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A model study of nuclear structure functions*

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Abstract

We calculate the structure function for a deuteron using the hadronic quark cluster decomposition. By assuming that nuclei might be composed of Quasi Deuterons we study their structure functions. The procedure enables a quantum mechanical parametrization of various scenarios, among them nuclear dynamics and nucleon swelling. Moreover it is specially suited to study Quark Exchange effects. We show, within a scheme where perturbative evolution effects are minimized, that the region around $x = 1$ is very sensitive to these two mechanisms and their effects can be disentangled.

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In order to study quark effects in nuclear systems we have constructed a model for the deuteron in terms of elementary quark constituents. We have further assumed, that deuteron type structures appear in heavier nuclei and that they are dominant in the two body correlations. From the nuclear dynamics point of view this formulation is not new in phenomenology, e.g. [1], moreover one could use convolution to obtain from it the properties of the nucleus. We shall consider, that in the region under study, no effects associated with the interaction between the deuteron clusters are relevant and therefore the probe just sees a number of times the same cluster. These deuteron type clusters, called hereafter Quasi Deuterons, might differ in size and even in the properties of the nucleons from those of the free deuteron.

The deuteron wave function can be written in terms of quarks (for the spin up component) in the general form [2]

$$\Psi_+(123456) = \mathcal{A}(d \uparrow (123;456)) \tag{1}$$

where $\mathcal{A}$ is the quark antisymmetrizer, 1,2, ... represent the quark degrees of freedom $(u, d, \uparrow, \downarrow, red, blue, white)$ and

$$d \uparrow (123;456) = \mathcal{R}(123;456)\frac{1}{\sqrt{2}}\{P \uparrow (123)N \uparrow (456) - N \uparrow (123)P \uparrow (456)\} \tag{2}$$

Here P and N characterize the internal proton and neutron wave function in terms of quarks respectively.

For simplicity and calculability we use a non-relativistic description with harmonic forces, i.e., for the spatial part of the wave function we take [2, 3].

$$\mathcal{R}(123;456) = \mathcal{S}exp[-Br_I^2 - Br_{II}^2 - A\sum_{i=1}^{6} r_i^2] \tag{3}$$

where $\mathcal{S}$ is the symmetrizer, $\vec{r}_i$ the radial coordinates of the quarks and $\vec{r}_I$ and $\vec{r}_{II}$ the coordinates of the center of mass of the clusters. The parameters A and B have a well defined meaning: A determines the size of the baryons, B the strenght of the nuclear harmonic force.

In terms of A and B the rms radius of the nucleon and deuteron are given by:

$$< r^2 >_{nuc} = \frac{1}{2A} \tag{4}$$

$$< r^2 >_{deut} = \frac{3}{8B} + \frac{1}{2A} \quad . \tag{5}$$

Note that we do not have D-state admixture in our deuteron wave function but for our pourpose here is not relevant.

This model will be taken as a laboratory (deuteron at rest) partonic description of the deuteron at some low hadronic scale Q_0^2. From it the deep inelastic structure function can be calculated for mass-shell partons [4, 5]. For our spherically symmetric wave function the structure function in the scaling limit is calculated according to the formula (we note that the deuteron has equal number of u and d quarks)

$$F_2(x) = \frac{5}{9} m \int_{k_m^2}^{\infty} dk^2 n(k^2) \tag{6}$$

where

$$k_m = \frac{M\bar{x}}{2}\left|1 - \frac{m^2}{M^2\bar{x}^2}\right| \tag{7}$$

In Eqs. (6) and (7) m represents the constituent quark mass ($m = m_u = m_d$), M the deuteron mass, $\bar{x}$ the deuteron scaling variable ($\approx 2x$, where x will label the so called Bjorken variable) and $n(k^2)$, the quark momentum distribution in the deuteron system, can be obtained as [5]

$$n(\vec{k}) = \frac{1}{(2\pi)^3} \int d^3r d^3r' e^{i\vec{k}(\vec{r}-\vec{r}')} \rho(\vec{r}, \vec{r}') \tag{8}$$

$\rho(\vec{r}, \vec{r'})$ is the non-diagonal one-body density determined in terms of the model wave function by:

$$\rho(\vec{r}, \vec{r}') = 6 \int d^3r_2 \dots d^3r_6 \Psi^*(\vec{r}, \vec{r}_2, \dots \vec{r}_6) \Psi^*(\vec{r}', \vec{r}_2, \dots \vec{r}_6) \quad . \tag{9}$$

Following the notation of ref. [3] the calculated momentum distribution has the form:

$$n(\vec{k}) = 3\frac{n_1(\vec{k}) + \frac{2}{27}n_{21}(\vec{k}) + \frac{1}{27}n_{22}(\vec{k})}{N_1 + \frac{N_2}{9}} \tag{10}$$

where n_1 and N_1 refer to diagonal contributions and n_{21}, n_{22} and N_2, correspond to quark exchange terms . These last ones give non convolution contributions to the structure function and have their origin in the symmetrization principle at the quark level [6]

The the structure function Eq.(6) becomes

$$F_2(x) = \frac{15\pi m}{9} \left(\frac{F^D(x)}{N_1} \frac{1 + \frac{1}{27}\frac{F^E(x)}{F^D(x)}}{1 + \frac{1}{9}\frac{N_2}{N_1}} \right) \quad . \tag{11}$$

The direct term is given by

$$\frac{F^D(x)}{N_1} = \frac{1}{2\pi^{\frac{3}{2}}} \sqrt{\frac{3}{4A+B}} exp\left(-\frac{3}{4A+B} k_m^2\right) \tag{12}$$

and the exchange term, $F^E(x)$, has a similar structure [7]. The Quark Exchange terms contribute only a few percent to the structure function and in the present global discussion we will not single out their contribution.

The formulation just presented does not lead to the correct support. The following very simple transformation corrects for this defect [7]

$$F_2'(x) = \frac{1}{(1-\bar{x})^2} F_2\left(\frac{\bar{x}}{1-\bar{x}}\right) \tag{13}$$

Note that under this transformation k_m tends to k_m' given by

$$k_m' = \frac{M\bar{x}}{2(1-\bar{x})} \left| 1 - \left(\frac{m(1-\bar{x})}{M\bar{x}}\right)^2 \right| \tag{14}$$

which tends to infinity in the two extremes of the support, making the structure function to vanish in them. In Eq.(13) the so called flux factor, arising from relativistic kinematics, has been incorporated [8].

To get the structure function at high Q^2 we do perturbative QCD evolution starting from a low energy scale Q_0^2, to which we adscribe the model calculated structure function [4, 5]. We present results obtained with a formalism which uses the constancy , under certain hypothesis, of ratios of the momenta of the structure function and which is valid to all orders in perturbation theory. However, as a check, we have calculated the explicit evolution through the renormalization group equations in the next to leading order approximation [9, 10]. The qualitative features of the results are common to both calculations [7].

Since the deuteron is an isosinglet, an SU(2) approximation to the evolution[1], and the assumption of the non existence of gluons at Q_0^2, an ingredient of the present model, lead to [10]

$$M_n^{\Sigma}(Q^2) \equiv \int d\bar{x}\bar{x}^{n-2}(\bar{x}\Sigma(\bar{x}, Q^2)) = a_{11}(\alpha_s) M_n^{\Sigma}(Q_0^2) \tag{15}$$

$$M_n^{G}(Q^2) \equiv \int d\bar{x}\bar{x}^{n-2}(\bar{x}G(\bar{x}, Q^2)) = a_{21}(\alpha_s) M_n^{\Sigma}(Q_0^2) \tag{16}$$

where Σ and G label the singlet and the gluon distributions respectively and the matrix elements a_{11} and a_{21} are calculated perturbatively as an expansion in powers of the coupling constant α_s.

Two deuteron like systems ($D1$ and $D2$) will have the same evolution properties determined by their isosinglet nature and from Eq.(15) we obtain

[1]We neglect the small contribution of the strange, charmed,... sea.

$$M_n^{D1}(Q^2) = \frac{M_n^{D1}(Q_0^2)}{M_n^{D2}(Q_0^2)} M_n^{D2}(Q^2) \tag{17}$$

to all orders of perturbation theory.

To generate from Eq.(17) the structure function at high momentum for the quasi-deuteron system we take one of the deuteron structure functions as known and generate the other by constructing its moments from the equation. For example in the realistic case of the deuteron we take the experimental parametrization of ref. [11]

$$F_2^n(\bar{x}) + F_2^p(\bar{x}) \tag{18}$$

and consider it to be the structure function of a loosely bound Quasi Deuteron system (rms radius $> 5fm$). From Eq.(18) we obtain its moments at $Q^2 = 5GeV^2/c^2$. Let us label this Quasi Deuteron $D2$. We now choose any other Deuteron system characterized by its A and B parameters, which we label $D1$, and calculate the RHS of Eq.(17) using our model and the experimental data. In this way Eq.(17) determines the moments of $D1$ at $Q^2 = 5GeV^2/c^2$. From them we reconstruct the $D1$ structure function by the inverse Mellin transform [12].

In the realistic case the inverse Mellin tranform turns out to be appropriate for values of $x < 1$. For $x \geq 1$, the values of the structure function are very small, and we have not found an adequate procedure for reconstruction (the inverse Mellin transform oscillates wildly). In order to envisage the physics beyond $x = 1$ we have performed the analysis also with a *toy* structure function which contains many more high momentum partons and therefore allows the deuteron one to extend more beyond this point [7].

The procedure just described assumes the same evolution scale for different Quasi Deuteron systems and, consequently, the same evolution scale for all nuclei at least at the level of their Quasi Deuteron subclusters. From the physical point of view it imposes on the input data the parton momentum flow of the low energy model, avoiding effects associated with the truncation of the matrix elements expansion in powers of the coupling constant.

We see our model calculation as a quantum mechanical parametrization of sizes within a cluster approach. No microscopic mechanism for these parameters is priviledged. We take them as external inputs and study various extreme scenarios which one can envisage going from almost non-overlapping nucleons to a six-quark Quasi-deuteron (the six quarks in the same potential well, B = 3 A). We analize what these changes imply on the input data in the region where the two nucleon correlations become important, i.e., we investigate the behavior of the structure functions for x around 1. Other models of nuclear structure have been considered, whose emphasis however has not been in the two nucleon correlation region but in the observability of Quark Exchange effects [13].

Our first case studied consists in calculating the structure function for a Quasi Deuteron whose size we decrease keeping the sizes of the nucleons fixed at their experimental value.

The mechanism producing this variation should be associated with the properties of the nuclear force in the medium [1] and therefore corresponds to long range QCD effects, having nothing to do with perturbative phenomena.

The ratio of the structure functions shown in Fig. (1) indicates that as we bring the two nucleons closer together towards the six quark hadron, the partons acquire on the average higher momentum, and therefore the ratio increases for large x and decreases for low x values. The dramatic increase near the $x = 1$ boundary is due to the vanishing of the deuteron structure function close thereafter, while that of the Quasi-deuteron vanishes for greater x values. The absolute values close to the boundary are, as already mentioned, very small.

A second case studied is defined by changing the size of the confinement region of the nucleon [14], characterized by A, while keeping the size of the Quasi Deuteron fixed to the Deuteron radius. Again we do not dwell on which mechanism produces this effect, we simply parametrize it.

When the nucleon swells, the ratio drops to zero as x approaches 1, that is, the momentum flow goes from the high x region to the low x region (see Fig.(1)). It is important to note that the structure function of this Quasi Deuteron vanishes *before* the $x = 1$ boundary.

The qualitative signature of our calculation seems to be quite clear. The $x = 1$ point strongly separates the dynamical origin of the phenomena. Single nucleon phenomena, like swelling, die out before one crosses this boundary, while two body correlations remain even in the region beyond. Therefore an experimental study of the immediate vecinity of the region on both sides of the the boundary is extremely important.

In the present calculation we have omitted some features of any realistic calculation to better isolate the behavior of the two dynamical scenarios. These are, the Quasi-Deuteron binding energy and nuclear wave function (Quasi Deuteron Fermi motion) effects. However we have estimated them in order to check the validity of our conclusions [7]. The former enhances the behavior of the first scenario, while diminishing that of the second, i.e., it produces a depletion of the low x ratio and an enhancement of the high x ratio. The Quasi Deuteron Fermi motion effects, contribute significantly only for $x >> 1$ and therefore do not affect significantly our discussion.

Before our concluding remarks a caveat which should extend to those. Quark exchange contributions, which we have not discussed in detail here, survive in the two nucleon correlation region. Moreover their magnitude increases with x and one should expect up to 10% effects from them [6, 7].

The two mechanisms explored might be relevant to understand the behavior of nuclei when probed with leptons. The conventional EMC type experiments [15] limit the analysis of these due to the washing away of many effects at the nucleon elastic point. From our work, the region around $x = 1$ seems to be crucial because it permits a kinematical separation of the two dynamical scenarios. The most important feature of our calculation, which in some way is a quantum mechanical intuitive result, is that both mechanisms produce opposite effects in the vecinity of this point and therefore it should be possible to disentangle them phenomenologically. Our model calculation defines the appropriate strategy for this analysis. The data for $x > 1$ should serve to determine the nuclear dynamics, i.e., long range QCD effects. Thereafter the $x < 1$ data should determine the single particle mechanisms, i.e., short range QCD dynamics. Finally one should be aware that Quark Exchange effects, i.e., non convolution contributions, should be incorporated in the analysis.

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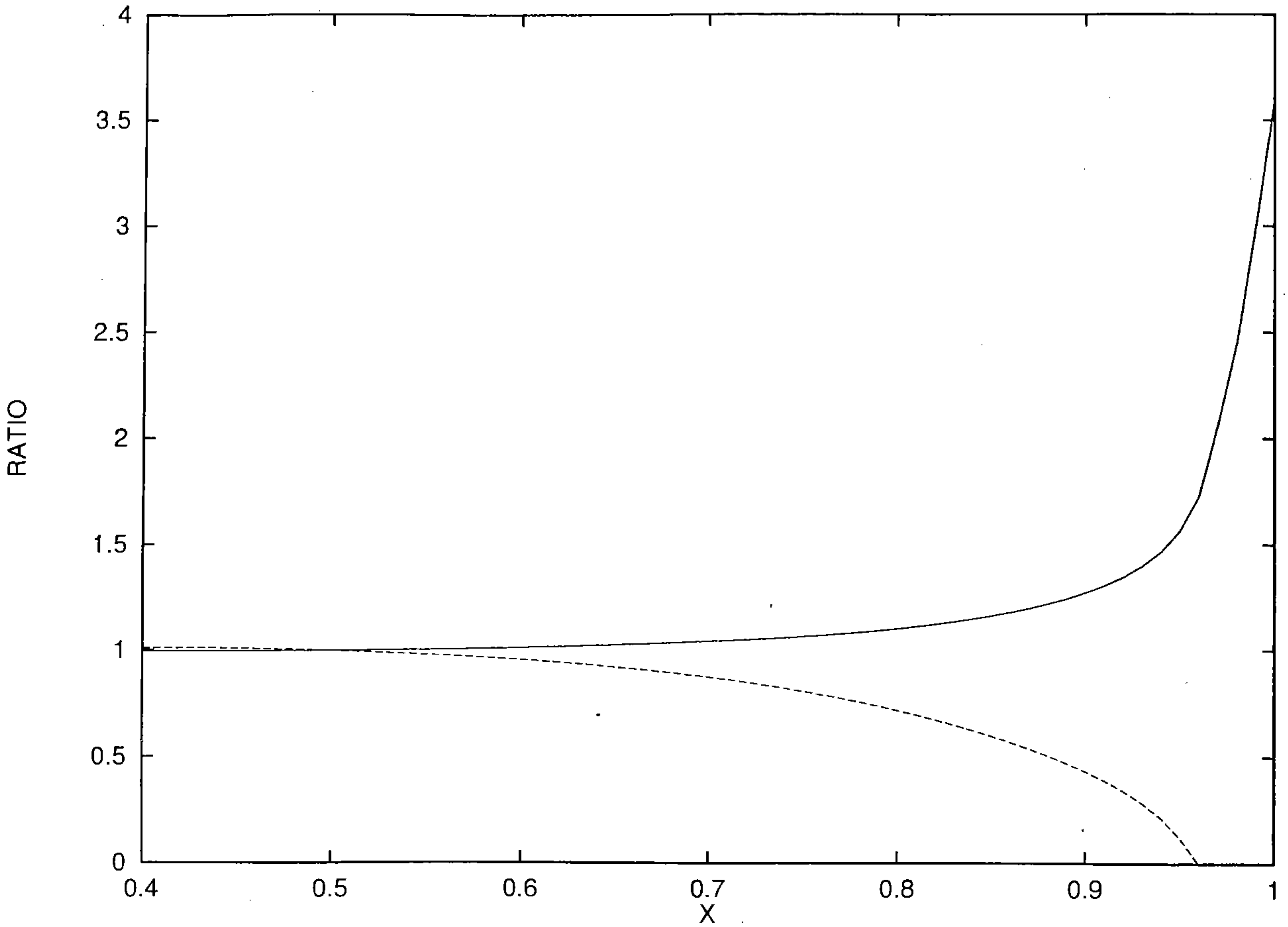
Figure Caption

Ratio of Quasi Deuteron over Deuteron structure functions. The deuteron is described by the following parameters: $A(fm^{-2})$=0.673, $B(fm^{-2})$=0.105. They produce the appropriate nucleon radius (rms 0.86 fm) and the appropriate deuteron radius (rms 2.08 fm). The solid line corresponds to a *small* Quasi Deuteron ($A(fm^{-2})$=0.673, $B(fm^{-2})$= 0.219 ; rms = 1.6 fm); the dashed line to a normal size Quasi Deuteron formed from swelled nucleon bubbles, i.e., *big* nucleons, ($A(fm^{-2})$=0.6, $B(fm^{-2})$=0.107, nucleon rms = 0.9 fm).


RATIO
4
3.5
3
2.5
2
1.5
1
0.5
0
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1
X