Flavour-Changing Neutral Currents in the Dualized Standard Model

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Abstract

The Dualized Standard Model which gives explanations for both fermion generations and Higgs fields has already been used to calculate fermion mass and mixing parameters with success. In this paper, we extend its application to low energy FCNC effects deriving bounds for various processes in terms of one single mass scale. Using then experimental information from $K_L - K_S$ mass difference and air showers beyond the GZK cut-off, these bounds are converted into rough, order-of-magnitude predictions. In particular, the estimates for the decay $K_L \to e^{\pm}\mu^{\mp}$ and for the mass difference between the neutral D-mesons seem accessible to experiment in the near future.

1 Introduction

Based on a nonabelian generalization of electric-magnetic duality [1] and a result on confinement of 't Hooft [2], a Dualized Standard Model (DSM) [3] was constructed which, though remaining strictly within the standard model framework in contrast to most other attempts with a similar purpose [4], yet offers an explanation for the existence of exactly 3 generations as broken dual colour and of Higgs fields as frame vectors in internal symmetry space. The symmetry breaking pattern inherent in the scheme is such that at tree-level only the highest generation fermions have a mass (mass hierarchy) while the Cabibbo-Kobayashi-Maskawa (CKM) matrix, whether for quarks or leptons, is the identity matrix. Loop corrections, however, give nonzero masses to the 2 lower generations and nonzero mixing both between the U- and D-type quarks and between the charged leptons and neutrinos. A calculation of these corrections to 1-loop level has already been performed [5, 6, 7] giving very encouraging results. By adjusting effectively only 3 real parameters, we have calculated the following 14 quantities: the 3 mixing parameters (except the CP-violating phase) in the quark CKM matrix V_{rs}, the 3 analogous parameters in the lepton CKM matrix U_{rs}, the 6 known fermion masses $m_c, m_s, m_\mu, m_u, m_d, m_e$, as well as estimates for the masses of the lightest neutrino m_{ν_1} and the 'right-handed' neutrino B. Of these 14 calculated quantities, 10 agree as well as can be expected with experiment (i.e. all except m_u and probably also U_{e2}, which are not good, and m_{ν_1} and B, which are experimentally unknown.)

Given these encouraging results it is natural to seek further tests for the DSM scheme in other directions. One obvious area which may yield interesting tests is in flavour-changing neutral current (FCNC) effects. The fact that generation has been identified to dual colour which is itself a (broken) gauge symmetry means that any fermion carrying a dual colour or generation index will interact by exchanging (electrically neutral) dual colour gauge bosons (dual gluons) leading thus to flavour-changing neutral currents, which will manifest themselves in rare FCNC decays and in the mass differences between for example, the neutral strange, charm and bottom mesons. The purpose of the present article is the evaluation of these FCNC effects as predicted by the DSM scheme and their comparison with the existing bounds obtained from experiment.

The predictions for the various FCNC effects in schemes of so-called 'horizontal symmetries' [8, 9], of which the DSM is one, depend on the masses of the exchanged gauge bosons and their couplings to the relevant fermion states. What is special for the DSM, however, is that, in view both of the intrinsic structure built into the scheme and of the calculations already performed which are referred to above, most of these quantities are now known. First, by virtue of the Dirac quantization condition [10]:

$$g_{3(2)}\tilde{g}_{3(2)} = 4\pi, \quad g_1\tilde{g}_1 = 2\pi, \tag{1}$$

the coupling strengths $\tilde{g}_i$ of the dual gauge bosons are derivable from the coupling strengths g_i of the ordinary colour and electroweak gauge bosons measured in present experiments. Secondly, the branching of these couplings $\tilde{g}_i$ into the various physical fermion states are given by the orientations of these physical states in generation or dual colour space, and these orientations are already determined in the calculation of CKM matrices [5, 6] mentioned above. Finally, in tree-level approximation, the masses of the dual gauge bosons are given in terms of the vacuum expectation values of the dual colour Higgs fields, the ratios between which are among the parameters determined in the calculation [5] by fitting the CKM matrix. Thus the only remaining unknown among the quantities required is the mass scale of the dual gauge bosons which, for reasons explained in [5], is not restrained significantly by the calculation reported there. That being the case, one can now calculate in the DSM scheme all FCNC effects in tree-level approximation in terms of this single scale parameter. Or, in other words, given the experimental bound on any one FCNC effect, one can evaluate the correlated bound on any other such effect.

Now, it turns out that in the tree-level spectrum found for the dual gauge bosons, one particular state has a much lower mass than the rest, so that the calculation of low energy FCNC effects becomes quite simple, being dominated by just the exchange of this one boson. In that case it happens that it is the mass difference between the K_L and K_S mesons which sets the tightest bound on the gauge boson mass. From this, one then obtains the correlated bounds on all other low energy FCNC effects.

At present, directly from the DSM scheme, one obtains only these bounds. However, going outside the DSM scheme proper there is one valuable piece of information borrowed from cosmic ray physics which, if taken seriously, converts the bounds mentioned above into order-of-magnitude predictions.

This concerns the small number of air showers with primary energy in excess of 10^{20} eV which have been observed over the last decades [11, 12]. These events are a mystery in that protons, which are thought to account for high energy showers, react readily via radiative pion-production with the 2.7 K microwave background and quickly degrade in energy. Indeed, according to Greisen, Zatsepin and Kuz'min [13] the spectrum of protons should cut off sharply at about 5×10^{19} eV, unless they originate at distances of less than 50 Mpc, which is thought unlikely, there being no known nearby sources capable of producing such high energy particles.

One suggestion [14, 15] was that these rare showers with energy above the GZK cut-off are due to neutrinos rather than protons. Neutrinos, being electrically neutral, could survive a long journey through the 2.7 K microwave background with their energy intact. However, neutrinos with only their presently known weak interactions will not have a large enough cross section with the air nucleus to produce air showers with the observed characteristics. The neutrino explanation, therefore, is not feasible unless, for some reason, neutrinos acquire a strong interaction at high energy. Now, the beauty in the DSM scheme is that this is exactly what has been predicted [3]. Neutrinos, like other fermions, occur in 3 generations, which are now identified with dual colour. This means that neutrinos will interact via the same exchanges of dual colour gauge bosons which give rise to the flavour-changing neutral currents considered above. Dual colour gauge bosons being massive, this new interaction for neutrinos will be suppressed at low energy like all FCNC effects and be at present unobservable. But at energies comparable to the dual colour gauge boson mass, it will come into its own, and is expected to be strong given that the dual colour coupling is constrained by the Dirac quantization condition [10]. Indeed, the analysis in [14] of this DSM neutrino hypothesis offers not only qualitative explanations for several outstanding mysteries surrounding post-GZK air showers but even some quantitative predictions testable by future experiments.[1]

As far as low energy FCNC effects are concerned, the relevance of the above discussion on post-GZK air showers lies in its giving a rough upper

[1] There has appeared in the literature [16] a claim that the DSM neutrino explanation along with most other particle physics explanations for post-GZK showers do not work. We find the arguments presented there, based only on first order perturbation theory, inadequate for such a sweeping claim. See [15] and section 4.2 of this paper for further discussion on this point.

bound on the dual colour gauge boson mass. The centre of mass energy for a neutrino with primary $E \sim 10^{20}$ eV impinging on a proton at rest is around 500 TeV, which ought to be comparable to the mass of the gauge boson exchanged before the resulting interaction becomes appreciable. Interestingly, as we shall see, this estimate is not far from the lower bounds obtained from present experiment on low energy FCNC effects. As a result, the correlated bounds deduced directly from the DSM scheme above become now absolute predictions, although still rather unsure ones to be trusted only at a rough order of magnitude level, given that the estimate for the scale from air showers is by necessity crude and affects the predictions sensitively. Even then, we think the predictions useful for planning future experiments since, as we shall see, there are several effects predicted with values only a couple of orders of magnitude below the existing experimental bounds.

It should be stressed, however, that the treatment in this paper suffers from one serious weakness in that it relies on the tree-level spectrum of the dual gauge bosons. The couplings involved being large, it is unclear how the spectrum will change under higher order corrections which, at the present stage of development of the DSM scheme, one does not know how to estimate. And if this change in spectrum turns out to be drastic, then it can affect our predictions even qualitatively, as we shall elucidate. We can only hope at present that this does not happen.

In section 2, we review briefly those features of the Dualized Standard Model relevant to our present study, and in section 3 construct its low energy effective Lagrangian. Its experimental consequences on low energy FCNC effects are analysed in section 4.

2 The Dualized Standard Model

According to [3], to which the reader is referred for details, the terms of the Lagrangian of interest to us here, including the Higgs potential and the couplings of the dual gauge bosons to the Higgs and fermions fields, can be cast into the form:

$$\begin{aligned} & L_{\phi,C} + L_{C,\psi} + V_\phi \\ = \; & \bar{\phi}\left[-\tilde{g}_3 C^b_{gauge,\mu}\frac{\lambda_b}{2} - \tilde{g}_1\left(n+\frac{1}{3}\right)B_{gauge,\mu}\right]^2\phi \end{aligned}$$

$$
\begin{aligned}
&+ \quad \bar{\psi}_L \left[-i\tilde{g}_3 C^b_{gauge,\mu} \frac{\lambda_b}{2} - i\tilde{g}_1 \left(n_L + \frac{1}{3} \right) B_{gauge,\mu} \right] \psi_L \\
&- \quad \mu \sum_{(a)} |\phi^{(a)}|^2 + \lambda \left[\sum_{(a)} |\phi^{(a)}|^2 \right]^2 + \kappa \sum_{(a)\neq(b)} |\bar{\phi}^{(a)}.\phi^{(b)}|^2,
\end{aligned}
\tag{2}
$$

with μ, κ and λ all positive, n and n_L odd integers, and $\tilde{g}_3$ and $\tilde{g}_1$ the dual gauge couplings related to the usual colour coupling g_3 and weak hypercharge coupling g_1 by the Dirac quantization condition (1). The Higgs ($\phi^{(a)}$, $a = 1, 2, 3$) and the left-handed fermion (ψ_L) fields are in the fundamental while the gauge boson fields ($C^b_{gauge,\mu}$, $b = 1, \ldots, 8$) are in the adjoint representation of the dual colour gauge group $\widetilde{SU}(3)$. The field $B_{gauge,\mu}$, which we shall also denote by $C^0_{gauge,\mu}$, represents the $\widetilde{U}(1)$ gauge field of dual (weak) hypercharge. The number $n_L + \frac{1}{3}$ is the dual hypercharge of the multiplet ψ_L. Note that we take a vanishing dual hypercharge for the right-handed fermion components in order to allow for a Majorana mass term for neutrinos in the Lagrangian [6]. With this choice of hypercharges only the left-handed component contributes to the current in (2) giving a $V - A$ fermion-dual boson interaction term.

By virtue of the Higgs potential in (2) the Higgs fields acquire nonzero vacuum expectation values which, because of the symmetries of the Higgs potential, we are allowed to assign the following values:

$$
\phi^{(1)} = \zeta \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, \quad \phi^{(2)} = \zeta \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, \quad \phi^{(3)} = \zeta \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}, \tag{3}
$$

where we have normalized the three fields according to:

$$
x^2 + y^2 + z^2 = 1, \quad \zeta^2 = \frac{\mu}{2\lambda}.
$$

As a result, the symmetry $\widetilde{SU}(3) \times \widetilde{U}(1)$ is completely broken, giving masses to all the dual bosons and to the fermions. The mass terms for the dual bosons added to the gauge boson-fermion interaction term, give the piece of the Lagrangian (2) which is relevant for our purpose:

$$
\begin{aligned}
&L_{mass} + L_{C,\psi} = \\
&C^b_{gauge,\mu} M_{b,b'} C^{b'}_{gauge,\mu} + \bar{\psi}_L \left[-i\tilde{g}_3 C^b_{gauge,\mu} \frac{\lambda_b}{2} - i\tilde{g}_1 \left(n_L + \frac{1}{3} \right) C^0_{gauge,\mu} \right] \psi_L.
\end{aligned}
\tag{4}
$$

The 9×9 mass matrix $M_{b,b'}$ has a piece which is already diagonal in the original gauge basis, namely that corresponding to the bosons C^b_{gauge} with $b = 1, 2, 4, 5, 6, 7$, which are thus already the physical mass eigenstates at tree-level. These have the masses:

$$
\begin{aligned}
M_b &= \zeta \frac{\tilde{g}_3}{2} \sqrt{x^2 + y^2}, \quad b = 1, 2; \\
M_b &= \zeta \frac{\tilde{g}_3}{2} \sqrt{z^2 + x^2}, \quad b = 4, 5; \\
M_b &= \zeta \frac{\tilde{g}_3}{2} \sqrt{y^2 + z^2}, \quad b = 6, 7; \qquad (5)
\end{aligned}
$$

The remaining dual bosons (C^b with $b = 3, 8, 0$), since they are diagonal in dual colour, mix with one another in the mass matrix:

$$
M^2 = \zeta^2 \begin{pmatrix} \frac{\tilde{g}_3{}^2}{4}(x^2+y^2) & \frac{\tilde{g}_3{}^2}{4\sqrt{3}}(x^2-y^2) & -\frac{\tilde{g}_3\tilde{g}_1}{3}(x^2-y^2) \\ \frac{\tilde{g}_3{}^2}{4\sqrt{3}}(x^2-y^2) & \frac{\tilde{g}_3{}^2}{12}(x^2+y^2+4z^2) & -\frac{\tilde{g}_3\tilde{g}_1}{3\sqrt{3}}(x^2+y^2-2z^2) \\ -\frac{\tilde{g}_3\tilde{g}_1}{3}(x^2-y^2) & -\frac{\tilde{g}_3\tilde{g}_1}{3\sqrt{3}}(x^2+y^2-2z^2) & \frac{4\tilde{g}_1{}^2}{9}(x^2+y^2+z^2) \end{pmatrix}, \qquad (6)
$$

where the ordering $3, 8, 0$ in the matrix elements has been followed.

We see thus that at tree-level the masses of the dual gauge bosons are given in terms of the vacuum expectation values of the Higgs fields, namely x, y, z. These parameters are, in principle, unknown but were determined in our previous paper [5] by fitting the experimental quark CKM matrix and the masses of the two heavier generations of quarks and charged leptons, giving:

$$
x = 1, \;\; y = 5 \times 10^{-5}, \;\; z = 1 \times 10^{-8}. \qquad (7)
$$

The manner in which this was done, together with the whole interesting question of the fermion mass matrices, is described in detail in [5] and will not be repeated here. Notice, however, that the parameter ζ which gives the mass scale is still unconstrained.

In addition, the above fit gave the rotation matrices transforming from the original gauge basis to the basis of the physical states for each fermion-type. We give here the transformation matrices for quarks and charged leptons which will be relevant later for the comparison to experimental data. With the notation:

$$
\psi^A_{gauge,L} = S^A \psi^A_{physical,L} \qquad (8)
$$

these rotation matrices are[2]:

$$S^U = \begin{pmatrix} 0.9999 & -0.0127 & 0.0045 \\ 0.0135 & 0.9163 & -0.4002 \\ 0.0009 & 0.4002 & 0.9164 \end{pmatrix},$$
$$S^D = \begin{pmatrix} 0.9983 & -0.0545 & 0.0215 \\ 0.0566 & 0.8044 & -0.5914 \\ 0.0149 & 0.5916 & 0.8061 \end{pmatrix},$$
$$S^L = \begin{pmatrix} 0.9961 & -0.0829 & 0.0293 \\ 0.0829 & 0.7740 & -0.6277 \\ 0.0294 & 0.6277 & 0.7779 \end{pmatrix}. \tag{9}$$

Notice that the CKM matrix is given in terms of these matrices as the combination $(S^U)^\dagger S^D$ and it was found to be in excellent agreement with the present experimental values [5].

3 One Dual Gluon Exchange and FCNC

The dual colour gauge bosons (or dual gluons) are responsible for transitions between fermions of different generations but of the same type. Since the dual gluons carry no electric charge, this is equivalent to the existence of neutral currents with changes of flavour (FCNC). In this section we develop the formalism needed to study their phenomenological implications, the actual analysis of which, however, is postponed to the next section. We study in detail in this paper only one dual gluon exchange. As a check, we shall make estimates of the contributions from higher order corrections. However, the couplings being large by virtue of the Dirac quantization condition (1), we cannot really be confident in perturbative calculations beyond the tree-level. For that reason, the results we shall present in the next sections have to be regarded only as rough, order-of-magnitude, estimates.

The relevant term of the interaction Lagrangian is given in the usual formalism by:

$$L = -i \sum_A \left[\tilde{g}_3 J^{A,a}_{L,\mu} C^{\mu}_{gauge,a} + \tilde{g}_1 J^{A,0}_{L,\mu} C^{0,\mu}_{gauge} \right]. \tag{10}$$

[2]We arrange the matrix elements here in order of decreasing fermion mass in accordance with the convention adopted in [5].

The current is expressed using a triplet of left-handed gauge fermion fields and the sum over the index A runs over the different fermion-types (U, D, L, N). Explicitly it is given in terms of these fields and the Gell-Mann matrices by:

$$\begin{aligned} J^{A,a}_{L,\mu} &= \bar{\psi}^A_L \gamma_\mu \frac{\lambda^a}{2} \psi^A_L, \\ J^{A,0}_{L,\mu} &= \bar{\psi}^A_L \gamma_\mu \left(n^A_L + \frac{1}{3} \right) I \psi^A_L. \end{aligned} \tag{11}$$

Both the fermion and dual gauge boson fields are here given in the (gauge) basis where the vacuum expectation values of the Higgs fields appear as in (3). We need now to express them in terms of their physical states.

The mixing in the fermion sectors was discussed in section 2, and when the fermion fields are written in the physical basis, the current takes the form[3]:

$$J^{A,a}_{L,\mu} = \bar{\psi}^A_L \gamma_\mu (S^A)^\dagger \frac{\lambda^a}{2} S^A \psi^A_L. \tag{12}$$

It is worth noting that the unitary matrices S^A, being a transformation in generation space, do not commute with the Gell-Mann matrices. As a consequence, one cannot rotate away, even with massless neutrinos, the transformation in the leptonic sector and this has observable consequences. This fact is common to many models with horizontal symmetries.

Next, consider the physical states in the gauge boson sector. At tree-level these are given in (5) and by the diagonalization of the matrix (6). In the situation when the v.e.v.'s of the Higgs fields in (3) are hierarchical, namely $x \gg y \gg z$, which is indeed the case in the fit (7) obtained in [5], the matrix (6) can be diagonalized algebraically giving the eigenvalues as:

$$\begin{aligned} M_1^{physical} &= \frac{2\tilde{g}_1}{\left[1 + \frac{16\tilde{g}_1^2}{3\tilde{g}_3^2}\right]^{1/2}} \, \zeta z, \\ M_2^{physical} &= \frac{\tilde{g}_3}{2} \left[\frac{1 + \frac{16\tilde{g}_1^2}{3\tilde{g}_3^2}}{1 + \frac{4\tilde{g}_1^2}{3\tilde{g}_3^2}} \right]^{1/2} \zeta y, \\ M_3^{physical} &= \frac{\tilde{g}_3}{\sqrt{3}} \left[1 + \frac{4\tilde{g}_1^2}{3\tilde{g}_3^2} \right]^{1/2} \zeta x. \end{aligned} \tag{13}$$

[3]In the following we will suppress the index *physical* in the fermionic fields.

Comparing this result with the masses previously found (5), we see that there is one boson ($M_1^{physical}$) much lighter than the others, the next lightest having, according to (3), (5) and (13), masses some 2 orders of magnitude heavier. Hence, given that the FCNC effects we seek depend on the mass of the gauge boson exchanged as M^{-4}, one sees that in the sum over the boson fields in equation (10), one needs to keep only the contribution of this lightest boson. The corresponding physical field is given just by the dual boson $C^8_{gauge,\mu}$ with a small admixture of the dual hypercharge boson, namely:

$$|C^1_{physical}\rangle = \frac{1}{k_1}\left[|C^8_{gauge}\rangle + \frac{\sqrt{3}\tilde{g}_3}{4\tilde{g}_1}|C^0_{gauge}\rangle\right], \tag{14}$$

where k_1 is a normalization factor.

For completeness, we give the full diagonalizing matrix for the matrix (6) in the limit $x \gg y \gg z$, with the convention $M_D^2 = U^\dagger M^2 U$:

$$U = \begin{pmatrix} 0 & -\frac{1}{k_2}\frac{1}{\sqrt{3}}\left(1+\frac{16\tilde{g}_1^2}{3\tilde{g}_3^2}\right) & \frac{1}{k_3} \\ \frac{1}{k_1} & \frac{1}{k_2} & \frac{1}{\sqrt{3}k_3} \\ \frac{1}{k_1}\frac{\sqrt{3}\tilde{g}_3}{4\tilde{g}_1} & -\frac{1}{k_2}\frac{4\tilde{g}_1}{\sqrt{3}\tilde{g}_3} & -\frac{1}{k_3}\frac{4\tilde{g}_1}{3\tilde{g}_3} \end{pmatrix}, \tag{15}$$

where k_i are the normalization constants of the respective column vectors.

After performing the rotation to the physical bases in both the fermionic and bosonic sectors, we end up with the interaction Lagrangian which we write as:

$$L = -i\sum_A \bar{\psi}^A_L \gamma_\mu g^{A,b} \psi^A_L C^\mu_{physical,b} \tag{16}$$

where

$$g^A_b = (S^A)^\dagger \left[\tilde{g}_3 \frac{\lambda_8}{2} U_{8,b} + \tilde{g}_1 (n^A_L + \frac{1}{3}) U_{0,b} I\right] S^A, \tag{17}$$

is a matrix in generation space, with the index A taking the values U, D, L, N and the index b running over $1, 2, 3$. As we shall see, as far as processes with change of flavour is concerned, the second term in (17) dependent on the dual hypercharge, and hence proportional to the identity matrix in generation space, does not contribute.

Next we change to the language of effective Lagrangians to study further the flavour changing effects induced by (16). The effective Lagrangian will have the current×current structure and will give, in first order, the same result as the interaction Lagrangian (10) in second order when the masses of the intermediate bosons are much heavier than any other energy scale involved in the process. Although the two formalisms are equivalent, the effective Lagrangian language is more c onvenient when considering low energy effects of a theory broken at a high scale [9].

Since only one intermediate boson (14) need to be considered, the effective Lagrangian is proportional to the inverse square of the lowest Higgs v.e.v. ζz:

$$L_{eff} = \frac{1}{2\zeta^2 z^2} \sum_{A,B} (J_A^\mu)^\dagger J_{\mu,B}. \tag{18}$$

Equivalently, if preferred, it can be written in terms of an effective coupling and the mass of the exchanged gauge boson:

$$\frac{g_{eff}^2}{M^2} = \frac{1}{\zeta^2 z^2}, \tag{19}$$

where

$$g_{eff}^2 = \frac{3}{4} \frac{\tilde{g}_3^2}{1 + \frac{3}{16}\frac{\tilde{g}_3^2}{\tilde{g}_1^2}}. \tag{20}$$

The fermionic current in (18) has the same form as in (16) but contains now only the hypercharge component proportional to the identity matrix and the component proportional to the Gell-Mann matrix λ_8. Explicitly, the couplings are given as:

$$g^A = (S^A)^\dagger \left[\frac{1}{\sqrt{3}} \lambda_8 + \frac{1}{2}(n_L^A + \frac{1}{3}) I \right] S^A, \tag{21}$$

from which the explicit dependence of the original gauge couplings $\tilde{g}_i$ cancel.

We notice in (21) that although the quantity inside the brackets is diagonal in the gauge basis, it is not diagonal in the physical fermion basis since λ_8 does not commute with the fermion mixing matrices S^A. This means that (18) will give transitions between physical fermion states of different flavours but of the same type. In order words it will give rise to the so-called flavour-changing neutral current effects that we seek. It will give transitions, of course, also between physical states with the same flavours, but we shall not consider here such 'diagonal processes' since they appear also in the Standard Model and, at present, are not likely to give experimentally distinctive signatures. We shall therefore extract from (18) only that piece relevant for flavour-changing transitions. The result will then be just the product of, firstly, 2 fermionic currents each one made out of single left-handed fermion fields with a definite physical flavour, secondly, the effective coupling strength which is essentially the inverse square of the Higgs v.e.v ζz, and, thirdly, a group factor ($f^{A,B}_{\alpha,\beta;\alpha',\beta'}$) coming from the rotation between gauge and physical states in generation or dual colour space. Explicitly, the final form of the effective Lagrangian which we shall use for the phenomenological analysis in the next section is:

$$L_{eff} = \frac{1}{2\zeta^2 z^2} \sum_{A,B} f^{A,B}_{\alpha,\beta;\alpha',\beta'} (J^{\mu\dagger}_A)^{\alpha,\beta} (J_{\mu,B})^{\alpha',\beta'} \; , \tag{22}$$

with currents of the usual $V-A$ form:

$$(J^A_\mu)_{\alpha,\beta} = \bar{\psi}^A_{L,\alpha} \gamma_\mu \psi^A_{L,\beta}, \tag{23}$$

and the group factor reduced to:

$$f^{A,B}_{\alpha,\beta;\alpha',\beta'} = S^{A*}_{3,\alpha} S^A_{3,\beta} S^{B*}_{3,\alpha'} S^B_{3,\beta'} \tag{24}$$

The simple structure of (22) comes from the fact that the exchange of only one dual gauge boson need be considered. In the formula for the group factor (24), we note that for some of the leptonic decays of mesons to be considered in the next section, there are in general additional terms diagonal in one of the fermion vertices which depend on the dual weak hypercharges of the left-handed fermions. However, for the minimal choice $-2/3$ advocated in [3] corresponding to $n^A_L = -1$ in (21), these terms cancel.

Notice that the group factors $f^{A,B}_{\alpha,\beta;\alpha',\beta'}$ are given entirely in terms of the matrices $S^A_{\alpha,\beta}$ of (9) determined in our earlier work [5] from fitting the CKM matrix, so that the only remaining free parameter in (22) is the mass scale ζz which has yet to be estimated from experiment.

4 Phenomenological Implications

In this section we apply the formalism developed above to phenomenology. Our first concern is to deduce bounds on the remaining unknown parameter ζz. As explained in the introduction, a lower bound will come from low energy flavour-changing neutral current effects, while an upper bound will arise in high energy air showers from cosmic rays. It turns out that the tightest lower bound one can obtain at present comes from the $K_L - K_S$ mass difference, which is what we shall now consider.

4.1 $K_L - K_S$ Mass Difference

The relevant piece of the effective Lagrangian (18) contributing to the mass difference in the $K_L - K_S$ system is:

$$L_{eff}^{D,D} = \frac{1}{2\zeta^2 z^2} |f_{2,3;2,3}^{D,D}| \, (\bar{s}_L \gamma^\mu d_L) \, (\bar{s}_L \gamma_\mu d_L) \, , \tag{25}$$

which gives a contribution to the mass-splitting from dual gluon exchange of the form:

$$\Delta m_K = \frac{1}{\zeta^2 \; z^2} |f_{2,3;2,3}^{D,D}| \langle K^0 | \, [\bar{s}_L \gamma^\mu d_L]^2 \, |\bar{K}^0\rangle . \tag{26}$$

Evaluating the matrix elements in the vacuum insertion approximation one arrives at the following expression for the mass-splitting:

$$\Delta m_K = \frac{1}{\zeta^2 \; z^2} |f_{2,3;2,3}^{D,D}| \frac{f_K^2 \; m_K}{3} \tag{27}$$

Taking $f_K^2 = 1.4 \times 10^{-8}$ TeV^2 for the K decay constant, and $m_K = 498$ MeV for the K mass, a direct comparison of (27) to the experimental value[4] $\Delta m_K (K_L - K_S) = 3.5 \, \times \, 10^{-12}$ MeV gives a lower bound:

$$\zeta \; z \geq 400 \text{ TeV}. \tag{28}$$

We could instead compare (27) with the Standard Model contribution of the charm quark to the $K_L - K_S$ mass splitting as advocated by [8]. Using then the Gaillard-Lee [17] effective Lagrangian for the charm contribution, one obtains that the bound on ζz is increased up to 600 TeV. On the other hand, it is by now generally accepted that the vacuum insertion approximation used in deriving the above bounds overestimates the matrix element and ought to be reduced by a factor of the order of 0.7 [18], in which case

[4] All experimental values of rare decay branching ratios and mass differences used in the analysis of this section are taken from [19].

the bound on ζz can be lowered down to around 300 TeV. To be specific, the estimates in this paper are calculated using the bound (28), but the corresponding estimates for other values of ζz are easily deduced given their $(\zeta z)^{-4}$ dependence.

The tree level relation (13) yields then a lower bound for the lightest dual gauge boson mass:

$$M_1^{physical} \geq 3000 \text{ TeV}. \tag{29}$$

Here we have used for the dual couplings $\tilde{g}_3$ and $\tilde{g}_1$ the Dirac conditions (1) where we have taken the running couplings of QED and QCD at the scale of the dual symmetry breaking (ζz). This last assumption is something which can be questioned and a second possibility, less conservative, is to take the QED and QCD couplings at a lower scale given by the energies relevant in the process, in which case the bound would be less severe.

4.2 Air Showers beyond the GZK Cut-off

As explained in the introduction, the suggestion that air showers with energy higher than the GZK cut-off [13] at 5×10^{19}eV are due to neutrinos having acquired a strong interaction from dual gauge boson exchange [14, 15] puts a rough upper bound on the dual gauge boson mass. At low energy, the effects due to dual gauge boson exchanges are suppressed generically by a factor $(\tilde{g}_{eff}\omega/M)^{-4} \sim (\omega/\zeta z)^{-4}$, where ω is the C.M. energy and M the mass of the lightest dual gauge boson. Therefore, for neutrinos to acquire a strong interaction from this dual gauge boson exchange, the C.M. energy should be such as to give $\omega/\zeta z$ of order unity. Now a neutrino with incoming energy 10^{20}eV impinging on a nucleon in an air nucleus at rest in the atmosphere has around 500 TeV C.M. energy. Since a neutrino is assumed at that sort of energies to produce air showers, its cross section with the air nucleus has to be hadronic and its interaction therefore strong. From this, one can deduce then the following rough upper bound:

$$\zeta z \leq 500 \text{ TeV}. \tag{30}$$

It is clear that this bound is rather crude since it is arguable whether $\tilde{g}_{eff}\omega/M$ need be as large as unity for the cross section to be hadronic, nor is it obvious that it is the C.M. energy for a neutrino-nucleon collision that should be taken rather than that for a neutrino-nucleus collision. It is nevertheless interesting that provided one accepts that air showers beyond the GZK cut-off are neutrino induced then one does obtain an upper bound for the scale parameter ζz which is not far from the lower bound deduced above from the $K_L - K_S$ mass difference. This is the only upper bound on the scale parameter that we are aware of at present.

In the literature, there has appeared a claim [16] that the suggested neutrino explanation for post-GZK air showers does not work. We do not think this claim is valid. The arguments presented there, leading to their conclusion that the neutrino cross section will be too small, are based only on first order perturbation theory which is far from adequate for dealing with the problem of (soft) hadronic cross sections addressed here which is a highly nonperturbative phenomenon. Indeed, with the same arguments, one would not obtain hadronic-sized cross sections even for proton-proton collisions. It is said in that paper that its conclusion is a consequence of s-wave unitarity but no justification nor reference for this statement is given[5]. Our own searches in the literature also have not found any reference to that result. We are sceptical that s-wave unitarity can give serious bounds on high energy hadron cross sections which involve many partial waves. Even from full (s-channel) unitarity, apart from the geometric constraints already dealt with in [14], the bounds on the rate of increase of the cross section that we are aware of are asymptotic bounds which are not relevant for the case in hand where there are still new thresholds opening[6]. Our conclusion is thus that the objection raised in [16] is ill-founded and in no way affects the feasibility of the proposal [14, 15]. Nevertheless, of course, the proposal must be subjected to much more stringent tests than it has so far undergone before it can be taken seriously. Indeed, apart from the proposed tests in air shower physics itself as given in [14], the best tests suggested are in the low energy FCNC effects considered in this paper and detailed below.

[5]One of us (CHM) thanks Francis Halzen for a series of email exchanges on this question, which has not, unfortunately, despite much effort on our part, succeeded in clarifying the basis of their assertion that s-wave unitarity implies the claim they made.

[6]We are deeply grateful to André Martin for correspondence and for his expert advice on this matter.

4.3 Rare K Decays

Taking into account the quark content of the K-meson $(\bar{s}d)$, one finds that the piece of the effective Lagrangian (22) for semileptonic K decays is:

$$L_{eff}^{D,L} = \frac{1}{\zeta^2 z^2} |f_{2,3;\alpha,\beta}^{D,L}| \left(\bar{s}_L \gamma^\mu d_L\right) \left(\bar{l}_{\alpha,L} \gamma_\mu l_{\beta,L}\right) \tag{31}$$

where $l_\alpha(l_\beta)$ stand for two different leptons of the same type. To minimise the errors from uncertainties in the hadron structure we take quotients between the rare and Standard Model-allowed processes which contain the same hadronic matrix elements. The result then depend on the ratio between the v.e.v.'s of the conventional (electroweak) and the new (dual colour) Higgs fields, as well as the group factors from the fermionic sector (24). For instance, for K^+ rare decays one has:

$$\frac{Br\left(K^+ \to \pi^+ l_\alpha l_\beta\right)}{Br\left(K^+ \to \pi^0 \nu_\mu \mu^+\right)} = |f_{2,3;\alpha,\beta}^{D,L}|^2 \left(\frac{v}{\zeta\ z}\right)^4 \frac{2}{\sin^2\theta_c}, \tag{32}$$

where $v = \frac{0.246}{\sqrt{2}}$ TeV and θ_c is the Cabibbo angle, $\sin\theta_c = 0.23$.

Similarly, for the leptonic decays of the neutral K-mesons, one can write:

$$\frac{\Gamma\left(K^0_{L(S)} \to l_\alpha l_\beta\right)}{\Gamma\left(K^+ \to \mu^+ \nu_\mu\right)} = |f_{2,3;\alpha,\beta}^{D,L}|^2 \left(\frac{v}{\zeta\ z}\right)^4 \frac{1}{\sin^2\theta_c}, \tag{33}$$

from which, given the total widths of K_L and K_S and the width of $K^+ \to \mu^+\nu_\mu$ measured in experiment, one can easily calculate the branching ratios of the various leptonic modes of K_L and K_S.

In Table 1 we have collected the tree-level predictions of the Dualized Standard Model for the rare semileptonic and leptonic decay modes of K_L, K_S and K^+, and compared them with the limits reached by present experiments. From the table we see that for the two modes $K_L \to e^\pm\mu^\mp$ and $K_L \to e^+e^-$, the predicted branching ratios are roughly a couple of orders of magnitude down from the present experimental limits and may be accessible in the near future. The other modes appear out of reach for some time.

	Theoretical Estimate	*Experimental Limit*
$Br(K^+ \to \pi^+ e^+ e^-)$	4×10^{-15}	2.7×10^{-7}
$Br(K^+ \to \pi^+ \mu^+ \mu^-)$	2×10^{-15}	2.3×10^{-7}
$Br(K^+ \to \pi^+ e^+ \mu^-)$	2×10^{-15}	7×10^{-9}
$Br(K^+ \to \pi^+ e^- \mu^+)$	2×10^{-15}	2.1×10^{-10}
$Br(K^+ \to \pi^+ \nu \bar{\nu})$	2×10^{-14}	2.4×10^{-9}
$Br(K_L \to e^+ e^-)$	2×10^{-13}	4.1×10^{-11}
$Br(K_L \to \mu^+ \mu^-)$	7×10^{-14}	7.2×10^{-9}
$Br(K_L \to e^\pm \mu^\mp)$	1×10^{-13}	3.3×10^{-11}
$Br(K_S \to \mu^+ \mu^-)$	1×10^{-16}	3.2×10^{-7}
$Br(K_S \to e^+ e^-)$	3×10^{-16}	2.8×10^{-6}
$Br(K_S \to e^\pm \mu^\mp)$	2×10^{-16}	?

Table 1: Branching ratios for semileptonic and leptonic rare K decays. The two columns show respectively the tree-level predictions of the Dualized Standard Model and the present experimental limits. The lowest v.e.v. ζz of the Higgs fields is taken at 400 TeV.

4.4 Mass Difference in the Neutral D-Meson and B-Meson Systems

Using the lower bound on the Higgs v.e.v. ζz obtained from the mass difference of neutral K's, one can deduce lower bounds also on the mass differences in the neutral D and B systems.

To study the D system we proceed as before. The relevant piece in the effective Lagrangian (22) is:

$$L_{eff}^{U,U} = \frac{1}{2\zeta^2 z^2} |f_{2,3;2,3}^{U,U}| \, (\bar{c}_L \gamma^\mu u_L) \, (\bar{c}_L \gamma_\mu u_L) \, , \tag{34}$$

and the contribution of the dual gauge boson to the mass splitting is:

$$\Delta m_D = \frac{m_D}{\zeta^2 z^2} \frac{f_D^2}{3} |f_{2,3;2,3}^{U,U}|. \tag{35}$$

Taking the values $f_D^2 = 10^{-8}$ TeV^2 for the D-meson coupling and $m_D = 1.865$ GeV for the mass we have:

$$\Delta m_D = 5 \times 10^{-12} \text{ MeV}. \tag{36}$$

This is one and a half orders of magnitude below the present experimental limits $\Delta m_D \leq 1.4 \times 10^{-10}$ MeV and could be accessible to planned experiments in the near future.

Applying the same procedure to the mass-splitting between the neutral B-mesons, one finds that the contribution from dual gauge boson exchange is 6 orders of magnitude smaller than that from the Standard Model as can be seen by comparing the coupling in L_{eff} (22) to the current $\bar{b}_L \gamma^\mu s_L$:

$$\frac{|f_{2,1;2,1}^{D,D}|}{\zeta^2 z^2} = 5 \times 10^{-10} \ \text{TeV}^{-2}, \tag{37}$$

and its counterpart in the Standard Model:

$$\sin^4 \theta_c \left(\frac{m_t}{37 \text{ GeV}} \right)^2 \frac{\alpha_{EM}}{4\pi v^2} \approx 10^{-3} \ \text{TeV}^{-2}. \tag{38}$$

The small contribution coming from the dual sector, apart from the large value of the intermediate dual boson mass, is due to the smallness of the dual colour group factor which here involves the small element (1,3) (the torsion term, according to [7]) of the mixing matrix (9).

4.5 Rare D and B Decays

We perform here the same exercise for the semileptonic decays of mesons which contain the quarks c and b as we did for K.

D-meson rare decays, such as $D \to \pi l_\alpha l_\beta$, are controlled by the term of the effective Lagrangian:

$$L_{eff}^{U,L} = \frac{1}{2\zeta^2 z^2} |f_{2,3;\alpha,\beta}^{U,L}| \left(\bar{c}_L \gamma^\mu u_L\right) \left(\bar{l}_{\alpha,L} \gamma_\mu l_{\beta,L}\right). \tag{39}$$

Therefore, in the spectator quark model, which roughly predicts the correct magnitudes for D-meson lifetimes, the decay width for the process is given by:

$$\Gamma\left(D^+ \to \pi^+ l_\alpha l_\beta\right) = |f_{2,3;\alpha,\beta}^{U,L}|^2 \frac{m_c^5}{192\pi^3} \frac{1}{8\zeta^4 z^4}, \tag{40}$$

which, for a total width of $\Gamma = 9.5 \times 10^{11} s^{-1}$ and a mass for the c quark $m_c = 1.3$ GeV gives an upper bound for the branching ratio for semileptonic rare decays:

$$Br\left(D^+ \to \pi^+ l_\alpha l_\beta\right) = |f_{2,3;\alpha,\beta}^{U,L}|^2 \; 5 \times 10^{-15}. \tag{41}$$

The analogous expression for leptonic decays of the neutral D-meson is:

$$Br\left(D^0 \to l_\alpha l_\beta\right) = |f_{2,3;\alpha,\beta}^{U,L}|^2 \; 2 \times 10^{-15}, \tag{42}$$

where the only change comes from the difference in total widths between the charged and neutral mesons.

In Table 2 we list the numerical values for the various modes. They are far from the present experimental bounds and seem inaccessible in the near future.

The same calculation for B-meson decays gives the branching ratio for the modes with the d quark in the final state as:

$$Br\left(B^+ \to \pi^+ l_\alpha l_\beta\right) = |f_{1,3;\alpha,\beta}^{U,L}|^2 \; 2 \times 10^{-12} \tag{43}$$

and with the s quark in the final state as:

$$Br\left(B^+ \to K^+ l_\alpha l_\beta\right) = |f_{1,2;\alpha,\beta}^{U,L}|^2 \; 2 \times 10^{-12}. \tag{44}$$

As in the case of the neutral B mass difference, the (1,3) element of the mixing mass matrix reduces the theoretical values by 6 orders of magnitude. Thus, the present experimental limits in this case (being in the interval 10^{-3}–10^{-5}) are far from the values predicted in the Dualized Standard Model.

	Theoretical Estimate	*Experimental Limit*
$Br(D^+ \to \pi^+ e^+ e^-)$	3×10^{-16}	6.6×10^{-5}
$Br(D^+ \to \pi^+ \mu^+ \mu^-)$	1×10^{-16}	1.8×10^{-5}
$Br(D^+ \to \pi^+ e^\pm \mu^\mp)$	2×10^{-16}	3.3×10^{-3}
$Br(D^0 \to e^+ e^-)$	1×10^{-16}	1.3×10^{-5}
$Br(D^0 \to \mu^+ \mu^-)$	4×10^{-17}	7.6×10^{-6}
$Br(D^0 \to e^\pm \mu^\mp)$	6×10^{-17}	1.9×10^{-5}

Table 2: Branching ratios for semileptonic and leptonic rare D decays. The two columns show respectively the tree-level predictions of the Dualized Standard Model and the present experimental limits. The v.e.v. of the Higgs field taken at 400 TeV.

5 Remarks

We have evaluated above the predictions of the Dualized Standard Model in tree-level approximation to a number of flavour-changing effects based on bounds to the dual gauge boson mass scale obtained from $K_L - K_S$ mass difference and from extremely high energy air showers. These, though interesting and hopefully useful to experimenters planning experiments, are subject to a number of reservations.

First, as has already been made clear, the estimate for the mass scale $\zeta z \sim 400$ TeV is rather crude, especially the upper bound from post-GZK air showers, even if the proposed neutrino explanation for these is accepted as valid. Given that the FCNC effects we seek are unfortunately dependent on this scale to the fourth power, the absolute normalization of our predictions can easily be wrong by say an order of magnitude. Ultimately, the best way to determine the scale would be to measure the actual magnitude of one of the FCNC effects studied in this paper.

The relative sizes of the effects, on the other hand, depend at tree-level only on the rotation matrices S of (9) which are fitted quite sensitively to the CKM matrices for quarks and seem in agreement with experiment for leptons. They are thus considered more reliable. Given that the fits to the lowest generation quark and lepton masses [5] are not perfect, future adjustments are probably necessary but are felt unlikely to affect the matrices

S too drastically.

Next, all the above analyses have been performed only at tree-level. However, the dual gauge couplings being large by virtue of the Dirac quantization condition (1) one has to bear in mind the possible effects of higher order terms. Assuming still the tree-level spectrum for the dual gauge boson states, and using the effective Lagrangian formalism above, we have calculated some higher order corrections, in particular the most important ones coming from the box diagram with the exchange of two gauge dual bosons. This contribution changes the effective coupling by a factor:

$$\frac{\tilde{g}_3^2}{4\pi^2}\frac{1}{k_1}\frac{3}{16}n_L^A n_L^B, \tag{45}$$

which directly depends on the hypercharge of the fermion-types involved, following [3], we take $n_L^A = -1$ for all of them. Because of the unitarity properties of the mixing matrices (9) there are no additional group factors of the kind (24). The factor in (45) can be estimated relating the coupling constant $\frac{\tilde{g}_3^2}{4\pi}$ to the QCD running coupling constant (1). At the energies relevant to the meson decays considered here, it is of the order of unity and, as a result, the correction due to the box diagram is of order 20% of the tree-level contribution. If so, it is still not too bad.

On the other hand, the effect of higher order corrections to the tree-level dual gauge boson spectrum itself is more worrisome and harder to handle. At tree-level, the dual gauge boson masses are given in terms of just the v.e.v.'s of the Higgs fields, and from the values of these deduced from our earlier fits to the CKM matrix [5] we identified the boson with the lowest mass which in the end dominates over all others in giving FCNC effects. This conclusion arose from the diagonalization of the matrix (6) with its rather special property, namely that for $y = z = 0$ the matrix factorizes, hence giving it two near-zero eigenvalues for $x \gg y \gg z$. We do not know whether this property will be preserved by loop corrections, and if not then our predictions for FCNC effects may be greatly altered. Let us consider as an illustration a scenario at the opposite extreme to that we have considered, namely when all dual gauge bosons have roughly equal mass. In that case, the sum in (10) runs over the whole range with all dual gauge bosons contributing at the same level to flavour-changing processes. One gets then, for processes with change of flavour in at least one of the fermion vertices:

$$f^{A,B}_{\alpha,\beta;\alpha',\beta'} = \left(S^{A\,\dagger} S^{B}\right)_{\alpha,\beta'} \left(S^{B\,\dagger} S^{A}\right)_{\alpha',\beta}. \tag{46}$$

Notice that, between fermions of the same type, i.e. $A = B$, this factor vanishes. There will then be no contribution to $K_L - K_S$ mass difference and the most stringent lower bound on dual gauge boson mass will arise instead from the K decay $K_L \to e^+e^-$. One obtains then from the present experimental limit on this decay a bound on the dual gauge bosons mass which translates to a Higgs v.e.v. of only 20 TeV as compared with the 400 TeV obtained before, giving upper bounds for rare decay processes some 10^4 times higher than the values given in Table 1.

However, it should be stressed that such an extreme situation need not happen, nor do we have to expect necessarily large loop corrections in spite of the large dual gauge couplings $\tilde{g}_i$. As emphazised in [3], the dual gauge bosons do not represent a different degree of freedom from the ordinary gauge bosons so that corrections from dual gauge boson loops really make no sense as an addition to ordinary gauge bosons loops and these latter need not be large. At present, however, we have to admit we do not know how to proceed beyond the tree-level and await a better understanding of the phenomenon called 'metamorphosis' in [3] which is under investigation.

Despite the limitations outlined above, the analysis presented is interesting in giving for, we believe, the first time, a full calculation of all FCNC effects in terms of only one scale parameter, and even this is constrained by experiment if the neutrino explanation of post-GZK air showers [14, 15] is to be believed. This is made possible by the ability of the Dualized Standard Model to identify the rotation matrices S in (9) relating the gauge and physical fermion states of which we have some confidence. Even if the tree-level spectrum for the dual gauge bosons turns out to be invalid because of loop corrections, one can imagine using in future some of the FCNC effects to fix the spectrum and then apply the S matrices to deduce the rest of FCNC effects. At present, however, the tree-level is the best that we can do, but we still think it should be useful as a guide.

The various FCNC effects investigated above mostly give estimates a long way below the present experimental bounds. Of the few that appear possibly accessible to experiment in the near future, the 2 decays $K_L \to e^+e^-$ and $K_L \to \mu^+\mu^-$ can occur also by second order weak interaction, with branching ratios estimated by a recent paper [20] to be respectively around 9×10^{-12} and 1.2×10^{-9}, which are above the estimates obtained here by dual gauge boson exchange, and are not therefore useful for testing the latter. The mode $K_L \to e^{\pm}\mu^{\mp}$, on the other hand, which also gives an estimate close to the present experimental limit, cannot proceed by second order weak interaction if neutrinos do not mix. Even if neutrinos do mix, the contribution from second order weak interaction will be suppressed by the square of the mixing angle which will make it comparable to the estimate obtained here from dual gauge boson exchange. We think, therefore, that this decay is a particularly good angle to attack, since a postive result here would give a confirmation either of neutrino mixing or of the dual gauge boson exchange process we have here advocated. Another effect worth noting, apart of course from the $K_L - K_S$ mass difference, is the mass difference of the neutral D-mesons, for which our estimate is only an order of magnitude below the present experimental bound.

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