Dynamical Symmetry Breaking in Warped Compactifications

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Abstract

We study dynamical electroweak symmetry breaking in the Randall-Sundrum scenario. We show that one extra dimension is enough to give the correct pattern of electroweak symmetry breaking in a simple model with gauge bosons and the right-handed top quark in the bulk. The top quark mass is also in agreement with experiment. Furthermore, we propose an extended scenario with all Standard Model gauge bosons and fermions propagating in the bulk, which naturally accommodates the fermion mass hierarchies. No new fields or interactions beyond the observed in the Standard Model are required.

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1   Introduction

The origin of the electroweak symmetry breaking is one of the most important questions in particle physics. In the Standard Model (SM) it is achieved by a nonzero vacuum expectation value of a fundamental scalar Higgs field. However, the squared-mass of a fundamental scalar field receives quadratically divergent radiative corrections, hence suffers from the so-called hierarchy problem if the cutoff scale is much higher than the electroweak scale. This is indeed the case in the SM, where a large hierarchy exists between the electroweak scale (~ 100 GeV) and the Planck scale ($M_P \sim 10^{18}$ GeV).

It has been recently realized that this hierarchy of scales could have its origin in the presence of extra dimensions with nontrivial space-time geometries[1, 2]. If we live in $D = 4 + \delta$ dimensions, there is the possibility that the Planck scale M_P is actually an effective four dimensional scale determined by the fundamental scale of the $(4 + \delta)$-theory, M, and the geometry of space-time. An explicit example of this is the Randall and Sundrum model [2], where the hierarchy problem is solved by introducing a warped extra dimension. The space-time is a slice of AdS_5 with one extra dimension compactified on an orbifold, S^1/Z_2, of radius r_c. The metric is given by

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{1}$$

where y is the fifth coordinate, $0 \leq y \leq \pi r_c$, μ, ν are four dimensional indices, k is the AdS_5 curvature of order of the Planck scale, and $e^{-2k|y|}$ is called the warp factor. The Randall-Sundrum scenario consists of two three-branes located at the orbifold fix points $y_* = 0, \pi r_c$. From (1) one can see that the warp factor determines the physical energy scale at the position y from the point of view of a 4D observer. Thus assuming $kr_c \sim 12$, the physical scale of the brane located at $y_* = \pi r_c$ is given by $ke^{-\pi kr_c} \sim (100 - 1000)$ GeV. This sets the scale on the brane to be the electroweak scale and solves the hierarchy problem.

The hierarchy problem can also be avoided if the Higgs is a composite object rather than a fundamental field, and ceases to be a dynamical degree of freedom not much above the electroweak scale. Since the top quark is the heavier fermion in the standard model, it has been the first candidate to form the composite Higgs, bound out of the third generation weak doublet ψ_L and the right-handed top field, t_R. Such composite Higgs arises naturally in the presence of some strongly coupled four-quark operators. In four dimensional top condensate models, though, there is a problem when one tries to accommodate the top quark mass in the experimental range as well as electroweak symmetry breaking at the correct scale, because the large top Yukawa coupling gives a top quark too heavy [3],[4]. This problem can be solved if a new vector-like fermion is introduced with the same quantum numbers of t_R, which becomes the appropriate constituent of the Higgs boson together with ψ_L [5]. While this mechanism neatly accommodates both the measured top quark mass and a Higgs vev of the electroweak scale, the drawback is that one has to include additional structures to originate the non-renormalizable four-quark interactions.

In this work we will show that in the Randall-Sundrum scenario it is possible to construct a phenomenologically successful effective theory with no fundamental scalars, including just the

symmetry group $SU(3)_C \times SU(2)_W \times U(1)_Y$ and the SM fermion and gauge boson fields. Dynamical electroweak symmetry breaking in the presence of flat extra dimensions has already been considered in the literature [6]-[9]. In particular, it has been noticed that the ingredients needed for dynamical electroweak symmetry breaking are naturally present in the SM, provided the gauge bosons and some fermions propagate in extra dimensions compactified at the $\sim$ TeV scale [8, 9]. Four-quark operators are always induced by QCD in compact dimensions, via the exchange of the Kaluza-Klein (KK) excitations of the gluons. Moreover, by allowing the right-handed top quark to live in the bulk, its KK modes will naturally play the role of the required vector-like quark.

The mechanism outlined above does not work in $D = 5$ flat space because the interchange of gluon KK excitations is not strong enough to form the quark condensate. It is necessary to assume a higher dimensionality ($D \geq 6$), thus the degeneracy of the gluon KK modes makes the relevant four-quark operator stronger. However, in the Randall-Sundrum scenario the coupling between KK gauge bosons and fermions (both living in the bulk and on the TeV brane) can be larger than in the flat space case [11], so one extra dimension is enough to trigger dynamical electroweak symmetry breaking without introducing fundamental scalar fields.

In section 2 we present a minimal set-up which breaks correctly the electroweak symmetry, and we briefly review the KK decomposition of massless gauge boson and fermion fields in the Randall-Sundrum scenario. In the next section, we use these results to construct an effective theory with four-fermion operators, and we argue that the binding strength of such operators is large enough to form bound states, one of which will be identified as the Higgs field. We compute the effective scalar Lagrangian in section 4. Finally, we discuss fermion masses both in the simplest model (section 5) and in an alternative scenario which provides an explanation of the observed fermion mass hierarchies (section 6). We conclude in section 7.

2   The simplest set-up

In the Randall-Sundrum model [2] only gravity propagates in the 5D bulk, while the SM fields are confined on the TeV brane. Subsequently, the phenomenological consequences of placing the SM fields in the bulk have been extensively studied [10]-[17].

In this section, we describe a minimal set-up which leads to dynamical electroweak symmetry breaking without the need for a fundamental Higgs field. We begin by studying a toy model with one generation of fermions, the third one, and postpone the discussion of flavor symmetry breaking to section 5. We consider that gluons live in the 5D bulk, so their KK modes strongly coupled to quarks can induce the formation of bound states. As we will see in section 5, in order to obtain the correct value of the top mass also the right-handed top quark should propagate in 5D. For simplicity, we assume that the remaining third generation fermions are confined on the TeV brane, $y_* = \pi r_c$.

Since the right-handed top carries hypercharge, the $U(1)_Y$ gauge boson propagates in the bulk, while the $SU(2)_W$ gauge bosons can either reside on the TeV brane or propagate in the bulk, because

we do not require the components of weak doublet fermions to be in different places along the fifth dimension. For definiteness we consider that the $SU(2)_W$ gauge bosons also live in 5D, but our conclusions are completely independent of this assumption.

We are going to study the dynamical generation of masses through the condensation of a pair quark-antiquark, so we shall use the KK decomposition of 5D massless fields in the Randall-Sundrum model.

2.1 Fermion field

Consider a 5D massless fermion field $\Psi(x, y)$. Compactifying on an orbifold S^1/Z_2 we can choose the zero mode to be a left- or right-handed fermion. Imposing that the bulk fermion is even under this compactification,

$$\gamma_5 \ \Psi(x, -y) = +\Psi(x, y) \ , \tag{2}$$

only the right-handed zero mode survives. We will identify this zero mode with the right-handed top quark t_R. The Kaluza Klein decomposition for a bulk fermion with boundary conditions (2) can be written as [13]

$$\Psi(x, y) = \sum_n \left[\Psi_{\text{L,B}}^{(n)}(x) \, \xi_n(y) + \Psi_{\text{R,B}}^{(n)}(x) \, \eta_n(y) \right] \tag{3}$$

where

$$
\begin{aligned}
\xi_n(y) &= \sqrt{\frac{2k}{1 - e^{-\pi k r_c}}} \, e^{-\frac{k}{2}|\pi r_c - y|} e^{\frac{3}{2}k|y|} \sin\left\{ \frac{m_n}{k} \left(e^{k|y|} - 1 \right) \right\} \ , \\
\eta_n(y) &= \sqrt{\frac{2k}{1 - e^{-\pi k r_c}}} \, e^{-\frac{k}{2}|\pi r_c - y|} e^{\frac{3}{2}k|y|} \cos\left\{ \frac{m_n}{k} \left(e^{k|y|} - 1 \right) \right\} \ ,
\end{aligned}
\tag{4}
$$

and $m_n = n\pi k/(e^{\pi k r_c} - 1) \neq 0$. For the zero mode

$$\xi_0(y) = 0, \qquad \eta_0(y) = \sqrt{\frac{k}{1 - e^{-\pi k r_c}}} \, e^{-\frac{k}{2}|\pi r_c - y|} e^{\frac{3}{2}k|y|} \ . \tag{5}$$

Note that massless bulk fermions are localized near the TeV brane, $y_* = \pi r_c$, due to the factor $e^{-k|\pi r_c - y|}$ of the wave function for all modes.

A 5D Lorentz invariant gauge theory has no chiral anomalies because the fermion representation is vector-like. However, the boundary conditions imposed above prevent the existence of the Ψ_L zero-mode and we have to worry about anomalies in the bulk. This problem can be solved by including a Chern-Simons term in the action, which makes the scenario presented in this paper anomaly-free [8].

2.2 Gauge bosons

The bulk gauge bosons in the Randall-Sundrum scenario have been discussed in [11]. We work in the gauge $\partial^\mu A_\mu = 0$ and $A_5 = 0$, with orbifold conditions

$$\partial_5 A_\mu(x, y{=}y_*) = 0 = A_5(x, y{=}y_*) \ . \tag{6}$$

Then the KK decomposition of $A_\mu(x, y)$ is given by

$$A_\mu(x, y) = \sum_n A_\mu^{(n)}(x)\chi_n(y) \ , \tag{7}$$

where $\chi_n(y)$ is a linear combination of first order Bessel functions, J_1 and Y_1,

$$\chi_n(y) = \frac{\sqrt{2k}\,e^{k|y|}}{N_n}\left[J_1\Big(\lambda_n e^{k|y|}\Big) + \alpha_n Y_1\Big(\lambda_n e^{k|y|}\Big)\right] \ , \tag{8}$$

with $\lambda_n \equiv M_n/k \neq 0$. The mass eigenvalues M_n are determined by the condition $\partial_y \chi_n(\pi r_c) = 0$, which leads to the equation

$$J_0(\lambda_n)\,Y_0\Big(\lambda_n e^{\pi k r_c}\Big) = Y_0(\lambda_n)\,J_0\Big(\lambda_n e^{\pi k r_c}\Big) \ . \tag{9}$$

The masses M_n grow linearly with n, being the first excited modes in the TeV range ($M_1 \sim 2.5\,k e^{-\pi k r_c}$, $M_2 \sim 5.6\,k e^{-\pi k r_c}$, ...).

Continuity of $\partial_y \chi_n(y)$ at $y = 0$ gives

$$\alpha_n = -\frac{J_0(\lambda_n)}{Y_0(\lambda_n)} \tag{10}$$

and N_n is a normalization constant which in the limit $M_n \ll k$ and $e^{\pi k r_c} \gg 1$ can be approximated by

$$N_n \sim e^{k\pi r_c} J_1\Big(\lambda_n e^{k\pi r_c}\Big) \ . \tag{11}$$

Several comments are in order. The wave function of the zero mode is $\chi_0 = 1/\sqrt{\pi r_c}$, independent of the fifth coordinate, so it couples equally to both boundaries with strength $g = g_{5D}/\sqrt{\pi r_c}$, being g_{5D} the 5D gauge coupling. On the contrary, the excited modes are localized near the TeV boundary and have different couplings to fermions located on different branes, since these couplings are determined by the eigenfunctions $\chi_n(y)$ near the boundaries. At the $y_* = \pi r_c$ boundary the term in (8) proportional to J_1 dominates while at the other brane the eigenfunction can be well approximated by the Y_1 term. Within these approximations, the coupling of a gauge boson KK mode n to 4D fermions is (for $kr_c \simeq 12$):

$$\begin{aligned} g^{(n)}/g &\simeq \quad 8.4 \quad \text{for the TeV boundary} \\ g^{(n)}/g &\simeq \quad .2/\sqrt{n} \quad \text{for the } M_P \text{ boundary} \end{aligned} \tag{12}$$

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The strong coupling of the KK gauge modes to fermions located on the TeV brane puts a restrictive constraint on this set-up. In the limit where the KK tower exchanges can be described as a set of contact interactions, they lead to dimension six operators which can be constrained by electroweak precision data, yielding a bound on the mass of the first gauge boson KK mode of order 20 TeV [11, 13]. Such a large scale may be a problem for the consistency of the theory, and seems to disfavor this scenario. However it is interesting to consider it further, because the simplicity of the model allows to make definite calculations that illustrate generic features of dynamical symmetry breaking in warped compactifications.

We have seen that the KK excitations of the SM gauge bosons couple strongly to the fermions located at the TeV brane and thus can produce bound states. The analysis for bulk fermions is more involved, however it has been shown that the fermion zero-mode couples strongly to the first KK gauge boson excitation, with $g^{(1)}/g \simeq 4.1$ (the coupling for higher n is weaker) [13]. Although this result is obtained in the effective 4D theory, it seems to indicate that the coupling of bulk SM gauge bosons to the bulk fermion Ψ may be also strong enough to form composite states involving the latter.

3 Bound states

In this section we study the formation of bound states in detail. The 5D gauge field theory is non renormalizable, and it remains weakly coupled beneath a local cut-off which depends on the position in the extra dimension, of order $\sim ke^{-ky}$ [15]. Moreover, at energies somewhat larger than this scale the excited gravitons are strongly coupled and string/M-theoretic excitations should appear, which lie outside the domain of the 5D field theory. Therefore near the TeV brane we expect the compositeness scale, Λ, to be given approximately by the cut-off of the effective 5D theory, not far above the scale of the first gauge boson KK excitations. Below the compositeness scale we integrate out the heavy gauge boson KK modes and approximate the resulting effective action by local four-fermion operators. These operators involve both, fermions $\psi(x)$ confined at the TeV brane and the bulk fermion $\Psi(x, y)$.

In order to construct the effective 5D theory which contains four-fermion operators we need the propagator for the bulk gauge bosons, given by

$$\langle 0|A_\mu(x', y')A_\nu(x, y)|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{2k}{N_n^2} e^{ip^\mu(x-x')_\mu} e^{(k|y|+k|y'|)} \sum_n T_n(y') \frac{-ig^{\mu\nu}}{p^\mu p_\mu - M_n^2} T_n(y) \qquad (13)$$

where we have defined

$$T_n(y) \equiv J_1\Big(\lambda_n e^{k|y|}\Big) + \alpha_n Y_1\Big(\lambda_n e^{k|y|}\Big) \ . \qquad (14)$$

Let us first consider the action corresponding to the exchange of gauge boson KK excitations between fermions confined at the TeV brane, $\psi(x)$, which reads

$$\mathcal{S}^{5D} = g_{5D}^2 \int d^4x \int d^4x' \int dy \int dy' \int \frac{d^4q}{(2\pi)^4} \sum_n e^{iq^\mu(x-x')_\mu} \frac{2k}{N_n^2} e^{(k|y|+k|y'|)} T_n(y') \otimes$$

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$$\frac{g^{\mu\nu}}{q^2 - M_n^2} T_n(y)(\bar{\psi}\gamma_\mu T^r\psi)_{x',y'}\,\delta(y' - \pi r_c)\,(\bar{\psi}\gamma_\nu T^r\psi)_{x,y}\,\delta(y - \pi r_c) \tag{15}$$

where the five dimensional gauge coupling is related to the effective four dimensional one g by $g_{5\mathrm{D}} = g\sqrt{\pi r_c}$.

At scales Λ above M_1 the action contains both light gauge boson KK modes and four-fermion operators obtained by integrating out the KK gauge bosons heavier than Λ, given by

$$
\begin{aligned}
\mathcal{S}_{eff}^{5D} &= -\sum_n g_{5\mathrm{D}}^2 \frac{1}{M_n^2}\int d^4x \int dy\, 2k\, e^{k|y|}\frac{T_n(y)}{N_n}(\bar{\psi}\gamma^\mu T^r\psi)_{x,y}\,\delta(y - \pi r_c)\otimes \\
&\quad \left\{\int dy'\, e^{k|y'|}\frac{T_n(y')}{N_n}\,(\bar{\psi}\gamma_\mu T^r\psi)_{x,y'}\,\delta(y' - \pi r_c)\right\}\,,
\end{aligned}
\tag{16}
$$

with the sum over KK modes starting at $M_n \gtrsim \Lambda$. The integration in y' is trivial due to the delta function. Since we evaluate $T_n(y)$ at the TeV brane, we can neglect the term proportional to Y_1 in (14). Using the approximate form of the normalization constant N_n given in eq. (11) and after Fierz transform, we obtain the familiar form of a Nambu-Jona-Lasinio interaction

$$\mathcal{L}_{eff}^{5D} \supset \frac{c}{\Lambda^2}\,\delta(y - \pi r_c)(\bar{\psi}_R\psi_L)(\bar{\psi}_L\psi_R)\,, \tag{17}$$

where we have approximated the sum over KK modes by

$$\sum_n \frac{1}{M_n^2} \sim \frac{1}{\Lambda^2}\,. \tag{18}$$

For the $SU(N)$ gauge groups, the coefficient c is given by

$$c = 3k\pi r_c g_N^2 \left\{C_2(\bar{\psi}_L) + C_2(\psi_R) - C_2(\bar{\psi}_L\psi_R)\right\} \tag{19}$$

where g_N is the 4D coupling constant of $SU(N)$ and $C_2(r)$ the second Casimir invariant for the representation r of the gauge group. For $U(1)_Y$,

$$c = 3k\pi r_c g_1^2 Y_{\bar{\psi}_L} Y_{\psi_R} \tag{20}$$

being g_1 the 4D $U(1)_Y$ coupling and Y_r the hypercharge of the fermion r.

Obviously, four-fermion operators involving quarks (the left-handed $SU(2)_W$ doublet ψ_L^3 and the right-handed bottom d_R^3) are more strongly coupled and therefore more likely to form bound states. The most attractive channels are scalars: an $SU(2)_W$ doublet $\bar{\psi}_L^3 d_R^3$ and a charged color triplet $\bar{\psi}_L^3\psi_R^{3c}$, with binding strength proportional to $3k\pi r_c(\frac{4}{3}g_3^2 - \frac{1}{18}g_1^2)$ and $3k\pi r_c(\frac{2}{3}g_3^2 + \frac{3}{4}g_2^2 - \frac{1}{36}g_1^2)$, respectively.

In [3, 4] it was shown that there is a critical value of the four-quark operator coefficient above which this attractive interaction gives rise to a bound state. In the large N_c limit, the critical value

is $8\pi^2/N_c$. For $kr_c \sim 11-12$, we easily obtain four-quark operator couplings larger than the critical one, thus there are bound states made of quarks located on the TeV brane. This is an expected result, because we have seen that the coupling of the excited modes of bulk gauge bosons to TeV brane fermions is stronger than the zero mode coupling.

Notice that the binding strength of the four-quark operators is supercritical for a wide range of the gauge couplings g_i. This is reassuring, because it means that the formation of bound states is not very sensitive to the running of the couplings above the TeV scale, which is still an open question in the Randall-Sundrum scenario [15],[18].

If these bound states acquire vevs we have to face some phenomenological problems: $\langle \bar{\psi}_L^3 \psi_L^{3c} \rangle$ would break charge and color, and since the Yukawa coupling of the doublet $\bar{\psi}_L^3 d_R^3$ to its constituents is typically large, the bottom quark would be too heavy, as it occurs in four dimensional top condensate models [3, 4, 8]. We will address these issues in the next section.

Let us consider now the action corresponding to the exchange of gauge boson KK excitations among fermions confined at the TeV brane and the bulk fermion $\Psi(x, y)$,

$$\mathcal{S}^{5D} = g_{5\mathrm{D}}^2 \int d^4x \int d^4x' \int dy \int dy' \int \frac{d^4q}{(2\pi)^4} \sum_n e^{iq^\mu(x-x')_\mu} \frac{2k}{N_n^2} e^{(k|y|+k|y'|)} \otimes$$

$$T_n(y') \frac{g^{\mu\nu}}{q^2 - M_n^2} T_n(y) (\bar{\Psi}\gamma_\mu T^r \Psi)_{x',y'} \, (\bar{\psi}\gamma_\nu T^r \psi)_{x,y} \, \delta(y - \pi r_c) \tag{21}$$

In order to approximate this non-local interaction by four-fermion operators we use an interesting property of the Randall-Sundrum model: the warp factor produces a shift of a bulk field wave function depending on its 5D mass, so that massless fermionic fields are localized near the TeV brane, as can be seen from the 4D KK decomposition of fermion fields (4). Thus we can approximate the integral over the fifth coordinate y' by a delta function $\delta(y' - \pi r_c)$. This is just an approximation, but it is justified by the warp factor, a feature not present in flat space scenarios.

Analogously to the brane fermion case, the effective action at the scale Λ is obtained by integrating out the gauge boson KK modes heavier than Λ, and after Fierz transform we get

$$\mathcal{L}_{eff}^{5D} \supset \frac{c}{k\,\Lambda^2} \, \delta(y - \pi r_c) \, (\bar{\Psi}_R \psi_L^3)(\bar{\psi}_L^3 \Psi_R) \, , \tag{22}$$

with c as defined in eqs. (19) and (20). The factor $1/k$ appears for dimensional reasons, but it will be compensated by the normalization constant of the bulk fermion field, eqs. (3)-(5).

The light bound states would be four-dimensional scalars, namely $H \propto \bar{\psi}_L^3 \Psi_R$, which we will identify with the Higgs doublet, and $\bar{\Psi}_R d_L^{3c}$. The binding strength of the corresponding four-quark operators is $3k\pi r_c(\frac{4}{3}g_3^2 + \frac{1}{9}g_1^2)$ and $3k\pi r_c(\frac{2}{3}g_3^2 + \frac{2}{9}g_1^2)$, so H is more deeply bound than the $SU(2)_W$ singlet $\bar{\Psi}_R d_L^{3c}$. The Ψ field in eq.(22) is five dimensional (although the delta function forces $y = \pi r_c$), so it is not possible to apply the result of [4] on the critical coupling in a straightforward manner, because it was obtained by solving the gap equation in four dimensions. However, since the bulk

fermion is localized in the region where the first KK mode of the gauge bosons couples strongly to the fermion zero-mode, we expect that these bound states are also produced. We assume that they are, and proceed with the analysis of the phenomenological implications of this scenario.

Finally, there is also the possibility of forming a five-dimensional gauge singlet composite scalar, $\bar{\Psi}\Psi$. However since its wave function vanishes at the TeV boundary, there is no quartic coupling involving this singlet and the previously discussed four-dimensional bound states, and thus it has no effect in the low energy effective theory.

Therefore in the Randall-Sundrum scenario, with only one extra dimension, strongly coupled four-quark operators are naturally induced and give rise to 4D bound states. The next step is to compute the parameters of the composite fields produced by this condensation of quark pairs.

4 Effective theory in 5D

To derive the low energy effective Lagrangian we follow the procedure of ref.[4]. We use the auxiliary field method to construct scalar fields from a pair of quarks and we assume that they become propagating degrees of freedom below the compositeness scale, Λ. We consider just the most deeply-bound channels, which lead to the lightest scalars, more likely to get negative squared-masses and acquire vevs. Thus we define

$$H = -\frac{1}{\Lambda^2}\sqrt{\frac{c_H}{k}}(\bar{\psi}_L^3 \Psi_R) \tag{23}$$

and

$$\phi = -\frac{1}{\Lambda^2}\sqrt{c_\phi}(\bar{\psi}_L^3 d_R^3) \ , \tag{24}$$

where $c_H = 3\pi k r_c(\frac{4}{3}g_3^2 + \frac{1}{9}g_1^2)$ and $c_\phi = 3\pi k r_c(\frac{4}{3}g_3^2 - \frac{1}{18}g_1^2)$. Introducing these definitions in (17) and (22) we obtain the following effective Lagrangian at the compositeness scale

$$\mathcal{L}^{5D}[\Lambda] = -\delta(y - \pi r_c)\left\{\sqrt{\frac{c_H}{k}}(\bar{\Psi}_R \psi_L^3)H + \Lambda^2 H^\dagger H - \sqrt{c_\phi}(\bar{d}_R^3 \psi_L^3)\phi + \Lambda^2 \phi^\dagger \phi\right\} \ . \tag{25}$$

At scales $\mu < \Lambda$, the Yukawa interactions will induce kinetic terms for the scalars, as well as an effective potential which includes mass and quartic terms:

$$\mathcal{L}^{5D}[\mu] = \delta(y - \pi r_c)\left\{Z_H(\mu)D^\nu H^\dagger D_\nu H + Z_\phi(\mu)\partial^\nu \phi^\dagger \partial_\nu \phi - \sqrt{\frac{c_H}{k}}H\bar{\psi}_L^3 \Psi_R - \sqrt{c_\phi}(\bar{d}_R^3 \psi_L^3)\phi - V(\mu)\right\} \tag{26}$$

where

$$V(\mu) = \delta(y - \pi r_c)\left\{M_H^2 H^\dagger H + \frac{\lambda_H}{2}(H^\dagger H)^2 + m_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2}(\phi^\dagger \phi)^2 + \lambda_{\phi H} H^\dagger H \phi^\dagger \phi\right\} \tag{27}$$

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We compute the parameters of the effective Lagrangian $\mathcal{L}^{5D}[\mu]$ in the large N_c limit, where only one fermion loop contributes. We thus need the bulk fermion propagator, given by

$$
\begin{aligned}
\langle 0|\Psi(x',y')\bar{\Psi}(x,y)|0\rangle &= \int \frac{d^4p}{(2\pi)^4} e^{ip^\mu(x-x')_\mu} \frac{2k}{1-e^{-\pi kr_c}} e^{-\frac{k}{2}(|\pi r_c - y| + |\pi r_c - y'|)} e^{-\frac{3}{2}(k|y|+k|y'|)} \frac{i}{1+\delta_{n0}} \otimes \\
&\quad \sum_n (c_n(y')P_R + s_n(y')P_L) \frac{\gamma^\mu p_\mu + \gamma_5 m_n}{p^\mu p_\mu - m_n^2} (s_n(y)P_R + c_n(y)P_L) \qquad (28)
\end{aligned}
$$

where $c_n(y) = \cos\left\{\frac{m_n}{k}\left(e^{k|y|}-1\right)\right\}$ and $s_n(y)$ is defined analogously. At leading order in the $1/N_c$ expansion the mass parameters are

$$
M_H^2 = \Lambda^2 - N_c c_H \frac{4}{1-e^{-\pi kr_c}} \sum_n \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k_\nu k^\nu - m_n^2} \qquad (29)
$$

$$
m_\phi^2 = \Lambda^2 - 4N_c c_\phi \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k_\nu k^\nu} \qquad (30)
$$

The above integrals are defined with a cutoff at the compositeness scale Λ. Assuming that the number of KK modes at the scale Λ, $n_{KK}(\Lambda) = (e^{\pi kr_c}-1)\Lambda/\pi k$, is large enough, we can approximate the sum over KK modes by an integral, $\sum_n \to \int dn$. Using that

$$
\frac{-i}{8\pi^2} \int_0^{n_{KK}} dn \int_0^1 dp \frac{p}{p^2 + (m_n/\Lambda)^2} = -\frac{in_{KK}}{16\pi^2} f_1 \qquad (31)
$$

where

$$
f_1 = \int_0^1 dq \int_0^1 p^2 dp^2 \frac{1}{p^2 + q^2} \sim 0.63 . \qquad (32)
$$

we obtain

$$
M_H^2 = \Lambda^2 \left(1 - n_{KK}(\Lambda)N_c \frac{c_H}{4\pi^2} \frac{f_1}{1-e^{-\pi kr_c}}\right) \qquad (33)
$$

$$
m_\phi^2 = \Lambda^2 \left(1 - N_c \frac{c_\phi}{4\pi^2}\right) \qquad (34)
$$

From eqs. (33),(34) we see that the Higgs mass M_H^2 can be negative while m_ϕ^2 remains positive, because the contribution of the bulk fermion KK modes in the loop decreases M_H^2. Since the compositeness scale Λ is above the electroweak scale, c_H should be close to the critical value for which M_H^2 becomes negative. In this region, H acquires a vev and provides a positive contribution to m_ϕ^2 through their coupling, $\lambda_{\phi H}$. The resulting mass m_ϕ^2 is then expected to stay positive and large. By the same argument, other bound states less deeply-bound than H and ϕ will also have positive squared-masses, preventing charge and color breaking, as well as a too heavy bottom quark. Therefore, naturally we obtain that the only composite field which acquires a vev is $H \propto \bar{\psi}_L^3 \Psi_R$,

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due to the KK excitations of one of its constituents. These excitations will give also a natural framework for a light top mass, as we will see in the next section.

The wave function renormalization and self-coupling of the Higgs in the large N_c limit are given by

$$Z_H = N_c c_H \frac{2}{1 - e^{-\pi k r_c}} \sum_n \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{k_\mu k^\mu (k_\nu k^\nu - m_n^2)} \tag{35}$$

$$\lambda_H = 8 N_c c_H^2 \left(\frac{1}{1 - e^{-\pi k r_c}} \right)^2 \sum_{n_1, n_2} \int \frac{d^4 k}{(2\pi)^4} (-1)^{n_1 + n_2} \frac{-i}{(k_\nu k^\nu - m_{n_1}^2)(k_\nu k^\nu - m_{n_2}^2)} \, . \tag{36}$$

Again, we approximate the sums over KK modes by integrals and we find

$$Z_H = N_c c_H \frac{1}{1 - e^{-\pi k r_c}} \frac{n_{KK}(\Lambda)}{8\pi^2} f_2 \tag{37}$$

$$\lambda_H = n_{KK}(\Lambda)^2 \frac{N_c}{2\pi^2} \left(\frac{c_H}{1 - e^{-\pi k r_c}} \right)^2 f_3 \tag{38}$$

where [1]

$$f_2 = \int_0^1 dq \int_0^1 dp^2 \frac{1}{p^2 + q^2} \sim 2.26 \tag{39}$$

$$f_3 = \int_0^1 dq \int_0^1 dq' \int_0^1 p^2 dp^2 \frac{1}{(p^2 + q^2)(p^2 + q'^2)} \sim 2.71 \, . \tag{40}$$

Redefining $H \to \sqrt{Z_H} H$ to obtain a canonical kinetic term, the Higgs mass becomes

$$\bar{M}_H^2 = \frac{M_H^2}{Z_H} = \frac{2\Lambda^2}{f_2} \left(\frac{4\pi^2 (1 - e^{-\pi k r_c})}{N_c c_H n_{KK}} - f_1 \right) \tag{41}$$

and the quartic coupling

$$\bar{\lambda}_H = \frac{\lambda_H}{Z_H^2} = \frac{32\pi^2}{N_c} \frac{f_3}{f_2^2} \sim 60 \, . \tag{42}$$

In this scenario we have a heavy Higgs boson with a large (non-perturbative) quartic coupling, which is generic from top mode scenarios. Having a heavy Higgs boson is not in contradiction with data, nor with triviality bounds because the cut-off of the effective theory, the compositeness scale, is very low. Regarding the unitarity bound from the longitudinal WW scattering cross section, the

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[1] Note that these integrals are the same of [8] with $f_1 = F_3(L), f_2 = F_1(L), f_3 = F_5(L)$, where $L \sim (TeV)^{-1}$ is the length of the fifth dimension

Higgs mass we obtain at tree level, $m_H = v/\sqrt{\lambda_H}$, is above this bound. However, with such a large quartic self coupling we cannot trust the tree level relation between the mass and the vev of the Higgs, and we can only take it as a rough estimate of the Higgs mass.

Next-to-leading order corrections in the N_c expansion (i.e., contributions from gauge bosons and composite scalar loops) could be included by evolving the couplings from the compositeness scale Λ down to the electroweak scale. To do so, we would need the β-functions for the four-dimensional SM couplings in the Randall-Sundrum scenario, which have not been computed yet. However we do not expect that next-to-leading effects would change qualitatively our results.

5   Fermion Masses

In this section we discuss the generation of fermion masses within the simplest set-up described in sec. 2. First, we calculate the top quark mass and then we include the other two generations in the model.

5.1   Top Mass

To compute the top quark mass we have to canonically normalize the Higgs kinetic term in the effective 5D Lagrangian (26) and integrate $\mathcal{L}_{eff}^{5D}$ over the fifth dimension y to obtain the 4D effective Lagrangian. We find

$$\mathcal{L}^{4D} \supset -y_t H \bar{\Psi}_R(x, \pi r_c)\psi_L^3(x) \tag{43}$$

where the top Yukawa coupling y_t is given by

$$y_t = \sqrt{\frac{c_H}{Z_H(1 - e^{-\pi k r_c})}} = \frac{2\sqrt{2}\pi}{\sqrt{N_c n_{KK} f_1}} \ . \tag{44}$$

Recall that n_{KK} is the number of KK modes produced at the compositeness scale Λ. Thus we obtain that the top Yukawa coupling is suppressed by the factor $\sqrt{n_{KK}}$, and turns out to be order one for $n_{KK} \sim 10$, i.e., $\Lambda \sim 10\,M_1$, with M_1 the mass of the first gauge boson KK excitation. This result is also generic in top condensate models with flat extra dimensions [8, 9], avoiding a too heavy top quark typical of four dimensional top condensate models.

The zero mode of $\Psi_R(x, \pi r_c)$ becomes the four-dimensional right-handed top quark, t_R. Taking into account the KK decomposition of the bulk fermion (4), when the Higgs acquires a vev $\langle H^0 \rangle = v/\sqrt{2}$ we obtain the following mass matrix for the t_L component of ψ_L and the Ψ KK modes:

$$
\begin{pmatrix} \bar{t}_L & \bar{\Psi}_L^{(1)} & \bar{\Psi}_L^{(2)} & \ldots \end{pmatrix}
\begin{pmatrix}
y_t \frac{v}{\sqrt{2}} & y_t v & y_t v & y_t v & \ldots \\
0 & \frac{\pi k}{e^{\pi k r_c}-1} & 0 & 0 & \ldots \\
0 & 0 & \frac{2\pi k}{e^{\pi k r_c}-1} & 0 & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots
\end{pmatrix}
\begin{pmatrix} t_R \\ \Psi_R^{(1)} \\ \Psi_R^{(2)} \\ \ldots \end{pmatrix}
\tag{45}
$$

The diagonal entries are the usual KK mass terms. The top mass is given by the lowest eigenvalue of this matrix. To compute it, we can perturbatively diagonalize the mass matrix, using that $m_n = n\pi k e^{-\pi k r_c} > v$ because KK excitations have not been observed at the moment. Thus, an expansion in $\left(\frac{v}{\pi k e^{-\pi k r_c}}\right)^2 \ll 1$ is justified and within this approximation the lowest eigenvalue is

$$m_t = \frac{y_t v}{\sqrt{2}} \left[1 - \mathcal{O}\left(\frac{v}{\pi k e^{-\pi k r_c}}\right)^2 \right] , \tag{46}$$

which gives easily a top mass in the experimental range ~ 170 GeV for $n_{KK} \sim 10$.

5.2 Flavor Symmetry Breaking

Once we have accommodated the electroweak symmetry breaking scale and the top mass, the next step is to generate masses for the remaining quarks and leptons of the Standard Model. The simplest possibility is that the fermions of the first two generations are confined at the TeV brane. Then, there would be three degenerate Higgs doublets $H^i \propto \bar{\psi}_L^i \Psi_R$, $(i = 1, 2, 3)$, which obtain a vev and break the $U(2)_\psi \times U(2)_u$ flavor symmetry, leading to two Nambu-Goldstone bosons, besides the one eaten by the electroweak gauge bosons. Obviously, it is necessary a source of flavor symmetry breaking. When fermions are located near the TeV brane, higher-dimensional operators are suppressed by the multi-TeV scale, so generically there would be four-quark operators of the type [8, 9]

$$\frac{\eta_i}{\Lambda^2}(\bar{\psi}_L^i \Psi_R)(\bar{\Psi}_R \psi_L^i) , \tag{47}$$

where the coefficients η_i are $\mathcal{O}(1)$ and can be treated perturbatively. We have seen that the squared-mass of a composite field depends on the strength of the interaction between its constituents (see eq.(33)), thus assuming that the coefficient η_3 is slightly larger than the others, it is possible that only H^3 gets a vev, giving a mass of the electroweak scale just to the top quark. The rest of bound states may be quite heavy, even with small differences among the flavor symmetry breaking coefficients η_i.

Of course, if all higher-dimensional operators consistent with the SM symmetry are only suppressed by the multi-TeV scale, flavor-changing effects and proton decay become a problem, much as in the original Randall-Sundrum set-up. We do not attempt to solve these problems here, and we just assume that dangerous flavor-changing and baryon number violating operators are suppressed by some mechanism of the underlying fundamental theory.

If the only light composite scalar is H^3, to produce the observed pattern of fermion masses and mixings we shall consider the presence of the effective operators

$$\frac{1}{\Lambda^2}(\bar{\Psi}_R \psi_L^3)\left(\lambda_{ij}^u \bar{\psi}_L^i u_R^j + \lambda_{ij}^d \bar{\psi}_L^i i\sigma_2 d_R^j + \lambda_{ij}^e \bar{l}_L^i i\sigma_2 e_R^j\right) , \tag{48}$$

which would lead to Yukawa couplings of the H^3 doublet to the SM fermions. As usual, large hierarchies in the coefficients λ_{ij} should be assumed in order to explain the observed fermion masses.

6 The Standard Model in the Bulk

The minimal model studied in the previous sections should be regarded as a 'working proof' of dynamical symmetry breaking in the Randall-Sundrum scenario, but it is not by any means unique. A more natural possibility is that all SM fermions propagate in the 5D bulk. In this case, the approximation of the higher-dimensional gauge interactions by local four-fermion operators is questionable and the issue of whether dynamical symmetry breaking really takes place or not is highly non trivial [19]. A careful study of such dynamical problem is beyond the scope of this paper, but given the strong coupling between gauge boson KK modes and the zero mode of fermions localized near the TeV brane, we argue that the condensation of 5D massless quarks seems very likely.

The localization of fermion fields living in a slice of AdS_5 depends on its 5D masses [14]. Left-handed (right-handed) zero modes of fermions with bulk mass terms $M_{5D}/k < 1/2$ $(M_{5D}/k > -1/2)$ (in particular massless fermions, as we have seen) live near the TeV brane, while the left-handed (right-handed) zero modes of fermions with $M_{5D}/k > 1/2$ $(M_{5D}/k < -1/2)$ are localized near the Planck boundary. For left-handed (right-handed) zero modes $M_{5D}/k = 1/2$ $(M_{5D}/k = -1/2)$ corresponds to the conformal limit, and in this case the zero mode is flat. On the other hand, KK modes of gauge bosons are always located near the TeV brane and therefore they will couple more strongly to light fermions [2]. We thus assume that the left-handed third generation quarks Ψ^3 have $M_{5D}/k \lesssim 1/2$ and the right-handed top quark U^3 has $M_{5D}/k \gtrsim -1/2$, and due to the strong coupling to gauge boson KK modes they form bound states. On the contrary, if the remaining SM left-handed (right-handed) fermions have 5D mass terms $M_{5D}/k \gtrsim 1/2$ $(M_{5D}/k \lesssim -1/2)$, their zero modes live near the Planck brane, where the coupling to gauge boson KK modes is too weak to produce composite states.

Under these assumptions, we expect that the SM gauge interactions in the bulk will induce only condensates of third generation quarks Ψ^3 and U^3. Using the most attractive channel analysis [9] one finds that the channel $H = \bar{\Psi}^3 U^3$ is the most attractive one among those which transform non-trivially under the SM group, but there would be gauge-singlet scalars more tightly bound than H, namely $\bar{\Psi}^3 \Psi^3$ and $\bar{U}^3 U^3$. As a consequence, these singlets would acquire vevs which do not break any gauge symmetry and provide a positive contribution to the squared-mass of the $SU(2)_W$ doublet H. In this case, it is not clear whether the scalar effective potential is minimized by a nonzero vev of H; moreover, simple estimates are not reliable due to the non-perturbative nature of the quark condensation. To avoid this problem we could incorporate more Z_2 symmetries during the orbifold projection, preventing the existence of singlet zero modes in the KK decomposition [9]. Thus the lightest modes of the singlets would have squared masses less negative, allowing electroweak symmetry breaking.

If the composite Higgs H acquires a vev, it will provide electroweak scale masses for the gauge bosons and the top quark. It is important to note that since the composite Higgs has a 5D mass

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[2]In particular, at the conformal limit KK gauge bosons do not couple to the fermion zero mode, because the five-momentum is conserved in this limit.

of order TeV, it is also localized near the TeV boundary. In order to explain the SM fermionic spectrum we have to assume the presence of four-fermion operators of the type (48). However now the 5D Yukawa couplings can be all of order $\lambda_{ij}^{(5)} k \sim 1$, because the hierarchy of masses is originated by the exponentially small overlap of the fermions localized near the Planck brane with the composite Higgs field, much as in the model of ref.[14] with a fundamental SM Higgs localized on the TeV brane. There, it is shown that the spectrum of the SM fermionic sector can be naturally generated just assuming small splittings in the bulk fermion masses, all in the range $|M_{5D}|/k \sim 1$. We refer the reader to [14] for details. This idea of using the warp factor to explain fermion mass hierarchies has also been consider in [12] in the context of neutrino masses.

Finally, we should point out that in this scenario FCNC dangerous operators are safely suppressed, but proton decay is still a problem [14].

7   Conclusions

We have studied dynamical electroweak symmetry breaking in the Randall-Sundrum scenario and we have shown that it is possible to break the electroweak symmetry using the ideas of top condensate mode in the context of one warped extra dimension, with no fundamental fields beyond the SM gauge bosons and fermions living in the 5D bulk.

In warped compactifications, the coupling of gauge boson KK excitations to fermions depends on the position of the fermion field in the bulk. In particular, there is a sizeable enhancement of their couplings to fermions localized near the TeV brane, as compared with the gauge boson zero mode coupling. Such a strong interaction produces quark pair condensates, some of which can acquire vevs and break dynamically the electroweak symmetry. Contrary to what happens in flat extra dimensions [8], we do not have to rely on the degeneracy of KK modes in $D \geq 6$ dimensions to trigger dynamical symmetry breaking.

We have considered a minimal model, where only gauge bosons and the right-handed top quark propagate in the 5D bulk, while all the other SM fermions are confined on the TeV brane. The Higgs boson emerges as a bound state of the left-handed third generation quarks and the right-handed top. The simplicity of the model allows to perform reliable calculations within the four-fermion operator approximation. We have computed, at leading order in the N_c expansion, the parameters of the composite scalar effective theory and the top quark mass. The reason why at least the right-handed top quark must live in the bulk is twofold. First, its KK excitations active at the compositeness scale give a large and negative contribution to the squared-mass of the $SU(2)_W$ doublet composite Higgs, which then acquires a vev and breaks the electroweak symmetry. Second, the number of its active KK excitations suppresses the top quark Yukawa coupling, leading to a top mass within the experimental range. Although our calculation only includes the leading N_c contribution, it shows that dynamical electroweak symmetry breaking is feasible in the Randall-Sundrum model, and we do not expect that next-to-leading order corrections will invalidate this conclusion.

We have also considered a more natural scenario, in which all SM fermions as well as the gauge

bosons propagate in the bulk. In this case, a local four-fermion approximation of the strong interactions mediated by KK gauge boson modes is not justified, and we can only describe qualitatively the expected features of the model. Interestingly enough, just assuming that the light fermions are localized near the Planck brane and the heavy ones near the TeV boundary, we argue that only composite states bound out of third generation quarks are produced and we can explain the fermion mass hierarchies. The question of whether the composite Higgs doublet does acquire a nonzero vev requires further investigation.

Acknowledgments

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