FTUV/98-23
IFIC/98-23
MZ-TH/98-01
May 1998

Electroweak Phase Transition in Left-Right Symmetric Models

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Abstract

We study the finite-temperature effective potential of minimal left-right symmetric models containing a bidoublet and two triplets in the scalar sector. We perform a numerical analysis of the parameter space compatible with the requirement that baryon asymmetry is not washed out by sphaleron processes after the electroweak phase transition. We find that the spectrum of scalar particles for these acceptable cases is consistent with present experimental bounds.

1 Introduction

The origin of the observed baryon asymmetry of the Universe (BAU) remains an interesting open question in particle physics. In 1967, Sakharov [1] established the three basic requirements for obtaining this baryon asymmetry as a result of particle interactions in the early universe: a) Baryon number violation, b) C and CP violation, c) departure from thermal equilibrium. These conditions are fulfilled in grand unification theories, in which the baryon asymmetry is generated by the out-of-equilibrium B-violating decay of some superheavy boson. However, this scenario presents the problem that anomalous processes can partially or totally erase the baryon asymmetry generated at extremely high energies.

It was realized in [2] that Sakharov conditions may also be satisfied at weak scale temperatures, if the electroweak phase transition is first order. In a strongly first order electroweak phase transition, bubbles of the true ground state (broken phase) nucleate and expand until they fill the Universe; local departure from thermal equilibrium occurs in the vicinity of the expanding bubble walls. C and CP are known to be violated by the electroweak interactions, and anomalous baryon number violation is fast at high temperatures in the symmetric phase. Moreover, electroweak baryogenesis provides an explanation of the observed BAU in terms of experimentally accessible physics and hence much attention has been devoted to the study of this possibility [3].

In principle, the Standard Model (SM) contains all the necessary ingredients for electroweak baryogenesis, but it has two problems: the CP asymmetry induced by the Kobayashi-Maskawa phase is far too small to account for the observed n_B/s ratio [4], and the phase transition appears too weakly first order for the Higgs mass experimentally allowed [5]. To avoid the erasure of the baryon asymmetry produced during the phase transition, the sphaleron processes need to be sufficiently suppressed in the broken phase and this in turn is directly related to the strength of the phase transition. Quantitatively, the requirement is that the ratio of the vacuum expectation value of the Higgs field at the critical temperature to the critical temperature must be larger than one,

$$\frac{v(T_c)}{T_c} > 1 \ . \tag{1}$$

In the SM, this imposes an upper bound on the Higgs mass which is below the present experimental bound $m_H \geq 88$ GeV [6].

However these two problems may be absent in simple extensions of the SM, which contain additional sources of CP violation and more scalars than the SM. The larger parameter space in the scalar sector allows for a stronger first-order phase transition without such a light Higgs [7]. Several possibilities have been analyzed in detail: two Higgs models with a strong CP phase [8]-[11], heavy neutrinos [12], and supersymmetric models [13].

In the present paper, we consider one of the most attractive extensions of the SM, based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [14, 15]. Various different models employing this gauge group are possible, depending on which Higgs and fermion spectrum is chosen, and on whether or not exact discrete left-right symmetry is imposed. We are interested in the class of left-right symmetric models described in [14, 16]. Besides the original idea of explaining the observed parity violation of the weak interaction at low energies, these models provide also an explanation for the lightness of the ordinary neutrinos, via the so-called see-saw mechanism.

There are two possible scenarios for baryogenesis in left-right symmetric models. In the first one, the BAU is generated at the scale where local $B-L$ is broken. This in turn can occur during the right phase transition (if it is first-order), due to the reflection of right-handed neutrinos on walls of broken phase bubbles (i.e. $SU(2)_L \otimes U(1)_Y$ symmetric) [17], and/or via out-of-equilibrium decay of right handed Majorana neutrinos [18]. In both cases, a lepton number asymmetry is produced and subsequently converted into a baryon number one through the rapid $(B+L)$-violating anomalous processes above the electroweak phase transition temperature. However, the right-handed scale needed to account for the BAU ($\mathcal{O}(10$ TeV) in the first case and above $10^6 - 10^7$ GeV in the latter) is too high to have any low-energy observable implication. In the second scenario, baryogenesis takes place at the lower electroweak scale, mainly due to the reflection of the top quark on walls of the true vacuum (i.e. $U(1)_{em}$ symmetric) bubbles. There are estimates [18, 17, 19] of the baryon asymmetry produced in left-right symmetric models using this mechanism, although they neglect effects that are now understood to be important, such as diffusion [9], [20] and thermal scattering [4].

We focus on the possibility of electroweak baryogenesis. We perform an analysis of the electroweak phase transition in phenomenologically acceptable left-right symmetric models with a relatively low right scale, which have interesting implications in present and planned experiments [22] [1].

It has been shown recently that, contrary to previous belief, spontaneous CP violation can occur in the minimal left-right symmetric model considered here [23]. However baryogenesis with (only) spontaneous breakdown of CP presents severe cosmological problems, due to the formation of domain walls as a result of the breaking of a discrete symmetry. Although this problem can be solved, in order to generate the BAU the scale of spontaneous CP violation and the scale at which baryogenesis takes place must be different [19]; otherwise, an equal amount of matter and anti-matter is generated. In the minimal left-right symmetric model with spontaneous CP-violation both scales coincide and therefore electroweak baryogenesis is not feasible.

The remainder of this paper is structured as follows. In section 2 we describe the model, while the effective potential at finite temperature is calculated in section 3. The order of the electroweak phase transition is analyzed in section 4 and we conclude in section 5.

2 Left-right symmetric model

We consider the minimal $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model with a left-right discrete symmetry [16, 24]. This model is formulated so that parity is a spontaneously broken symmetry: the Lagrangian is left-right symmetric but the vacuum is not invariant under the parity transformation. Thus, the observed V-A structure of the weak interactions is only a low energy phenomenon, which should disappear when one reaches energies of order v_R, where v_R is the vacuum expectation value of some right-handed scalar.

According to the left-right symmetry requirements, quarks (and similarly leptons) are placed in left and right doublets,

$$\Psi_{iL} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \equiv \left(2, 1, \frac{1}{3}\right), \quad \Psi_{iR} = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R \equiv \left(1, 2, \frac{1}{3}\right), \tag{2}$$

where $i = 1, 2, 3$ is the generation index and the representation content with respect to the gauge group is explicitly given. Taking advantage of the fact that the weak interactions observed at low energies involve only the left handed helicity components, the electric charge formula can also be written in a

[1]A similar study in a left-right symmetric model with a simpler scalar content has been done in ref. [21].

left-right symmetric form as

$$Q = I_{3L}^W + I_{3R}^W + \frac{B-L}{2} \tag{3}$$

where I_3^W denotes the third component of the weak isospin.

Regarding the bosons, gauge vector bosons consist of two triplets $\mathbf{W}_L^\mu \equiv (3,1,0)$, $\mathbf{W}_R^\mu \equiv (1,3,0)$, and a singlet $B^\mu \equiv (1,1,0)$.

The Higgs sector of the model is dictated by two requirements,the choice of the symmetry breaking chain and the desire to reproduce the phenomenologically observed light masses of the known neutrinos via the so-called see-saw mechanism. The best candidates for these purposes seem to be

$$\begin{aligned}
\Phi &= \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \equiv \left(\frac{1}{2}, \frac{1}{2}^*, 0\right) && (4)\\
\Delta_L &= \begin{pmatrix} \frac{\delta_L^+}{\sqrt{2}} & \delta_L^{++} \\ \delta_L^0 & -\frac{\delta_L^+}{\sqrt{2}} \end{pmatrix} \equiv (1,0,2) && (5)\\
\Delta_R &= \begin{pmatrix} \frac{\delta_R^+}{\sqrt{2}} & \delta_R^{++} \\ \delta_R^0 & -\frac{\delta_R^+}{\sqrt{2}} \end{pmatrix} \equiv (0,1,2) && (6)
\end{aligned}$$

where the scalar fields have been written in a convenient representation given by 2×2 matrices.

Let us now discuss the form of the Lagrangian. We require the Lagrangian to be invariant under the discrete left-right symmetry defined by

$$\Psi_L \leftrightarrow \Psi_R \qquad \Delta_L \leftrightarrow \Delta_R \qquad \Phi \leftrightarrow \Phi^\dagger \tag{7}$$

where Ψ denotes any fermion. We assume that the global phases allowed to appear in the transformations above are absorbed by proper redefinition of the fields.

The most general renormalizable Lagrangian consistent with the above discrete symmetry and gauge invariance can be written as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_f + \mathcal{L}_{Higgs} \tag{8}$$

where the gauge field part of the Lagrangian contains the kinetic energy terms for the gauge bosons corresponding to the gauge groups $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. The gauge coupling constants for the gauge groups $SU(2)_L$ and $SU(2)_R$ are the same and we denote it by g, while that of the $U(1)_{B-L}$ is denoted by g'. The fermionic part of the Lagrangian, $\mathcal{L}_f$, contains the kinetic energy terms for the fermions and the Yukawa couplings, which are given by

$$-\mathcal{L}_Y = \sum_{a,b} h_{ab} \overline{\Psi}_{aL} \Phi \Psi_{bR} + \tilde{h}_{ab} \overline{\Psi}_{aL} \tilde{\Phi} \Psi_{bR} + i f_{ab} \left[\Psi_{aL}^T C \tau_2 \Delta_L \Psi_{bL} + (L \leftrightarrow R) \right] + \text{h.c.} \tag{9}$$

where $\tilde{\Phi} = \tau_2 \Phi^* \tau_2$, C is the Dirac charge-conjugation matrix and a, b label different generations.

The Higgs part of the Lagrangian contains the kinetic energy terms for the fields $\Delta_{L,R}$ and Φ and the scalar interaction terms, i.e. the most general scalar potential. This potential cannot have trilinear terms: because the nonzero $B-L$ quantum numbers of the Δ_L and Δ_R triplets, these must always appear in the quadratic combinations $\Delta_L^\dagger \Delta_L$, $\Delta_R^\dagger \Delta_R$, $\Delta_L^\dagger \Delta_R$ or $\Delta_R^\dagger \Delta_L$. These can never be combined with a single bidoublet Φ in such a way as to form $SU(2)_L$ and $SU(2)_R$ singlets. Nor can three bidoublets be combined so as to yield a singlet. However, quartic combinations of the form $\mathrm{Tr}(\Delta_L^\dagger \Phi \Delta_R \Phi^\dagger)$ are in general allowed by the left-right symmetry. According to these strict conditions, the most general form of the Higgs potential is

$$\mathbf{V} = \mathbf{V}_\Phi + \mathbf{V}_\Delta + \mathbf{V}_{\Phi\Delta} \tag{10}$$

with

$$\begin{aligned}
\mathbf{V}_\Phi \;=\; & -\mu_1^2 \,\mathrm{Tr}(\Phi^\dagger \Phi) \,-\, \mu_2^2 \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\right] \,+\, \lambda_1 \left[\mathrm{Tr}(\Phi\Phi^\dagger)\right]^2 + \\
& \lambda_2 \left\{\left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\right]^2 + \left[\mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\right]^2\right\} \,+\, \lambda_3 \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\right] + \\
& \lambda_4 \left\{\mathrm{Tr}(\Phi^\dagger \Phi) \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
\mathbf{V}_\Delta \;=\; & -\mu_3^2 \left[\mathrm{Tr}(\Delta_L \Delta_L^\dagger) + \mathrm{Tr}(\Delta_R \Delta_R^\dagger)\right] \,+\, \rho_1 \left\{\left[\mathrm{Tr}(\Delta_L \Delta_L^\dagger)\right]^2 + \right. \\
& \left.\left[\mathrm{Tr}(\Delta_R \Delta_R^\dagger)\right]^2\right\} \,+\, \rho_2 \left[\mathrm{Tr}(\Delta_L \Delta_L)\mathrm{Tr}(\Delta_L^\dagger \Delta_L^\dagger) + \right. \\
& \left.\mathrm{Tr}(\Delta_R \Delta_R)\mathrm{Tr}(\Delta_R^\dagger \Delta_R^\dagger)\right] \,+\, \rho_3 \left[\mathrm{Tr}(\Delta_L \Delta_L^\dagger)\mathrm{Tr}(\Delta_R \Delta_R^\dagger)\right] + \\
& \rho_4 \left[\mathrm{Tr}(\Delta_L \Delta_L)\mathrm{Tr}(\Delta_R^\dagger \Delta_R^\dagger) + \mathrm{Tr}(\Delta_L^\dagger \Delta_L^\dagger)\mathrm{Tr}(\Delta_R \Delta_R)\right]
\end{aligned}$$

$$\begin{aligned}
\mathbf{V}_{\Phi\Delta} \;=\; & \alpha_1 \left\{\mathrm{Tr}(\Phi^\dagger \Phi) \left[\mathrm{Tr}(\Delta_L \Delta_L^\dagger) + \mathrm{Tr}(\Delta_R \Delta_R^\dagger)\right]\right\} \,+\, \alpha_2 \left[\mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\mathrm{Tr}(\Delta_R \Delta_R^\dagger) \right. \\
& \left. + \mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\Delta_L \Delta_L^\dagger)\right] + \alpha_2^* \left[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\Delta_R \Delta_R^\dagger) + \right. \\
& \left.\mathrm{Tr}(\tilde{\Phi}^\dagger \Phi)\mathrm{Tr}(\Delta_L \Delta_L^\dagger)\right] \,+\, \alpha_3 \left[\mathrm{Tr}(\Phi\Phi^\dagger \Delta_L \Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger)\right] + \\
& \beta_1 \left[\mathrm{Tr}(\Phi \Delta_R \Phi^\dagger \Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger \Delta_L \Phi \Delta_R^\dagger)\right] \,+\, \beta_2 \left[\mathrm{Tr}(\tilde{\Phi}\Delta_R \Phi^\dagger \Delta_L^\dagger) + \right. \\
& \left.\mathrm{Tr}(\tilde{\Phi}^\dagger \Delta_L \Phi \Delta_R^\dagger)\right] + \beta_3 \left[\mathrm{Tr}(\Phi \Delta_R \tilde{\Phi}^\dagger \Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger \Delta_L \tilde{\Phi} \Delta_R^\dagger)\right]
\end{aligned}$$

where we have written out each term completely to display the full parity symmetry. Note that as a consequence of the discrete left-right symmetry all terms in the potential are self-conjugate, except for the α_2 one; therefore α_2 is the only parameter which may be complex. The potential (10) is not invariant under the exchange of the fields $\phi_1^0 \leftrightarrow \phi_2^{0*}$. One can restore this symmetry by setting $\beta_2 = \beta_3$, $\alpha_3 = 0$ and α_2 real. Then all the parameters in the scalar potential have to be real and it is CP conserving. In that case, spontaneous CP violation can occur even with the minimal scalar sector described above [23]. However, as explained in the introduction this model can not lead to successful electroweak baryogenesis and we shall not consider it here.

Since we will not discuss the CP violation aspect of the BAU generation, in our analysis we assume CP conservation and take α_2 to be real. It can be shown [24] that, without fine tuning, the β_i terms spoil the seesaw mechanism by inducing a direct Majorana mass term for the left-handed neutrino, unless $|\beta_i| \leq 10^{-7} - 10^{-8}$ or the right scale is very large. As a result, in realistic left-right symmetric models with a low right scale ($v_R \sim 1$ TeV) and no fine tuning, the effects of such terms will be negligible and it is a good approximation to assume that they vanish. Therefore, we set $\beta_i = 0$ in the rest of the paper This choice will also avoid the unwanted presence of too large FCNC which could enter in conflict with experimental data.

Only the neutral components of the scalar fields, $\phi_1^0, \phi_2^0, \delta_L^0, \delta_R^0$, can acquire vevs without violating electric charge. If Δ_L or Δ_R acquire a vev, then $B - L$ is necessarily broken, and if $\langle \Delta_L \rangle \neq \langle \Delta_R \rangle$ parity breakdown is also ensured. Thus the correct pattern of symmetry breaking is achieved by

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix} \quad , \quad \langle \Delta_{L,R} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix} \tag{11}$$

where k_1, k_2, v_L and v_R are real, and phenomenologically the hierarchy $v_R \gg k_1, k_2 \gg v_L$ is required. Moreover, when the β parameters in the scalar potential vanish then $v_L = 0$ [15, 24], so we neglect v_L in the following.

In the tree-level scalar potential we have thus 14 free parameters, plus the zero temperature vevs. Three of these parameters can be fixed by minimizing the zero-temperature tree-level potential, i.e. imposing the vanishing of the first derivatives of $\mathbf{V}$ at (k_1, k_2, v_R), which leads to the relations

$$\begin{aligned}
\mu_1^2 &= \frac{\alpha_1}{2} v_R^2 - \frac{\alpha_3}{2} \frac{k_2^2 v_R^2}{(k_1^2 - k_2^2)} + \lambda_1 (k_1^2 + k_2^2) + 2\lambda_4 k_1 k_2 \\
\mu_2^2 &= \frac{\alpha_2}{2} v_R^2 + \frac{\alpha_3}{4} \frac{k_1 k_2 v_R^2}{(k_1^2 - k_2^2)} + (2\lambda_2 + \lambda_3) k_1 k_2 + \frac{\lambda_4}{2} (k_1^2 + k_2^2) \\
\mu_3^2 &= \frac{\alpha_1}{2} (k_1^2 + k_2^2) + \rho_1 v_R^2 + 2\alpha_2 k_1 k_2 + \frac{\alpha_3}{2} k_2^2
\end{aligned} \tag{12}$$

Before writing down the finite temperature effective potential, let us discuss briefly the values that can be taken by the α_i $(i = 1, 2, 3)$ parameters. From the minimization conditions (12), one can see that to obtain μ_1 and μ_2 of order of the weak scale (and hence phenomenologically acceptable values of $k \equiv \sqrt{k_1^2 + k_2^2}$) the α_i should be of order $\mathcal{O}(k^2/v_R^2) \ll 1$. Otherwise, μ_1, μ_2 would naturally be of order of the right scale, v_R. But this is by no means an artificial fine tuning, as the α_i parameters govern the doublet-triplet mixing and therefore we do expect them to be of that order. We shall take advantage of this fact to obtain an approximate analytic expression for the finite temperature effective potential in the next section.

The masses for the relevant degrees of freedom of the theory in the background of the fields k_1, k_2, v_R are given in the appendices.

3 Finite Temperature Effective Potential

The main tool for the study of the electroweak phase transition in the left-right symmetric model described above is the one-loop, daisy improved finite-temperature effective potential of the model. We are actually interested in the dependence of the potential on $k_1 = \mathrm{Re}\ \phi_1^0/\sqrt{2}, k_2 = \mathrm{Re}\ \phi_2^0/\sqrt{2}$ and $v_R = \mathrm{Re}\ \delta_R^0/\sqrt{2}$. It can be readily computed by the usual methods [25] and is given by

$$V_{eff}(k_i, v_R, T) = V(k_i, v_R) + V_1(k_i, v_R, T) + V_{daisy}(k_i, v_R, T) \tag{13}$$

where $V(k_i, v_R)$ is the tree-level potential (10),

$$V_1(k_i, v_R, T) = \frac{T^4}{2\pi^2} \sum_i n_i J_i \left[\frac{m_i^2(k_i, v_R)}{T^2} \right] , \tag{14}$$

$$V_{daisy}(k_i, v_R, T) \;=\; -\frac{T}{12\pi}\sum_i n_i\left[\overline{m}_i^3(k_i, v_R, T) - m_i^3(k_i, v_R)\right] \; . \tag{15}$$

The sum runs over all the particles in the model, n_i is the corresponding number of degrees of freedom, taken negative for fermions, and $m_i^2(k_i, v_R)$ is the tree-level mass of the particle i in presence of the background fields k_1, k_2, v_R. The functions $J_i = J_+(J_-)$ for bosons (fermions) are given by

$$J_\pm(y^2) = \int_0^\infty dx x^2 \; \log\left(1 \mp e^{-\sqrt{x^2+y^2}}\right) \tag{16}$$

The last term in (13), $V_{daisy}(k_i, v_R, T)$, is a correction coming from the resummation of the leading infrared divergent higher-loop contributions, associated with the so-called daisy diagrams. The sum runs over bosons only. The masses $\overline{m}_i^2(k_i, v_R, T)$ are obtained from the $m_i^2(k_i, v_R)$ by adding the leading T-dependent self-energy contributions, which are proportional to T^2. In the contribution of the longitudinal gauge boson degrees of freedom, there is a suppression due to the temperature dependent Debye mass. A simple treatment is just to drop the longitudinal contribution [26], and we follow this prescription.

For values of the fields such that $m_i(k_i, v_R)/T < 1$, we can expand $J_\pm$ as [25] [2]

$$J_+(m^2/T^2) \;=\; -\frac{\pi^4}{45} + \frac{\pi^2 m^2}{12T^2} - \frac{\pi}{6}\left(\frac{m^2}{T^2}\right)^{3/2} + \mathcal{O}\left(\frac{m^4}{T^4}\log\frac{m}{T}\right) \tag{17}$$

$$J_-(m^2/T^2) \;=\; \frac{7\pi^4}{360} - \frac{\pi^2 m^2}{24T^2} + \mathcal{O}\left(\frac{m^4}{T^4}\log\frac{m}{T}\right) \tag{18}$$

In left-right symmetric models one expects two phase transitions [21]: one at $T = T_R = \mathcal{O}(v_R) \sim 1$ TeV [27], where $SU(2)_R$ is spontaneously broken, and the other at $T = T_L = \mathcal{O}(k) \sim 250$ GeV. Hence at temperatures much higher than T_R down to $T = T_R$, $v_R = k = 0$ will be the minimum of the effective potential (13). At $T = T_R$, two degenerate minima exist: one for $v_R = k = 0$ and a new one at $k = 0$, $v_R = v_R(T_R)$. For temperatures $T < T_R$, the right triplet field v_R will settle down near the minimum given by $v_R = v_R(T_R)$, which will slowly evolve to its zero temperature value. In our analysis we focus on the left-sector phase transition, and we just assume that by the time it occurs, equilibrium has again been attained after the right phase transition, so that we can reliably use the finite temperature effective potential (13).

Near the electroweak phase transition temperature, T_L, the high-temperature expansion of $J_\pm$ is not valid for particles with mass of order $v_R \gg T_L$. The contribution due to these particles is Boltzmann suppressed, and in the limit $m_i(k_i, v_R) \gg T$ it reduces to [7]

$$V_1(k_i, v_R, T) \sim \sum_i \frac{n_i T^2}{(2\pi)^{3/2}} m_i^2 \sqrt{\frac{T}{m_i}} e^{-m_i/T}\left[1 + \frac{15T}{8m_i} + O\left(\frac{T^2}{m_i^2}\right)\right] \; . \tag{19}$$

The exponential factor in the previous expression allows us, within good approximation, to neglect the effect of particles with masses of the v_R scale. We then have to identify the heavy $\mathcal{O}(v_R)$ degrees of freedom, by diagonalizing the mass matrices given in the appendices, and remove their contribution from eq. (13).

[2] In fact, it may be shown numerically that the m/T expansion is a good approximation up to $m/T \sim 2.2(1.6)$ for bosons (fermions) [7].

In the case of the gauge bosons, we see from eqs. (A.7), (A.8) that only the W_1 and Z_1 bosons should be included in eq. (13), since W_2 and Z_2 get masses of order v_R. In the limit $k^2 \ll v_R^2$, the W_1 mass is just given by the (11) entry of the mass matrix (A.4), while the Z_1 mass can be approximated by

$$M_{Z_1}^2 \simeq \frac{g^2(g^2+2g'^2)}{4(g^2+g'^2)}(k_1^2+k_2^2). \tag{20}$$

Since the electric charge formula, eq.(3), implies that

$$\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2}, \tag{21}$$

a Standard Model type relation for the light gauge bosons is also valid in the left-right symmetric model, $M_{Z_1}^2 = M_{W_1}^2/\cos^2\theta_W$, with $cos^2\theta_W = (g^2+g'^2)/(g^2+2g'^2)$.

With respect to the fermions, only the right-handed neutrinos get masses of order v_R. The contribution to the effective potential of quarks and charged leptons is proportional to the Yukawa couplings $(h, \tilde{h})$, so we neglect all of them except for the third generation of quarks. The effect of the light neutrinos is even more suppressed, since their masses are of order k^2/v_R due to the see-saw mechanism.

Let us finally discuss the scalar sector spectrum. Both doubly charged scalar fields, $\delta_R^{++}, \delta_L^{++}$, acquire masses of order v_R and hence decouple. In the singly charged Higgs sector, δ_L^+ is an eigenstate with mass of $\mathcal{O}(v_R)$, while $\phi_1^+, \phi_2^+, \delta_R^+$ mix among themselves. However, the mass matrix elements which relate the triplet with the doublet components are of the type $\alpha_i k v_R$, which are negligible with respect to the terms $\mathcal{O}(\lambda_i k^2)$, since $\alpha_i = \mathcal{O}(k^2/v_R^2)$ and $\lambda_i = \mathcal{O}(1)$. Thus, neglecting the doublet-triplet mixing, δ_R^+ can be identified with the pseudo-Goldstone boson eaten by the W_R, which in a general R_ξ gauge will not contribute to the effective potential (13) near T_L. The two remaining charged scalar fields get electroweak scale masses, which can be calculated analytically by diagonalizing the corresponding 2×2 submatrix, given in appendix B.

Since we have assumed that the tree-level scalar potential is CP conserving, in the neutral Higgs sector scalars and pseudo-scalars do not mix, and we are left with two 4×4 mass matrices. From the one corresponding to the imaginary parts of the neutral fields, we see that neither δ_R^i nor δ_L^i contribute to $V_{eff}(k_i, v_R, T)$; the former is the pseudo-Goldstone boson eaten by the Z', while the latter acquires a right scale mass. Concerning the real parts, δ_L^r decouples because it is also a heavy eigenstate, and neglecting terms of the type $\alpha_i k v_R$ so does δ_R^r. The field dependent masses of the light eigenstates can be found in appendix B.

Then, the field-dependent part of the finite temperature effective potential near the electroweak phase transition may be approximated by:

$$\begin{aligned} V_{eff}(k_i, v_R, T) &= V(k_i, v_R) + \frac{T^2}{24}\Big\{2(2\alpha_1+\alpha_3)v_R^2 + \left[24\lambda_4 + 12h\tilde{h}\right]k_1k_2 \\ &\quad + \left[10\lambda_1 + 4\lambda_3 + \frac{3}{2}g^2 + \frac{3}{4}\frac{g^2(g^2+2g'^2)}{(g^2+g'^2)} + 3(h^2+\tilde{h}^2)\right](k_1^2+k_2^2)\Big\} \\ &\quad - \frac{T}{12\pi}\left\{\sum_{j=1}^{8}[\overline{m}_j^2(k_i, v_R, T)]^{3/2} + 4\left[\frac{g^2}{4}(k_1^2+k_2^2)\right]^{3/2} + 2\left[\frac{g^2(g^2+2g'^2)}{4(g^2+g'^2)}(k_1^2+k_2^2)\right]^{3/2}\right\} \end{aligned} \tag{22}$$

where the sum runs only over the bidoublet scalar degrees of freedom.

Notice that, within reasonable approximations, we have found that in the scalar sector only the bidoublet Φ can give a sizeable contribution to the finite temperature effective potential near T_L, together with the fermions and SM gauge bosons. The effective theory at temperatures of order T_L contains then the same degrees of freedom as a two Higgs doublet model. However, a careful look at the part of the tree-level potential involving the bidoublet shows that some of the scalar couplings of the most general two Higgs model in the left-right symmetric model are constrained or vanish, while the coupling λ_4, usually taken to be zero in two Higgs models, is present in our case. Therefore, although the electroweak phase transition in two Higgs doublet models has been extensively studied in the literature [28], the results can not be extrapolated to the left-right symmetric model in a straight forward way, and it is worth to perform a new analysis within the (different) parameter space relevant for this case.

4 Numerical results

We shall now use the effective potential (22) to calculate the critical temperature and the location of the minimum at the critical temperature. We define the critical temperature T_c as the value of T at which the determinant of the second derivatives of $V_{eff}(k_i, v_R, T)$ at $k = 0$ vanishes:

$$\det \left[\frac{\partial^2 V_{eff}(k_i, v_R, T_c)}{\partial k_i \partial k_j} \right]_{k=0} = 0 \ . \tag{23}$$

In fact, the phase transition starts at $T = T_D$, where T_D is the temperature at which there are two degenerate minima, by tunnelling. At T_c there is no longer any barrier in some direction between what was the minimum at the origin and the new minimum away from the origin, and condensation of the scalar fields can progress rapidly without any suppression from a tunnelling factor.

The effective potential $V_{eff}(k_i, v_R, T_c)$ is a function of the three temperature-dependent vevs, $k_1(T), k_2(T), v_R(T)$; however we expect $v_R(T_c) \simeq v_R(T = 0)$ and therefore we approximate $v_R(T_c)$ by its zero temperature value. Within this approximation, we solve numerically eq. (23), and once T_c is determined we minimize (numerically) the potential $V_{eff}(k_i, v_R, T_c)$ and find the minimum $[k_1(T_c), k_2(T_c)]$. Then we compute the quantity of interest concerning the strength of the electroweak phase transition, that is, the ratio $k(T_c)/T_c$ where $k(T_c) \equiv \sqrt{k_1^2(T_c) + k_2^2(T_c)}$.

Our procedure is the following: we use the minimization conditions at zero temperature (12) to fix three of the unknown parameters in the effective potential (22). The experimental constraint on the weak scale

$$k^2 \equiv k_1^2 + k_2^2 = (246\text{GeV})^2 \tag{24}$$

eliminates one more, while the Yukawa couplings $h, \tilde{h}$ can be determined from the masses of the third generation of quarks, for which we take $m_{top} = 175$ GeV and $m_b = 4.5$ GeV.

The finite temperature effective potential (22) still depends on a large number of free parameters in the scalar sector. However, only some of them are relevant to determine the critical temperature and the position of the minimum, namely $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, $k_1(T = 0)$ and α_3, which only appears in the combination $\alpha_3 v_R^2$. We generate randomly values of these parameters in the ranges $|\lambda_i| \leq 1/2$ (so that the use of the perturbative effective potential is reasonable), $|k_1(T = 0)| \leq 246$ GeV and $|\alpha_3 v_R^2| \leq (246\text{GeV})^2/10$, which takes into account that $\alpha_i \sim \mathcal{O}(k^2/v_R^2)$ for realistic left-right symmetric models.

There are further restrictions on this parameter space, due to zero temperature requirements. First, the potential must be bounded from below, which leads to the set of constraints:

$$\lambda_1 > 0 \quad \lambda_1 - |\lambda_4| > 0 \quad 2\lambda_2 + \lambda_3 - \lambda_1 > 0 \tag{25}$$

Finally, in order to obtain the correct symmetry breaking pattern at zero temperature, we also require that the scalar vevs do not break electromagnetism and that the squared masses of fluctuations about these vevs are positive. That is, the eigenvalues of the light scalar mass matrices at zero temperature (see appendix B) should be positive, once the relations (12) have been used. For any random set $(\lambda_1, \lambda_2, \lambda_3, \lambda_4, k_1(T = 0), \alpha_3 v_R^2)$ satisfying the above conditions, we calculate the critical temperature T_c according to the definition (23), minimize the effective potential at T_c with respect to $k_1(T_c), k_2(T_c)$, and obtain $k(T_c)/T_c$.

Since we use the high temperature expansion of the one-loop effective potential, we need to verify that such approximation is valid at T_c. Thus, once the vevs $k_i(T_c)$ have been determined, we compute the value of all the masses which enter in the effective potential (22) and impose the condition

$$\frac{m(T_c)}{T_c} < 1.6 \tag{26}$$

For the sets of parameters excluded by this condition, the high temperature expansion used would be questionable.

In Fig. 1 we plot the ratio $k(T_c)/T_c$ against the lightest scalar mass m_1, corresponding to a sample of 500 points in the parameter space which passed our selection criterion. As we see, there is a sizeable fraction which satisfies the condition for preserving the baryon asymmetry, $k(T_c)/T_c > 1$, and corresponds to experimentally allowed values of the lightest scalar mass, $m_1 > 50$ GeV [29]. We find this result to be particularly interesting, given the relatively large number of potential signatures of such a model in future experiments [22] and the small number of free parameters to adjust the remaining phenomenology [30].

In Figs. 2, 3, 4 and 5 we show the frequency of occurrence of the (zero temperature) masses corresponding to the light physical scalars, in the allowed baryon preserving region for a sample of 9000 points. They range from about 50 GeV to 250 GeV. The masses of the lightest neutral scalar and the charged ones are peaked about 110 GeV, while the pseudo-scalar and heavy neutral scalar mass distributions are broader and centered in a somehow higher value ~ 150 GeV. So we conclude that there is no significant contradiction with experimental bounds in the baryon preserving cases found.

5 Conclusions

We have analyzed the electroweak phase transition in left-right symmetric models with a scalar sector consisting of a bidoublet and two triplets. Within reasonable simplifying assumptions about the scalar couplings, we find regions of parameter space which are consistent with the present experimental bound on the Higgs mass and with a sufficiently strong first-order electroweak phase transition, eq. (1). We have also obtained the scalar spectrum for these phenomenologically acceptable values of the parameters.

In this paper we have focused on the requirement that the sphaleron processes be sufficiently suppressed after the electroweak phase transition, to preserve the produced baryon asymmetry. Once we have shown that the transition can be strongly enough first order, a detailed calculation of the baryon asymmetry generated during the electroweak phase transition in the framework of left-right symmetric models would be very interesting. As mentioned in the introduction, there are estimates of this quantity in the literature [17]-[19] but they do not include some relevant effects and lead to different results. In principle, the baryon asymmetry in the class of left-right symmetric models considered here will be generated in much the same manner as in two Higgs doublet models, where it has been computed by several groups [8]-[11]. Some of these calculations seem to indicate that enough baryon asymmetry can be generated, so we expect that this will also be the case in left-right symmetric models.

Acknowledgements

We thank D. Comelli, J. Moreno, M. Pietroni, M. Quirós and G. Senjanovic for useful discussions, and E. Roulet for comments on the manuscript. This work was supported in part by the CICYT contract AEN-96/1718, the DGICYT contract PB95-1077 and by EEC under the TMR contract ERBFMRX-CT96-0090. One of us (G.B.) acknowledges a post-doctoral fellow of the Teilchen und Mittelrnergiephysik Graguiertenkolleg of the University of Mainz.

A Gauge boson eigenstates

For the sake of completeness in this appendix we derive physical gauge boson eigenstates (eigenstates of mass matrices) and their eigenvalues. Remember that our scalar sector consists of one bidoublet and one set of left-right symmetric lepton-number carrying triplets with the pattern of symmetry breaking given in eq. (11).

The piece of the Lagrangian containing their covariant derivatives is

$$\mathcal{L}_D = \mathrm{Tr}(D_\mu\Delta_{_L})^\dagger(D^\mu\Delta_{_L}) + \mathrm{Tr}(D_\mu\Delta_{_R})^\dagger(D^\mu\Delta_{_R}) + \mathrm{Tr}(D_\mu\Phi)^\dagger(D^\mu\Phi) \tag{A.1}$$

where

$$\begin{aligned} D_\mu\Delta_{_L} &= \partial_\mu\Delta_{_L} + \frac{1}{2}ig\left[\vec{\tau}\cdot\vec{W}_L\Delta_{_L} - \Delta_{_L}\vec{\tau}\cdot\vec{W}_L\right] + \frac{1}{2}ig'B\Delta_{_L}, \\ D_\mu\Delta_{_R} &= \partial_\mu\Delta_{_R} + \frac{1}{2}ig\left[\vec{\tau}\cdot\vec{W}_R\Delta_{_R} - \Delta_{_R}\vec{\tau}\cdot\vec{W}_R\right] + \frac{1}{2}ig'B\Delta_{_R}, \\ D_\mu\Phi &= \partial_\mu\Phi + \frac{1}{2}ig(\vec{\tau}\cdot\vec{W}_L\Phi - \Phi\vec{\tau}\cdot\vec{W}_R) \end{aligned} \tag{A.2}$$

Then in this model there are seven gauge bosons: four charged ones, the $W^1_{L,R}$ and $W^2_{L,R}$ and three neutral ones, $W^3_{L,R}$ and B. When the Higgs multiplets acquire their vevs (see eq. (11)) the interaction bosons get their masses.

By inspecting the Lagrangian, it is easy to see that the mass terms for the charged bosons are

$$\mathcal{L}^c_{mass} = \begin{pmatrix} W_L^+ & W_R^+ \end{pmatrix} M^c \begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} \tag{A.3}$$

where $W^\pm$ are defined by $W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp W^2)$ and M^c is

$$M^c = \frac{g^2}{4}\begin{pmatrix} k_1^2+k_2^2 & -2k_1k_2 \\ -2k_1k_2 & v_R^2+k_1^2+k_2^2 \end{pmatrix} \tag{A.4}$$

While that of the neutral sector has the form

$$\mathcal{L}^n_{mass} = \frac{1}{2}\begin{pmatrix} W_L^3 & W_R^3 & B \end{pmatrix} M^n \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix} \tag{A.5}$$

where the M^n is given by

$$M^n = \frac{1}{4}\begin{pmatrix} g^2(k_1^2+k_2^2) & -g^2(k_1^2+k_2^2) & 0 \\ -g^2(k_1^2+k_2^2) & g^2(v_R^2+k_1^2+k_2^2) & -gg'v_R^2 \\ 0 & -gg'v_R^2 & g'^2v_R^2 \end{pmatrix} \tag{A.6}$$

The diagonalization of (A.4) and (A.6) gives the masses of the charged $W^\pm_{1,2}$ and neutral A and $Z_{1,2}$ physical fields, they are

$$M^2_{W_{1,2}} = \frac{g^2}{8}\left[v_R^2 + 2(k_1^2+k_2^2) \mp \sqrt{v_R^4 + 16(k_1k_2)^2}\right] \tag{A.7}$$

$$M^2_{Z_{1,2}} = C \mp \sqrt{C^2 - 4D} \tag{A.8}$$

with

$$C = \frac{1}{8}[(g^2+g'^2)v_R^2 + 2g^2(k_1^2+k_2^2)]$$

$$D = \frac{1}{64}g^2(g^2+2g'^2)(k_1^2+k_2^2)v_R^2$$

and

$$M_A = 0 \tag{A.9}$$

B Higgs masses

Here we give a variety of useful result for the mass-squared matrices of the various Higgs sectors before the first derivative constraints have been substituted. The mass matrices are symmetric.

B.1 Neutral scalar mass matrix

We first compute the mass matrix corresponding to the real components of the neutral scalar fields in the $\{\phi_1^r\,,\,\phi_2^r\,,\,\delta_R^r\,,\,\delta_L^r\}$ basis.

$$\begin{aligned}
\mathcal{M}_{11}^{\mathrm{Re}^2} &= -\mu_1^2 + \lambda_1(3k_1^2 + k_2^2) + 4\lambda_2 k_2^2 + 2\lambda_3 k_2^2 + 6\lambda_4 k_1 k_2 + \frac{1}{2}\alpha_1 v_R^2 \\
\mathcal{M}_{12}^{\mathrm{Re}^2} &= -2\mu_2^2 + k_1 k_2\,(2\lambda_1 + 8\lambda_2 + 4\lambda_3) + 3\lambda_4(k_2^2 + k_1^2) + \alpha_2 v_R^2 \\
\mathcal{M}_{13}^{\mathrm{Re}^2} &= \alpha_1 k_1 v_R + 2\alpha_2 k_2 v_R \\
\mathcal{M}_{14}^{\mathrm{Re}^2} &= 0 \\
\mathcal{M}_{22}^{\mathrm{Re}^2} &= -\mu_1^2 + \lambda_1\left(3k_2^2 + k_1^2\right) + 2k_1^2(2\lambda_2 + \lambda_3) + 6\lambda_4 k_1 k_2 + \frac{1}{2}(\alpha_1 + \alpha_3)v_R^2 \\
\mathcal{M}_{23}^{\mathrm{Re}^2} &= 2\alpha_2 k_1 v_R + \alpha_1 k_2 v_R + \alpha_3 k_2 v_R \\
\mathcal{M}_{24}^{\mathrm{Re}^2} &= 0 \\
\mathcal{M}_{33}^{\mathrm{Re}^2} &= -\mu_3^2 + 3\rho_1 v_R^2 + 2\alpha_2 k_1 k_2 + \frac{1}{2}\alpha_1(k_1^2 + k_2^2) + \frac{1}{2}\alpha_3 k_2^2 \\
\mathcal{M}_{34}^{\mathrm{Re}^2} &= 0 \\
\mathcal{M}_{44}^{\mathrm{Re}^2} &= -\mu_3^2 + \frac{1}{2}\rho_3 v_R^2 + \frac{1}{2}\alpha_1(k_1^2 + k_2^2) + 2\alpha_2 k_1 k_2 + \frac{1}{2}\alpha_3 k_2^2
\end{aligned} \tag{B.1}$$

As explained in section 3, near the electroweak phase transition temperature only the light states contribute to the effective potential in eq. (13) and are relevant for our analysis. Within the approximation $k^2 \ll v_R^2$, those are the bidoublet components $\phi_1^r\,,\,\phi_2^r$, and the corresponding mass matrix is just the 2×2 submatrix obtained from the entries (11),(12) and (22) above.

B.2 Neutral pseudo-scalar mass matrix

In a manner similar to the previous section, we compute the mass matrix corresponding to the imaginary components of the neutral scalars, in the basis $\{\phi_1^i\,,\,\phi_2^i\,,\,\delta_R^i\,,\,\delta_L^i\}$.

$$\begin{aligned}
\mathcal{M}_{11}^{\mathrm{Im}^2} &= -\mu_1^2 + \lambda_1(k_1^2 + k_2^2) - 4\lambda_2 k_2^2 + 2\lambda_3 k_2^2 + 2\lambda_4 k_1 k_2 + \frac{1}{2}\alpha_1 v_R^2 \\
\mathcal{M}_{12}^{\mathrm{Im}^2} &= 2\mu_2^2 - 8\lambda_2 k_1 k_2 - \lambda_4(k_2^2 + k_1^2) - \alpha_2 v_R^2 \\
\mathcal{M}_{13}^{\mathrm{Im}^2} &= 0 \\
\mathcal{M}_{14}^{\mathrm{Im}^2} &= 0 \\
\mathcal{M}_{22}^{\mathrm{Im}^2} &= -\mu_1^2 + \lambda_1\left(k_2^2 + k_1^2\right) + 2k_1^2(-2\lambda_2 + \lambda_3) + 2\lambda_4 k_1 k_2 + \frac{1}{2}(\alpha_1 + \alpha_3)v_R^2 \\
\mathcal{M}_{23}^{\mathrm{Im}^2} &= 0 \\
\mathcal{M}_{24}^{\mathrm{Im}^2} &= 0 \\
\mathcal{M}_{33}^{\mathrm{Im}^2} &= -\mu_3^2 + \rho_1 v_R^2 + 2\alpha_2 k_1 k_2 + \frac{1}{2}\alpha_1(k_1^2 + k_2^2) + \frac{1}{2}\alpha_3 k_2^2 \\
\mathcal{M}_{34}^{\mathrm{Im}^2} &= 0
\end{aligned}$$

$$\mathcal{M}^{\mathrm{Im}^2}_{44} = -\mu_3^2 + \frac{1}{2}\rho_3 v_R^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 + \frac{1}{2}\alpha_3 k_2^2 \tag{B.2}$$

Again, in the limit $k^2 \ll v_R^2$, the light states are the bidoublet components ϕ_1^i , ϕ_2^i, and their mass matrix is given by the entries (11), (12) and (22) of $\mathcal{M}^{\mathrm{Im}^2}$.

B.3 Singly charged Higgs mass matrix

The singly charged Higgs mass matrix, in the $\{\phi_1^+ , \phi_2^+ , \delta_R^+ , \delta_L^+\}$ basis, is

$$\begin{aligned}
\mathcal{M}^{+2}_{11} &= -\mu_1^2 + \lambda_1(k_1^2+k_2^2) + 2\lambda_4 k_1 k_2 + \frac{1}{2}(\alpha_1+\alpha_3)v_R^2 \\
\mathcal{M}^{+2}_{12} &= -\alpha_2 v_R^2 + 2\mu_2^2 - \lambda_4(k_1^2+k_2^2) - 2k_1k_2\,(\lambda_3+2\lambda_2) \\
\mathcal{M}^{+2}_{13} &= \frac{1}{2\sqrt{2}}\alpha_3 k_1 v_R \\
\mathcal{M}^{+2}_{14} &= 0 \\
\mathcal{M}^{+2}_{22} &= -\mu_1^2 + \frac{1}{2}\alpha_1 v_R^2 + \lambda_1(k_1^2+k_2^2) + 2\lambda_4 k_1 k_2 \\
\mathcal{M}^{+2}_{23} &= \frac{1}{2\sqrt{2}}\alpha_3 k_2 v_R \\
\mathcal{M}^{+2}_{24} &= 0 \\
\mathcal{M}^{+2}_{33} &= -\mu_3^2 + \frac{1}{2}\,(\alpha_1+\alpha_3)\,(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 + \rho_1 v_R^2 \\
\mathcal{M}^{+2}_{34} &= 0 \\
\mathcal{M}^{+2}_{44} &= -\mu_3^2 + \frac{1}{2}\,(\alpha_1+\alpha_3)\,(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 + \frac{1}{2}\rho_3 v_R^2
\end{aligned} \tag{B.3}$$

In the limit $k^2 \ll v_R^2$, the light mass eigenstates coincide with ϕ_1^+ , ϕ_2^+, and their mass matrix is given by the entries (11), (12) and (22) of $\mathcal{M}^{+^2}$.

B.4 Doubly charged Higgs mass matrix

We now present the doubly charged Higgs mass matrix components in the $\{\delta_R^{++} , \delta_L^{++}\}$ basis .

$$\begin{aligned}
\mathcal{M}^{++2}_{11} &= -\mu_3^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 + \rho_1 v_R^2 + 2\rho_2 v_R^2 + \frac{1}{2}\alpha_3 k_1^2 \\
\mathcal{M}^{++2}_{12} &= 0 \\
\mathcal{M}^{++2}_{22} &= -\mu_3^2 + \frac{1}{2}\alpha_1(k_1^2+k_2^2) + 2\alpha_2 k_1 k_2 + \frac{1}{2}\rho_3 v_R^2 + \frac{1}{2}\alpha_3 k_1^2
\end{aligned} \tag{B.4}$$

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Figure captions

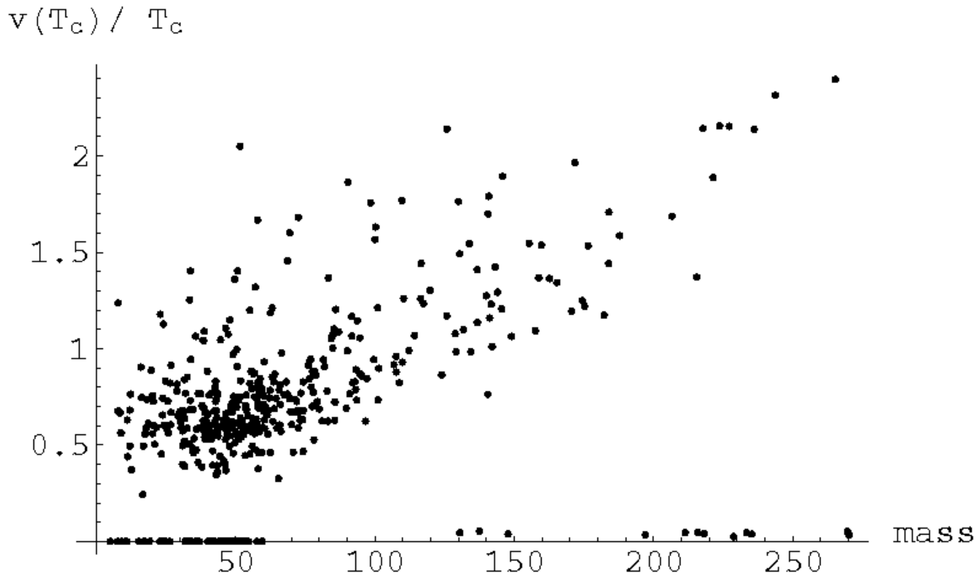
Fig. 1. $v(T_c)/T_c$ ratio vs the ligthest scalar mass (in Gev).

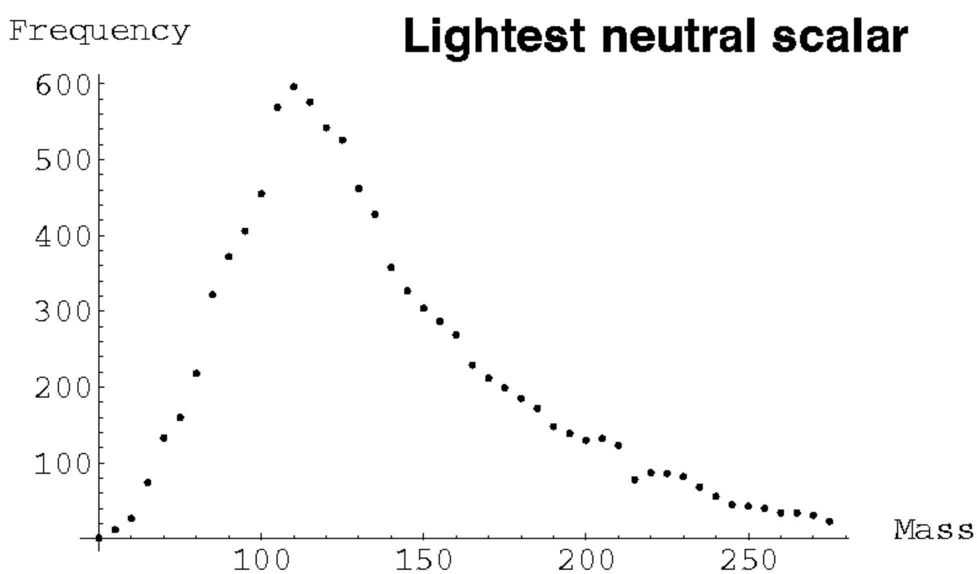
Fig. 2. Frequency distribution of the lightest neutral scalar mass (in Gev) for the baryon preserving cases.

Fig. 3. Frequency distribution of the charged scalar mass (in GeV) for the baryon preserving cases.

Fig. 4. Frequency distribution of the pseudoscalar mass (in Gev) for the baryon preserving cases.

Fig. 5. Frequency distribution of the heavy neutral scalar mass (in Gev) for the baryon preserving cases.


v(Tc)/ Tc
2
1.5
1
0.5
mass
50
100
150
200
250


Lightest neutral scalar
Frequency
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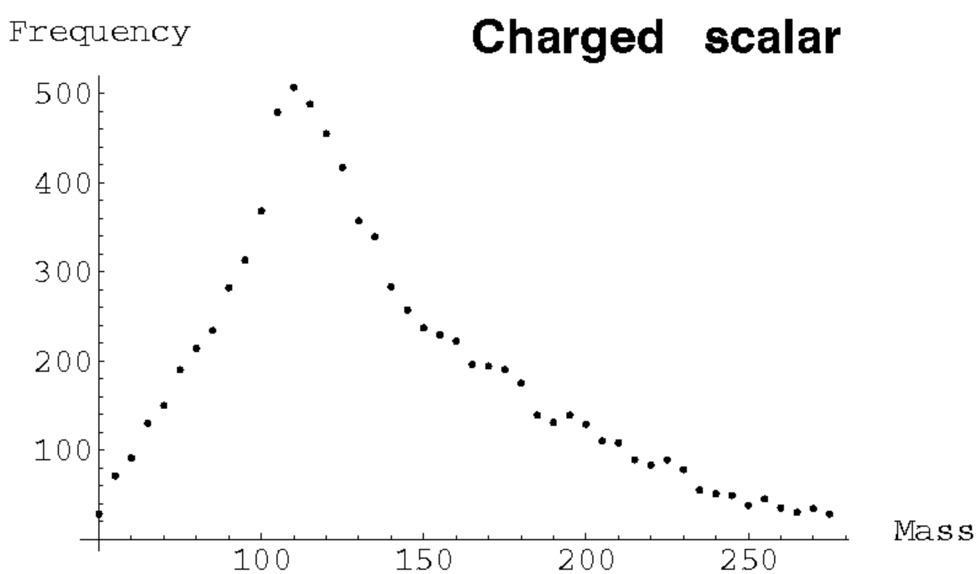
500
400
300
200
100
100
150
200
250
Mass


Charged scalar
Frequency
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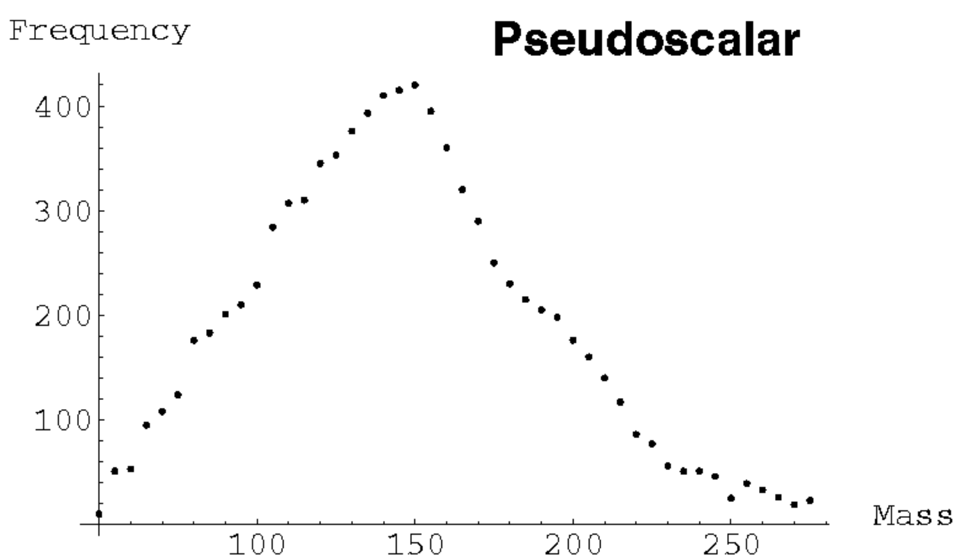
400
300
200
100
100
150
200
250
Mass


Pseudoscalar
Frequency
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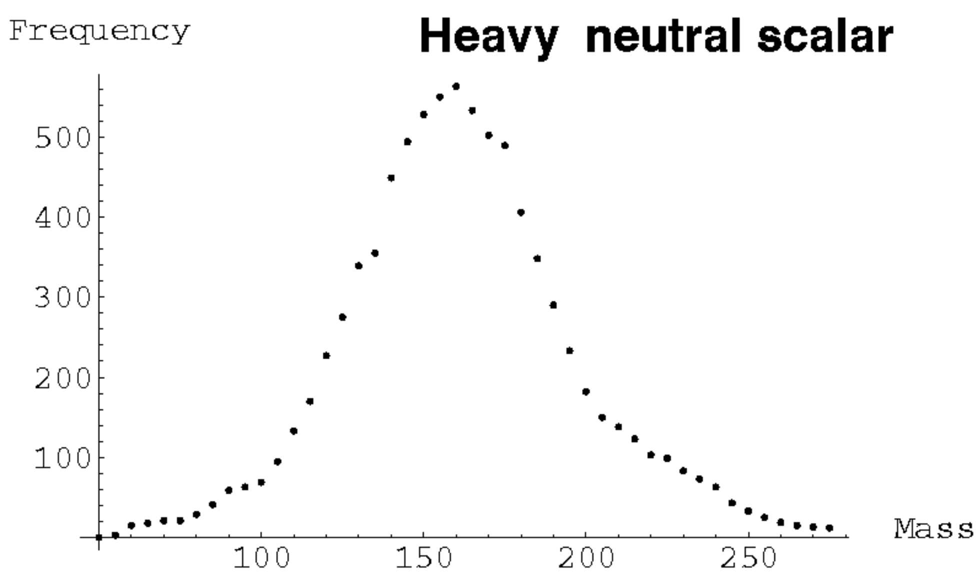
300
200
100
100
150
200
250
Mass


Frequency
Heavy neutral scalar
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400
300
200
100
100
150
200
250
Mass