Quark model description of the $NN^*(1440)$ potential

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We derive a $NN^*(1440)$ potential from a non-relativistic quark-quark interaction and a chiral quark cluster model for the baryons. By making use of the Born-Oppenheimer approximation we examine the most important features of this interaction in comparison to those obtained from meson-exchange models.

1. INTRODUCTION

Baryonic resonances play a major role in the understanding of reactions that take place in nucleons and nuclei in the so-called intermediate energy regime [1]. In particular, the low-lying nucleonic resonances $\Delta(1232)$ and $N^*(1440)$, can be now analyzed in more detail due to the development of specific experimental programs in TJNAF, Uppsala...

In this context the transition, $NN \to NR$ (R : resonance), and direct $NR \to NR$ and $RR \to RR$ interactions should be understood. Usually these interactions have been written as straightforward extensions of some pieces of the $NN \to NN$ potential with the modification of the values of the coupling constants, extracted from their decay widths. Though this procedure can be appropriate for the very long-range part of the interaction, it is under suspicion at least for the short-range part for which the detailed structure of the baryons may determine to some extent the form of the interaction. This turns out to be the case for the $NN \to N\Delta$ and $N\Delta \to N\Delta$ potentials previously analyzed elsewhere [2]. It seems therefore convenient to proceed to a derivation of these potentials based on the more elementary quark-quark interaction.

This is the purpose of this talk: starting from a quark-quark non-relativistic interaction, we implement the baryon structure through technically simple gaussian wave functions and we calculate the potential at the baryonic level in the static Born-Oppenheimer approach. The $N^*(1440)$, the Roper resonance, considered as a radial excitation of the nucleon, is taken as a stable particle. For dynamical applications its width should be implemented through the coupling to the continuum.

We center our attention in the $NN^* \to NN^*$ potential where a complete parallelism with the $NN \to NN$ case can be easily established. Notice that the quark-quark interaction parameters are fixed (from the $NN \to NN$ case) and are kept independent of the baryons involved in the interaction. This eliminates the bias introduced in models at the baryonic level by a different choice of effective parameters according to the baryon-baryon interaction considered (this effectiveness of the parameters may hide distinct physical effects).

2. THE NN^*(1440) WAVE FUNCTION

The wave function of a two-baryon system, B_1 and B_2, with a definite symmetry under the exchange of the baryon quantum numbers is written as [3]:

$$\begin{aligned}\Psi^{ST}_{B_1B_2}(\vec{R}) &= \frac{\mathcal{A}}{\sqrt{1+\delta_{B_1B_2}}}\sqrt{\frac{1}{2}}\Bigg\{ \left[B_1\left(123;-\frac{\vec{R}}{2}\right) B_2\left(456;\frac{\vec{R}}{2}\right)\right]_{ST} \\ &+ (-1)^f \left\{\left[B_2\left(123;-\frac{\vec{R}}{2}\right) B_1\left(456;\frac{\vec{R}}{2}\right)\right]_{ST}\right\},\end{aligned} \tag{1}$$

being $\mathcal{A}$ the six-quark antisymmetrizer given by:

$$\mathcal{A} = (1-\sum_{i=1}^{3}\sum_{j=4}^{6} P_{ij})(1-\mathcal{P})\,, \tag{2}$$

where $\mathcal{P}$ exchanges the three quarks between the two clusters and P_{ij} exchanges quarks i and j.

If one projects on a state of definite orbital angular momentum L, due to the $(1-\mathcal{P})$ operator in the antisymmetrizer the wave function $\Psi^{ST}_{B_1B_2}(\vec{R})$ vanishes unless:

$$L + S_1 + S_2 - S + T_1 + T_2 - T + f = \text{odd}\,. \tag{3}$$

Since $S_1 = \frac{1}{2} = S_2$, $T_1 = \frac{1}{2} = T_2$, this fixes the relative phase between the two components of the wave function at Eq. (1) to be:

$$f = S + T - L + \text{odd}\,. \tag{4}$$

It is important to realize that for the NN system f is necessarily even in order to prevent the vanishing of the wave function. No such restriction exists for NN^*. Therefore, there are NN^* channels (f odd) with no counterpart in the NN case.

We will assume the three-quark wave function for the quark clusters at a position $\vec{R}$ to be given by

$$|N\rangle = |[3](0s)^3\rangle\,, \tag{5}$$

$$|N^*\rangle = \sqrt{\frac{2}{3}}|[3](0s)^2(1s)\rangle - \sqrt{\frac{1}{3}}|[3](0s)(op)^2\rangle\,, \tag{6}$$

explicitly,

$$N(\vec{r}_1,\vec{r}_2,\vec{r}_3;\vec{R}) = \prod_{n=1}^{3}\left(\frac{1}{\pi b^2}\right)^{3/4} e^{-\frac{(\vec{r}_n-\vec{R})^2}{2b^2}} \otimes [3]_{ST}\otimes[1^3]_c\,, \tag{7}$$

and

$$N^*(\vec{r}_1,\vec{r}_2,\vec{r}_3;\vec{R}) = (\sqrt{\frac{2}{3}}\phi_1 - \sqrt{\frac{1}{3}}\phi_2) \otimes [3]_{ST} \otimes [1^3]_C\,, \tag{8}$$

being

$$\phi_1 = \frac{\sqrt{2}}{3}\left(\frac{1}{\pi b^2}\right)^{9/4} \sum_{k=1}^{3}\left[\frac{3}{2} - \frac{(\vec{r}_k-\vec{R})^2}{b^2}\right] \prod_{i=1}^{3} e^{-\frac{(\vec{r}_i-\vec{R})^2}{2b^2}}\,, \tag{9}$$

and

$$\phi_2 = -\frac{2}{3}\left(\frac{1}{\pi^{\frac{9}{4}} b^{\frac{13}{2}}}\right) \sum_{j<k=1}^{3} (\vec{r}_j-\vec{R})\cdot(\vec{r}_k-\vec{R}) \prod_{i=1}^{3} e^{-\frac{(\vec{r}_i-\vec{R})^2}{2b^2}}\,, \tag{10}$$

where $[3]_{ST}$ and $[1^3]_c$ stand for the spin-isospin and color part, respectively. The validity of the harmonic oscillator wave functions to calculate the two-baryon interaction has been discussed in ref. [4].

The quark-quark potential we use can be written in terms of the interquark distance $\vec{r}_{ij}$ as:

$$V_{qq}(\vec{r}_{ij}) = V_{CON}(\vec{r}_{ij}) + V_{OGE}(\vec{r}_{ij}) + V_{OPE}(\vec{r}_{ij}) + V_{OSE}(\vec{r}_{ij})\,, 3 \tag{11}$$

where V_{CON} stands for the confining potential, and V_{OGE}, V_{OPE}, and V_{OSE} for one-gluon, one-pion and one-sigma exchange potentials, respectively. The expression of these potentials has been very much detailed elsewhere [5].

The baryon-baryon potential is obtained as the expectation value of the energy of the six-quark system minus the self-energy of the two clusters. The presence of the anti-symmetrization in the two-baryon wave function has also an important dynamical effect, the baryon-baryon potential contains quark-exchange contributions where the interaction takes place between two baryons that exchange a quark.

3. RESULTS

In Figure 1 we show the results for the NN^* potential in terms of the interbaryon distance R for two channels: ${}^1S_0(T=0)$, which is forbidden in the NN system, and the ${}^1S_0(T=1)$, which is allowed in the NN system. In this last case, the result is quite close to the corresponding channel in the NN system, a consequence of the near to identity similarity of N and N^*. As can be seen, the behavior in the two previous channels is completely different such that it could not be obtained by a simple rescaling of the vertex coupling constants from one case to the other. In order to emphasize the effects of quark antisymmetrization, we have compared to a direct potential without quark-exchange contributions. We have also separated the contribution of the different terms of the quark-quark potential in Eq.

As general features of the results we may remark that the OPE interaction determines the very long range behavior (R>4 fm), the OPE altogether with the OSE are responsible for the long-range par (1.5 fm $<$ R $<$ 4 fm), and OPE, OSE and OGE added to quark-exchange determine the attractive or repulsive character of the interaction at the intermediate- and short-range.

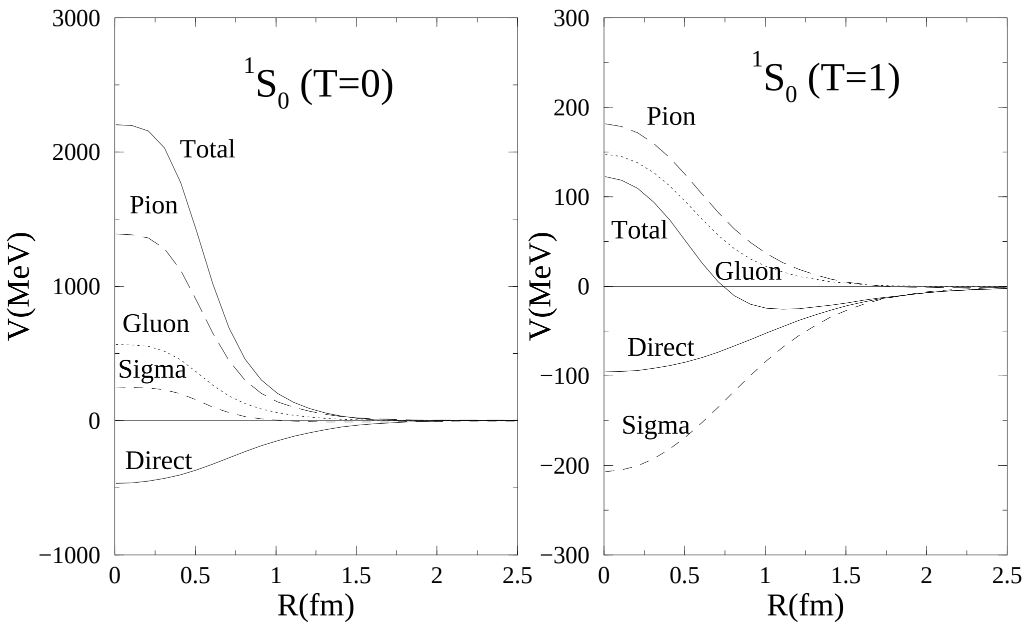

Figure 1. $^1S_0(T=0)$ and $^1S_0(T=1)$ NN^* potentials.

Certainly data on $NN^* \rightarrow NN^*$ phase shifts can be only obtained indirectly and no direct experimental test of our results can actually be performed. Nonetheless, our results should help to a better understanding of baryonic processes at a microscopic level and serve as a guide when dealing with reactions where some indicative predictions are needed in theoretical as well as in experimental studies. The elastic πd scattering above the Roper threshold as well as the breakup of the deuteron into NN^* channels, although not available for the moment, should serve as a test of the results we have derived.

A transition potential $NN \rightarrow NN^*$ can also be derived within the same framework. Althought this transition does not show forbidden channels, the quantum numbers are fixed by the NN system, the quark model provides a parameter-free prediction. This potential can be tested in several reactions [6]. We have determined the $NN^*(1440)$ probability on the deuteron by means of a multichannel calculation including: $^3S_1^{NN}$, $^3D_1^{NN}$, $^3S_1^{\Delta\Delta}$, $^3D_1^{\Delta\Delta}$, $^7D_1^{\Delta\Delta}$, $^7G_1^{\Delta\Delta}$,$^3S_1^{NN^*(1440)}$, and $^3D_1^{NN^*(1440)}$, finding for the Roper components 0.003% and 0.024%, respectively, much lower than the $\Delta\Delta$ ones ($\approx 0.25\%$).

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REFERENCES

1. E. Oset, H. Toki, and W. Weise, Phys. Rep. **82**, 282 (1982).
2. A Valcarce, F. Fernández, P. González, and V. Vento, Phys. Rev. **C52**, 38 (1995).
3. A Valcarce, F. Fernández, and P. González, Phys. Rev. **C56**, 3026 (1997).
4. A Valcarce, F. Fernández, P. González, and V. Vento, Phys. Lett. **B367**, 35 (1996).
5. F. Fernández, A. Valcarce, U. Straub, and, A. Faessler, J. Phys. **G19**, 2013 (1993).
6. H. P. Morsch and P. Zupranski, Phys. Rev. **C61**, 024002 (1999).