

# Finite temperature effects on CP violating asymmetries

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## Abstract

We compute the  $CP$  violating decay asymmetries relevant for baryogenesis scenarios involving the out of equilibrium decays of heavy particles, including the finite temperature effects arising from the background of light thermal particles which are present during the decay epoch. Thermal effects can modify the size of  $CP$  violation by a sizeable fraction in the decay of scalar particles, but we find interesting cancellations in the thermal corrections affecting the asymmetries in the decays of fermions, as well as in the decay of scalars in supersymmetric theories. We also estimate the effects which arise from the motion of the decaying particles with respect to the background plasma.

# 1 Introduction

A classic, and still very attractive, scenario for the generation of the baryon asymmetry of the Universe, is based on the fact that very massive particles fall out of equilibrium as the temperature of the Universe drops below their mass and their equilibrium density becomes Boltzmann suppressed. If they decay at that epoch through baryon ( $B$ ) violating channels which also violate  $CP$ , a net baryon asymmetry will result.

Usually, in these scenarios the  $CP$  violation results from a one-loop decay diagram, whose interference with the tree level process allows the phases of the complex coupling constants to show up in the decay asymmetries.

In addition to the complex couplings, to have a non-zero partial decay rate asymmetry requires the loop integral to develop an absorptive part. This actually happens whenever the loop diagram can be cut in such a way that the particles in the cut lines can be produced on-shell, and hence these particles need to be lighter than the decaying one. In the proposed baryogenesis scenarios of this kind, the light particles in the loop are just standard quarks, leptons or Higgs bosons, so that the appearance of a non-vanishing absorptive part is guaranteed.

At the high temperatures at which the heavy particles decay, the light standard particles are in equilibrium with the hot plasma present, and hence a question arises on whether the existence of the background particles has any effect in the evaluation of the  $CP$  violating asymmetries. Indeed, some time ago Takahashi [1] showed that the thermal effects could modify the predictions for baryogenesis in  $SU(5)$  models by up to  $\sim 40\%$  with respect to the  $T = 0$  results, and hence these effects may need to be taken into account in the proper computation of the resulting baryon asymmetry in specific models.

The main effect of the background can be taken into account by employing finite temperature propagators in the computation of the loop. In this way it is possible to consider simultaneously both the ‘direct’ propagation of a particle between two vertices in the loop and the absorption by the medium of a particle from the first vertex combined with the emission of another one towards the second. These two alternatives are actually indistinguishable in a thermal bath.

Another implication of the finite density background is the modification of the final state phase space density distributions, which take into account the stimulation of the decays into bosons and the Pauli blocking of the decays into fermions, and this may eventually also affect the rates.

## 2 $SU(5)$ triplet decays

In order to discuss these issues, let us start by reanalysing the scalar decays into two fermions, which is relevant in the case of heavy Higgs boson triplet decays in  $SU(5)$ . We will generalise the computation in ref. [1] by including also the  $CP$  violating diagrams arising from mixing among different heavy states [2, 3, 4, 5]. Notice that in order to have  $CP$  violation at one loop in the  $SU(5)$  GUT it is necessary to have more than one heavy state, and also the existence of more than one heavy state is natural in other models, such as the leptogenesis scenarios to be discussed later, and hence the contribution from the mixing among the heavy states is generally important. For simplicity we will consider the case in which the masses of the heavy states are significantly splitted, since the study of the  $CP$  violation in the near degenerate situation present additional complications [6, 7].

One generally expects that only the decay of the lightest of the heavy states,  $T_1$ , will

be the one leading to a net  $B$ , since any asymmetry produced at earlier times through the decay of heavier states would be erased by  $B$  violating processes which could still be in equilibrium<sup>1</sup>. Hence, one has to compute the asymmetry resulting from the  $T_1$  decay, and in the case we will consider in which there is a hierarchy among the masses of the heavy states ( $M_k \gg M_1$ , with  $k > 1$ ), it will be natural to assume that the heavier states  $T_k$  have already decayed, and hence have a negligible density, at temperatures  $T \leq M_1$  when the lighter one is falling out of equilibrium.

Let us consider the  $SU(5)$  lagrangian involving several scalar five-plets  $\Phi_i = (T_i, H_i)$ , containing, together with Higgs doublets  $H_i$ , the heavy color triplets  $T_i$  :

$$\mathcal{L} = f_i \Phi_{i\alpha} (\bar{\Psi}^{\alpha\beta} \chi_\beta) + \frac{g_i}{8} \Phi_{i\alpha} (\epsilon^{\alpha\beta\gamma\delta\epsilon} \bar{\Psi}_{\beta\gamma}^c \Psi_{\delta\epsilon}) + h.c., \quad (1)$$

where the gauge indices are denoted by greek letters. The matter fields are in the decuplet and the fiveplet representations as usual,  $\Psi = (q, u^c, e^c)$  and  $\chi = (d, l^c)$ . Since they are vectors in flavour space, the Yukawa couplings  $f_i$  and  $g_i$  should be thought as  $3 \times 3$  matrices, but for simplicity the flavour indices are not displayed.

The  $CP$  violating  $B$  asymmetry arising from the decay of a  $T_1$  and  $\bar{T}_1$  pair is

$$\epsilon = \frac{\sum_f B_f [\Gamma(T_1 \rightarrow F_f) - \Gamma(\bar{T}_1 \rightarrow \bar{F}_f)]}{\sum_f [\Gamma(T_1 \rightarrow F_f) + \Gamma(\bar{T}_1 \rightarrow \bar{F}_f)]}, \quad (2)$$

with  $B_f$  the baryon number of the final states  $F_f = \bar{q}l^c, \bar{u}e^c, \bar{u}^c d, \bar{q}^c q$ . From the interactions in eq. (1), at zero temperature we have

$$\epsilon = \frac{4 \sum_k \left[ -\text{Im} \left\{ \text{Tr} \left( g_k^\dagger g_1 f_k f_1^\dagger \right) \right\} \text{Im} \{ I_t(x_k) \} + \frac{1}{1-x_k} \text{Im} \left\{ \text{Tr} \left( f_k f_1^\dagger \right) \text{Tr} \left( g_k^\dagger g_1 \right) \right\} \text{Im} \{ I_s \} \right]}{\left[ 4 \text{Tr} \left( f_1 f_1^\dagger \right) + 3 \text{Tr} \left( g_1^\dagger g_1 \right) \right]}, \quad (3)$$

where  $x_k$  is the heavy-to-light ratio of squared masses:

$$x_k \equiv \frac{M_k^2}{M_1^2} \quad (4)$$

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<sup>1</sup>However, it is possible to imagine the situation in which a heavier triplet has the smallest decay rate, if the effect of the larger phase space is compensated by the smallness of the relevant couplings.

Notice that the asymmetry is null if there is a single triplet field. In this case, the result (3) depends on the hermitian matrices  $g_1^\dagger g_1$  and  $f_1 f_1^\dagger$ ; therefore their trace has no imaginary part, and the trace of the product is equal to half the trace of the anticommutator, again an hermitian matrix. The terms in the denominator correspond to the tree level decay rate; those in the numerator to the interference between the tree level and the absorptive part of the loop amplitudes<sup>2</sup>. In particular,  $I_t$  arises from the loop integral in the ‘vertex’ contribution, in which the  $T_k$  is exchanged in the  $t$ -channel (see fig. 1a), and  $I_s$  comes from the loop integral in the ‘wave’ contribution, in which the  $T_k$  is exchanged in the  $s$ -channel (fig. 1b), i.e.

$$\bar{u}(p_1)v(p_2)I_t(x_k) = \bar{u}(p_1) \int \frac{d^4q}{(2\pi)^4} S(p-q, 0)S(-q, 0)D(q-p_2, M_k)v(p_2), \quad (5)$$

$$M_1^2 I_s = -\frac{i}{2} \int \frac{d^4q}{(2\pi)^4} \text{Tr}(S(-q, 0)S(p-q, 0)), \quad (6)$$

with  $S(p, m)$  ( $D(p, m)$ ) being the propagator of a fermion (scalar) with mass  $m$  and momentum  $p$ .

From these expressions one gets the following absorptive parts:

$$\text{Im}\{I_t(x)\} = \frac{1}{16\pi} \left[ 1 - x \ln \left( 1 + \frac{1}{x} \right) \right], \quad (7)$$

and

$$\text{Im}\{I_s\} = \frac{1}{16\pi}. \quad (8)$$

We consider now the finite temperature effects on the  $CP$  violating asymmetry  $\epsilon$ . The main effect comes from using instead of the usual  $T = 0$  propagators, the finite  $T$  ones.

We choose to work in the real time formalism (RTF) of thermal field theory [10]. The

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<sup>2</sup>The apparent difference with respect to refs. [8, 1] in the denominator is just due to a different normalization of the field  $\Psi$ , and hence of the Yukawa couplings  $f_i$ . However, our result for the numerator differs from ref. [3] in the sign of the first term (in agreement with [9, 4]) and a factor 2 in the second one.

Green's functions computed in this formalism are directly the time ordered ones, unlike in the imaginary time formalism where different analytical extensions to real momenta lead to different Green's functions (retarded, advanced, etc.) [11]. The RTF involves the introduction of a ghost field dual to each physical field, which leads to a doubling of the degrees of freedom. The thermal propagator has then a  $2 \times 2$  matrix structure: the (11) component refers to the physical field, the (22) component to the corresponding ghost field and the off-diagonal (12) and (21) components mixing them. However, since we are working only at one loop and the external legs are physical, i.e. type-1 fields, we just need the (11) component of the propagators, which are, for fermions and bosons respectively,

$$S_{11}(p, m) = (\not{p} + m) \left[ \frac{i}{p^2 - m^2 + i0^+} - 2\pi n_F(p \cdot u) \delta(p^2 - m^2) \right], \quad (9)$$

$$D_{11}(p, m) = \left[ \frac{i}{p^2 - m^2 + i0^+} + 2\pi n_B(p \cdot u) \delta(p^2 - m^2) \right], \quad (10)$$

with  $u$  the 4-velocity of the medium ( $u = (1, 0, 0, 0)$  in the medium rest frame), and

$$n_{F,B}(x) = \frac{1}{\exp(|x|/T) \pm 1}. \quad (11)$$

We drop the (11) subindex in the propagators from now on.

As discussed previously, we neglect the background density of the heavy triplets<sup>3</sup>  $T_k$  (assuming  $M_k \gg M_1$ ), and as a first approximation we assume that the decaying particle  $T_1$  is at rest (particle motion effects will be considered in section 4). Using the well known property,

$$\frac{1}{x \pm i0^+} = \mathcal{P} \left( \frac{1}{x} \right) \mp i\pi \delta(x) \quad (12)$$

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<sup>3</sup>Notice that the exchange of  $T_1$  in the one-loop diagrams does not contribute to  $\epsilon$ , so that only  $k > 1$  are relevant.

we get for the absorptive part of the vertex loop integral <sup>4</sup>

$$\text{Im}\{I_t^T(x_k)\} = \text{Im}\{I_t(x_k)\} \left[1 - 2\bar{n}_F + 2\bar{n}_F^2\right] - \frac{x_k}{8\pi} \int_{x_k}^{\infty} \frac{du}{u+1} n_F\left(\frac{M_1 u}{2}\right), \quad (13)$$

where

$$\bar{n}_{F,B} \equiv n_{F,B}\left(\frac{M_1}{2}\right). \quad (14)$$

This is similar to the result in ref. [1], except for the relative sign between the  $T = 0$  part and the finite temperature corrections, implying that the temperature effects tend to reduce (instead of enhancing) the  $CP$  violation. The integral term in the r.h.s. of eq. (13) arises from the absorption of particles from the background which are energetic enough so as to put the intermediate state  $T_k$  on-shell and hence make it contribute to the absorptive part. This term becomes then extremely small in the case in which the heavy masses have a significant hierarchy, i.e. for  $M_k \gg M_1$ , since the amount of background particles which are energetic enough is Boltzmann suppressed.

For the finite temperature ‘wave’ contribution we obtain

$$\text{Im}\{I_s^T\} = \text{Im}\{I_s\} \left[1 - 2\bar{n}_F + 2\bar{n}_F^2\right], \quad (15)$$

so that the temperature dependence is similar to the one in the leading term of the vertex part. To give a quantitative idea of the effect, we notice that if the temperature is taken to be 1, 1/3 or 1/10 of the lightest triplet mass,  $M_1$ , the overall factor in (15) including the temperature effects is 0.53, 0.70 or 0.99 respectively. The physical interpretation of this effect is simple: due to the thermal background, the two light fermions exchanged in the loop are subject to a Pauli blocking, and this leads to a reduction of the amount of  $CP$  violation.

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<sup>4</sup>We have checked that the same result can be obtained by applying the Cutkosky cutting rules at finite temperature in the real time formalism [12].

In this scenario there is no effect on  $\epsilon$  resulting from the final state blocking, since this just leads to overall factors  $(1 - \bar{n}_F)^2$  multiplying the rates, and hence these factors cancel in the ratio in eq. (2). Other thermal effects, such as thermal masses or wave function renormalization, are higher order in the coupling constants and hence we neglect them.

### 3 Leptogenesis scenarios

The  $SU(5)$  model discussed in the previous section, in spite of being the prototype for the ‘out of equilibrium decay’ scenarios of baryogenesis, has the drawback that it generates no net  $B - L$  asymmetry (a characteristic of  $SU(5)$ ), and hence the  $B$  generation is vulnerable to the anomalous  $B$  violating processes of the Standard Model [13] (which only leave  $B - L$  unaffected), with the consequence that all asymmetries generated within this model will be eventually erased.

A very interesting way out to this problem [14, 15] is based on the generation of a lepton ( $L$ ) asymmetry at early times, by the out of equilibrium decay of heavy isosinglet neutrinos (the usual ones appearing in see-saw models for neutrino masses and naturally present in GUT models such as  $SO(10)$ ).

In this section we discuss temperature effects in this kind of models, first under the minimal assumption that the standard model spectrum is enlarged to include heavy right-handed neutrinos, and then, considering the supersymmetric extension of the model.

#### 3.1 Non-supersymmetric case

The interactions of the heavy neutrinos  $N_i$ , in the basis in which their mass matrix is diagonal, are given by the following Lagrangian

$$\mathcal{L} = -\lambda_{ai}\epsilon_{\alpha\beta}\bar{N}_i P_L \ell_a^\alpha H^\beta + h.c. \quad (16)$$



where  $\ell_a = (\nu_a, l_a^-)$  and  $H = (H^+, H^0)$  are the lepton and Higgs Standard Model doublets ( $a = e, \mu, \tau, i = 1, 2, 3$ , and  $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$ , with  $\epsilon_{12} = +1$ ).

The complete  $T = 0$   $CP$  violating  $L$  asymmetry was computed for this model in ref. [5], resulting in

$$\epsilon = 2 \sum_{k>1} \mathcal{I}_{k1} \left[ \text{Im}\{J_t(x_k)\} + 2 \frac{\sqrt{x_k}}{x_k - 1} \text{Im}\{J_s\} \right], \quad (17)$$

where  $x_k$  is defined analogously to eq. (4) and

$$\mathcal{I}_{k1} \equiv \frac{\text{Im}\left\{\left(\lambda^\dagger \lambda\right)_{k1}^2\right\}}{\left(\lambda^\dagger \lambda\right)_{11}}. \quad (18)$$

Notice that, in full analogy with the  $SU(5)$  case, a single right handed neutrino would be unable to generate any asymmetry.

The loop integrals are given in this case by

$$\bar{u}(p_1) P_R u(p) J_t(x_k) = \bar{u}(p_1) P_R \int \frac{d^4 q}{(2\pi)^4} S(q, 0) S(q - p_2, M_k) D(p - q, 0) P_L u(p) \quad (19)$$

and

$$\bar{u}(p_1) u(p) M_1 J_s = -i \bar{u}(p_1) \int \frac{d^4 q}{(2\pi)^4} S(q, 0) D(p - q, 0) u(p), \quad (20)$$

so that the absorptive parts result

$$\text{Im}\{J_t(x)\} = \frac{1}{16\pi} \sqrt{x} \left[ 1 - (1+x) \ln \left( 1 + \frac{1}{x} \right) \right], \quad (21)$$

and

$$\text{Im}\{J_s\} = -\frac{1}{32\pi}. \quad (22)$$

The computation of the finite temperature contribution to these quantities is similar to the one in the previous section, and leads to

$$\begin{aligned} \text{Im}\{J_t^T(x_k)\} &= \text{Im}\{J_t(x_k)\} [1 - \bar{n}_F + \bar{n}_B - 2\bar{n}_F \bar{n}_B] \\ &+ \frac{\sqrt{x_k}}{16\pi} \int_{x_k}^{\infty} \frac{du}{u+1} \left[ n_F \left( \frac{M_1 u}{2} \right) (u - x_k) + n_B \left( \frac{M_1 u}{2} \right) (x_k + 1) \right], \end{aligned} \quad (23)$$

$$\text{Im}\{J_s^T\} = \text{Im}\{J_s\} [1 - \bar{n}_F + \bar{n}_B - 2\bar{n}_F \bar{n}_B],$$

where for instance in this last expression the different contributions in the r.h.s. clearly separate into the pieces coming from the  $T = 0$  propagators, the one from the thermal correction to the fermion ( $\ell$ ) propagator, the one from the thermal piece of the boson ( $H$ ) propagator and the product of these two corrections. However, it is easy to check that the Bose and Fermi distributions satisfy

$$n_B(E) - n_F(E) = 2n_B(E)n_F(E), \quad (24)$$

so that the main temperature correction cancels out (only the small integral term in  $\text{Im}\{J_t\}$  survives). This is due to the opposite effects resulting from the Pauli blocking of the loop fermion line and the stimulation of the bosonic loop line. On the other hand, here again the final state statistical factors  $(1 - \bar{n}_F)(1 + \bar{n}_B)$  cancel in the expression for  $\epsilon$ , and as a result no significant temperature dependent effect is found.

### 3.2 Supersymmetric case

This scenario has also received considerable attention within a supersymmetric framework [16, 17, 5], in particular because the scalar partner of the heavy neutrino  $\tilde{N}_1$  is a good candidate for being the inflaton field, in which case the  $L$  asymmetry could be produced during the process of reheating of the Universe as  $\tilde{N}_1$  decays [17]. In this case, as we will now show, another interesting cancellation is found in the asymmetry produced by  $\tilde{N}_1$  decay.

The scalar neutrino can decay either into two scalars ( $\tilde{L}H$ ) or into two fermions ( $\ell\tilde{h}$ ), and the contribution to  $CP$  violation in one channel is obtained from the loop involving the particles of the other channel [5]. The thermal effects will modify the asymmetries corresponding to each channel, in a way similar as they did in the case of  $SU(5)$ . For instance, in the  $\tilde{L}H$  channel, if we ignore the small integral piece coming from the vertex

(equivalent to the last term in the r.h.s. of eq. (13)), the asymmetry will be

$$\epsilon^T(\tilde{N} \rightarrow \tilde{L}H) \simeq \epsilon^{T=0}(\tilde{N} \rightarrow \tilde{L}H) [1 - 2\bar{n}_F(1 - \bar{n}_F)], \quad (25)$$

and similarly

$$\epsilon^T(\tilde{N} \rightarrow \tilde{\ell}\tilde{h}) \simeq \epsilon^{T=0}(\tilde{N} \rightarrow \tilde{\ell}\tilde{h}) [1 + 2\bar{n}_B(1 + \bar{n}_B)]. \quad (26)$$

However, due to the effects of the final state phase space factors  $(1 \mp \bar{n}_{F,B})$  entering into the partial decay rates, the branching ratios of the two different channels will no longer be equal (as is the case at  $T = 0$ ). One has instead that

$$BR(\tilde{N} \rightarrow \tilde{\ell}\tilde{h}) = \frac{(1 - \bar{n}_F)^2}{(1 - \bar{n}_F)^2 + (1 + \bar{n}_B)^2} = 1 - BR(\tilde{N} \rightarrow \tilde{L}H). \quad (27)$$

The total asymmetry produced in the  $\tilde{N}$  decay is

$$\epsilon_{\tilde{N}}^T = BR(\tilde{N} \rightarrow \tilde{\ell}\tilde{h})\epsilon^T(\tilde{N} \rightarrow \tilde{\ell}\tilde{h}) + BR(\tilde{N} \rightarrow \tilde{L}H)\epsilon^T(\tilde{N} \rightarrow \tilde{L}H), \quad (28)$$

with the surprising result that the main corrections arising from thermal effects actually cancel out, leading to

$$\epsilon_{\tilde{N}}^T \simeq \epsilon_{\tilde{N}}^{T=0}. \quad (29)$$

Notice that there are also new supersymmetric diagrams contributing to the  $CP$  violating asymmetry in the heavy neutrino decays. However, since the particles in the loop as well as the external ones are always one fermion and one boson, both massless, the cancellation found in the previous subsection will also occur in the new channels, and therefore there are no significant thermal corrections to the zero temperature result of ref. [5].

## 4 Effects of particle motion

In the discussion so far we have always considered the decay rate of a particle at rest in the thermal bath. However, the decaying particle will in general be moving through the

background with non zero velocity  $\vec{\beta} = \vec{v}/c$ . Since now the Lorentz symmetry is explicitly broken by the plasma, this motion can in principle affect the thermal corrections to the decay asymmetry. When the leading thermal corrections cancel, as in the leptogenesis scenarios discussed in Section 3, the effects of the motion of the decaying particle will provide the main thermal corrections, and hence it is worth to quantify them. To estimate the size of these effects, we will consider here the case of the heavy neutrino decay in the non-supersymmetric model. The other cases can be analysed similarly.

It is convenient to compute the decay rate asymmetries in the rest frame of the decaying particle, where the medium will be characterised by a non trivial 4-velocity  $u = (\gamma, -\gamma\vec{\beta})$ , with  $\gamma = 1/\sqrt{1-\beta^2}$  as usual. In this system, the effect of the motion will reflect in a modification of the equilibrium distributions appearing in the thermal propagators, eqs. (9–10). The decay rate will also depend on  $\vec{\beta}$  through the final state phase space factors, which for the case of fermion decays is

$$[1 - n_F(k_\ell \cdot u)] [1 + n_B(k_H \cdot u)] = \frac{2}{1 - \exp(-M_1\gamma/T)} P_\beta(\cos\theta), \quad (30)$$

where  $\theta$  is the angle between  $\vec{\beta}$  and the momentum of the final state lepton, and we have introduced

$$P_\beta(z) \equiv \frac{1}{2} \left\{ 1 - \sinh\left(\frac{z\beta\gamma M_1}{2T}\right) / \sinh\left(\frac{\gamma M_1}{2T}\right) \right\}^{-1}. \quad (31)$$

We see that now there is a privileged direction selected by the plasma spatial velocity and therefore the decay process is no more isotropic: the rate depends on the angle of the decay product trajectories with respect to this direction. Due to statistics the fermions (bosons) are preferentially emitted in the direction parallel (antiparallel) to the plasma velocity, which corresponds to the less (more) occupied region of the thermal distribution. Anyway, since we are interested in the total decay rate, the angular dependence is integrated out and we are left only with a  $\beta^2$  dependence<sup>5</sup>.

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<sup>5</sup>The integrated decay rate can depend only on the Lorentz invariants  $p^2 = M_1^2$  and  $p \cdot u = M_1\gamma$  [18].

Let us define  $\epsilon_\beta$  as the integrated asymmetry generated in the decay of a heavy neutrino moving with velocity  $\beta$ . The decaying particles will actually have a distribution of velocities with occupation numbers  $n(E)$ , where  $E = M_1\gamma$ . To estimate the overall effect of the particle motion we may just approximate this distribution with the Fermi Dirac distribution,  $n(E) = n_F(E)$ , and compute the average asymmetry

$$\langle \epsilon \rangle = \frac{1}{N_1} \int \frac{d^3p}{(2\pi)^3} n(\gamma M_1) \epsilon_\beta = \frac{1}{N_1} \int_0^1 d\beta \frac{dN_1}{d\beta} \epsilon_\beta, \quad (32)$$

where  $N_1$  is the particle's volume density and

$$\frac{dN_1}{d\beta} = \frac{M_1^3}{2\pi^2} \beta^2 \gamma^5 n(\gamma M_1). \quad (33)$$

It is clear that  $\langle \epsilon \rangle$ , computed for a given temperature  $T$ , is just the asymmetry that would result if the initial thermal population of heavy neutrinos went out of equilibrium and decayed all simultaneously at temperature  $T$ . We will use the asymmetry  $\langle \epsilon \rangle$  as an indicator of the possible effects of the particle motion, although clearly to obtain the exact impact of this into the final lepton asymmetry would require to integrate the whole Boltzmann equations, a task beyond our scopes.

To obtain the decay asymmetry at a fixed velocity  $\epsilon_\beta$ , it is convenient to separate the angular dependences arising from the final state phase space factor  $P_\beta(\cos \theta)$  and the one arising from the one loop integrals (via the anisotropic background density in the particle rest frame). This last will be included in the factor  $L_\beta(\cos \theta, x_k)$  defined through

$$\epsilon_\beta^c = \frac{2}{\int_{-1}^1 dz P_\beta(z)} \sum_k \mathcal{I}_{k1} \int_{-1}^1 dz L_\beta^c(z, x_k) P_\beta(z). \quad (34)$$

Here the supraindex  $c$  labels the two different contributions to the CP asymmetry in the decay, i.e. the ‘‘wave’’ ( $\epsilon^s$ ) and the ‘‘vertex’’ ( $\epsilon^t$ ) pieces. Clearly at zero velocity we have, according to eq. (17),

$$L_0^s(z, x_k) = \frac{2\sqrt{x_k}}{x_k - 1} \text{Im}\{J_s\}, \quad (35)$$

$$L_0^t(z, x_k) = \text{Im}\{J_t(x_k)\}, \quad (36)$$

which are actually independent of  $z$  as expected.

At  $\beta \neq 0$  we find after direct computation of the interference terms

$$L_\beta^s(z, x_k) = L_0^s(z, x_k)[f_\beta^{(0)} - z f_\beta^{(1)}], \quad (37)$$

where the loop functions  $f_\beta^{(n)}$

$$f_\beta^{(n)} = \int_{-1}^1 dy y^n P_\beta(y) \quad (38)$$

arise after integrating over the angle of the momenta in the loop. Notice that the phase space factor  $P_\beta$  appears also in these integrals, arising from the statistical factors in the thermal loops. In the limit of zero velocity, we have  $f_0^{(0)} = 1$  and  $f_0^{(1)} = 0$ .

Now we can write in a simple form the final expression for  $\epsilon_\beta^s$ ; namely, putting together eqs. (32), (34) and (37) we get:

$$\langle \epsilon^s \rangle = \epsilon_0^s \frac{1}{N_1} \int_0^1 d\beta \frac{dN_1}{d\beta} f_\beta^{(0)} \left[ 1 - \left( \frac{f_\beta^{(1)}}{f_\beta^{(0)}} \right)^2 \right]. \quad (39)$$

It is not possible to integrate analytically this expression, but expanding the loop functions to first order in  $\beta^2$  we obtain

$$f_\beta^{(0)} \simeq 1 + \frac{1}{3} \beta^2 \left( \frac{M_1/2T}{\sinh(M_1/2T)} \right)^2, \quad (40)$$

$$f_\beta^{(1)} \simeq \frac{1}{3} \beta \frac{M_1/2T}{\sinh(M_1/2T)}. \quad (41)$$

Substituting in eq. (39), we get

$$\langle \epsilon^s \rangle \simeq \epsilon_0^s \left[ 1 + \frac{2}{9} \langle \beta^2 \rangle \left( \frac{M_1/2T}{\sinh(M_1/2T)} \right)^2 \right]. \quad (42)$$

For  $M_1/T = 1, 3, 10$  we have  $\langle \epsilon^s \rangle / \epsilon_0^s \simeq 1.168, 1.057, 1.000$ ; we see therefore that the effect is small compared to the  $\beta = 0$  piece. In the previous evaluation we have used

for the average velocity just a simple approximation, which is accurate to the 10% level, writing  $\langle \beta^2 \rangle \simeq \langle \beta \rangle_{Boltz}^2$ , with the average velocity for a Boltzmann distribution of massive relativistic particles being (with  $x \equiv M/T$ )

$$\langle \beta \rangle_{Boltz} = \frac{2(1+x)\exp(-x)}{x^2 K_2(x)}. \quad (43)$$

Computing numerically eq. (39) we find, for the same reference values of  $M_1/T$  as before, that  $\langle \epsilon^s \rangle / \epsilon_0^s = 1.231, 1.054$  and  $1.000$ , so that the approximate result is acceptable.

Let us now consider the ‘‘vertex’’ contribution. In this case the loop integration is more involved and we get the following expression (neglecting the small integral piece which is Boltzmann suppressed):

$$L_\beta^t(z, x_k) = \frac{\sqrt{x_k}}{16\pi} \left[ f_\beta^{(0)} - g_\beta(x_k, z) \right], \quad (44)$$

where

$$g_\beta(x_k, z) = \int_{-1}^1 dy P_\beta(y) \frac{2(1+x_k)}{\sqrt{[y+z(1+2x_k)]^2 + 4x_k(x_k+1)(1-z^2)}}. \quad (45)$$

For  $\beta = 0$  the function  $g$  actually does not depend on  $z$  and is

$$g_0(x_k) = (1+x_k) \ln \left( 1 + \frac{1}{x_k} \right), \quad (46)$$

so that we recover exactly eq. (36).

Substituting the result of the loop integration in eq. (34) we finally obtain

$$\epsilon_\beta^t = 2 \sum_k \mathcal{I}_{k1} \frac{\sqrt{x_k}}{16\pi} \left[ f_\beta^{(0)} - \frac{\int_{-1}^1 dz g_\beta(x_k, z) P_\beta(z)}{f_\beta^{(0)}} \right]. \quad (47)$$

Again we can evaluate this function analytically only for small  $\beta$ , by using the expansion

$$g_\beta(x_k, z) \simeq g_0(x_k) + \beta z \frac{M_1/2T}{\sinh(M_1/2T)} h(x_k) + \beta^2 \left( \frac{M_1/2T}{\sinh(M_1/2T)} \right)^2 \left[ j(x_k) - z^2 l(x_k) \right], \quad (48)$$

where

$$h(x) = 2(1+x) - (1+2x)g_0(x), \quad (49)$$

$$j(x) = (1+x)(1+2x) - 2x(1+x)g_0(x), \quad (50)$$

$$l(x) = 3(1+x)(1+2x) - (1+6x+6x^2)g_0(x). \quad (51)$$

So, the average CP violation is given by

$$\langle \epsilon^t \rangle \simeq 2 \sum_k \mathcal{I}_{k1} \text{Im}\{J_t(x_k)\} \left[ 1 + \frac{1}{3} \langle \beta^2 \rangle \left( \frac{M_1/2T}{\sinh(M_1/2T)} \right)^2 \left( \frac{1}{g_0(x_k) - 1} - 2x_k \right) \right]. \quad (52)$$

We then see that the effect due to the particle motion is again to increase the vertex CP asymmetry, as in the case of the wave part. However, the effect depends also on  $x_k$ , i.e. on the hierarchy between the particle masses. In the limit of large  $x_k$  we can use the expansion  $(g_0(x_k) - 1)^{-1} \simeq 2x_k + 2/3$ , to obtain

$$\langle \epsilon^t \rangle \simeq \epsilon_0^t \left[ 1 + \frac{2}{9} \langle \beta^2 \rangle \left( \frac{M_1/2T}{\sinh(M_1/2T)} \right)^2 \right], \quad (53)$$

so that the overall factor coincides with the one in eq. (42) for  $\langle \epsilon^s \rangle$ .

To estimate the goodness of the approximate result in eq. (52) we evaluated the term in square brackets there, taking  $x_k = 5$  for definiteness, and we obtained 1.163, 1.055 and 1.000 for  $M_1/T = 1, 3$  and 10 respectively. A numerical evaluation of the exact correction due to the velocity, using eq. (47), leads for the same factor the values 1.222, 1.052, 1.000 respectively, so that again the accuracy is reasonable, and we see that for  $T < M_1$  the velocity dependent correction to  $\epsilon$  is not large.

## 5 Conclusion

The baryon asymmetry of the Universe is probably the most important manifestation of the existence of  $CP$  violation. In the classic scenarios for baryogenesis through the



out of equilibrium decay of heavy particles in the early stages of rapid expansion of the Universe, the asymmetries in the  $B$  (or  $L$ ) violating decay rates to conjugate final states arise at the one loop level, involving the virtual exchange of light particles, such as quarks, leptons or Higgs bosons. We have studied in this paper the effects of the thermal background of standard particles present during the decay epoch in the evaluation of the  $CP$  violating asymmetries. We first reconsidered the triplet scalar decay in  $SU(5)$ , finding that the asymmetry is reduced (contrary to an earlier result), as could be expected on the basis of the Pauli exclusion principle applied to the virtual fermionic lines in the loop. We also included the  $CP$  violation produced by the mixing among different heavy states. Confronting with the  $T = 0$  results, the modification produced by the thermal effects could be as large as 50% for  $T \simeq M_1$ , but diminishes with decreasing temperature, becoming negligible for  $T < M_1/10$ .

The realization that standard model anomalous  $B$  and  $L$  violating processes are in equilibrium at temperatures above the electroweak phase transition one, has made the  $SU(5)$  scenario just of academic interest, since no net  $B - L$  asymmetry (the only one unaffected by anomalous processes) is generated within it. However, the same anomalous processes have allowed some new very attractive possibilities, including the baryogenesis at the electroweak scale itself (although its practical implementation faces several difficulties). The most simple and promising scenario seems to be the leptogenesis, in which heavy right handed neutrinos generate a lepton asymmetry in their decay, which is then reprocessed into a baryon asymmetry by the standard model anomalous processes. We showed that in these scenarios the leading thermal corrections canceled among themselves, due to the opposing effects produced by the bosons and fermions involved in the loop. Also in the supersymmetric version of leptogenesis the thermal corrections to the heavy scalar neutrino decay were shown to vanish, once the thermal modification of the

branching ratios to the different final states are included. In view of this, we studied the correction due to the fact that in general the decaying particle is not at rest in the thermal bath. We showed that the  $CP$  asymmetry depends on the particle motion, since the background density distribution, and hence the thermal corrections, are now modified in the rest frame of the decaying particle. This effect can however change the decay asymmetries only by at most  $\sim 20\%$  with respect to the usual  $T = 0$  results, and hence these last can be safely employed.

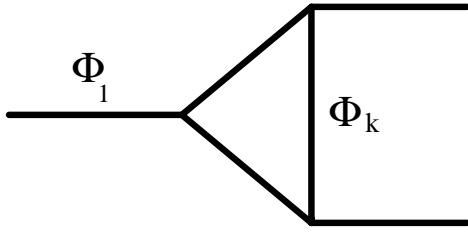
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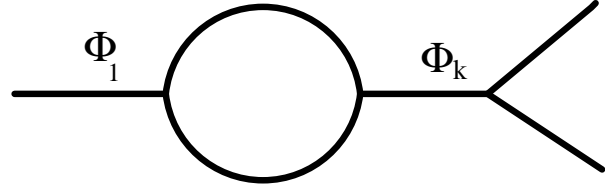
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(a)



(b)

Figure captions:

Figure 1: Diagrams which interfere with the tree one to produce the  $CP$  violation in the heavy particle decay. Fig. 1a gives the so called vertex contribution while Fig. 1b gives the wave function one.