states are derived in Sec. 4. Finally in Sec. 5 we present our quantitative results. In appendix A

we recompute the asymmetry using a quantum mechanics approach, based on an effective (non hermitic) Hamiltonian [10, 11].

## 2. Inverse Seesaw Mechanism

In this type of models [13], the lepton sector of the Standard Model is extended with two electroweak singlet two-component leptons per generation, i.e.,

$$L^i = \left( \begin{array}{c} \nu_L^i \\ e_L^i \end{array} \right), e_R^i, \nu_R^i, s_L^i \quad (2.1)$$

We assign lepton number  $L = 1$  to the singlets  $s_L^i$  and  $\nu_R^i$ . In the original formulation of the model, the singlets  $s_L^i$  were superstring inspired E(6) singlets, in contrast to the right-handed neutrinos  $\nu_R^i$ , which are in the spinorial representation. More recently this mechanism has also arisen in the context of left-right symmetry [19] and SO(10) unified models [20].

The  $(9 \times 9)$  mass matrix of the neutral lepton sector in the  $\nu_L, \nu_R^c, s_L$  basis is given by

$$\mathcal{M} = \left( \begin{array}{ccc} 0 & m_D & 0 \\ m_D^T & 0 & M^T \\ 0 & M & \mu \end{array} \right) \quad (2.2)$$

where  $m_D, M$  are arbitrary  $3 \times 3$  complex matrices in flavour space and  $\mu$  is complex symmetric. In models where lepton number is spontaneously broken by a vacuum expectation value  $\langle \sigma \rangle$ ,  $\mu = \lambda \langle \sigma \rangle$  [21]. The matrix  $\mathcal{M}$  can be diagonalized by a unitary transformation, leading to nine mass eigenstates  $n_a$ : three of them correspond to the observed light neutrinos, while the other three pairs of two component leptons combine to form three quasi-Dirac leptons.

In this ‘‘inverse seesaw’’ scheme, assuming  $m_D, \mu \ll M$  the effective Majorana mass matrix for the light neutrinos is approximately given by

$$m_\nu = m_D^T M^{T-1} \mu M^{-1} m_D, \quad (2.3)$$

while the three pairs of heavy neutrinos have masses of order  $M$ , and the admixture among singlet and doublet  $SU(2)$  states is suppressed by  $m_D/M$ . Although  $M$  is a large mass scale suppressing the light neutrino masses, in contrast to the Majorana mass ( $\Delta L = 2$ ) of the right-handed neutrinos in the standard seesaw mechanism, it is a Dirac mass ( $\Delta L = 0$ ), and it can be much smaller, since the suppression in Eq. (2.3) is quadratic and moreover light neutrino masses are further suppressed by the small parameter  $\mu$  which characterizes the lepton number violation scale.

Notice that in the  $\mu \rightarrow 0$  limit lepton number conservation is restored. Then, the three light neutrinos are massless Weyl particles and the six heavy neutral leptons combine exactly into three Dirac fermions.

We are going to consider the supersymmetric version of this model (for an analysis of lepton flavour violation in this case see [22]). In this case, the above neutral lepton mass

matrix (2.2) is described by the following superpotential:

$$W = Y_{ij}N_iL_jH + \frac{1}{2}\mu_{ij}S_iS_j + M_{ij}S_iN_j , \quad (2.4)$$

where  $i, j = 1, 2, 3$  are flavour indices,  $H, L_i, N_i, S_i$  are the superfields corresponding to the  $SU(2)$  up-Higgs and lepton doublets, and  $\nu_R^i$  and  $s_L^i$  singlets, respectively, and  $Y_{ij}$  denote the neutrino Yukawa couplings. Thus, after spontaneous electroweak symmetry breaking, the neutrino Dirac masses are given by

$$(m_D)_{ij} = Y_{ij}\langle H \rangle \quad (2.5)$$

The relevant soft supersymmetry breaking terms are the bilinear and trilinear scalar couplings involving the singlet sneutrino fields, that provide new sources of lepton number and CP violation. From now on we consider a simplified one generation of  $(N, S)$  model because a single generation of singlet sneutrinos is sufficient to generate the CP asymmetry. Indeed in the three-generation case, the relevant out of equilibrium decays are usually those of the lightest heavy singlet states while the decay of the heavier (if heavier enough) give no effect. Thus with our simplified single generation model we refer to the lightest of the three heavy singlet sneutrinos which we number as 1. Consequently we label  $M = M_{11}$  and  $\mu = \mu_{11}$ . Also, for simplicity, we will assume proportionality of the soft trilinear terms.

$$-L_{soft} = AY_{1i}\tilde{L}_i\tilde{N}H + \tilde{m}_S^2\tilde{S}\tilde{S}^\dagger + \tilde{m}_N^2\tilde{N}\tilde{N}^\dagger + \tilde{m}_{S_N}^2\tilde{S}\tilde{N}^\dagger + B_S\tilde{S}\tilde{S} + B_{S_N}\tilde{S}\tilde{N} + h.c. \quad (2.6)$$

With our lepton number assignments, the soft SUSY breaking terms which violate L are  $\tilde{m}_{S_N}^2$  and  $B_S$ . The sneutrino interaction Lagrangian is then:

$$\begin{aligned} L = & (Y_{1i}L_iNH + Y_{1i}L_i\tilde{N}h + Y_{1i}\tilde{L}_iNh + h.c.) + \\ & + (Y_{1i}M^*\tilde{L}_i\tilde{S}^\dagger H + AY_{1i}\tilde{L}_i\tilde{N}H + h.c.) + \\ & + (\mu SS + MSN + h.c.) + \\ & + (|\mu|^2 + \tilde{m}_S^2 + |M|^2)\tilde{S}^\dagger\tilde{S} + (\tilde{m}_N^2 + |M|^2)\tilde{N}\tilde{N}^\dagger + (\mu M^* + \tilde{m}_{S_N}^2)\tilde{S}\tilde{N}^\dagger + h.c.) + \\ & + (B_S\tilde{S}\tilde{S} + B_{S_N}\tilde{S}\tilde{N} + h.c.) \end{aligned} \quad (2.7)$$

This Lagrangian has three independent physical CP violating phases:  $\phi_B$  which can be assigned to  $B_{S_N}$ ,  $\phi_A$  which is common to the three terms with  $AY_{1i}$ , and  $\phi_{MY}$  which is common to the three terms with  $MY_{1i}^*$ , and are given by:

$$\begin{aligned} \phi_B &= \arg(B_{S_N}B_S^*\tilde{M}_{S_N}^2) \\ \phi_A &= \arg(AB_S^*M^2\mu^*(\tilde{M}_{S_N}^2)^2) \\ \phi_{MY} &= \arg(\tilde{M}_{S_N}^2M^*\mu) , \end{aligned} \quad (2.8)$$

where we have defined  $\tilde{M}_{S_N}^2 \equiv \mu M^* + \tilde{m}_{S_N}^2$ . These phases provide the CP violation necessary to generate dynamically a lepton asymmetry, even with a single generation of sneutrinos. They can also contribute to lepton electric dipole moments [23].

From the Lagrangian in Eq. (2.7) we obtain the sneutrino mass matrix in the interaction basis ( $\tilde{F}_i \equiv \tilde{N}, \tilde{N}^\dagger, \tilde{S}, \tilde{S}^\dagger$ ):

$$\begin{pmatrix} \tilde{m}_N^2 + |M|^2 & 0 & \widetilde{M}_{SN}^{2*} & B_{SN} \\ 0 & \tilde{m}_N^2 + |M|^2 & B_{SN}^* & \widetilde{M}_{SN}^2 \\ \widetilde{M}_{SN}^2 & B_{SN} & |\mu|^2 + \tilde{m}_S^2 + |M|^2 & 2B_S \\ B_{SN}^* & \widetilde{M}_{SN}^{2*} & 2B_S^* & |\mu|^2 + \tilde{m}_S^2 + |M|^2 \end{pmatrix} \quad (2.9)$$

Notice that in the most general case it is not possible to remove all CP phases from the sneutrino mass matrix. With our choice of basis,  $B_{SN}$  is the only complex parameter,  $B_{SN} = |B_{SN}|e^{i\phi_B}$ .

Although one can easily obtain the analytic expressions for the corresponding mass eigenvalues and eigenvectors, for the general case they are lengthy and we do not give them here. Under the assumption that all the entries are real, i.e.,  $\phi_B = 0$ , we obtain the following mass eigenvalues:

$$\begin{aligned} M_1^2 &= M'^2 + B_S - \frac{1}{2}\sqrt{4(B_{SN} + \widetilde{M}_{SN}^2)^2 + (2B_S - \tilde{m}_N^2 + \tilde{m}_S^2 + |\mu|^2)^2} \\ M_2^2 &= M'^2 - B_S + \frac{1}{2}\sqrt{4(B_{SN} - \widetilde{M}_{SN}^2)^2 + (2B_S + \tilde{m}_N^2 - \tilde{m}_S^2 - |\mu|^2)^2} \\ M_3^2 &= M'^2 - B_S - \frac{1}{2}\sqrt{4(B_{SN} - \widetilde{M}_{SN}^2)^2 + (2B_S + \tilde{m}_N^2 - \tilde{m}_S^2 - |\mu|^2)^2} \\ M_4^2 &= M'^2 + B_S + \frac{1}{2}\sqrt{4(B_{SN} + \widetilde{M}_{SN}^2)^2 + (2B_S - \tilde{m}_N^2 + \tilde{m}_S^2 + |\mu|^2)^2}, \end{aligned} \quad (2.10)$$

where we have defined  $M'^2 \equiv |M|^2 + \tilde{m}_N^2 + \tilde{m}_S^2 + |\mu|^2$ .

Furthermore, if we assume conservative values of the soft breaking terms:

$$\begin{aligned} A &\sim \mathcal{O}(m_{SUSY}) \\ \tilde{m}_N &\sim \tilde{m}_S \sim \tilde{m}_{SN} \sim \mathcal{O}(m_{SUSY}) \\ B_S &\sim \mathcal{O}(m_{SUSY}\mu) \\ B_{SN} &\sim \mathcal{O}(m_{SUSY}M) \end{aligned} \quad (2.11)$$

with both,  $\mu, m_{SUSY} \ll M$ , we see that  $B_S, \tilde{m}_N^2, \tilde{m}_S^2, \tilde{m}_{SN}^2 \ll B_{SN}$  and  $\widetilde{M}_{SN}^2 \sim \mu M^*$ . Neglecting these small soft terms, there is still one physical CP violating phase,

$$\phi = \phi_A - \phi_B = \arg(AB_{SN}^*M). \quad (2.12)$$

In this limit, we choose for simplicity a basis where  $A = |A|e^{i\phi}$  is the only complex parameter. Then we diagonalize to first order in two expansion parameters,

$$\epsilon = \frac{|\mu|}{2|M|}, \quad \tilde{\epsilon} = \frac{|B_{SN}|}{2|M|^2} \sim \mathcal{O}(m_{SUSY}/M) \quad (2.13)$$

To this order the mass eigenvalues are:

$$\begin{aligned} M_1^2 &= M^2 - M\mu - B_{SN} \\ M_2^2 &= M^2 - M\mu + B_{SN} \\ M_3^2 &= M^2 + M\mu - B_{SN} \\ M_4^2 &= M^2 + M\mu + B_{SN} \end{aligned} \quad (2.14)$$

and the eigenvectors:

$$\begin{aligned}
\tilde{N}_1 &= \frac{1}{2} (\tilde{S}^\dagger - \tilde{N}^\dagger) + \frac{1}{2} (\tilde{S} - \tilde{N}) \\
\tilde{N}_2 &= \frac{i}{2} (\tilde{S}^\dagger - \tilde{N}^\dagger) - \frac{i}{2} (\tilde{S} - \tilde{N}) \\
\tilde{N}_3 &= \frac{i}{2} (\tilde{S}^\dagger + \tilde{N}^\dagger) - \frac{i}{2} (\tilde{S} + \tilde{N}) \\
\tilde{N}_4 &= \frac{1}{2} (\tilde{S}^\dagger + \tilde{N}^\dagger) + \frac{1}{2} (\tilde{S} + \tilde{N})
\end{aligned} \tag{2.15}$$

Note that in this limit the mass degeneracy among the four sneutrino states is removed by both the  $L$ -violating mass  $\mu$  and  $L$ -conserving supersymmetry breaking term  $B_{SN}$ . Together with the trilinear  $A$  term they also provide a source of CP violation, and the mixing among the four sneutrino states leads to a CP asymmetry in their decay.

Another interesting limit is to diagonalize the sneutrino mass matrix (2.9) neglecting only the  $B_S$  entry, which may be appropriate if  $\mu \ll m_{SUSY}$  and the order of magnitude of the soft breaking terms is as given by (2.11). In this limit the mass matrix can also be taken real and the mass eigenvalues can be read from Eq. (2.10), just setting  $B_S = 0$ . Now there are two non zero CP violating phases,  $\phi_{YM}$  and  $\phi'_A = \arg(AB_{SN}^* M^2 \mu^* \tilde{M}_{SN}^2)$ . However the combination that is relevant for the CP asymmetry in sneutrino decays is the same as in the previous case,  $\phi = \phi_{YM} + \phi'_A = \arg(AB_{SN}^* M)$ .

As we will see in Sec. 5, the total CP asymmetry in the singlet sneutrino decays turns out to be sizable for very small values of the soft term  $B_{SN} \ll M m_{SUSY}$ . Neglecting the  $B_{SN}$  term in the Lagrangian there are still two CP violating phases,  $\phi_{MY}$  and  $\phi_A$ , but again the sneutrino mass matrix can be taken real, so that the mass eigenvalues are as given by Eq.(2.10) with  $B_{SN} = 0$ . The phase relevant for the CP asymmetry in the singlet sneutrino decays is now  $\phi' = \phi_{YM} + \phi_A = \arg(AB_S^* M \tilde{M}_{SN}^2)$ .

Finally, neglecting supersymmetry breaking effects, the total singlet sneutrino decay width is given by

$$\Gamma = \frac{\sum_i |M| |Y_{1i}|^2}{8\pi} . \tag{2.16}$$

### 3. The CP Asymmetry

In this section we compute the CP asymmetry in the singlet sneutrino decays. As discussed in Ref. [11], when  $\Gamma \gg \Delta M_{ij} \equiv M_i - M_j$ , the four singlet sneutrino states are not well-separated particles. In this case, the result for the asymmetry depends on how the initial state is prepared. In what follows we will assume that the sneutrinos are in a thermal bath with a thermalization time  $\Gamma^{-1}$  shorter than the typical oscillation times,  $\Delta M_{ij}^{-1}$ , therefore coherence is lost and it is appropriate to compute the CP asymmetry in terms of the mass eigenstates Eq.(2.15).

The CP asymmetry produced in the decay of the state  $\tilde{N}_i$  is given by (see sec.4):

$$\epsilon_i = \frac{\sum_f \Gamma(\tilde{N}_i \rightarrow f) - \Gamma(\tilde{N}_i \rightarrow \bar{f})}{\sum_f \Gamma(\tilde{N}_i \rightarrow f) + \Gamma(\tilde{N}_i \rightarrow \bar{f})}, \quad (3.1)$$

where  $f = \tilde{L}_k H, L_k h$ . We also define the fermionic and scalar CP asymmetries in the decay of each  $\tilde{N}_i$  as

$$\epsilon_{s_i} = \frac{\sum_k |\hat{A}_i(\tilde{N}_i \rightarrow \tilde{L}_k H)|^2 - |\hat{A}_i(\tilde{N}_i \rightarrow \tilde{L}_k^\dagger H^\dagger)|^2}{\sum_k |\hat{A}_i(\tilde{N}_i \rightarrow \tilde{L}_k H)|^2 + |\hat{A}_i(\tilde{N}_i \rightarrow \tilde{L}_k^\dagger H^\dagger)|^2} \quad (3.2)$$

$$\epsilon_{f_i} = \frac{\sum_k |\hat{A}_i(\tilde{N}_i \rightarrow L_k h)|^2 - |\hat{A}_i(\tilde{N}_i \rightarrow \bar{L}_k \bar{h})|^2}{\sum_k |\hat{A}_i(\tilde{N}_i \rightarrow L_k h)|^2 + |\hat{A}_i(\tilde{N}_i \rightarrow \bar{L}_k \bar{h})|^2}. \quad (3.3)$$

Notice that  $\epsilon_{s_i}$  and  $\epsilon_{f_i}$  are defined in terms of decay amplitudes, without the phase-space factors which, as we will see, are crucial to obtain a non-vanishing CP asymmetry, much as in the standard seesaw case [10, 11]. The total asymmetry  $\epsilon_i$  generated in the decay of the singlet sneutrino  $\tilde{N}_i$  can then be written as

$$\epsilon_i = \frac{\epsilon_{s_i} c_s + \epsilon_{f_i} c_f}{c_s + c_f}, \quad (3.4)$$

where  $c_s, c_f$  are the phase-space factors of the scalar and fermionic channels, respectively.

Since the scale of lepton number and supersymmetry breaking are  $\mu, m_{SUSY} \ll M$ , there is an enhancement of the CP violation in mixing (wave-function diagrams), so we only include this leading effect and neglect direct CP violation in the decay (vertex diagrams).

We compute the CP asymmetry following the effective field theory approach described in [25], which takes into account the CP violation due to mixing of nearly degenerate states by using resummed propagators for unstable (mass eigenstate) particles. The decay amplitude  $\hat{A}_i^f$  of the unstable external state  $\tilde{N}_i$  defined in Eq. (2.15) into a final state  $f$  is described by a superposition of amplitudes with stable final states:

$$\hat{A}_i(\tilde{N}_i \rightarrow f) = A_i^f - \sum_{j \neq i} A_j^f \frac{i\Pi_{ij}}{M_i^2 - M_j^2 + i\Pi_{jj}}, \quad (3.5)$$

where  $A_i^f$  are the tree level decay amplitudes and  $\Pi_{ij}$  are the absorptive parts of the two-point functions for  $i, j = 1, 2, 3, 4$ . The amplitude for the decay into the conjugate final state is obtained from (3.5) by the replacement  $A_i^f \rightarrow A_i^{f*}$ .



The decay amplitudes can be read off from the interaction Lagrangian (2.7), after performing the change from the current to the mass eigenstate basis:

$$\begin{aligned}
-\mathcal{L} = & \frac{1}{2}\tilde{N}_1[-Y_{1k}L_k h + (Y_{1k}M^* - AY_{1k})\tilde{L}_k H] \\
& + \frac{i}{2}\tilde{N}_2[-Y_{1k}L_k h - (Y_{1k}M^* + AY_{1k})\tilde{L}_k H] \\
& + \frac{i}{2}\tilde{N}_3[Y_{1k}L_k h - (Y_{1k}M^* - AY_{1k})\tilde{L}_k H] \\
& + \frac{1}{2}\tilde{N}_4[Y_{1k}L_k h + (Y_{1k}M^* + AY_{1k})\tilde{L}_k H] + h.c.
\end{aligned} \tag{3.6}$$

Neglecting supersymmetry breaking in vertices, up to an overall normalization we obtain that the decay amplitudes into scalars,  $A_i^{sk} = A(\tilde{N}_i \rightarrow \tilde{L}_k H)$ , verify  $A_2^{sk} = A_3^{sk} = iA_1^{sk} = iA_4^{sk}$ . Correspondingly the decay amplitudes into fermions  $A_i^{fk} = A(\tilde{N}_i \rightarrow L_k h)$ , verify  $A_2^{fk} = -A_3^{fk} = -iA_1^{fk} = iA_4^{fk}$ .

Keeping only the lowest order contribution in the soft terms,

$$\Pi_{ii} = M\Gamma \quad i = 1, \dots, 4 \tag{3.7}$$

$$\Pi_{12} = \Pi_{21} = -\Pi_{34} = -\Pi_{43} = |A|\Gamma \sin \phi \tag{3.8}$$

Altogether we can then write the fermionic and scalar CP asymmetries as:

$$\epsilon_{s_i} = \sum_{j \neq i} \frac{2(M_i^2 - M_j^2)\Pi_{ji} \sum_k \text{Im}(A_i^{sk*} A_j^{sk})}{[(M_i^2 - M_j^2)^2 + \Pi_{jj}^2] \sum_k |A_i^{sk}|^2} \tag{3.9}$$

$$\epsilon_{f_i} = \sum_{j \neq i} \frac{2(M_i^2 - M_j^2)\Pi_{ji} \sum_k \text{Im}(A_i^{fk*} A_j^{fk})}{[(M_i^2 - M_j^2)^2 + \Pi_{jj}^2] \sum_k |A_i^{fk}|^2} \tag{3.10}$$

Inserting the values of the amplitudes  $A_i^f$  and the absorptive parts of the two-point functions (3.7) we obtain the final expression for the scalar and fermionic CP asymmetries at  $T = 0$ :

$$\epsilon_{s_i} = -\epsilon_{f_i} = \bar{\epsilon}_i = -\frac{4|B_{SN} A|\Gamma}{4|B_{SN}|^2 + |M|^2\Gamma^2} \sin \phi \tag{3.11}$$

and the total CP asymmetry generated in the decay of the sneutrino  $\tilde{N}_i$  is then:

$$\epsilon_i(T) = \bar{\epsilon}_i \frac{c_s - c_f}{c_s + c_f} . \tag{3.12}$$

As long as we neglect the zero temperature lepton and slepton masses and small Yukawa couplings, the phase-space factors of the final states are flavour independent. After including finite temperature effects they are given by:

$$c_f = (1 - x_L - x_h)\lambda(1, x_L, x_h) [1 - f_L^{eq}] [1 - f_h^{eq}] \tag{3.13}$$

$$c_s = \lambda(1, x_H, x_{\tilde{L}}) [1 + f_H^{eq}] [1 + f_{\tilde{L}}^{eq}] \tag{3.14}$$

where

$$f_{H,\tilde{L}}^{eq} = \frac{1}{\exp[E_{H,\tilde{L}}/T] - 1} \quad (3.15)$$

$$f_{h,L}^{eq} = \frac{1}{\exp[E_{h,L}/T] + 1} \quad (3.16)$$

are the Bose-Einstein and Fermi-Dirac equilibrium distributions, respectively, and

$$E_{L,h} = \frac{M}{2}(1 + x_{L,h} - x_{h,L}), \quad E_{H,\tilde{L}} = \frac{M}{2}(1 + x_{H,\tilde{L}} - x_{\tilde{L},H}) \quad (3.17)$$

$$\lambda(1, x, y) = \sqrt{(1 + x - y)^2 - 4x}, \quad x_a \equiv \frac{m_a(T)^2}{M^2} \quad (3.18)$$

The thermal masses for the relevant supersymmetric degrees of freedom are [26]:

$$m_H^2(T) = 2m_h^2(T) = \left( \frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 + \frac{3}{4}\lambda_t^2 \right) T^2, \quad (3.19)$$

$$m_{\tilde{L}}^2(T) = 2m_L^2(T) = \left( \frac{3}{8}g_2^2 + \frac{1}{8}g_Y^2 \right) T^2. \quad (3.20)$$

Here  $g_2$  and  $g_Y$  are gauge couplings and  $\lambda_t$  is the top Yukawa, renormalized at the appropriate high-energy scale.

As we will see in the next section, from Eq. (4.46), if the initial distributions of all four states  $\tilde{N}_i$  are equal, their total contribution to the total lepton number can be factorized as:

$$\epsilon(T) \equiv \sum_i \frac{\sum_f \Gamma(\tilde{N}_i \rightarrow f) - \Gamma(\tilde{N}_i \rightarrow \bar{f})}{\sum_f \Gamma(\tilde{N}_i \rightarrow f) + \Gamma(\tilde{N}_i \rightarrow \bar{f})}. \quad (3.21)$$

Several comments are in order. We find that this leptogenesis scenario presents many features analogous to soft leptogenesis in seesaw models [10–12]: (i) The CP asymmetry (3.12) vanishes if  $c_s = c_f$ , because then there is an exact cancellation between the asymmetry in the fermionic and bosonic channels. Finite temperature effects break supersymmetry and make the fermion and boson phase-spaces different  $c_s \neq c_f$ , mainly because of the final state Fermi blocking and Bose stimulation factors. (ii) It also displays a resonance behaviour: the maximum value of the asymmetry is obtained for  $2B_{SN}/M \sim \Gamma$ . (iii) The CP asymmetry is due to the presence of supersymmetry breaking and irremovable CP violating phases, thus it is proportional to  $|B_{SN} A| \sin \phi$ .

As seen from Eq.(3.11) we obtain that the CP asymmetry is not suppressed by the lepton number violating scale  $\mu$ . This may seem counterintuitive. However if  $\mu = 0$  the four sneutrino states are pair degenerate, and we can choose a lepton number conserving mass basis, made of the ( $L = 1$ ) states

$$\begin{aligned} \tilde{N}'_1 &= \frac{1}{\sqrt{2}} (\tilde{S}^\dagger - \tilde{N}) \\ \tilde{N}'_2 &= \frac{1}{\sqrt{2}} (\tilde{S}^\dagger + \tilde{N}) \end{aligned} \quad (3.22)$$

and their hermitian conjugates, with  $L = -1, \tilde{N}_1^\dagger, \tilde{N}_2^\dagger$ . Although there is a CP asymmetry in the decay of these sneutrinos, it is not a lepton number asymmetry (since in the limit  $\mu = 0$  total lepton number is conserved) but just a redistribution of the lepton number stored in heavy sneutrinos and light lepton and slepton  $SU(2)$  doublets. At very low temperatures,  $T \ll M$ , when no heavy sneutrinos remain in the thermal bath, all lepton number is in the light species and obviously if we started in a symmetric Universe with no lepton number asymmetry it will not be generated.

In other words, the total lepton number generated for the case with no lepton number violation is zero but it cannot be recovered by taking the limit  $\mu \rightarrow 0$  of Eq.(3.11) because in the derivation of Eq.(3.11) it is assumed implicitly that the four singlet sneutrino states are non-degenerate and consequently it is only valid if  $\mu$  (or some of the other  $L$  violating parameters) is non zero.

In appendix A we recompute the asymmetry using a quantum mechanics approach, based on an effective (non hermitic) Hamiltonian [10,11], and we get the same parametric dependence of the result, which differs only by numerical factors. Both expressions agree in the limit  $\Gamma \ll |B_{SN}/M|$ .

As discussed at the end of the previous section there may be other interesting ranges of parameters beyond Eq.(2.13). Thus in order to verify the stability of the results to departures from this expansion we have redone the computation of the CP asymmetry keeping all the entries in the sneutrino mass matrix, and just assuming that it is real. The expressions are too lengthy to be given here but let us simply mention that we have found that, in the general case, the CP asymmetries generated in the decay of each of the four singlet sneutrino states are not equal but they can always be written as:

$$\epsilon_i = -\frac{4|B_{SN}A|\Gamma}{4|B_{SN}|^2 + |M|^2\Gamma^2} \sin \phi + f_i(B_S, \mu, \tilde{M}_{SN}^2) \quad (3.23)$$

where the functions  $f_i$  verify that for  $|\mu|^2 \ll |B_{SN}|$  and to any order in  $|B_S|$ :

$$\sum_i f_i(B_S, \mu, \tilde{M}_{SN}^2) \propto |B_{SN}|. \quad (3.24)$$

In the limiting case  $|B_{SN}| \ll |B_S|, m_{SUSY}^2, |\mu^2|$  the dominant term in the CP asymmetry at leading order in  $|B_S| \sim |M|\Gamma \ll |\tilde{M}_{SN}^2|$  is

$$\sum_i \epsilon_i = \frac{8|B_S A|\Gamma}{(4|B_S|^2 + |M|^2\Gamma^2)^2} \frac{|\mu^2| + \tilde{m}_S^2 - \tilde{m}_N^2}{|\tilde{M}_{SN}^2|} (4|B_S|^2 - |M|^2\Gamma^2) \sin \phi'. \quad (3.25)$$

It also exhibits a resonant behaviour, described now by  $|B_S|\Gamma/(4|B_S|^2 + |M|^2\Gamma^2)^2$ , however the total CP asymmetry in this limit is further suppressed by a factor of order  $(|\mu^2|, m_{SUSY}^2)/|\tilde{M}_{SN}^2|$ .

Finally, let's comment that in the previous derivation we have neglected thermal corrections to the CP asymmetry from the loops, i.e., we have computed the imaginary part of the one-loop graphs using Cutkosky cutting rules at  $T = 0$ . These corrections are the same for scalar and fermion decay channels, since only bosonic loops contribute to the wave-function diagrams in both cases, so they are not expected to introduce significant changes to our results.

## 4. Boltzmann Equations

We next write the relevant Boltzmann equations describing the decay, inverse decay and scattering processes involving the sneutrino states.

As mentioned above we assume that the sneutrinos are in a thermal bath with a thermalization time shorter than the oscillation time. Under this assumption the initial states can be taken as being the mass eigenstates in Eq. (2.15) and we write the corresponding equations for those states and the scalar and fermion lepton numbers. The  $CP$  fermionic and scalar asymmetries for each  $\tilde{N}_i$  defined at  $T = 0$  are those given in Eq. (3.11).

Let's notice that the  $CP$  asymmetries as defined in Eq. (3.11) verify  $\epsilon_{s_i} = -\epsilon_{f_i} \equiv \bar{\epsilon}_i$ . However in order to better trace the evolution of the scalar and fermion lepton numbers separately we will keep them as two different quantities in writing the equations.

Using  $CPT$  invariance and the above definitions for the  $CP$  asymmetries and including all the multiplicative factors we have:

$$\begin{aligned}
\sum_k |\hat{A}(\tilde{N}_i \rightarrow \tilde{L}_k H)|^2 &= \sum_k |\hat{A}(\tilde{L}_k^\dagger H^\dagger \rightarrow \tilde{N}_i)|^2 \simeq \frac{1 + \epsilon_{s_i}}{2} \sum_k |A_i^{s_k}|^2, \\
\sum_k |\hat{A}(\tilde{N}_i \rightarrow \tilde{L}_k^\dagger H^\dagger)|^2 &= \sum_k |\hat{A}(\tilde{L}_k H \rightarrow \tilde{N}_i)|^2 \simeq \frac{1 - \epsilon_{s_i}}{2} \sum_k |A_i^{s_k}|^2, \\
\sum_k |\hat{A}(\tilde{N}_i \rightarrow L_k h)|^2 &= \sum_k |\hat{A}(\bar{L}_k \bar{h} \rightarrow \tilde{N}_i)|^2 \simeq \frac{1 + \epsilon_{f_i}}{2} \sum_k |A_i^{f_k}|^2, \\
\sum_k |\hat{A}(\tilde{N}_i \rightarrow \bar{L}_k \bar{h})|^2 &= \sum_k |\hat{A}(L_k h \rightarrow \tilde{N}_i)|^2 \simeq \frac{1 - \epsilon_{f_i}}{2} \sum_k |A_i^{f_k}|^2.
\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
\sum_k |A_i^{s_k}|^2 &= \sum_k \frac{|Y_{1k} M|^2}{4}, \\
\sum_k |A_i^{f_k}|^2 &= \sum_k \frac{|Y_{1k} M|^2}{4} \frac{M_i^2}{M^2}.
\end{aligned} \tag{4.2}$$

The Boltzmann equations describe the evolution of the number density of particles in the plasma:

$$\begin{aligned}
\frac{dn_X}{dt} + 3Hn_X &= \sum_{j,l,m} \Lambda_{lm\dots}^{Xj\dots} [f_l f_m \dots (1 \pm f_X)(1 \pm f_j) \dots W(lm\dots \rightarrow Xj\dots) - \\
&\quad - f_X f_j \dots (1 \pm f_l)(1 \pm f_m) \dots W(Xj\dots \rightarrow lm\dots)]
\end{aligned}$$

where,

$$\Lambda_{lm\dots}^{Xj\dots} = \int \frac{d^3 p_X}{(2\pi)^3 2E_X} \int \frac{d^3 p_j}{(2\pi)^3 2E_j} \dots \int \frac{d^3 p_l}{(2\pi)^3 2E_l} \int \frac{d^3 p_m}{(2\pi)^3 2E_m} \dots,$$

and  $W(lm\dots \rightarrow Xj\dots)$  is the squared transition amplitude summed over initial and final spins. In what follows we will use the notation of Ref. [24] and we will assume that the Higgs and higgsino fields are in thermal equilibrium with distributions given in Eqs. (3.15)

and (3.16) respectively, while the leptons and sleptons are in kinetic equilibrium and we introduce a chemical potential for the leptons,  $\mu_f$ , and sleptons,  $\mu_s$ :

$$\begin{aligned}
f_L &= \frac{1}{\exp[(E_L + \mu_f)/T] + 1}, \\
f_{\bar{L}} &= \frac{1}{\exp[(E_{\bar{L}} - \mu_f)/T] + 1}, \\
f_{\tilde{L}} &= \frac{1}{\exp[(E_{\tilde{L}} + \mu_s)/T] - 1}, \\
f_{\tilde{L}^\dagger} &= \frac{1}{\exp[(E_{\tilde{L}^\dagger} - \mu_s)/T] - 1}.
\end{aligned} \tag{4.3}$$

Furthermore in order to eliminate the dependence in the expansion of the Universe we write the equations in terms of the abundances  $Y_X$ , where  $Y_X = n_X/s$ .

We are interested in the evolution of sneutrinos  $Y_{\tilde{N}_i}$ , and the fermionic  $Y_{\mathcal{L}}$  and scalar  $Y_{\tilde{\mathcal{L}}}$  lepton number, defined as  $Y_{\mathcal{L}} = (Y_L - Y_{\bar{L}})/2$ ,  $Y_{\tilde{\mathcal{L}}} = (Y_{\tilde{L}} - Y_{\tilde{L}^\dagger})/2$ . The number density of sneutrinos is regulated through its decays and inverse decays, defined in the *D – terms*, while to compute the evolution of the fermionic and scalar lepton number we also need to consider the scatterings where leptons and sleptons are involved. The scattering terms are defined in the *S – terms*.

$$\frac{dY_{\tilde{N}_i}}{dt} = -D_i - \bar{D}_i - \tilde{D}_i - \tilde{D}_i^\dagger \tag{4.4}$$

$$\frac{dY_{\mathcal{L}}}{dt} = \sum_i (D_i - \bar{D}_i) - 2S - S_{L\tilde{L}^\dagger} + \bar{S}_{L\tilde{L}^\dagger} - S_{L\tilde{L}} + \bar{S}_{L\tilde{L}} \tag{4.5}$$

$$\frac{dY_{\tilde{\mathcal{L}}}}{dt} = \sum_i (\tilde{D}_i - \tilde{D}_i^\dagger) - 2\tilde{S} - S_{L\tilde{L}^\dagger} + \bar{S}_{L\tilde{L}^\dagger} + S_{L\tilde{L}} - \bar{S}_{L\tilde{L}} \tag{4.6}$$

where

$$\begin{aligned}
sD_i &= \Lambda_{\tilde{N}_i}^{12} \left[ f_{\tilde{N}_i} (1 - f_L) (1 - f_h^{eq}) \sum_k |\hat{A}(\tilde{N}_i \rightarrow L_k h)|^2 - \right. \\
&\quad \left. - f_L f_h^{eq} (1 + f_{\tilde{N}_i}) \sum_k |\hat{A}(L_k h \rightarrow \tilde{N}_i)|^2 \right],
\end{aligned} \tag{4.7}$$

$$\begin{aligned}
s\bar{D}_i &= \Lambda_{\tilde{N}_i}^{12} \left[ f_{\tilde{N}_i} (1 - f_{\bar{L}}) (1 - f_h^{eq}) \sum_k |\hat{A}(\tilde{N}_i \rightarrow \bar{L}_k \bar{h})|^2 - \right. \\
&\quad \left. - f_{\bar{L}} f_h^{eq} (1 + f_{\tilde{N}_i}) \sum_k |\hat{A}(\bar{L}_k \bar{h} \rightarrow \tilde{N}_i)|^2 \right],
\end{aligned} \tag{4.8}$$

$$\begin{aligned}
s\tilde{D}_i &= \Lambda_{\tilde{N}_i}^{12} \left[ f_{\tilde{N}_i} (1 + f_{\tilde{L}}) (1 + f_H^{eq}) \sum_k |\hat{A}(\tilde{N}_i \rightarrow \tilde{L}_k H)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}} f_H^{eq} (1 + f_{\tilde{N}_i}) \sum_k |\hat{A}(\tilde{L}_k H \rightarrow \tilde{N}_i)|^2 \right],
\end{aligned} \tag{4.9}$$

$$\begin{aligned}
s\tilde{D}_i^\dagger &= \Lambda_{\tilde{N}_i}^{12} \left[ f_{\tilde{N}_i} (1 + f_{\tilde{L}^\dagger}) (1 + f_H^{eq}) \sum_k |\hat{A}(\tilde{N}_i \rightarrow \tilde{L}_k^\dagger H^\dagger)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}^\dagger} f_H^{eq} (1 + f_{\tilde{N}_i}) \sum_k |\hat{A}(\tilde{L}_k^\dagger H^\dagger \rightarrow \tilde{N}_i)|^2 \right], \tag{4.10}
\end{aligned}$$

and

$$\begin{aligned}
sS &= \Lambda_{34}^{12} \left[ f_L f_h^{eq} (1 - f_{\tilde{L}}) (1 - f_h^{eq}) \sum_{k,k'} |M_{sub}(L_k h \rightarrow \bar{L}_{k'} \bar{h})|^2 - \right. \\
&\quad \left. - f_{\tilde{L}} f_h^{eq} (1 - f_L) (1 - f_h^{eq}) \sum_{k,k'} |M_{sub}(\bar{L}_k \bar{h} \rightarrow L_{k'} h)|^2 \right], \tag{4.11}
\end{aligned}$$

$$\begin{aligned}
s\tilde{S} &= \Lambda_{34}^{12} \left[ f_{\tilde{L}} f_H^{eq} (1 + f_{\tilde{L}^\dagger}) (1 + f_H^{eq}) \sum_{k,k'} |M_{sub}(\tilde{L}_k H \rightarrow \tilde{L}_{k'}^\dagger H^\dagger)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}^\dagger} f_H^{eq} (1 + f_{\tilde{L}}) (1 + f_H^{eq}) \sum_{k,k'} |M_{sub}(\tilde{L}_k^\dagger H^\dagger \rightarrow \tilde{L}_{k'} H)|^2 \right], \tag{4.12}
\end{aligned}$$

$$\begin{aligned}
sS_{L\tilde{L}^\dagger} &= \Lambda_{34}^{12} \left[ f_L f_h^{eq} (1 + f_{\tilde{L}^\dagger}) (1 + f_{H^\dagger}) \sum_{k,k'} |M_{sub}(L_k h \rightarrow \tilde{L}_{k'}^\dagger H^\dagger)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}^\dagger} f_H^{eq} (1 - f_L) (1 - f_h^{eq}) \sum_{k,k'} |M_{sub}(\tilde{L}_k^\dagger H^\dagger \rightarrow L_{k'} h)|^2 \right], \tag{4.13}
\end{aligned}$$

$$\begin{aligned}
s\bar{S}_{L\tilde{L}^\dagger} &= \Lambda_{34}^{12} \left[ f_{\tilde{L}} f_h^{eq} (1 + f_{\tilde{L}}) (1 + f_H^{eq}) \sum_{k,k'} |M_{sub}(\bar{L}_k \bar{h} \rightarrow \tilde{L}_{k'} H)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}} f_H^{eq} (1 - f_{\tilde{L}}) (1 - f_h^{eq}) \sum_{k,k'} |M_{sub}(\tilde{L}_k H \rightarrow \bar{L}_{k'} \bar{h})|^2 \right], \tag{4.14}
\end{aligned}$$

$$\begin{aligned}
sS_{L\tilde{L}} &= \Lambda_{34}^{12} \left[ f_L f_h^{eq} (1 + f_{\tilde{L}}) (1 + f_H^{eq}) \sum_{k,k'} |M_{sub}(L_k h \rightarrow \tilde{L}_{k'} H)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}} f_H^{eq} (1 - f_L) (1 - f_h^{eq}) \sum_{k,k'} |M_{sub}(\tilde{L}_k H \rightarrow L_{k'} h)|^2 \right], \tag{4.15}
\end{aligned}$$

$$\begin{aligned}
s\bar{S}_{L\tilde{L}} &= \Lambda_{34}^{12} \left[ f_{\tilde{L}} f_h^{eq} (1 + f_{\tilde{L}^\dagger}) (1 + f_H^{eq}) \sum_{k,k'} |M_{sub}(\bar{L}_k \bar{h} \rightarrow \tilde{L}_{k'}^\dagger H^\dagger)|^2 - \right. \\
&\quad \left. - f_{\tilde{L}^\dagger} f_H^{eq} (1 - f_{\tilde{L}}) (1 - f_h^{eq}) \sum_{k,k'} |M_{sub}(\tilde{L}_k^\dagger H^\dagger \rightarrow \bar{L}_{k'} \bar{h})|^2 \right]. \tag{4.16}
\end{aligned}$$

The  $S$ -terms are defined in terms of subtracted amplitudes, since the on-shell contribution is already taken into account through the decays and inverse decays in the  $D$ -terms. So for example:

$$|M_{sub}(L_k h \rightarrow \bar{L}_{k'} \bar{h})|^2 = |M(L_k h \rightarrow \bar{L}_{k'} \bar{h})|^2 - |M_{os}(L_k h \rightarrow \bar{L}_{k'} \bar{h})|^2, \quad (4.17)$$

where,

$$|M_{os}(L_k h \rightarrow \bar{L}_{k'} \bar{h})|^2 = \left| \hat{A}(L_k h \rightarrow \tilde{N}_i) \right|^2 \frac{\pi \delta(s - m_{\tilde{N}_i}^2)}{m_{\tilde{N}_i} \Gamma_{\tilde{N}_i}} \left| \hat{A}(\tilde{N}_i \rightarrow \bar{L}_{k'} \bar{h}) \right|^2. \quad (4.18)$$

In writing Eqs.(4.4)–(4.6) we have not included the  $\Delta L = 1$  processes. They do not contribute to the out of equilibrium condition. However they can lead to a dilution of the generated  $\mathcal{L}_{total}$ . Therefore they are relevant in the exact computation of the  $\kappa$  factor defined in Eq. (5.1).

In order to compute the  $D$ -terms, we use the following relation between the equilibrium densities:

$$\begin{aligned} f_{\tilde{L}} f_h^{eq} (1 + f_{\tilde{N}_i}^{eq}) &= f_{\tilde{N}_i}^{eq} (1 - f_{\tilde{L}}) (1 - f_h^{eq}) e^{\mp \mu_f/T} \simeq \\ &\simeq f_{\tilde{N}_i}^{eq} (1 - f_L^{eq}) (1 - f_h^{eq}) (1 \mp Y_{\mathcal{L}}), \end{aligned} \quad (4.19)$$

$$\begin{aligned} f_{\tilde{L}^{(\dagger)}} f_H^{eq} (1 + f_{\tilde{N}_i}^{eq}) &= f_{\tilde{N}_i}^{eq} (1 + f_{\tilde{L}^{(\dagger)}}) (1 + f_H^{eq}) e^{\mp \mu_s/T} \simeq \\ &\simeq f_{\tilde{N}_i}^{eq} (1 + f_{\tilde{L}}^{eq}) (1 + f_H^{eq}) (1 \mp Y_{\tilde{\mathcal{L}}}), \end{aligned} \quad (4.20)$$

with

$$f_{\tilde{N}_i}^{eq} = \frac{1}{\exp[E_{\tilde{N}_i}/T] - 1}. \quad (4.21)$$

One gets

$$\begin{aligned} D_i + \bar{D}_i &= \frac{1}{s} \Lambda_{\tilde{N}_i}^{12} \left[ f_{\tilde{N}_i} (1 - f_L^{eq}) (1 - f_h^{eq}) \left( \frac{1 + \epsilon_{f_i}}{2} + \frac{1 - \epsilon_{f_i}}{2} \right) \sum_k |A_i^{f_k}|^2 \right. \\ &\quad \left. - f_{\tilde{N}_i}^{eq} \frac{1 + f_{\tilde{N}_i}}{1 + f_{\tilde{N}_i}^{eq}} (1 - f_L^{eq}) (1 - f_h^{eq}) \left[ (1 - Y_{\mathcal{L}}) \frac{1 - \epsilon_{f_i}}{2} + (1 + Y_{\mathcal{L}}) \frac{1 + \epsilon_{f_i}}{2} \right] \sum_k |A_i^{f_k}|^2 \right] = \\ &= \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^f \rangle - Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\mathcal{L}} \epsilon_{f_i} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \end{aligned}$$

where in order to write the equations in the closest to the standard notation we have

defined the following average widths:

$$n_{\tilde{N}_i}^{eq} \langle \Gamma_{\tilde{N}_i}^f \rangle = \Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i}^{eq} (1 - f_L^{eq})(1 - f_h^{eq}) \sum_k |A_i^{fk}|^2 \quad (4.22)$$

$$n_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^f \rangle = \Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i} (1 - f_L^{eq})(1 - f_h^{eq}) \sum_k |A_i^{fk}|^2 \quad (4.23)$$

$$n_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle = \Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i}^{eq} \frac{(1 + f_{\tilde{N}_i})}{(1 + f_{\tilde{N}_i}^{eq})} (1 - f_L^{eq})(1 - f_h^{eq}) \sum_k |A_i^{fk}|^2 \quad (4.24)$$

$$n_{\tilde{N}_i}^{eq} \langle \Gamma_{\tilde{N}_i}^s \rangle = \Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i}^{eq} (1 + f_{\tilde{L}}^{eq})(1 + f_H^{eq}) \sum_k |A_i^{sk}|^2 \quad (4.25)$$

$$n_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^s \rangle = \Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i} (1 + f_L^{eq})(1 + f_H^{eq}) \sum_k |A_i^{sk}|^2 \quad (4.26)$$

$$n_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle = \Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i}^{eq} \frac{(1 + f_{\tilde{N}_i})}{(1 + f_{\tilde{N}_i}^{eq})} (1 + f_{\tilde{L}}^{eq})(1 + f_H^{eq}) \sum_k |A_i^{sk}|^2 \quad (4.27)$$

which verify that in equilibrium

$$\langle \Gamma_{\tilde{N}_i}^{f(s)} \rangle = \langle \Gamma_{\tilde{N}_i}^{f(s)} \rangle = \langle \tilde{\Gamma}_{\tilde{N}_i}^{f(s)} \rangle \quad (4.28)$$

Equivalently for the rest of terms :

$$D_i + \bar{D}_i = \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^f \rangle - Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\mathcal{L}} \epsilon_{f_i} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \quad (4.29)$$

$$D_i - \bar{D}_i = \epsilon_{f_i} \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^f \rangle + Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\mathcal{L}} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \quad (4.30)$$

$$\tilde{D}_i + \tilde{D}_i^\dagger = \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^s \rangle - Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\tilde{\mathcal{L}}} \epsilon_{s_i} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \quad (4.31)$$

$$\tilde{D}_i - \tilde{D}_i^\dagger = \epsilon_{s_i} \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^s \rangle + Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\tilde{\mathcal{L}}} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \quad (4.32)$$

Concerning *S - terms*, in order to evaluate, for example, the on-shell contribution  $|M_{os}(L_k h \rightarrow \bar{L}_{k'} \bar{h})|^2$  we use the following relation between the equilibrium densities

$$(1 - f_L^{eq})(1 - f_h^{eq}) = f_{\tilde{N}_i}^{eq} e^{E_N/T} [(1 - f_L^{eq})(1 - f_h^{eq}) - f_L^{eq} f_h^{eq}] , \quad (4.33)$$

and the identity,

$$1 = \int d^4 p_N \delta^4(p_{\tilde{N}_i} - p_L - p_h) . \quad (4.34)$$

They allow us to write the on-shell contribution to the scattering terms at the required order in  $\epsilon$  as:

$$\begin{aligned} & \Lambda_{12}^{34} f_L f_h^{eq} (1 - f_{\tilde{L}})(1 - f_h^{eq}) \sum_k |\hat{A}(L_k h \rightarrow \tilde{N}_i)|^2 \frac{\pi \delta(s - m_{\tilde{N}_i})}{m_{\tilde{N}_i} \Gamma_{\tilde{N}_i}^{th}} \sum_{k'} |\hat{A}(\tilde{N}_i \rightarrow \bar{L}_{k'} \bar{h})|^2 = \\ & = \int \frac{d^3 p_L}{(2\pi)^3 2E_L} \frac{d^3 p_h}{(2\pi)^3 2E_h} (2\pi)^4 \delta^4(p_{\tilde{N}_i} - p_L - p_h) f_L^{av} f_h^{eq} \left( \frac{1 - \epsilon_{f_i}}{2} \right)^2 \sum_k |A_i^{fk}|^2 \times \\ & \quad \int \frac{d^4 p_{\tilde{N}_i}}{(2\pi)^4} \frac{2\pi \delta(s - m_{\tilde{N}_i})}{2m_{\tilde{N}_i} \Gamma_{\tilde{N}_i}^{th}} \int \frac{d^3 p_L}{(2\pi)^3 2E_L} \frac{d^3 p_h}{(2\pi)^3 2E_h} (2\pi)^4 \delta^4(p_{\tilde{N}_i} - p_L - p_h) \times \\ & \quad f_{\tilde{N}_i}^{eq} e^{E_N/T} [(1 - f_L^{eq})(1 - f_h^{eq}) - f_L^{eq} f_h^{eq}] \sum_{k'} |A_i^{fk'}|^2 \end{aligned} \quad (4.35)$$



Now we define the thermal width into fermion and scalars as:

$$\begin{aligned}
\Gamma_{\tilde{N}_i}^{th,f} &= \frac{1}{2m_{\tilde{N}_i}} \int \frac{d^3 p_L}{(2\pi)^3 2E_L} \frac{d^3 p_h}{(2\pi)^3 2E_L} (2\pi)^4 \delta^4(p_{\tilde{N}_i} - p_L - p_h) \\
&\quad [(1 - f_L^{eq})(1 - f_h^{eq}) - f_L^{eq} f_h^{eq}] \sum_k |A_i^{fk}|^2, \\
\Gamma_{\tilde{N}_i}^{th,s} &= \frac{1}{2m_{\tilde{N}_i}} \int \frac{d^3 p_{\tilde{L}}}{(2\pi)^3 2E_{\tilde{L}}} \frac{d^3 p_H}{(2\pi)^3 2E_H} (2\pi)^4 \delta^4(p_{\tilde{N}_i} - p_{\tilde{L}} - p_H) \\
&\quad [(1 + f_{\tilde{L}}^{eq})(1 + f_H^{eq}) - f_{\tilde{L}}^{eq} f_H^{eq}] \sum_k |A_i^{sk}|^2.
\end{aligned} \tag{4.36}$$

so that the thermal width of sneutrinos is  $\Gamma_{\tilde{N}_i}^{th} = \Gamma_{\tilde{N}_i}^{th,f} + \Gamma_{\tilde{N}_i}^{th,s}$ .

Using that  $\int \frac{d^4 p_{\tilde{N}_i}}{(2\pi)^4} 2\pi \delta(p_{\tilde{N}_i}^2 - m_{\tilde{N}_i}^2) = \int \frac{d^3 p_{\tilde{N}_i}}{(2\pi)^3 2E_{\tilde{N}_i}}$  we can write Eq. (4.35) as:

$$\Lambda_{\tilde{N}_i}^{12} f_{\tilde{N}_i}^{eq} (1 - f_L^{eq})(1 - f_h^{eq}) \left( \frac{1 - \epsilon_{f_i}}{2} \right)^2 \sum_k |A_i^{fk}|^2 \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}} = n_{\tilde{N}_i}^{eq} \frac{(1 - \epsilon_{f_i})^2}{4} \langle \Gamma_{\tilde{N}_i}^f \rangle \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}}.$$

Altogether we find that

$$S = \sum_{k,k'} \langle \sigma(L_k h \rightarrow \bar{L}_{k'} \bar{h}) - \sigma(\bar{L}_{k'} \bar{h} \rightarrow L_k h) \rangle + Y_{\tilde{N}_i}^{eq} \epsilon_{f_i} \langle \Gamma_{\tilde{N}_i}^f \rangle \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}}. \tag{4.37}$$

The rest of on-shell contributions can be evaluated similarly:

$$\tilde{S} = \sum_{k,k'} \langle \sigma(\tilde{L}_k H \rightarrow \tilde{L}_{k'}^\dagger H^\dagger) - \sigma(\tilde{L}_{k'}^\dagger H^\dagger \rightarrow \tilde{L}_k H) \rangle + Y_{\tilde{N}_i}^{eq} \epsilon_{s_i} \langle \Gamma_{\tilde{N}_i}^s \rangle \frac{\Gamma_{\tilde{N}_i}^{th,s}}{\Gamma_{\tilde{N}_i}^{th}} \tag{4.38}$$

$$\begin{aligned}
S_{L\tilde{L}^\dagger} &= \sum_{k,k'} \langle \sigma(L_k h \rightarrow \tilde{L}_{k'}^\dagger H^\dagger) - \sigma(\tilde{L}_{k'}^\dagger H^\dagger \rightarrow L_k h) \rangle + Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} + \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^f \rangle \frac{\Gamma_{\tilde{N}_i}^{th,s}}{\Gamma_{\tilde{N}_i}^{th}} = \\
&= \sum_{k,k'} \langle \sigma(L_k h \rightarrow \tilde{L}_{k'}^\dagger H^\dagger) - \sigma(\tilde{L}_{k'}^\dagger H^\dagger \rightarrow L_k h) \rangle + Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} + \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^s \rangle \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}}
\end{aligned} \tag{4.39}$$

$$\begin{aligned}
\tilde{S}_{L\tilde{L}^\dagger} &= \sum_{k,k'} \langle \sigma(\bar{L}_k \bar{h} \rightarrow \tilde{L}_{k'} H) - \sigma(\tilde{L}_{k'} H \rightarrow \bar{L}_k \bar{h}) \rangle - Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} + \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^f \rangle \frac{\Gamma_{\tilde{N}_i}^{th,s}}{\Gamma_{\tilde{N}_i}^{th}} = \\
&= \sum_{k,k'} \langle \sigma(\bar{L}_k \bar{h} \rightarrow \tilde{L}_{k'} H) - \sigma(\tilde{L}_{k'} H \rightarrow \bar{L}_k \bar{h}) \rangle - Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} + \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^s \rangle \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}}
\end{aligned} \tag{4.40}$$

$$\begin{aligned}
S_{L\tilde{L}} &= \sum_{k,k'} \langle \sigma(L_k h \rightarrow \tilde{L}_{k'} H) - \sigma(\tilde{L}_{k'} H \rightarrow L_k h) \rangle + Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} - \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^f \rangle \frac{\Gamma_{\tilde{N}_i}^{th,s}}{\Gamma_{\tilde{N}_i}^{th}} = \\
&= \sum_{k,k'} \langle \sigma(L_k h \rightarrow \tilde{L}_{k'} H) - \sigma(\tilde{L}_{k'} H \rightarrow L_k h) \rangle - Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} - \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^s \rangle \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}}
\end{aligned} \tag{4.41}$$

$$\begin{aligned}
\bar{S}_{L\tilde{L}} &= \sum_{k,k'} \langle \sigma(\bar{L}_k \bar{h} \rightarrow \tilde{L}_{k'}^\dagger H^\dagger) - \sigma(\tilde{L}_{k'}^\dagger H^\dagger \rightarrow \bar{L}_k \bar{h}) \rangle - Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} - \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^f \rangle \frac{\Gamma_{\tilde{N}_i}^{th,s}}{\Gamma_{\tilde{N}_i}^{th}} = \\
&= \sum_{k,k'} \langle \sigma(\bar{L}_k \bar{h} \rightarrow \tilde{L}_{k'}^\dagger H^\dagger) - \sigma(\tilde{L}_{k'}^\dagger H^\dagger \rightarrow \bar{L}_k \bar{h}) \rangle - Y_{\tilde{N}_i}^{eq} \frac{\epsilon_{f_i} - \epsilon_{s_i}}{2} \langle \Gamma_{\tilde{N}_i}^{s,eq} \rangle \frac{\Gamma_{\tilde{N}_i}^{th,f}}{\Gamma_{\tilde{N}_i}^{th}} \quad (4.42)
\end{aligned}$$

Altogether we can write the Boltzmann equations for the sneutrinos and leptonic numbers <sup>†</sup>:

$$\begin{aligned}
\frac{dY_{\tilde{N}_i}}{dt} &= -Y_{\tilde{N}_i} \left( \langle \Gamma_{\tilde{N}_i}^f \rangle + \langle \Gamma_{\tilde{N}_i}^s \rangle \right) + Y_{\tilde{N}_i}^{eq} \left( \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle + \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \right) \\
&\quad - Y_{\tilde{N}_i}^{eq} \left( Y_{\mathcal{L}} \epsilon_{f_i} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle + Y_{\tilde{\mathcal{L}}} \epsilon_{s_i} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \right) \quad (4.43)
\end{aligned}$$

$$\begin{aligned}
\frac{dY_{\mathcal{L}}}{dt} &= \sum_i \left[ \epsilon_{f_i} \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^f \rangle + Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle - 2Y_{\tilde{N}_i}^{eq} \langle \Gamma_{\tilde{N}_i}^{f,eq} \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\mathcal{L}} \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \right] \\
&\quad - \langle 2\sigma(Lh \rightarrow \bar{L}\bar{h}) + \sigma(Lh \rightarrow \tilde{L}^\dagger H^\dagger) + \sigma(Lh \rightarrow \tilde{L}H) \rangle \\
&\quad + \langle 2\sigma(\bar{L}\bar{h} \rightarrow Lh) + \sigma(\bar{L}\bar{h} \rightarrow \tilde{L}H) + \sigma(\bar{L}\bar{h} \rightarrow \tilde{L}^\dagger H^\dagger) \rangle \\
&\quad - \langle \sigma(\tilde{L}H \rightarrow \bar{L}\bar{h}) - \sigma(\tilde{L}H \rightarrow Lh) \rangle + \\
&\quad + \langle \sigma(\tilde{L}^\dagger H^\dagger \rightarrow Lh) - \sigma(\tilde{L}^\dagger H^\dagger \rightarrow \bar{L}\bar{h}) \rangle \quad (4.44)
\end{aligned}$$

$$\begin{aligned}
\frac{dY_{\tilde{\mathcal{L}}}}{dt} &= \sum_i \left[ \epsilon_{s_i} \left( Y_{\tilde{N}_i} \langle \Gamma_{\tilde{N}_i}^s \rangle + Y_{\tilde{N}_i}^{eq} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle - 2Y_{\tilde{N}_i}^{eq} \langle \Gamma_{\tilde{N}_i}^{s,eq} \rangle \right) + Y_{\tilde{N}_i}^{eq} Y_{\tilde{\mathcal{L}}} \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle \right] \\
&\quad - \langle 2\sigma(\tilde{L}H \rightarrow \tilde{L}^\dagger H^\dagger) + \sigma(\tilde{L}H \rightarrow \bar{L}\bar{h}) + \sigma(\tilde{L}H \rightarrow Lh) \rangle \\
&\quad + \langle 2\sigma(\tilde{L}^\dagger H^\dagger \rightarrow \tilde{L}H) + \sigma(\tilde{L}^\dagger H^\dagger \rightarrow Lh) + \sigma(\tilde{L}^\dagger H^\dagger \rightarrow \bar{L}\bar{h}) \rangle \\
&\quad - \langle \sigma(Lh \rightarrow \tilde{L}^\dagger H^\dagger) - \sigma(Lh \rightarrow \tilde{L}H) \rangle \\
&\quad + \langle \sigma(\bar{L}\bar{h} \rightarrow \tilde{L}H) - \sigma(\bar{L}\bar{h} \rightarrow \tilde{L}^\dagger H^\dagger) \rangle \quad (4.45)
\end{aligned}$$

The out of equilibrium condition is verified since using Eq. (4.28) the first term of Eq. (4.43) and the  $\epsilon$  terms of Eqs.(4.44) and (4.45) cancel out in thermal equilibrium.

The Boltzmann equation for the total lepton number can be written as (here we use  $\epsilon_{s_i} = -\epsilon_{f_i} = \bar{\epsilon}_i$ ):

$$\begin{aligned}
\frac{dY_{\mathcal{L}_{total}}}{dt} &= \sum_i \bar{\epsilon}_i \left[ Y_{\tilde{N}_i} \left( \langle \Gamma_{\tilde{N}_i}^s \rangle - \langle \Gamma_{\tilde{N}_i}^f \rangle \right) + Y_{\tilde{N}_i}^{eq} \left( \langle \tilde{\Gamma}_{\tilde{N}_i}^s \rangle - \langle \tilde{\Gamma}_{\tilde{N}_i}^f \rangle \right) - 2Y_{\tilde{N}_i}^{eq} \left( \langle \Gamma_{\tilde{N}_i}^{s,eq} \rangle - \langle \Gamma_{\tilde{N}_i}^{f,eq} \rangle \right) \right] \\
&\quad + Y_{\tilde{N}_i}^{eq} \left( Y_{\mathcal{L}} \tilde{\Gamma}_{\tilde{N}_i}^f + Y_{\tilde{\mathcal{L}}} \tilde{\Gamma}_{\tilde{N}_i}^s \right) + \text{scattering terms} \\
&\simeq \sum_i \left[ \langle \Gamma_{\tilde{N}_i}^{s,eq} \rangle \left( Y_{\tilde{N}_i} - Y_{\tilde{N}_i}^{eq} \right) \epsilon_i^{eff}(T) + Y_{\tilde{N}_i}^{eq} \left( Y_{\mathcal{L}} \tilde{\Gamma}_{\tilde{N}_i}^{f,eq} + Y_{\tilde{\mathcal{L}}} \tilde{\Gamma}_{\tilde{N}_i}^{s,eq} \right) \right] + s. t. \quad (4.46)
\end{aligned}$$

with  $\langle \Gamma_{\tilde{N}_i}^{s,eq} \rangle = \langle \Gamma_{\tilde{N}_i}^f \rangle + \langle \Gamma_{\tilde{N}_i}^s \rangle$ .

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<sup>†</sup>Here we have suppressed flavour indices in the two body  $\Delta L = 2$  scattering terms for the sake of simplicity, but a sum over all flavours in initial and final states should be understood.

In the last line we have used that at  $\mathcal{O}(\epsilon)$  we can neglect the difference between  $f_{\tilde{N}_i}$  and  $f_{\tilde{N}_i}^{eq}$  in the definitions of the thermal average widths and we have defined the effective  $T$  dependent total asymmetry:

$$\epsilon_i^{eff}(T) = \bar{\epsilon}_i \frac{\langle \Gamma_{\tilde{N}_i}^s \rangle - \langle \Gamma_{\tilde{N}_i}^f \rangle}{\langle \Gamma_{\tilde{N}_i}^f \rangle + \langle \Gamma_{\tilde{N}_i}^s \rangle}, \quad (4.47)$$

which in the approximate decay at rest takes the form Ref. [10, 11]

$$\epsilon_i(T) = \bar{\epsilon}_i \frac{c_s - c_f}{c_s + c_f}. \quad (4.48)$$

## 5. Results

We now quantify the conditions on the parameters which can be responsible for a successful leptogenesis.

The final amount of  $\mathcal{B} - \mathcal{L}$  asymmetry  $Y_{\mathcal{B}-\mathcal{L}} = n_{\mathcal{B}-\mathcal{L}}/s$  generated by the decay of the four light singlet sneutrino states  $\tilde{N}_i$  assuming no pre-existing asymmetry and thermal initial sneutrino densities can be parameterized as

$$Y_{\mathcal{B}-\mathcal{L}} = -\kappa \sum_i \epsilon_i(T_d) Y_{\tilde{N}_i}^{eq}(T \gg M_i). \quad (5.1)$$

$\epsilon_i(T)$  is given in Eq.(3.12) and  $T_d$  is the temperature at the time of decay defined by the condition that the decay width is equal to the expansion rate of the universe:  $\Gamma = H(T_d)$ , where the Hubble parameter  $H = 1.66 g_*^{1/2} \frac{T^2}{m_{pl}}$ ,  $m_{pl} = 1.22 \cdot 10^{19}$  GeV is the Planck mass and  $g_*$  counts the effective number of spin-degrees of freedom in thermal equilibrium,  $g_* = 228.75$  in the MSSM. Furthermore  $Y_{\tilde{N}_i}^{eq}(T \gg M_i) = 90\zeta(3)/(4\pi^4 g_*)$ .

In Eq. (5.1)  $\kappa \lesssim 1$  is a dilution factor which takes into account the possible inefficiency in the production of the singlet sneutrinos, the erasure of the generated asymmetry by  $L$ -violating scattering processes and the temperature dependence of the CP asymmetry  $\epsilon_i(T)$ . The precise value of  $\kappa$  can only be obtained from numerical solution of the Boltzmann equations. Moreover, in general, the result depends on how the lepton asymmetry is distributed in the three lepton flavours [5]. For simplicity we will ignore flavour issues. Furthermore, in what follows we will use an approximate constant value  $\kappa = 0.2$ .

After conversion by the sphaleron transitions, the final baryon asymmetry is related to the  $\mathcal{B} - \mathcal{L}$  asymmetry by

$$\frac{n_{\mathcal{B}}}{s} = \frac{24 + 4n_H}{66 + 13n_H} \frac{n_{\mathcal{B}-\mathcal{L}}}{s}, \quad (5.2)$$

where  $n_H$  is the number of Higgs doublets. For the MSSM:

$$\frac{n_{\mathcal{B}}}{s} = -8.4 \times 10^{-4} \kappa \sum_i \epsilon_i(T_d) \quad (5.3)$$

This has to be compared with the WMAP measurements that in the  $\Lambda$ CDM model imply [27]:

$$\frac{n_{\mathcal{B}}}{s} = (8.7_{-0.4}^{+0.3}) \times 10^{-11} \quad (5.4)$$

Altogether we find that (for maximal CP violating phase  $\sin \phi = 1$ ):

$$\frac{4|B_{SN} A| \Gamma}{4|B_{SN}|^2 + |M|^2 \Gamma^2} \frac{c_s(T_d) - c_f(T_d)}{c_s(T_d) + c_f(T_d)} \gtrsim 2.6 \times 10^{-7} \quad (5.5)$$

Further constraints arise from the timing of the decay. First, successful leptogenesis requires the singlet sneutrinos to decay out of equilibrium: its decay width must be smaller than the expansion rate of the Universe  $\Gamma < H|_{T=M}$ , with  $\Gamma$  given in Eq. (2.16),

$$\frac{M \sum_k |Y_{1k}|^2}{8\pi} < 1.66 g_*^{1/2} \frac{M^2}{m_{pl}}. \quad (5.6)$$

This condition gives an upper bound:

$$\sum_k |Y_{1k}|^2 \left( \frac{10^8 \text{ GeV}}{M} \right) < 5 \times 10^{-9} \quad (5.7)$$

Second, in order for the generated lepton asymmetry to be converted into a baryon asymmetry via the B-L violating sphaleron processes, the singlet sneutrino decay should occur before the electroweak phase transition

$$\Gamma > H(T \sim 100 \text{ GeV}) \Rightarrow M \sum_k |Y_{1k}|^2 \geq 2.6 \times 10^{-13} \text{ GeV} \quad (5.8)$$

The combination of Eqs.(5.7) and (5.8) determines a range for the possible values of  $\sum |Y_{1k}|^2$  for a given  $M$ :

$$2.6 \times 10^{-21} \left( \frac{10^8 \text{ GeV}}{M} \right) < \sum_k |Y_{1k}|^2 < 5 \times 10^{-9} \left( \frac{M}{10^8 \text{ GeV}} \right) \quad (5.9)$$

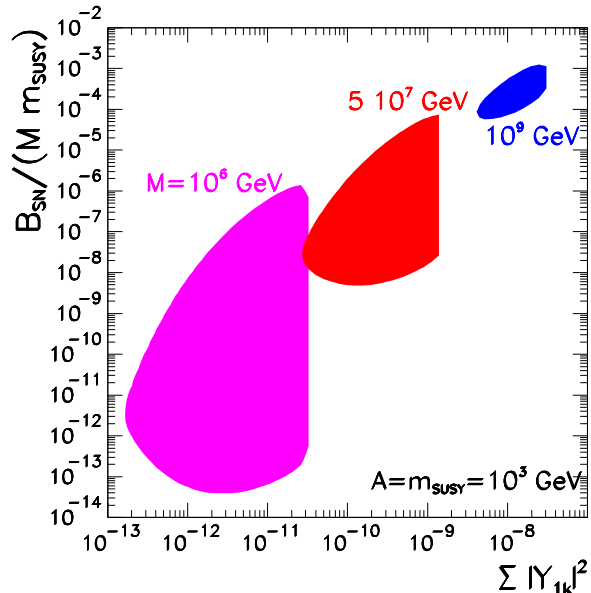
We now turn to the consequences that these constraints may have for the neutrino mass predictions in this scenario. Without loss of generality one can work in the basis in which  $M_{ij}$  is diagonal. In that basis the light neutrino masses, Eq. (2.3), are:

$$m_{\nu ij} = 3 \times 10^{-3} \text{ eV} \left( \frac{v}{175 \text{ GeV}} \right)^2 \sum_{kl} Y_{li} \frac{10^8 \text{ GeV}}{M_l} \frac{10^8 \text{ GeV}}{M_k} \frac{\mu_{kl}}{\text{GeV}} Y_{kj} \quad (5.10)$$

where  $v = \langle H \rangle$  is the Higgs vev.

It is clear from Eq. (5.10) that the out of equilibrium condition, Eq. (5.7), implies that the contribution of the lightest pseudo-Dirac singlet neutrino generation to the neutrino mass is negligible. Consequently, to reproduce the observed mass differences  $\Delta m_{\odot}^2$  and  $\Delta m_{atm}^2$ , the dominant contribution to the neutrino masses must arise from the exchange of the heavier singlet neutrino states.

This can be easily achieved, for example, in the single right-handed neutrino dominance mechanism (SRHND) [28]. These models naturally explain the strong hierarchy in the masses and the large mixing angle present in the light neutrino sector. In particular, in the simple case in which the matrix  $\mu$  and  $M$  are simultaneously diagonalizable, the results



**Figure I:**  $\sum_k |Y_{1k}|^2 - B_{SN}$  regions in which enough  $CP$  asymmetry can be generated (Eq. (5.5)) and the non-equilibrium condition in the sneutrino decay, Eq. (5.6) and decay before the electroweak phase transition, Eq. (5.8) are verified. We take  $A = m_{SUSY} = 10^3$  GeV. The regions correspond to  $M = 10^6$ ,  $5 \times 10^7$ , and  $10^9$  GeV, from left to right.

in Ref. [28] imply that for the inverse see-saw model with three generations of singlet neutrinos, the SRHND condition is attained if there is a strong hierarchy:

$$\mu_3 \frac{Y_{3k} Y_{3k'}}{M_3} \gg \mu_2 \frac{Y_{2l} Y_{2l'}}{M_2} \gg \mu_1 \frac{Y_{1l} Y_{1l'}}{M_1}. \quad (5.11)$$

Generically this means that the out of equilibrium condition requires the neutrino mass spectrum to be strongly hierarchical,  $m_1 \ll m_2 < m_3$ .

Conversely this implies that the measured neutrino masses do not impose any constraint on the combination of Yukawa couplings and sneutrino masses which is relevant for the generation of the lepton asymmetry which can be taken as an independent parameter in the evaluation of the asymmetry.

Finally we plot in Fig. I the range of parameters  $\sum |Y_{1k}|^2$  and  $B_{SN}$  for which enough asymmetry is generated, Eq. (5.5), and the out of equilibrium and pre-electroweak phase transition decay conditions, Eq. (5.9) are verified. We show the ranges for three values of  $M$  and for the characteristic value of  $A = m_{SUSY} = 10^3$  GeV.

From the figure we see that this mechanism works for relatively small values of  $M$  ( $< 10^9$  GeV). The smaller is  $M$ , the smaller are the yukawas  $\sum |Y_{1k}|^2$ . Also, in total analogy with the standard seesaw [10], [11], the value of the soft supersymmetry-breaking bilinear  $B_{SN}$ , is well below the expected value  $M m_{SUSY}$ . The reason is that, in order to generate an asymmetry large enough  $B_{SN} \sim M \Gamma$ , but  $\Gamma$  is very small if the sneutrinos

decay out of equilibrium,  $\Gamma \leq 1 \text{ GeV} \left(\frac{M}{10^9 \text{ GeV}}\right)^2$ . A small  $B$  term with large CP phase can be realized naturally for example within the framework of gauge mediated supersymmetry breaking [29] or in warped extra dimensions [30].

Given the small required values of  $B_{SN}$  one can question the expansion in the small parameters in Eq. (2.13). As described at the end of Sec. 3 in order to verify the stability of the results we have redone the computation of the CP asymmetry keeping all the entries in the sneutrino mass matrix, and just assuming that it is real. We have found that as long as  $|B_{SN}| \gg |B_S|, |\mu|^2$  the total CP asymmetry is always proportional to  $B_{SN}$ , and presents the same resonant behaviour, so that it is still significant only for  $B_{SN} \ll M m_{SUSY}$ .

In summary in this work we have studied the conditions for successful soft leptogenesis in the context of the supersymmetric inverse seesaw mechanism. In this model the lepton sector is extended with two electroweak singlet superfields to which opposite lepton number can be assigned. This scheme is characterized by a small lepton number violating Majorana mass term  $\mu$  with the effective light neutrino mass being  $m_\nu \propto \mu$ . The scalar sector contains four single sneutrino states per generation and, after supersymmetry breaking, their interaction lagrangian contains both  $L$ -conserving and  $L$ -violating soft supersymmetry-breaking bilinear  $B$ -terms which together with the  $\mu$  parameter give a small mass splitting between the four singlet sneutrino states of a single generation. In combination with the trilinear soft supersymmetry breaking terms they also provide new CP violating phases needed to generate a lepton asymmetry in the singlet sneutrino decays.

We have computed the relevant lepton asymmetry and we find in that, as long as the  $L$ -conserving  $B$ -term,  $B_{SN}$ , is not much smaller than the  $L$ -violating couplings, the asymmetry is proportional to  $B_{SN}$  and it is not suppressed by any  $L$ -violating parameter. As in the standard see-saw case, the asymmetry displays a resonance behaviour with the maximum value of the asymmetry being obtained when the largest mass splitting,  $2B_{SN}/M$ , is of the order of the singlet sneutrinos decay width,  $\Gamma$ . Consequently we find that this mechanism can lead to successful leptogenesis only for relatively small values of  $B_{SN}$ . The right-handed neutrino masses are low enough to elude the gravitino problem. Also, the out of equilibrium decay condition implies that the Yukawa couplings involving the lightest of the right-handed neutrinos are constrained to be very small which, for the naturally small values of the  $L$ -violating parameter  $\mu$ , implies that the neutrino mass spectrum has to be strongly hierarchical.

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## A. CP Asymmetry in Quantum Mechanics

The four sneutrino system is completely analogous to the  $K^0 - \bar{K}^0$  system, so here we compute the CP asymmetry generated in their decay using the same formalism. In order to compare with the effective field theory approach described in Sec. 3 we consider only the simplified case  $B_S, \tilde{m}_N^2, \tilde{m}_S^2, \tilde{m}_{SN}^2 \ll B_{SN}$  and  $\tilde{M}_{SN}^2 \sim \mu M^*$ . In this limit, we have chosen for simplicity a basis where  $A = |A|e^{i\phi}$  is the only complex parameter with  $\phi$  given in Eq.(2.12).

The evolution of the system is then determined by the effective Hamiltonian,

$$H = \hat{M} - i\frac{\hat{\Gamma}}{2} \quad (\text{A.1})$$

where, in the interaction basis and at leading order in the expansion parameters  $\epsilon, \tilde{\epsilon}$  defined in Eq. (2.13),

$$\hat{M} = M \begin{pmatrix} 1 & 0 & \epsilon & \tilde{\epsilon} \\ 0 & 1 & \tilde{\epsilon} & \epsilon \\ \epsilon & \tilde{\epsilon} & 1 & 0 \\ \tilde{\epsilon} & \epsilon & 0 & 1 \end{pmatrix} \quad (\text{A.2})$$

and

$$\hat{\Gamma} = \Gamma \begin{pmatrix} 1 & 0 & 0 & \frac{A}{M} \\ 0 & 1 & \frac{A^*}{M} & 0 \\ 0 & \frac{A}{M} & 1 & 0 \\ \frac{A^*}{M} & 0 & 0 & 1 \end{pmatrix} \quad (\text{A.3})$$

with  $\Gamma$  given in Eq.(2.16).

It is convenient to write the effective Hamiltonian in the mass eigenstate basis Eq.(2.15), because in such basis the four sneutrino system decouples in two subsystems of two sneutrinos, with the resulting width matrix:

$$\Gamma = \Gamma \begin{pmatrix} 1 - \epsilon_A \cos \phi & \epsilon_A \sin \phi & 0 & 0 \\ \epsilon_A \sin \phi & 1 + \epsilon_A \cos \phi & 0 & 0 \\ 0 & 0 & 1 - \epsilon_A \cos \phi & -\epsilon_A \sin \phi \\ 0 & 0 & -\epsilon_A \sin \phi & 1 + \epsilon_A \cos \phi \end{pmatrix} \quad (\text{A.4})$$

where  $\epsilon_A = \frac{|A|}{|M|}$ .

The eigenvectors of the effective Hamiltonian  $H$  are:

$$\begin{aligned} \tilde{N}'_1 &= \frac{1}{\sqrt{2+2\frac{|\epsilon_+|}{|\epsilon_-|}}} \left[ e^{i(\phi_- - \phi_+)/4} (\tilde{S}^\dagger - \tilde{N}^\dagger) + \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/4} (\tilde{S} - \tilde{N}) \right] \\ \tilde{N}'_2 &= \frac{i}{\sqrt{2+2\frac{|\epsilon_+|}{|\epsilon_-|}}} \left[ e^{i(\phi_- - \phi_+)/4} (\tilde{S}^\dagger - \tilde{N}^\dagger) - \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/4} (\tilde{S} - \tilde{N}) \right] \\ \tilde{N}'_3 &= \frac{i}{\sqrt{2+2\frac{|\epsilon_+|}{|\epsilon_-|}}} \left[ e^{i(\phi_- - \phi_+)/4} (\tilde{S}^\dagger + \tilde{N}^\dagger) - \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/4} (\tilde{S} + \tilde{N}) \right] \\ \tilde{N}'_4 &= \frac{1}{\sqrt{2+2\frac{|\epsilon_+|}{|\epsilon_-|}}} \left[ e^{i(\phi_- - \phi_+)/4} (\tilde{S}^\dagger + \tilde{N}^\dagger) + \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/4} (\tilde{S} + \tilde{N}) \right] \end{aligned}$$

and the eigenvalues

$$\begin{aligned}
\nu_1 &= |M| - |\mu|/2 - i\Gamma/2 - e^{i(\phi_- + \phi_+)/2} \sqrt{|\epsilon_+|} \sqrt{|\epsilon_-|} \\
\nu_2 &= |M| - |\mu|/2 - i\Gamma/2 + e^{i(\phi_- + \phi_+)/2} \sqrt{|\epsilon_+|} \sqrt{|\epsilon_-|} \\
\nu_3 &= |M| + |\mu|/2 - i\Gamma/2 - e^{i(\phi_- + \phi_+)/2} \sqrt{|\epsilon_+|} \sqrt{|\epsilon_-|} \\
\nu_4 &= |M| + |\mu|/2 - i\Gamma/2 + e^{i(\phi_- + \phi_+)/2} \sqrt{|\epsilon_+|} \sqrt{|\epsilon_-|}
\end{aligned}$$

where

$$\begin{aligned}
\epsilon_- &= |\epsilon_-| e^{i\phi_-} = |B_{SN}/(2M)| - i\Gamma\epsilon_A/2 \\
\epsilon_+ &= |\epsilon_+| e^{i\phi_+} = |B_{SN}/(2M)| - i\Gamma\epsilon_A^*/2
\end{aligned}$$

We consider an initial state at  $t = 0$  with equal number densities of the four sneutrino interaction states  $\tilde{F}_i$ . Using that the time evolution for the hamiltonian eigenstates Eq.(A.5) is trivially  $|\tilde{N}'_i(t)\rangle = e^{-i\nu_i t} |\tilde{N}'_i\rangle$ , we obtain that at time  $t$  the interaction states are the following:

$$\begin{aligned}
|\tilde{N}(t)\rangle &= \\
&= \frac{g_{1+}(t) + g_{2+}(t)}{4} |\tilde{N}\rangle + \sqrt{\frac{|\epsilon_-|}{|\epsilon_+|}} e^{i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) - g_{2-}(t)}{4} |\tilde{N}^\dagger\rangle - \\
&- \frac{g_{1+}(t) - g_{2+}(t)}{4} |\tilde{S}\rangle - \sqrt{\frac{|\epsilon_-|}{|\epsilon_+|}} e^{i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) + g_{2-}(t)}{4} |\tilde{S}^\dagger\rangle \\
|\tilde{N}^\dagger(t)\rangle &= \\
&= \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) - g_{2-}(t)}{4} |\tilde{N}\rangle + \frac{g_{1+}(t) + g_{2+}(t)}{4} |\tilde{N}^\dagger\rangle - \\
&- \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) + g_{2-}(t)}{4} |\tilde{S}\rangle - \frac{g_{1+}(t) - g_{2+}(t)}{4} |\tilde{S}^\dagger\rangle \\
|\tilde{S}(t)\rangle &= \\
&= -\frac{g_{1+}(t) - g_{2+}(t)}{4} |\tilde{N}\rangle - \sqrt{\frac{|\epsilon_-|}{|\epsilon_+|}} e^{i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) + g_{2-}(t)}{4} |\tilde{N}^\dagger\rangle + \\
&+ \frac{g_{1+}(t) + g_{2+}(t)}{4} |\tilde{S}\rangle + \sqrt{\frac{|\epsilon_-|}{|\epsilon_+|}} e^{i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) - g_{2-}(t)}{4} |\tilde{S}^\dagger\rangle \tag{A.5} \\
|\tilde{S}^\dagger(t)\rangle &= \\
&= -\sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) + g_{2-}(t)}{4} |\tilde{N}\rangle - \frac{g_{1+}(t) - g_{2+}(t)}{4} |\tilde{N}^\dagger\rangle + \\
&+ \sqrt{\frac{|\epsilon_+|}{|\epsilon_-|}} e^{-i(\phi_- - \phi_+)/2} \frac{g_{1-}(t) - g_{2-}(t)}{4} |\tilde{S}\rangle + \frac{g_{1+}(t) + g_{2+}(t)}{4} |\tilde{S}^\dagger\rangle
\end{aligned}$$

The functions  $g_{1\pm}(t)$  and  $g_{2\pm}(t)$  containing the time dependence are given by:

$$g_{1\pm}(t) = e^{-i(|M| - \frac{|\mu|}{2} - i\frac{\Gamma}{2})t} [e^{i\Delta\nu t} \pm e^{-i\Delta\nu t}] , \tag{A.6}$$



$$g_{2\pm}(t) = e^{-i\left(|M| + \frac{|\mu|}{2} - i\frac{\Gamma}{2}\right)t} \left[ e^{i\Delta\nu t} \pm e^{-i\Delta\nu t} \right], \quad (\text{A.7})$$

with,

$$\Delta\nu = e^{i\left(\frac{\phi_- + \phi_+}{2}\right)} \sqrt{|\epsilon_-|} \sqrt{|\epsilon_+|}. \quad (\text{A.8})$$

We neglect soft supersymmetry-breaking terms, so that  $\tilde{S}$  only decay to scalars, and  $\tilde{N}$  to antifermions:

$$\begin{aligned} |A[\tilde{S} \rightarrow \tilde{L}_k H]|^2 &= |A[\tilde{S}^\dagger \rightarrow \tilde{L}_k^\dagger H^\dagger]|^2 = |Y_{1k} M|^2 \equiv |A_{\tilde{L}_k}|^2 \\ |A[\tilde{N} \rightarrow L_k^\dagger h^\dagger]|^2 &= |A[\tilde{N}^\dagger \rightarrow L_k h]|^2 = |Y_{1k}|^2 (s - m_L^2 - m_h^2) \equiv |A_{L_k}|^2 \end{aligned} \quad (\text{A.9})$$

We next write the time dependent decay amplitudes in terms of  $|A_{\tilde{L}_k}|^2$  and  $|A_{L_k}|^2$ :

$$|A[\tilde{N}(t) \rightarrow \tilde{L}_k H]|^2 = |A[\tilde{N}^\dagger(t) \rightarrow \tilde{L}_k^\dagger H^\dagger]|^2 = \frac{|g_{1+} - g_{2+}|^2}{16} |A_{\tilde{L}_k}|^2 \quad (\text{A.10})$$

$$|A[\tilde{N}(t) \rightarrow \tilde{L}_k^\dagger H^\dagger]|^2 = \left( \frac{|\epsilon_-|}{|\epsilon_+|} \right)^2 |A[\tilde{N}^\dagger(t) \rightarrow \tilde{L}_k H]|^2 = \frac{|\epsilon_-|}{|\epsilon_+|} \frac{|g_{1-} + g_{2-}|^2}{16} |A_{\tilde{L}_k}|^2 \quad (\text{A.11})$$

$$|A[\tilde{N}(t) \rightarrow L_k h]|^2 = \left( \frac{|\epsilon_-|}{|\epsilon_+|} \right)^2 |A[\tilde{N}^\dagger(t) \rightarrow L_k^\dagger h^\dagger]|^2 = \frac{|\epsilon_-|}{|\epsilon_+|} \frac{|g_{1-} - g_{2-}|^2}{16} |A_{L_k}|^2 \quad (\text{A.12})$$

$$|A[\tilde{N}(t) \rightarrow L_k^\dagger h^\dagger]|^2 = |A[\tilde{N}^\dagger(t) \rightarrow L_k h]|^2 = \frac{|g_{1+} + g_{2+}|^2}{16} |A_{L_k}|^2 \quad (\text{A.13})$$

$$|A[\tilde{S}(t) \rightarrow \tilde{L}_k H]|^2 = |A[\tilde{S}^\dagger(t) \rightarrow \tilde{L}_k^\dagger H^\dagger]|^2 = \frac{|g_{1+} + g_{2+}|^2}{16} |A_{\tilde{L}_k}|^2 \quad (\text{A.14})$$

$$|A[\tilde{S}(t) \rightarrow \tilde{L}_k^\dagger H^\dagger]|^2 = \left( \frac{|\epsilon_-|}{|\epsilon_+|} \right)^2 |A[\tilde{S}^\dagger(t) \rightarrow \tilde{L}_k H]|^2 = \frac{|\epsilon_-|}{|\epsilon_+|} \frac{|g_{1-} - g_{2-}|^2}{16} |A_{\tilde{L}_k}|^2 \quad (\text{A.15})$$

$$|A[\tilde{S}(t) \rightarrow L_k h]|^2 = \left( \frac{|\epsilon_-|}{|\epsilon_+|} \right)^2 |A[\tilde{S}^\dagger(t) \rightarrow L_k^\dagger h^\dagger]|^2 = \frac{|\epsilon_-|}{|\epsilon_+|} \frac{|g_{1-} + g_{2-}|^2}{16} |A_{L_k}|^2 \quad (\text{A.16})$$

$$|A[\tilde{S}(t) \rightarrow L_k^\dagger h^\dagger]|^2 = |A[\tilde{S}^\dagger(t) \rightarrow L_k h]|^2 = \frac{|g_{1+} - g_{2+}|^2}{16} |A_{L_k}|^2 \quad (\text{A.17})$$

We define the integrated  $CP$  asymmetries for the fermionic and scalar channels as:

$$\begin{aligned} \epsilon_f &= \frac{\int dt \sum_{i,k} \left[ |A[\tilde{F}_i(t) \rightarrow L_k + X]|^2 - |A[\tilde{F}_i(t) \rightarrow L_k^\dagger + X]|^2 \right]}{\int dt \sum_{i,k} \left[ |A[\tilde{F}_i(t) \rightarrow L_k + X]|^2 + |A[\tilde{F}_i(t) \rightarrow L_k^\dagger + X]|^2 \right]} \\ \epsilon_s &= \frac{\int dt \sum_{i,k} \left[ |A[\tilde{F}_i(t) \rightarrow \tilde{L}_k + X]|^2 - |A[\tilde{F}_i(t) \rightarrow \tilde{L}_k^\dagger + X]|^2 \right]}{\int dt \sum_{i,k} \left[ |A[\tilde{F}_i(t) \rightarrow \tilde{L}_k + X]|^2 + |A[\tilde{F}_i(t) \rightarrow \tilde{L}_k^\dagger + X]|^2 \right]} \end{aligned} \quad (\text{A.18})$$

Using the time-dependent amplitudes from Eq. (A), the time integrated asymmetries are:

$$\epsilon_s = -\epsilon_f = \bar{\epsilon} = -\frac{1}{2} \left( \frac{|\epsilon_-|}{|\epsilon_+|} - \frac{|\epsilon_+|}{|\epsilon_-|} \right) \chi, \quad (\text{A.19})$$

where the factor  $\frac{|\epsilon_-|}{|\epsilon_+|} - \frac{|\epsilon_+|}{|\epsilon_-|}$  vanishes if  $CP$  is conserved, and for  $\epsilon_A \ll 1$  is given by:

$$\frac{|\epsilon_-|}{|\epsilon_+|} - \frac{|\epsilon_+|}{|\epsilon_-|} \simeq \frac{4M\Gamma|\epsilon_A|}{B_{SN}} \sin \phi = \frac{4\Gamma A}{B_{SN}} \sin \phi \quad (\text{A.20})$$

The time dependence is encoded in  $\chi$ :

$$\chi = \frac{\int_0^\infty dt [ |g_{1-}(t)|^2 + |g_{2-}(t)|^2 ]}{\int_0^\infty dt [ |g_{1+}(t)|^2 + |g_{2+}(t)|^2 + |g_{1-}(t)|^2 + |g_{2-}(t)|^2 ]} . \quad (\text{A.21})$$

In the limit  $\epsilon_A \ll 1$  the time integrals are :

$$\int_0^\infty dt [ |g_{1-}|^2 + |g_{2-}|^2 ] \simeq 4 \frac{|B_{SN}|^2/\Gamma|M|^2}{\Gamma^2 + |B_{SN}|^2/|M|^2} , \quad (\text{A.22})$$

$$\int_0^\infty dt [ |g_{1+}|^2 + |g_{2+}|^2 + |g_{1-}(t)|^2 + |g_{2-}(t)|^2 ] \simeq \frac{8}{\Gamma} ; \quad (\text{A.23})$$

$$\epsilon_s = -\epsilon_f = \bar{\epsilon} = -\frac{\Gamma |B_{SN}| |A|}{\Gamma^2 |M|^2 + |B_{SN}|^2} \sin \phi \quad (\text{A.24})$$

The comparison between the asymmetry in Eq.(3.11) and Eq.(A.24) is in full analogy to the corresponding comparison in the standard see-saw case discussed in [10]. The asymmetry computed in the quantum mechanics approach, based on an effective (non hermitic) Hamiltonian, Eq.(A.24), agrees with the one obtained using a field-theoretical approach, Eq.(3.11) in the limit  $\Gamma \ll B_{SN}/M$ . When  $\Gamma \gg B_{SN}/M$ , the four sneutrino states become two pairs of not well-separated particles. In this case the result for the asymmetry can depend on how the initial state is prepared. If one assumes that the singlet sneutrinos are in a thermal bath with a thermalization time  $\Gamma^{-1}$  shorter than the typical oscillation times,  $\Delta M_{ij}^{-1}$ , coherence is lost and it is appropriate to compute the CP asymmetry in terms of the mass eigenstates Eq.(2.15) as done in Sec. 3 and one obtains Eq. (3.11). If, on the contrary, one assumed that the  $\tilde{N}, \tilde{S}$  states are produced in interaction eigenstates and coherence is not lost in their evolution, then it is appropriate to compute the CP asymmetry in terms of the interaction eigenstates Eq.(A.5) as done in this appendix and one obtains Eq.(A.24).

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