

# Supersymmetric Electroweak Baryogenesis

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## Abstract

We calculate the baryon asymmetry generated at the electroweak phase transition in the minimal supersymmetric standard model, using a new method to compute the CP-violating asymmetry in the Higgsino flux reflected into the unbroken phase. The method is based on a Higgs insertion expansion. We find that the CP asymmetry at leading order is proportional to the change in  $\tan\beta$  in the bubble wall, which is at most of order  $10^{-2}$ , while at next-to-leading order this suppression factor disappears. This result may enhance the final baryon asymmetry generated during the electroweak phase transition for small  $\Delta\beta$  ( $< 10^{-3}$ ).

# 1 Introduction

The baryon to entropy ratio in the observed part of the Universe is constrained to be  $n_B/s \sim 10^{-10}$  by primordial nucleosynthesis measurements [1]. Sakharov [2] established more than thirty years ago the three basic requirements for obtaining this baryon asymmetry as a result of particle interactions in the early universe, namely baryon number violation, C and CP violation and departure from thermal equilibrium. These conditions may be satisfied at weak scale temperatures, if the electroweak phase transition is first order [3]. Electroweak baryogenesis provides an explanation of the observed baryon asymmetry of the Universe (BAU) in terms of experimentally accessible physics, hence much attention has been devoted to the study of this possibility [4]-[16].

Although the Standard Model contains all the necessary ingredients for electroweak baryogenesis, the phase transition is too weakly first order to avoid the wash out of the generated baryon asymmetry, for the Higgs mass experimentally allowed [17]. Moreover, the CP asymmetry induced by the Kobayashi-Maskawa phase is far too small to account for the observed  $n_B/s$  ratio [6, 7]. Therefore, for the baryon asymmetry to be generated at the electroweak phase transition, new physics is required at the weak scale.

Among the different extensions of the Standard Model, low energy supersymmetry is a well motivated possibility, and thus several groups have recently studied under which conditions electroweak baryogenesis is feasible in the framework of the Minimal Supersymmetric Standard Model (MSSM). Regarding the strength of the phase transition, a region in the space of supersymmetric parameters has been found where the phase transition is strong enough to avoid the wash out of the generated baryon asymmetry [18]-[28]. Such region corresponds to a light Higgs boson and a light top squark, within the reach of LEP2 and Tevatron colliders. As for the baryon asymmetry, in the region of parameter space favored by the previous phase transition studies, it is mainly generated by charginos and neutralinos, provided they are not much heavier than the critical temperature ( $T_c \sim 100$  GeV)[11, 13].

The physics of the mechanism which produces the baryon asymmetry is essentially agreed upon, namely particles in the plasma interact with the bubble wall and due to CP-violation lead to a chiral asymmetry of fermions in front of the wall, which in turn biases sphaleron processes to produce the BAU. Several groups have estimated the baryon asymmetry generated at the electroweak phase transition in the MSSM: in refs.[11, 13], CP-violating source terms were computed and then inserted into a set of diffusion equations, with a prescription not motivated by first principles. Subsequently, the correct definition of the CP-violating sources was self-consistently derived in [16], using the closed time-path formalism to write down a set of quantum Boltzmann equations describing the local particle densities. The CP-violating sources were computed using a mass expansion [11] and a Higgs insertion expansion [13, 16]. In [15], the diffusion equations and the source terms were derived together within the WKB quasi-particle approximation.

In [11, 13, 16], the source terms are proportional to  $v_2(z)\partial_z v_1(z) - v_1(z)\partial_z v_2(z)$ , which is zero for  $\tan\beta = v_2/v_1$  constant in the bubble wall. It has been shown [29, 28] that the angle  $\beta$  varies at most by a few percent over the wall in the MSSM, thus it results in an important suppression factor. On the other hand, in [11] it was mentioned that for charginos and neutralinos such  $\Delta\beta$  dependence could disappear at higher orders in the mass expansion used to calculate the source terms, and in [14], CP asymmetries of the reflection and transmission amplitudes for charginos are computed numerically, and they are non-zero for constant  $\tan\beta$  <sup>1</sup>.

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<sup>1</sup>The authors of [15] also find a non-vanishing Higgsino source term for constant  $\tan\beta$ , but they consider a different

Motivated by this discrepancy and the possibility that it is a consequence of the lowest order approximation, we have performed a new computation of the baryon asymmetry generated by charginos at the electroweak phase transition within the MSSM. Although we introduce a different method to compute the CP-violating source terms, it is based on a Higgs insertion expansion similar to the one employed in [13, 16], thus we expect that our conclusion will also apply to that approach.

The remainder of the paper is structured as follows. In section 2 we describe the method used to compute the CP-violating asymmetries, and present the results for Higgsinos at leading and next-to-leading order in the Higgs insertion expansion. In section 3 we compute the baryon number induced by the Higgsino current, and we conclude in section 4.

## 2 CP asymmetry for Higgsinos

Baryogenesis is fueled by CP asymmetries induced by the advancing bubble wall. Unremovable CP-odd phases appear in the mass matrices due to either

a) CP-violating interactions in the thermal loops that correct the dispersion relations of particles propagating in the plasma [5].

b) space-time dependence of the scalar vevs inside the bubble wall (for more than one Higgs field), which induces space-dependent CP-violating phases. These phases cannot be rotated away in two adjacent points by the same unitary transformation [11].

When present, the second mechanism dominates over the first one, since in the first mechanism there are suppression factors coming from loops. While in the Standard Model only the first mechanism is possible, the second one controls the generation of the baryon asymmetry in all the extensions of the Standard Model proposed in the literature for electroweak baryogenesis. Thus, particle mass matrices acquire a non-trivial space dependence when bubbles of the broken phase nucleate and expand during a first order electroweak phase transition. This provides fast non-equilibrium CP-violating effects inside the bubble walls (thick walls) or in front of the bubble walls (thin walls), and may generate a baryon asymmetry through the anomalous ( $B + L$ )-violating processes when particles diffuse to the exterior of the bubble, in the unbroken phase.

In what follows, we will focus on CP-violating effects in the Higgsino current, since they make the dominant contribution in the region of the MSSM parameter space preferred by phase transition studies [13]. However the method presented here is completely general, and may be applied to any particle whose mass depends on the scalar vevs. We will perform the calculation in the thin wall regime ( $L_w \lesssim \ell$ , where  $L_w$  is the bubble wall width and  $\ell$  the mean free path of the particle). This is a reasonable approximation for weakly interacting particles, such as charginos. For bubble walls thinner than the mean free path, the incoming fermions interact with the bubble wall like quantum mechanical particles scattering from a potential barrier. CP-violating interactions with the scalar field result in different reflection probability for fermions of a given chirality and their corresponding antifermions, leading to a CP asymmetry in the reflected chiral number flux [4].

The chargino mass matrix is, in the basis of Winos and Higgsinos ( $\tilde{W}, \tilde{H}$ ),

$$M_\chi = \begin{pmatrix} m & gv_2/\sqrt{2} \\ gv_1/\sqrt{2} & \mu \end{pmatrix}, \quad (1)$$

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Higgsino density [30], namely the difference of the two helicity states instead of the sum, which is the one considered by all the other groups, including us.

with  $v_i$  the spatially varying Higgs field vevs and  $m, \mu$  the soft supersymmetry breaking parameters, which contain one physical CP violating phase ( $\mu = |\mu|e^{i\phi}$ ). In the symmetric phase ( $v_i = 0$ ), Winos and Higgsinos are the mass eigenstates, while in the bubble wall and the broken phase they mix to form the mass eigenstates, denoted  $\tilde{\chi}_i, i = 1, 2$ . We treat the wall as planar, and assume it has reached a static configuration in the wall rest frame, with the vevs of the scalar fields being functions only of the  $z$  coordinate.

The quantity of interest is the CP-violating asymmetry in the Higgsino current reflected into the unbroken phase. In the rest frame of the bubble wall, it is given by

$$\begin{aligned}
j_{CP} &= j_{\tilde{W} \rightarrow \tilde{H}}^r + j_{\tilde{H} \rightarrow \tilde{H}}^r + \sum_{i=1}^2 j_{\tilde{\chi}_i \rightarrow \tilde{H}}^{tr} \\
&= \int \frac{d^3 p}{(2\pi)^3} [ |R_{\tilde{W} \rightarrow \tilde{H}}^u(-p_m, p_\mu)|^2 - |\bar{R}_{\tilde{W} \rightarrow \tilde{H}}^u(-p_m, p_\mu)|^2 ] \frac{p_m}{E} \rho^u(E, p_m) \\
&+ \int \frac{d^3 p}{(2\pi)^3} [ |R_{\tilde{H} \rightarrow \tilde{H}}^u(-p_\mu, p_\mu)|^2 - |\bar{R}_{\tilde{H} \rightarrow \tilde{H}}^u(-p_\mu, p_\mu)|^2 ] \frac{p_\mu}{E} \rho^u(E, p_\mu) \\
&+ \int \frac{d^3 p}{(2\pi)^3} \sum_{i=1}^2 [ |T_{\tilde{\chi}_i \rightarrow \tilde{H}}^b(p_i, p_\mu)|^2 - |\bar{T}_{\tilde{\chi}_i \rightarrow \tilde{H}}^b(p_i, p_\mu)|^2 ] \frac{p_i}{E} \rho^b(E, p_i),
\end{aligned} \tag{2}$$

where

$$p_m = \sqrt{E^2 - p_{\parallel}^2 - m^2}, \quad p_\mu = \sqrt{E^2 - p_{\parallel}^2 - \mu^2}, \quad p_i = \sqrt{E^2 - p_{\parallel}^2 - m_i^2} \tag{4}$$

are the absolute value of the  $z$  component of the Wino, Higgsino and chargino momenta, and

$$\rho^u(E, p_z) = \frac{1}{e^{(E+v_w p_z)/T} + 1}, \quad \rho^b(E, p_z) = \frac{1}{e^{(E-v_w p_z)/T} + 1}, \tag{5}$$

are the thermal distributions of the charginos in the unbroken (u) and broken (b) phases, as seen from the rest frame of the wall.  $v_w$  is the wall velocity,  $v_w \sim 0.1$  in the MSSM. The use of equilibrium particle distributions is a good approximation, because any departure from thermal equilibrium is caused by the passage of the wall, and therefore is  $\mathcal{O}(v_w)$ . Since we will see that the final CP asymmetry is already linear in  $v_w$ , working with thermal equilibrium distribution functions amounts to ignore terms of higher order in  $v_w$  [11].

It is well known that one-loop self-energy corrections to the propagator modify the dispersion relations of the particles in the plasma. The main thermal effects are that particles propagating in the plasma acquire an effective mass (even if they are massless in vacuum) and have a finite life-time (damping), due to incoherent thermal scattering with the medium. The damping rate,  $\gamma$ , is defined as (minus) the imaginary part of the solution  $\omega = \omega(\mathbf{k})$  of the dispersion relation.

For the Higgsino and Wino, the effective plasma masses in the thermal bath may be well approximated by their value in the present vacuum ( $m_{\tilde{H}}^2(T) \simeq |\mu|^2, m_{\tilde{W}}^2(T) \simeq m^2$ ). However, it has been shown that the effects of damping can lead to a sizeable suppression of the CP asymmetry [6, 7], due to the loss of coherence of the wave function. The damping rate of Winos and Higgsinos has been estimated in [31] to be  $\gamma_{\tilde{H}} \simeq 0.025T, \gamma_{\tilde{W}} \simeq 0.065T$ , hence the mean free path  $\ell \sim 1/(2\gamma) \sim (10-20)/T$  is comparable to the wall width  $L_w \sim (20-30)/T$  [29], and decoherence effects may be relevant.

The effects of damping may be taken into account by including the imaginary part of the fermion self-energy in the dispersion relation [6, 7, 13]. In our case, this leads to the approximate dispersion relation

$$[\omega(\mathbf{k}) + i\gamma]^2 = \mathbf{k}^2 + m^2. \tag{6}$$

If we choose  $\omega$  to be real, the momenta must become complex in order to satisfy the dispersion relations, and propagation of particles in space is damped. We have taken  $\omega$  to be real because energy is conserved in the scattering off the wall, so the reflection and transmission probabilities are time independent. We could have satisfied the dispersion relations with real momenta and complex  $\omega$ , but then the reflection and transmission amplitudes would have an exponentially decaying time dependence, which would require us to study the time and space dependence of the particle scattering process. To obtain an estimate of the damping effects, from (6) we approximate

$$k \simeq \pm \left\{ \sqrt{\omega^2 - m^2} + i\gamma \frac{\omega}{\sqrt{\omega^2 - m^2}} \right\} \simeq \pm (\sqrt{\omega^2 - m^2} + i\gamma) , \quad (7)$$

where  $k = |\mathbf{k}|$ .

We thus calculate the reflection and transmission amplitudes at zero temperature (except for the damping), using the LSZ reduction formulae in terms of the propagator in the presence of the bubble wall [12]:

$$\begin{aligned} \mathcal{A} &= \int d^4x \int d^4y e^{-iq_i x} e^{iq_f y} \bar{u}(q_f) (i\vec{\partial} - \mu) S(y, x) (-i\vec{\partial} - m) u(q_i) \\ &= (2\pi)^3 \delta(q_f^x - q_i^x) \delta(q_f^y - q_i^y) \delta(E_f - E_i) A(q_i^z, q_f^z), \end{aligned} \quad (8)$$

with

$$S(y, x) \equiv \langle 0 | T[\Psi(y) \bar{\Psi}(x)] | 0 \rangle. \quad (9)$$

An analogous expression holds for antiparticles. The spinors in formula (8) are on-shell and normalized to unit flux in the  $z$  direction, i.e.

$$\bar{u} \gamma_z u = 1. \quad (10)$$

Momenta in the  $x$  and  $y$  directions are conserved, because the potential created by the bubble wall only depends on the  $z$  coordinate. The transmission and reflection amplitudes are then functions only of the momenta in the  $z$  direction and can be computed in a simpler way by first boosting to a frame where  $q_x, q_y = 0$ . With the proper normalization chosen for the spinors (10), the amplitude in the boosted frame is simply given by (8), with the propagator and incoming and outgoing momenta substituted by the boosted ones.

The expression of  $j_{CP}$  can be further simplified by using CPT and unitarity constraints, which imply

$$|R_{\tilde{W} \rightarrow \tilde{H}}^u|^2 + |R_{\tilde{H} \rightarrow \tilde{H}}^u|^2 + \sum_{i=1}^2 |T_{\tilde{\chi}_i \rightarrow \tilde{H}}^b|^2 = 1. \quad (11)$$

Substituting (11) in eq.(3), and expanding the Fermi distributions for small wall velocities we obtain:

$$\begin{aligned} j_{CP} &= \frac{v_w}{T} \int \frac{d^3p}{(2\pi)^3} \left\{ -(p_\mu + p_1) [ |R_{\tilde{H} \rightarrow \tilde{H}}^u(-p_\mu, p_\mu)|^2 - |\bar{R}_{\tilde{H} \rightarrow \tilde{H}}^u(-p_\mu, p_\mu)|^2 ] \right. \\ &\quad - (p_m + p_1) [ |R_{\tilde{W} \rightarrow \tilde{H}}^u(-p_m, p_\mu)|^2 - |\bar{R}_{\tilde{W} \rightarrow \tilde{H}}^u(-p_m, p_\mu)|^2 ] \\ &\quad \left. + (p_2 - p_1) [ |T_{\tilde{\chi}_2 \rightarrow \tilde{H}}^b(p_2, p_\mu)|^2 - |\bar{T}_{\tilde{\chi}_2 \rightarrow \tilde{H}}^b(p_2, p_\mu)|^2 ] \right\} \frac{p_m}{E} \rho(E) [1 - \rho(E)] , \end{aligned} \quad (12)$$

where  $\rho(E) = 1/(e^{E/T} + 1)$  is the Fermi distribution. This result explicitly shows that the out of equilibrium condition needed for baryogenesis is due to the expansion of the bubble wall through the

thermal bath. As anticipated, the CP violating current  $j_{CP}$  is linear in the wall velocity,  $v_w$ , and the use of the equilibrium distributions in the calculation is justified.

In order to compute the amplitudes in eq.(8) we would need the exact propagator in the presence of the wall (9). In ref.[11], the approach was to perform an expansion in powers of mass, which is effectively an expansion in  $M(z)/E$ , but this approximation is not justified, since the region of interest is always  $E \sim M$ . Instead, we perform a Higgs insertion expansion [13], which will be a good approximation at least close to the symmetric phase. By making a phase redefinition of the Higgsino field, we can write the mass matrix as

$$M_\chi(z) = M_\chi^0 + \delta M_\chi(z) \quad (13)$$

where

$$M_\chi^0 = \begin{pmatrix} m & 0 \\ 0 & |\mu| \end{pmatrix} \quad (14)$$

and

$$\delta M_\chi(z) = \begin{pmatrix} 0 & e^{-i\phi} u_2(z) \\ u_1(z) & 0 \end{pmatrix}, \quad (15)$$

with  $u_i(z) = gv_i(z)/\sqrt{2}$ . Now, expanding in  $\delta M_\chi(z)$ , the approximate result for the propagator is

$$S(x_2, x_1) = \int \prod_i dz_i S^{(0)}(x_2, z_1) \delta M_\chi(z_1) S^{(0)}(z_1, z_2) \delta M_\chi(z_2) \dots S^{(0)}(z_i, x_1), \quad (16)$$

where the integration is done over all  $z_i(-\infty, \infty)$  and  $S^{(0)}$  stands for the propagators of Higgsino and Wino in the symmetric phase, at zero temperature. When the approximate dispersion relation (7) is used,  $S^{(0)}$  includes also the damping. In the boosted frame ( $p_x = p_y = 0$ ), for the Wino it is given by

$$\begin{aligned} S^{(0)}(z_2, z_1) &= i \int \frac{dp_z}{2\pi} \frac{e^{ip_z(z_2-z_1)}}{(\omega + i\gamma_{\tilde{W}})^2 - p_z^2 - m^2} (\omega\gamma^0 - p_z\gamma^z + m) \\ &\simeq \frac{1}{2} \left\{ \theta(z_2 - z_1) e^{(ip_m - \gamma_{\tilde{W}})(z_2 - z_1)} \left( \frac{\omega}{p_m} \gamma^0 - \gamma^z + \frac{m}{p_m} \right) \right. \\ &\quad \left. + \theta(z_1 - z_2) e^{-(ip_m - \gamma_{\tilde{W}})(z_2 - z_1)} \left( \frac{\omega}{p_m} \gamma^0 + \gamma^z + \frac{m}{p_m} \right) \right\}. \end{aligned} \quad (17)$$

The Higgsino propagator has an analogous expression, changing  $m \rightarrow |\mu|$ ,  $p_m \rightarrow p_\mu$  and  $\gamma_{\tilde{W}} \rightarrow \gamma_{\tilde{H}}$ . Here,  $\omega$  is the energy in the boosted frame,  $\omega = \sqrt{p_m^2 + m^2} = \sqrt{p_\mu^2 + |\mu|^2}$  and we have only included the damping rate in the exponential factors, where the effect is expected to be more important.

## 2.1 Leading order

The leading CP violating contributions arise at second order in the Higgs insertion expansion,  $\mathcal{O}(v^2)$ , as expected from refs.[11, 13] where this was also the order of the CP violating currents. Since the perturbation matrix  $\delta M_\chi(z)$  is off-diagonal in the Wino-Higgsino basis, at lowest order only Winos can be reflected or transmitted into Higgsinos, and we get

$$R_{\tilde{W} \rightarrow \tilde{H}}^{(1)}(-p_m, p_\mu) = \bar{u}_{\tilde{H}}(q_f) \int_0^\infty dz e^{i(p_m + p_\mu)z} e^{-\gamma z} \{u_1(z)L + u_2(z)e^{i\phi}R\} u_{\tilde{W}}(q_i), \quad (18)$$

where  $\gamma \equiv \gamma_{\tilde{H}} + \gamma_{\tilde{W}}$ ,  $q_i = (\omega, -p_m - i\gamma_{\tilde{W}})$ ,  $q_f = (\omega, p_\mu + i\gamma_{\tilde{H}})$ ,  $\omega$  is the energy in the boosted frame ( $q_x = q_y = 0$ ) and  $L, R$  are the chiral projectors. The result for the transmission amplitude  $T_{\tilde{W} \rightarrow \tilde{H}}^{(1)}(p_m, p_\mu)$  can be obtained just changing  $q_i$  by  $q'_i = (\omega, p_m - i\gamma_{\tilde{W}})$  and  $p_m$  by  $-p_m$  in eq.(18).

Using standard techniques to compute Dirac matrix traces, and substituting the leading order reflection and transmission contributions in eq.(12), we obtain

$$j_{CP}^{(1)} = \frac{v_w}{T} \sin \phi \int \frac{d^3 p}{(2\pi)^3 E} F^{(1)}(p_m) \rho(E) [1 - \rho(E)] , \quad (19)$$

where  $F^{(1)}(p_m) = -(p_m + p_\mu)F_r^{(1)}(p_m) + (p_m - p_\mu)F_t^{(1)}(p_m)$ , being

$$F_r^{(1)}(p_m) = \frac{2m|\mu|}{p_\mu} \int dz_1 dz_2 e^{i(p_m + p_\mu)(z_1 - z_2)} e^{-\gamma(z_1 - z_2)} \{u_1(z_1)u_2(z_2) - u_1(z_2)u_2(z_1)\} \quad (20)$$

the contribution coming from the reflection, and

$$F_t^{(1)}(p_m) = F_r^{(1)}(-p_m) . \quad (21)$$

the transmission one. From the above equations, we see that the CP asymmetry  $j_{CP}^{(1)}$  vanishes if  $\tan \beta = u_2/u_1$  is constant along the bubble wall, in agreement with refs.[11, 13], where the computation of the CP violating currents was done to lowest nontrivial order in the mass and Higgs insertion expansions, respectively.

To obtain the final result we must still specify the shape and speed of the bubble wall. For the wall profile we take the following semirealistic approximation <sup>2</sup> :

$$\begin{aligned} v(z) &= \frac{v}{2} \left\{ 1 - \cos \left( \frac{z\pi}{L_w} \right) \right\} [\theta(z) - \theta(z - L_w)] + v \theta(z - L_w) \\ \beta(z) &= \frac{\Delta\beta}{2} \left\{ 1 - \cos \left( \frac{z\pi}{L_w} \right) \right\} [\theta(z) - \theta(z - L_w)] + \Delta\beta \theta(z - L_w) \end{aligned} \quad (22)$$

where  $v^2 = v_1^2 + v_2^2$  and  $v_{1,2}$  are the vevs of the two Higgs doublets in the broken phase, at the critical temperature. For the remaining parameters we take the bubble wall width  $L_w = 25/T$ , the wall velocity  $v_w = 0.1$  and  $\Delta\beta = 0.01$  [29, 28].

Fig. 1 shows  $F_t^{(1)}(p_m)$ , which is the dominant contribution, as a function of  $p_m$ , for  $v/T_c = 1$  and different values of the Higgsino and Wino masses, ( $|\mu|, m$ ). The weak coupling at the phase transition temperature is  $\alpha_w = 0.035$ . We can see an enhancement of the CP asymmetry for values of  $|\mu|$  close to  $m$ , i.e., when the Higgsino and Wino are nearly degenerate. This resonant behavior, also found in [13], is enhanced by the approximations made in our calculation, because the masses of the particles propagating in the plasma are those of the symmetric phase, and thus strong degeneracies occur when  $|\mu| \sim m$ .

Regarding the damping effects, as explained before the lifetime of the charginos  $\ell \sim \gamma^{-1}$  is of the same order of magnitude as the wall width,  $L_w$ . In this situation, *a priori* it is not clear whether the damping will have an important effect or not. To estimate it, we have repeated the leading order calculation neglecting the damping rate, i.e. setting  $\gamma = 0$ . The resulting  $j_{CP}$  is typically one order of

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<sup>2</sup>The difference in the final result using the ansatz (22) or the functional kinks is typically 5%, although for some values of the soft masses may be as large as 40%.

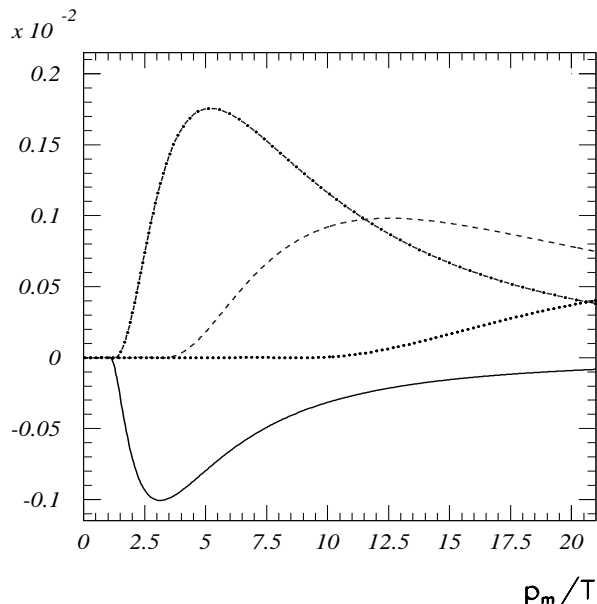


Figure 1:  $F_t^{(1)}(p_m)$  as a function of  $p_m$  for Higgsino mass  $|\mu| = 1.$  and Wino mass  $m = 0.5$  (solid),  $m = 1.5$  (dashed-dotted),  $m = 2.$  (dashed) and  $m = 3.$  (dotted). All in units of the temperature.

magnitude larger, independently of the Higgsino and Wino masses (provided they are not degenerate<sup>3</sup>). Therefore we conclude that although the suppression of the CP asymmetry due to the scattering of charginos with the plasma is not as enormous as in the case of strongly interacting particles (such as quarks in the Standard Model), decoherence effects are not negligible.

## 2.2 Next-to-leading order

In this section, we will compute the CP violating current  $j_{CP}$  at next order in the Higgs insertion expansion. Now both Wino and Higgsino can be reflected or transmitted into Higgsino and the CP asymmetry (12) can be written as

$$j_{CP}^{(2)} = \frac{v_w}{T} \sin \phi \int \frac{d^3 p}{(2\pi)^3} \frac{1}{E} F^{(2)}(p_m) \rho(E) [1 - \rho(E)] , \quad (23)$$

where the function  $F^{(2)}(p_m)$  is given in Appendix A.

Contrary to the lowest order result, we find that  $j_{CP}^{(2)} \neq 0$  even if  $\tan \beta$  remains constant in the bubble wall. Thus the  $\Delta\beta$  suppression disappears at higher order in the Higgs insertion expansion, as mentioned in [11], and in agreement with the numerical computation of [14].

For constant  $v_2/v_1$  we obtain  $F^{(2)}(p_m) = -(p_m + p_\mu)F_r^{(2)}(p_m) + (p_m - p_\mu)F_t^{(2)}(p_m)$ , with

$$F_r^{(2)}(p_m) = \frac{m|\mu|}{2p_\mu} \sin(4\beta) \text{Im} \left\{ \int_0^\infty dz_4 u(z_4) e^{-ip_s z_4} e^{-\gamma z_4} \right.$$

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<sup>3</sup>For  $|\mu| \sim m$ , since the transmission contribution behaves as  $1/(p_m - p_\mu)$  the computation without damping rate is divergent.



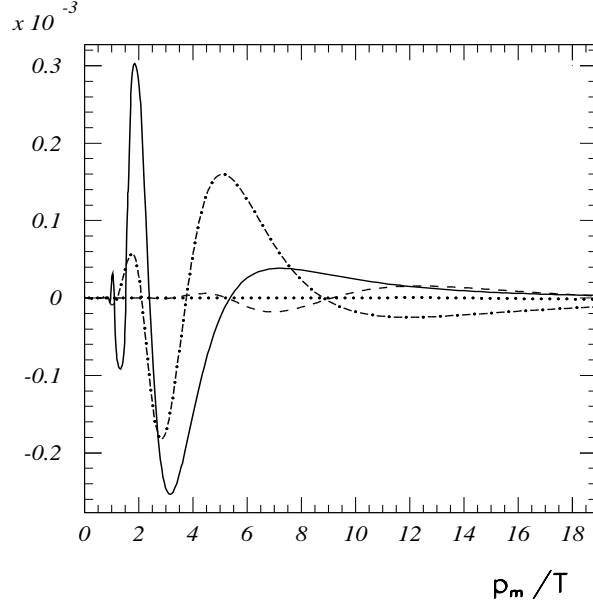


Figure 2:  $F_t^{(2)}(p_m)$  as a function of  $p_m$  for  $\tan\beta = 2$ ,  $|\mu| = 1$ . and  $m = 0.5$  (solid),  $m = 1.5$  (dashed-dotted),  $m = 2$ . (dashed) and  $m = 3$ . (dotted). All masses in units of the temperature.

$$\begin{aligned}
& \times \int_0^\infty dz_2 u(z_2) e^{ip_s z_2} \int_0^{z_2} dz_1 u(z_1) e^{ip_r z_1} \int_0^{z_2} dz_3 u(z_3) e^{-ip_r z_3} \\
& \times \left. e^{-\gamma_{\tilde{H}}(z_3+z_2-z_1)} e^{-\gamma_{\tilde{W}}(z_1+z_2-z_3)} \right\} \quad (24)
\end{aligned}$$

and

$$\begin{aligned}
F_t^{(2)}(p_m) &= -\frac{m|\mu|}{2p_\mu} \sin(4\beta) \text{Im} \left\{ \int_0^\infty dz_4 u(z_4) e^{ip_r z_4} e^{-\gamma z_4} \right. \\
& \times \int_0^\infty dz_3 u(z_3) e^{ip_s z_3} \int_0^{z_3} dz_2 u(z_2) e^{-ip_r z_2} \int_0^{z_2} dz_1 u(z_1) e^{-ip_s z_1} \\
& \times \left. e^{-\gamma_{\tilde{H}}(z_3+z_2-z_1)} e^{-\gamma_{\tilde{W}}(z_1+z_3-z_2)} \right\}, \quad (25)
\end{aligned}$$

where  $u(z) = gv(z)/\sqrt{2}$ ,  $p_s = p_m + p_\mu$ ,  $p_r = p_m - p_\mu$  and  $\gamma = \gamma_{\tilde{H}} + \gamma_{\tilde{W}}$ .

Again, the dominant contribution comes from the transmission amplitude. In Fig. 2 we plot  $F_t^{(2)}(p_m)$  as a function of  $p_m$ , for the same values and shape of the wall profile as in the previous section, except that now  $\Delta\beta = 0$  and we take  $\tan\beta = 2$  (favored by studies of the phase transition). Recall that for this choice of parameters, the CP asymmetry at lowest order vanishes, so this is the leading order result. We also find an enhancement of the CP asymmetry when  $|\mu| \sim m$ .

### 3 Baryon asymmetry

Now we have to solve a set of coupled differential equations describing the effects of diffusion [32], particle number changing reactions and CP asymmetries, to obtain the various particle densities in

the MSSM <sup>4</sup>. We follow the approach of refs.[11, 13] (see [11] for details). If the system is near equilibrium and the particles interact weakly, the particle number densities  $n_i$  are given by

$$n_i = k_i \mu_i \frac{T^2}{6}, \quad (26)$$

where  $\mu_i$  is the local chemical potential for particle species  $i$ , and  $k_i$  is a statistical factor of order 2 (1) for light bosons (fermions), while for particles much heavier than  $T$  it is Boltzmann suppressed.

The particle densities we need to include are the left-handed doublet,  $q_L \equiv (t_L + b_L)$ , the right handed top quark  $t \equiv t_R$ , the Higgs particles  $h = (H_1^0 + H_1^- + \bar{H}_2^0 + \bar{H}_2^+)$ , and their superpartners,  $\tilde{q}_L, \tilde{t}_R, \tilde{h}$ . The particle numbers of these species can change due to the top quark Yukawa interaction with rate  $\Gamma_t$ , the top quark mass interaction with rate  $\Gamma_m$ , the Higgs self-interactions with rate  $\Gamma_h$ , the strong sphaleron interactions with rate  $\Gamma_{ss}$ , the anomalous weak interactions with rate  $\Gamma_{ws}$  and the gauge interactions (which we shall assume that are in equilibrium). Then, the system may be described by the densities  $Q = q + \tilde{q}$ ,  $T = t + \tilde{t}$  and  $H = h + \tilde{h}$ . Alternatively, one could use the Higgs particle density  $H' = (H_1^0 + H_1^- + H_2^0 + H_2^+) +$  superpartners, which does not have the  $\Delta\beta$  suppression in the corresponding source term [15]. However, if Yukawa or helicity-flipping interactions were in equilibrium this would force  $H_1 = -H_2$ , leading to  $H' = Q = T = 0$ . Thus, the approximations we make below would give a vanishing baryon asymmetry, and a numerical solution of the diffusion equations for the three particle densities including all the relevant interaction rates is needed [30].

CP-violating interactions with the phase boundary produce an injected Higgsino flux, which we model as

$$J^{inj}(z) = \xi j_{CP} \delta(z - v_w t), \quad (27)$$

where  $j_{CP}$  is the net Higgsino flux reflected into the unbroken phase and  $\xi$  defines the persistence length of the current in the vicinity of the wall, i.e., it parameterizes our ignorance about how the injected flux thermalizes. This approximation is reasonable if the injected current thermalizes in a time  $\tau_{th}$  short in comparison to the time the particle spends diffusing before being recaptured by the wall, i.e., for small velocities of the wall ( $v_w \ll 1/\sqrt{3}$ ) [10]. We use the estimate for  $\xi$  of ref.[10],  $\xi \sim 6D_h \langle v \rangle$ , where  $D_h$  is the diffusion constant of the Higgsino, which can be approximated by the one of left handed leptons,  $D_h \sim 110/T$  [10], and  $\langle v \rangle$  is the average velocity of the Higgsinos in the reflected flux,

$$\langle v \rangle \equiv \frac{\int \frac{d^3 p}{(2\pi)^3} F^{(i)}(p_m) \frac{v_m}{E^2} \rho(E) [1 - \rho(E)]}{\int \frac{d^3 p}{(2\pi)^3} F^{(i)}(p_m) \frac{1}{E} \rho(E) [1 - \rho(E)]}. \quad (28)$$

with  $F^{(i)}(p_m)$  ( $i = 1, 2$ ) defined in eqs. (19) and (23), respectively, for the leading and next-to-leading order computations of the CP asymmetry.

In ref.[16], the closed time-path formalism was used to derive a set of quantum Boltzmann equations describing the local number density asymmetries of the particles involved in supersymmetric electroweak baryogenesis. In these diffusion equations the CP-violating sources which fuel baryogenesis are self-consistently incorporated. According to them, the CP-violating source term which should be inserted in the diffusion equation for the Higgs density is given by

$$\gamma_{\tilde{h}} \sim \frac{J^{inj}}{\tau}, \quad (29)$$

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<sup>4</sup>We neglect the Debye screening of induced gauge charges, since the effect on the baryon number produced is  $\mathcal{O}(1)$  [33].

where  $\tau = \gamma_{\bar{H}}^{-1}$  is the thermalization time of the Higgsino.

Assuming that the rates  $\Gamma_t$  and  $\Gamma_{ss}$  are fast, so that  $Q/k_Q - H/k_H - T/k_T = \mathcal{O}(1/\Gamma_t)$  and  $2Q/k_Q - T/k_T + 9(Q+T)/k_B = \mathcal{O}(1/\Gamma_{ss})$ , we obtain

$$\begin{aligned} Q &= H \frac{k_Q(9k_T - k_B)}{k_H(9k_T + k_B + 9k_Q)} + \mathcal{O}(1/\Gamma_{ss}, 1/\Gamma_t) , \\ T &= -H \frac{k_T(9k_B + 9k_Q)}{k_H(9k_T + k_B + 9k_Q)} + \mathcal{O}(1/\Gamma_{ss}, 1/\Gamma_t) . \end{aligned} \quad (30)$$

Substituting these expressions we find the equation for the Higgs density

$$\bar{D}H'' - v_w H' - \bar{\Gamma}H + \bar{\gamma} = 0 , \quad (31)$$

where  $\bar{D}$  is an effective diffusion constant,  $\bar{\Gamma}$  is an effective decay constant and  $\bar{\gamma}$  is an effective source term, given by <sup>5</sup>

$$\begin{aligned} \bar{D} &= \frac{D_q(9k_Q k_T + k_Q k_B + 4k_T k_B) + D_h(9k_T + k_B + 9k_Q)k_H}{9k_Q k_T + k_Q k_B + 4k_T k_B + k_H(9k_T + k_B + 2k_Q)} , \\ \bar{\Gamma} &= (\Gamma_h + \Gamma_m) \frac{9k_T + k_B + 9k_Q}{9k_Q k_T + k_Q k_B + 4k_T k_B + k_H(9k_T + k_B + 2k_Q)} , \\ \bar{\gamma} &= \gamma_{\bar{h}} \frac{k_H(9k_T + k_B + 2k_Q)}{9k_Q k_T + k_Q k_B + 4k_T k_B + k_H(9k_T + k_B + 2k_Q)} , \end{aligned} \quad (32)$$

with  $D_q$  ( $D_h$ ) the diffusion constant for quarks and squarks (Higgs and Higgsinos).

We are interested in an analytic solution to eq.(31) which satisfies the boundary conditions  $H(\pm\infty) = 0$ , and at the interphase  $z = 0$

$$H|_{\pm}^{\pm} = 0 , \quad \bar{D}H'|_{\pm}^{\pm} = -\frac{\xi}{\tau} \bar{j}_{CP} , \quad (33)$$

which are derived by integrating up the eq.(31) through  $z = 0$ , imposing the condition that  $H$  is at most step-like discontinuous across the wall. Here,  $\bar{j}_{CP} = (\bar{\gamma}/\gamma_{\bar{h}}) j_{CP}$ . We use a  $z$ - independent effective diffusion constant and a step function for the effective decay rate  $\bar{\Gamma} = \tilde{\Gamma}\theta(z)$ . The values of  $\bar{D}$  and  $\bar{\Gamma}$  depend on the supersymmetric parameters, for the considered range  $\bar{D} \sim 0.8\text{GeV}^{-1}$ ,  $\bar{\Gamma} \sim 1.7\text{ GeV}$  [13].

Then, the solution of eq.(31) in the symmetric phase ( $z < 0$ ) reads

$$H(z) = \mathcal{A} e^{zv_w/\bar{D}} , \quad (34)$$

with

$$\mathcal{A} = \frac{\xi \bar{j}_{CP}}{\tau \lambda_+ \bar{D}} \quad (35)$$

and  $\lambda_{\pm} = \frac{v_w \pm \sqrt{v_w^2 + 4\bar{D}\bar{\Gamma}}}{2\bar{D}}$ .

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<sup>5</sup>Our expressions slightly differ from those of ref.[11], but we have checked that this difference is numerically negligible.

In ref.[11], average current densities  $J(z)$  were computed in each point  $z$  of the bubble wall, and CP-violating sources  $\gamma_Q(z)$  associated with these currents constructed as

$$\gamma_Q(z) \sim J^0(z)/\tau . \quad (36)$$

In order to qualitatively understand the dependence of the produced baryon asymmetry on the various parameters, the sources were approximated as step functions of width  $L_w$ . In the thin wall limit ( $L_w \rightarrow 0$ ,  $L_w J^0 = \text{constant}$ ) the expression of the coefficient  $\mathcal{A}$  obtained in [11] coincides with ours with the substitution  $\xi j_{CP} \rightarrow L_w J^0$ . This result may help to understand the relation between the two approaches, in the regime when both are applicable.

From the form of (35) we see that the CP-violating densities are non zero for a time  $t \sim \bar{D}/v_w^2$ , thus the assumptions which lead to the equation (31) for the Higgs density are valid provided  $\Gamma_t, \Gamma_{ss} \gg v_w^2/\bar{D}$ .

What we aim to compute is the total baryon number density left inside the bubble, so now we need to consider the effect of the weak sphaleron processes, since they provide the only source for net baryon number. The equation satisfied by the baryon number density,  $n_B$  is

$$D_q n_B'' - v_w n_B' - \theta(-z) n_f \Gamma_{ws} n_L = 0 , \quad (37)$$

where  $n_L$  is the total number density of left handed weak doublet fermions,  $n_f = 3$  is the number of families and for the weak sphaleron rate we take  $\Gamma_{ws} = 6\kappa\alpha_w^4 T$  ( $\kappa \simeq 1$ ). It has recently been estimated as  $\Gamma_{ws} \sim C\alpha_w \log(1/g)(\alpha_w T)^4$  [34], but lattice measurements of the rate are consistent with  $C \sim 1/\alpha_w$  [35] so this does not affect the numerical value of our result.

Assuming that all squarks except  $\tilde{t}_{L,R}$  and  $\tilde{b}_L$  are heavy, there is no suppression due to the strong sphaleron and  $n_L$  is given by [11]

$$n_L = \frac{9k_Q k_T - 8k_T k_B - 5k_Q k_B}{k_H(9k_T + k_B + 9k_Q)} H = \frac{27}{82} H . \quad (38)$$

Substituting the Higgs density (34) in eq.(37), we obtain

$$\frac{n_B}{s} = - \frac{81\mathcal{A}\bar{D}\Gamma_{ws}}{82v_w^2 s} , \quad (39)$$

where  $s = 2\pi^2 g_* T^3/45$  is the entropy density, with  $g_* \sim 126$  the effective number of relativistic degrees of freedom.

In order to compare our approach with previous related work, we have calculated the final baryon asymmetry (39) using both, the leading and the next-to-leading order results for the CP asymmetry in the Higgsino current. When we use the lowest order CP asymmetry we obtain a baryon number to entropy ratio of the same order of magnitude as in [16]<sup>6</sup>, where the CP-violating sources were self-consistently incorporated in the diffusion equations. In previous calculations [11, 13], different definitions of the CP-violating currents were used without any first principles justification, leading to smaller numerical results for the final baryon asymmetry (about one order of magnitude in [11] and two orders of magnitude in [13]).

Fig. 3 shows our leading order results. From eq.(39), we see that the baryon number produced depends linearly on the phase of the Higgsino mass parameter,  $\phi$ . So in Fig. 3 we plot the value

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<sup>6</sup>We thank A. Riotto for pointing out this fact to us.

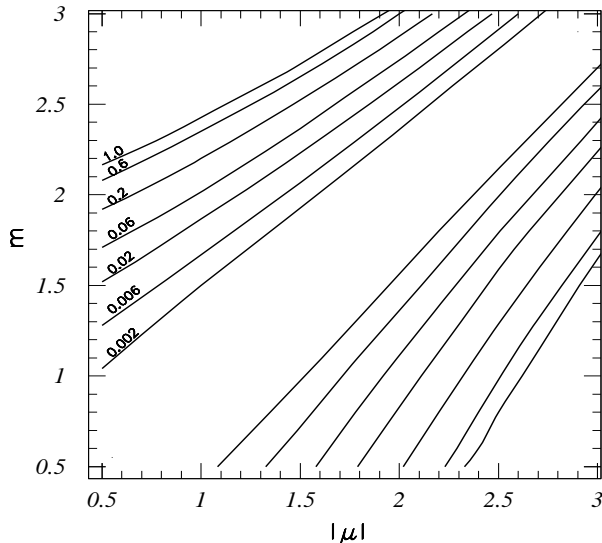


Figure 3: Contour plot of  $|\sin \phi|$  in the plane  $(|\mu|, m)$  for fixed  $n_B/s \simeq 4 \times 10^{-11}$ , according to our leading order result for the CP asymmetry in the Higgsino current. The masses are in units of the temperature.

of  $|\sin \phi|$  necessary to obtain  $n_B/s \simeq 4 \times 10^{-11}$ , in the  $|\mu| - m$  plane, using the leading order CP-asymmetry computation with  $\Delta\beta = 0.01$ . The shape of the curves is similar to the ones obtained in [13] but we need a smaller phase to explain the observed baryon asymmetry, as discussed above. We have also calculated the baryon number to entropy ratio at leading order for other values of the parameter  $\Delta\beta$ , namely  $\Delta\beta = 0.001, 0.005$ , and we find that it scales as  $n_B/s \propto \Delta\beta$ .

For  $\tan \beta$  constant in the bubble wall, the CP asymmetry at lowest order vanishes, and at next-to-leading order we obtain

$$\frac{n_B}{s} \sim (10^{-11} - 10^{-8}) \sin \phi, \quad (40)$$

where the largest values correspond to nearly degenerate Wino and Higgsino. In most cases,  $|\sin \phi|$  should be of order 0.1-1 to explain the observed baryon asymmetry. These results correspond to  $\tan \beta = 2$ , but they can easily be converted to other values of  $\tan \beta$ , since the  $\beta$  dependence is just a global factor (see eqs.(24) and (25)). The values above seem to indicate that next-to-leading order corrections are relevant only for very small  $\Delta\beta (< 0.001)$ .

## 4 Conclusions

We have computed the baryon asymmetry generated at the electroweak phase transition in the MSSM. The leading CP asymmetry which fuels baryogenesis is in the Higgsino current, provided that Higgsinos and gauginos are not much heavier than the electroweak critical temperature ( $T_c \sim 100$  GeV). The main motivation for the calculation was to settle the question of whether or not the  $\Delta\beta$  dependence of the CP-violating Higgsino current was a consequence of the approximations used in the computations

[11, 13, 16]. Given that  $\Delta\beta$  is at most  $2.5 \times 10^{-2}$  in the MSSM [29, 28], such dependence leads to a significant suppression of the produced baryon asymmetry.

We have done the calculation in the thin wall regime, using a new method to compute the CP asymmetry by expanding in the Higgs vevs. At leading order, our results show the same parametric dependence as previous related approaches [11, 13], in particular the CP asymmetry vanishes if  $\tan\beta = v_2/v_1$  is constant along the bubble wall. However, at next-to-leading order we find a non-zero CP-asymmetry even if  $\tan\beta$  is constant, in agreement with the numerical calculation of [14]. Although our approach differs from those of refs.[11, 13], all of them contain a Higgs insertion expansion and thus we believe that this result will also apply to their methods.

We have estimated the effects of damping in the CP asymmetry, using a simple model for decoherence. In the leading order calculation we find a suppression of order 0.1 due to the incoherent scattering of charginos with the particles in the plasma.

The subsequent solution of the diffusion equations using the leading order CP asymmetry leads to a baryon number to entropy ratio in agreement with observation, even for small CP-violating phase of the  $\mu$  parameter,  $|\sin\phi| \sim (10^{-2} - 10^{-4})$ . These results are comparable with those of ref.[16], for the same range of the relevant MSSM parameters. At this order in the Higgs insertion expansion, the dependence of  $n_B/s$  with  $\Delta\beta$  is approximately linear.

For  $\tan\beta$  constant in the bubble wall, the CP asymmetry at lowest order vanishes, and the next-to-leading order calculation gives a baryon asymmetry sufficiently large if the CP-violating phase is in the range  $|\sin\phi| \sim (0.1 - 1)$ . Smaller values are acceptable only in the region  $m \sim |\mu|$ .

The various approximations made in the current analysis lead to a sizeable uncertainty in the final result. Since the mean free path for charginos  $\ell \sim L_w$  (bubble wall width), we expect the thin wall limit to give an  $\mathcal{O}(1)$  estimate of the true solution. Regarding the Higgs insertion expansion, our next-to-leading order calculation shows that the leading order result is reliable, except for very small values of  $\Delta\beta (< 0.001)$ . We have also made a number of simplifications of the diffusion equations [11], which are valid provided  $\Gamma_t, \Gamma_{ss} \gg v_w^2/\bar{D}$ ,  $\Gamma_{ws} \ll v_w^2/\bar{D}$  and the scattering processes due to Yukawa couplings other than top are slow. Given our poor knowledge of the parameters involved (diffusion constants  $D$ , reaction rates  $\Gamma_{t,ss,ws}$  and wall velocity  $v_w$ ) it is not possible to ensure that these inequalities are satisfied, but the present estimates seem to indicate so.

Our most severe approximation was the insertion of the CP-violating source in the diffusion equation for the Higgs density (31). We have modeled the injected Higgsino flux as a delta function localized at  $z = v_w t$ . The largest uncertainty comes from our ignorance about how this injected flux thermalizes. Following [10], we have parameterized it by  $\xi$ , the persistence length of the injected current, estimated to be  $\xi \sim 6D_h \langle v \rangle$ , where  $6D_h$  is the velocity randomization time of a diffusing Higgsino and  $\langle v \rangle$  is the average velocity of the injected flux. To calculate this parameter more precisely involves going beyond the diffusion approximation to determine exactly how the reflected asymmetry enters in the diffusion equation [16]. Taking into account the above uncertainties and approximations, we estimate that our results for the baryon asymmetry are reliable to about one order of magnitude.

Finally, we want to comment on the differences between our approach and the one in refs.[13, 16], since both are based on a Higgs insertion expansion. Our approach is only applicable in the thin wall regime, and we perform a tree level computation of the reflection and transmission amplitudes to obtain the Higgsino current, much as in [11]. On the other hand, the method of [13, 16] is valid for any thickness of the bubble wall and they calculate directly CP-violating currents using the closed-time path formalism, which involves a one-loop computation at finite temperature. However, at least in the thin wall regime, both approaches should be somehow related, and it will be very interesting to

understand the connection between them.

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## Appendix A.

We present here the different contributions to the function  $F^{(2)}(p_m)$  which appears in the CP asymmetry at next-to-leading order in the Higgs insertion expansion,  $j_{CP}^{(2)}$  (23). Taking into account the unitarity relation (11), we only need the contributions from the reflection and transmission of Winos into Higgsinos, and the reflection of Higgsinos into themselves. We work in the boosted frame ( $q_x = q_y = 0$ ). In the reflection, the momenta of the initial particles are  $q_i = (\omega, -p_\mu - i\gamma_{\tilde{H}})$  for Higgsino and  $q_i = (\omega, -p_m - i\gamma_{\tilde{W}})$  for Wino, while  $q_f = (\omega, p_\mu + i\gamma_{\tilde{H}})$  is the momentum of the Higgsino reflected into the symmetric phase.

From  $\tilde{H} \rightarrow \tilde{H}$  reflection we obtain:

$$\begin{aligned}
F_{\tilde{H}}^{(2)}(p_m) &= -2 p_\mu F_{r,\tilde{H}}^{(2)}(p_m) \\
&= -2im|\mu| \int dz_1 \dots dz_4 \int \frac{dp_z}{2\pi} \frac{dp'_z}{2\pi} \frac{e^{(ip_\mu - \gamma_{\tilde{H}})(z_1+z_2)} e^{(-ip_\mu - \gamma_{\tilde{H}})(z_3+z_4)} e^{-ip_z(z_2-z_1)+ip'_z(z_4-z_3)}}{(p^2 - m^2 + i\gamma_{\tilde{W}})(p'^2 - m^2 - i\gamma_{\tilde{W}})} \\
&\times \{2m|\mu| \cos \phi [u_2(z_1)u_1(z_2)u_2(z_3)u_1(z_4) - u_1(z_1)u_2(z_2)u_1(z_3)u_2(z_4)] \\
&\quad + (q_f \cdot p') [u_2(z_1)u_1(z_2)u_2(z_3)u_2(z_4) - u_1(z_1)u_2(z_2)u_1(z_3)u_1(z_4)] \\
&\quad + (q_i \cdot p) [u_2(z_1)u_2(z_2)u_2(z_3)u_1(z_4) - u_1(z_1)u_1(z_2)u_1(z_3)u_2(z_4)] \\
&\quad + (q_f \cdot p) [u_1(z_1)u_1(z_2)u_2(z_3)u_1(z_4) - u_2(z_1)u_2(z_2)u_1(z_3)u_2(z_4)] \\
&\quad + (q_i \cdot p') [u_2(z_1)u_1(z_2)u_1(z_3)u_1(z_4) - u_1(z_1)u_2(z_2)u_2(z_3)u_2(z_4)] \}, \tag{A. 1}
\end{aligned}$$

where  $p^2 = \omega^2 - p_z^2$ ,  $p'^2 = \omega^2 - p_z'^2$ . By using symmetry arguments, it is easy to see that the above expression vanishes identically.

The contribution coming from  $\tilde{W} \rightarrow \tilde{H}$  transitions can be written as

$$F_{\tilde{W}}^{(2)}(p_m) = -(p_m + p_\mu)F_{r,\tilde{W}}^{(2)}(p_m) + (p_m - p_\mu)F_{t,\tilde{W}}^{(2)}(p_m), \tag{A. 2}$$

where

$$\begin{aligned}
F_{r,\tilde{W}}^{(2)}(p_m) &= \\
&= \frac{2m|\mu|}{p_\mu} \text{Im} \left( \int dz_1 \dots dz_4 \int \frac{dp_z}{2\pi} \frac{dp'_z}{2\pi} \frac{e^{(ip_\mu - \gamma_{\tilde{H}})z_3 + (ip_m - \gamma_{\tilde{W}})z_1} e^{[-i(p_\mu + p_m) - \gamma]z_4} e^{ip_z(z_3 - z_2) + ip'_z(z_2 - z_1)}}{(p^2 - m^2 + i\gamma_{\tilde{W}})(p'^2 - \mu^2 + i\gamma_{\tilde{H}})} \right. \\
&\times \{2m|\mu| \cos \phi [u_2(z_1)u_1(z_2)u_2(z_3)u_1(z_4) - u_1(z_1)u_2(z_2)u_1(z_3)u_2(z_4)]
\end{aligned}$$

$$\begin{aligned}
& +(q_f \cdot q_i)[u_2(z_1)u_1(z_2)u_2(z_3)u_2(z_4) - u_1(z_1)u_2(z_2)u_1(z_3)u_1(z_4)] \\
& +(q_f \cdot p)[u_2(z_1)u_1(z_2)u_1(z_3)u_1(z_4) - u_1(z_1)u_2(z_2)u_2(z_3)u_2(z_4)] \\
& +(q_i \cdot p')[u_1(z_1)u_1(z_2)u_2(z_3)u_1(z_4) - u_2(z_1)u_2(z_2)u_1(z_3)u_2(z_4)] \\
& +(p \cdot p')[u_2(z_1)u_2(z_2)u_2(z_3)u_1(z_4) - u_1(z_1)u_1(z_2)u_1(z_3)u_2(z_4)] \Big\} , \tag{A. 3}
\end{aligned}$$

with  $\gamma = \gamma_{\tilde{W}} + \gamma_{\tilde{H}}$ . The contribution of the transmission from the broken phase,  $F_{t,\tilde{W}}^{(2)}(p_m)$ , can be obtained from eq.(A. 3) changing  $q_i \rightarrow q'_i = (\omega, p_m - i\gamma_{\tilde{W}})$  and  $p_m \rightarrow -p_m$  in the exponentials. We have only included the damping rate in the exponential factors.

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