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Multipole compensation in the LHC low-β insertions

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Multipole compensation in the LHC low-β insertions

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Abstract

The LHC dynamic aperture in Physics conditions is determined by the field errors in the low-β quadrupoles and these errors set a lower limit to the value of β^*. The associated aberrations have been computed with the transfer matrix method which gives particularly simple and efficient formulae for the case of low-β insertions. These formulae have been applied to the LHC case to design a multipole compensation system. The efficiency of the method has been assessed by trajectory tracking.

1 INTRODUCTION

It has been shown that the multipole imperfections in the low-β quadrupoles are much more important than those in the main magnets of LHC in Physics [1]. Although the associated dynamic aperture is sufficient for the nominal optics with a β^* value of 0.5m, it might not be enough when the latter is reduced by a factor of two. This is why a compensation system has been foreseen.

The computation of the correctors has been done by cancelling the aberrations associated with the multipole imperfections by means of multipole corrector magnets. In what follows, the computation of the aberrations is shown first. Then the efficiency of the compensation system is then tested by trajectory tracking. Eventually a first estimation of effects of the edge multipole is done.

2 ABERRATION COMPUTATION

We consider each triplet of the insertion, on each side of the interaction point, as a transfer line. These triplets have antisymmetric gradients, which guarantees the existence of optics for both rings. The aberrations at the end of this line, i.e. the difference between the linear coordinates and the actual ones, are obtained by merely transferring the local effect of a multipole to the end of the line [2]. The low-β insertions are probably the best case for this approach as it gives simple and efficient closed formulae. We restrict ourselves here to the case where the transfer line is composed of thin multipoles separated by pure linear parts. The accuracy of such a model can be pushed as far as desired by increasing the number of elements representing either the central part of the actual magnets or their edges. More modern approaches have been proposed to compute transfer maps [3]. However the efficiency of any compensation method has to be tested by tracking and what matters is the result. The closed formulae obtained by the transfer matrix method show well the physics and guide safely the design.

The main feature of a low-β insertion is the small value of both phase advances in the low-β quadrupoles because of the large value of the β-functions. As a consequence the transfer matrices between quadrupoles depend only on the β-functions. This simplifies considerably the computations which can be carried out analytically.

In the frame of the thin lens model, the local effect of a multipole is a kick superimposed to that of the quadrupole it perturbs. The complication is in the expression of the coordinates used to compute the kick. For the case of a low-β insertion, it is possible to demonstrate that, if the field errors are less than 10^{-3} at the maximum amplitude of the transverse oscillations, the relative contribution of the non-linear kicks to the value of the transverse coordinates at any place of the low-β insertion is much less than 1%. Consequently the non-linear kicks are computed with sufficient precision by using the values of the linear transverse coordinates. The kick values are obtained merely by dividing the magnetic field errors by the magnetic rigidity of the particles. The field errors can be described in terms of multipole field errors b_n (normal) and a_n (skew), so that the kicks are given, for the element of index k, by:

$$\Delta x'_k = -(K_1 L R_0)_k \sum_{n=1}^{\infty} Re\left[(b_{n,k} + i a_{n,k})\left(\frac{x_k + i y_k}{R_0}\right)^{n-1}\right]$$

$$\Delta y'_k = (K_1 L R_0)_k \sum_{n=1}^{\infty} Im\left[(b_{n,k} + i a_{n,k})\left(\frac{x_k + i y_k}{R_0}\right)^{n-1}\right]$$

where K_1 is the quadrupole gradient, L its length, R_0 the radius where the field errors are measured and x_k and y_k the values of the linear coordinates at the element position. As we are interested in the maximum amplitudes and as the phase advance across the low-β quadrupoles can be neglected, we take : $x, y_k = \sqrt{E_{x,y}\beta_{x,y}}$ where $E_{x,y}$ is the emittance associated with the dynamic aperture. Summing over all elements, the geometrical aberrations at the end of the transfer line are given, for the un-coupled case, by :

$$\Delta x = \sum_k T_{12}^k \Delta x'_k, \quad \Delta x' = \textstyle\sum_k T_{22}^k \Delta x'_k$$

$$\Delta y = \sum_k T_{34}^k \Delta y'_k, \quad \Delta y' = \textstyle\sum_k T_{44}^k \Delta y'_k$$

where the T_{ij}^k's are the elements of the transfer matrix between the element of index k and the end of the transfer line. Substituting the coordinate expressions, we obtain the general expression of the aberrations associated with the

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multipole error of index n in all elements, to first order in multipole strengths :

$$\begin{aligned}
\Delta x' \;\simeq -& \sum_{k=1}^{m} \sqrt{\frac{1}{\beta_x}}(K_1 L R_0)_k \;\frac{b_{n,k}}{R_0^{n-1}} \;\Big[\epsilon_{x_k}^{\frac{n-1}{2}} \beta_{x_k}^{\frac{n}{2}} \\
&-\frac{(n-1)(n-2)}{2!} \;\epsilon_{x_k}^{\frac{n-3}{2}} \beta_{x_k}^{\frac{n-2}{2}} \epsilon_{y_k} \beta_{y_k} \\
&+\frac{(n-1)...(n-4)}{4!} \;\epsilon_{x_k}^{\frac{n-5}{2}} \beta_{x_k}^{\frac{n-4}{2}} \epsilon_{y_k}^{2} \beta_{y_k}^{2} - ...\Big] \\
\Delta y' \;\simeq\;& \sum_{k=1}^{m} \sqrt{\frac{1}{\beta_y}}(K_1 L R_0)_k \;\frac{b_{n,k}}{R_0^{n-1}} \\
&\Big[(n-1) \;\epsilon_{x_k}^{\frac{n-2}{2}} \beta_{x_k}^{\frac{n-2}{2}} \epsilon_{y_k}^{\frac{1}{2}} \beta_{y_k} \\
&-\frac{(n-1)...(n-3)}{3!} \;\epsilon_{x_k}^{\frac{n-4}{2}} \beta_{x_k}^{\frac{n-4}{2}} \epsilon_{y_k}^{\frac{3}{2}} \beta_{y_k}^{2} \\
&+\frac{(n-1)...(n-5)}{5!} \;\epsilon_{x_k}^{\frac{n-6}{2}} \beta_{x_k}^{\frac{n-6}{2}} \epsilon_{y_k}^{\frac{5}{2}} \beta_{y_k}^{3} - ...\Big]
\end{aligned} \quad (1)$$

These expressions contain terms very similar to those obtained for the tune-shifts due to the multipoles to first order in multipole strength [4].

We do not have to consider the displacement aberration at the end of the insertion since the phase advance is small. However this approximation can be checked by a mere computation of single pass trajectory at maximum amplitude and an evaluation of the normalised aberrations.

3 CORRECTION SYSTEM

It is clear from formulae (1) that it is of utmost importance to place correctors at locations where the β-functions are as large as possible.

An important issue is then the emittance values for which the correction has to be computed. As the aberrations are homogeneous polynomials in the emittances, the only parameter relevant for a compensation system is the emittance ratio since we are interested in the cancellation of the aberrations. Three cases have been tested with two correctors. For each case we obtain two equations to compute the corrector strengths by cancelling the two aberration kicks given by (1) (including the correctors contribution) at the end of a half insertion. A further complication arises from the fact that the system must work for both rings which have different optics. This requirement can be fulfilled by installing the correctors at places where the β values are swapped when changing the quadrupole signs. The cases for two correctors are :

- $E_x = E_y$ for both $\Delta x' = 0$ and $\Delta y' = 0$
- $E_x = 0$ for $\Delta x' = 0$ and $E_y = 0$ for $\Delta y' = 0$
- minimisation of the quadratic sum of the three emittance cases for each aberration

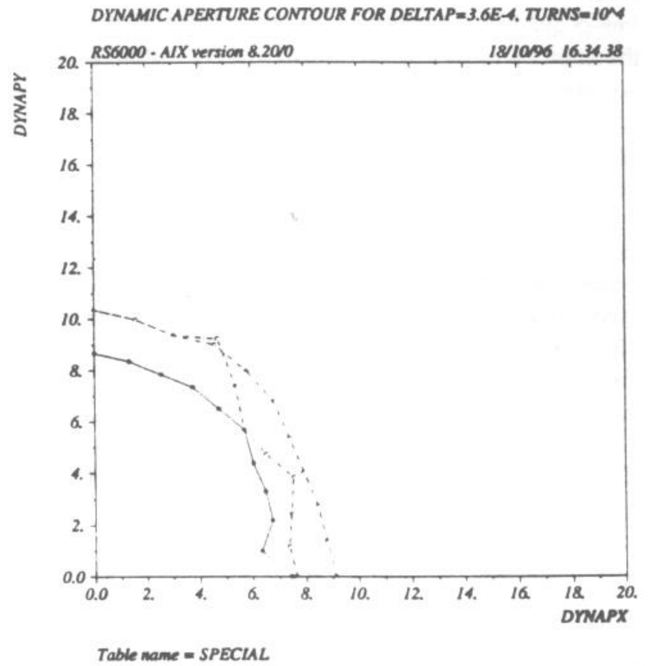
From the aberration computation with a one pass trajectory tracking, the best results are obtained for the second case.

Using three correctors is not possible because of the conditions imposed by the two rings. With four correctors the situation is similar to that of two. It is possible to write four equations which express the cancellation of the two kicks for two emittance ratios, respecting the two ring constraint.

4 TRACKING TEST

The dynamic aperture is defined as the maximum action in both planes simultaneously for which transverse betatron oscillations remain stable over 10^5 turns. Tracking the particle trajectories is done with the MAD program [5] element per element. The lattice is that of Version 4.2 described with thin lenses to preserve the symplecticity.

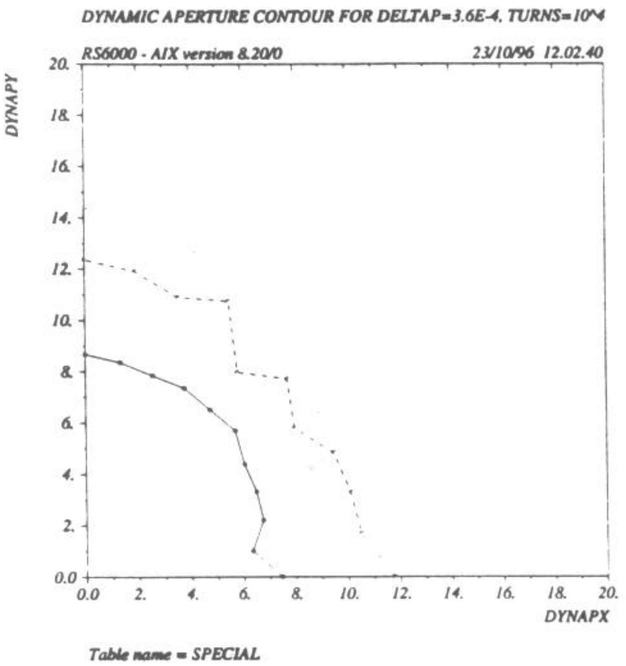
Figure 1: Solid line: Dynamic aperture for 10^5 turns (same as 10^4 turns) associated with a low-β quadrupole systematic error b_{10} compensated with two correctors. The other non-linearity is the chromaticity sextupoles. β^*= 0.25m. Solid line : no compensation. Dashed line : $E_x = E_y$. Dotted line: $E_x = 0$ for $\Delta x' = 0$ and $E_y = 0$ for $\Delta y' = 0$. Dot-dashed line: least square minimisation.



The tunes are Qx=63.28, Qy=63.31, Qs=0.001 (16MV RF voltage for an energy of 7TeV). Both tune derivatives are set to 1. Only the systematic multipole errors in the triplet quadrupoles have been considered since the other ones have a negligible effect [1]. The crossing angle which makes an orbit excursion in the low-β quadrupoles, producing a "multipole feed-down" and an horizontal dispersion is included. The trajectories start at the interaction point IP1 where both β^*'s are 0.25m. Both starting coordinates are equal, the initial relative momentum deviation is 3.6×10^{-4} and all canonical momenta are zero. Under these conditions, the initial amplitude (the same value in x and y) has to be multiplied by $\sqrt{2}$ in order to give the maximum radius in the {x,y} plane.

At first the best correction system described above has been confirmed. This is apparent from the results of the tracking, done for the three emittance ratios given above, shown on figure 1. The best case gives a dynamic aperture of about 10σ (radius of the mean circle passing through the tracking points), which is quite satisfactory. Using four correctors does not improve the situation as shown on figure 2. The additional correctors have been put at places where the β-functions have similar values and where the two-ring condition can be respected. The strengths are smaller than with two correctors.

Figure 2: Dynamic aperture for 10^5 turns (same result for 10^4 turns) for a b_{10} of -5.10^{-7} (R_0= 10mm) There are four low-β quadrupoles 5.5m long with normalised gradients of 0.009m^{-2}. β^* =0.25m. The other non-linearities of the machine are only the two chromaticity sextupole families. The solid line is without any compensation. The dashed line is with two multipole compensators and the dotted line with four compensators. These figures are still valid if all random errors are included.



5 FIRST ESTIMATE OF END EFFECTS

The decrease of the gradient at the ends of a quadrupole produces pseudo-multipole fields. In the hard edge approximation the expression of $\Delta x_k{}'$ is, to the lowest order in coordinates [6] :

$$\Delta x_k{}' = \frac{K_1 L}{4}\left[(x_k{}^2 + y_k{}^2)x_k{}' - 2x_k y_k y_k{}'\right]$$

The ratio between this kick and that due to a b_n multipole in the quadrupole core is, for y=0 and omitting k : $xx'/4Lb_n(x/R_0)^{n-2}$. The worst case is that of the central quadrupole where the slopes of the trajectories are maximum. It is $-\alpha\sqrt{\frac{E}{\beta}}$ at the entrance (all quantities for the horizontal plane). It changes sign from one side to the other one and the sign of K_1 changes as well so that both terms add-up and the ratio is eventually $\frac{E\alpha}{2Lb_n\left(\frac{E\beta}{R_0{}^2}\right)^{(n-2)/2}}$. It has the numerical value of 0.28. We can estimate that if the maximum amplitude is reduced by 4%, the sum of the kicks due to the end effects and b_{10} has the same value as that due to b_{10} alone. This qualitative argument shows that the effect of the edge multipoles is small. This has been confirmed by a numerical integration of the trajectories [7] and more work is going on this subject.

6 CONCLUSION

Closed formulae have been obtained to compute correctors for the LHC low-β insertions. The old method of transfer matrices has yet proved to be efficient. The correction of the most dangerous multipoles in the LHC low-β triplets has been done by cancelling the aberration kicks in both planes independently with two compensators. It opens the possibility to operate the machine with a β^* as low as 0.25m. Using four correctors is not significantly better.

An outcome of the present study is that the usual compensation in the core of the multipoles created by the end coils has to be reviewed : the mere cancellation of the field error integral is not correct. The cancellation of the aberrations under physics conditions is mandatory.

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