

Plausible explanation of the $\Delta_{5/2^+}(2000)$ puzzle

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From a Faddeev calculation for the $\pi - (\Delta\rho)_{N_{5/2^-(1675)}}$ system we show the plausible existence of three dynamically generated $I(J^P) = 3/2 (5/2^+)$ baryon states below 2.3 GeV whereas only two resonances, $\Delta_{5/2^+}(1905)(***)$ and $\Delta_{5/2^+}(2000)(**)$, are cataloged in the Particle Data Book Review. Our results give theoretical support to data analyses extracting two distinctive resonances, $\Delta_{5/2^+}(\sim 1740)$ and $\Delta_{5/2^+}(\sim 2200)$, from which the mass of $\Delta_{5/2^+}(2000)(**)$ is estimated. We propose that these two resonances should be cataloged instead of $\Delta_{5/2^+}(2000)$. This proposal gets further support from the possible assignment of the other baryon states found in the approach in the $I = 1/2, 3/2$ with $J^P = 1/2^+, 3/2^+, 5/2^+$ sectors to known baryonic resonances. In particular, $\Delta_{1/2^+}(1750)(*)$ is naturally interpreted as a $\pi N_{1/2^-(1650)}$ bound state.

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I. INTRODUCTION

Baryon spectroscopy (see Ref. [1] for a recent general review) is an essential tool to analyze the baryon structure. Data on baryon masses and transitions, regularly compiled in the Particle Data Book Review (PDG) [2], allow us when confronted with theoretical calculations to learn about the effective constituent degrees of freedom and their interactions inside the baryon. From the experimental point of view the information on baryonic resonances mainly comes from pion-nucleon (πN) scattering experiments. The photon nucleon (γN) reactions have led to advancement in the field, reconfirming many known resonances and claiming evidence for new ones. From the theoretical point of view it has become clear in the last years that the primitive quark model view of a baryon as formed by three effective valence quarks ($3q$) may require the implementation of higher Fock space terms, in the form of $4q1\bar{q}$, $5q2\bar{q}$... or meson-baryon, meson-meson-baryon... components to provide a satisfactory explanation of some baryonic resonances. Paradigmatic cases are the $\Lambda(1405) S_{01}$ and the $\Delta(1930) D_{35}$. For $\Lambda_{1/2^-(1405)}$ the relevance of $\bar{K}N$ was first pointed out in 1977 [3] (more recently the role of $\pi\Sigma$ has been also emphasized [4]). For $\Delta_{5/2^-(1930)}$ the important role of $\rho\Delta$ has been recognized [5]. These are particular examples of a more general situation where a $3q$ model calculation (providing a reasonable overall description of the whole spectrum) overestimates the mass of a resonance so that a meson-baryon threshold lies in between the calculated $3q$ value and the experimental data [6].

As a consequence, the meson-baryon component may be dominant when the meson-baryon interaction is attractive, and the dynamical generation of the resonance from these hadronic degrees of freedom may be more efficient than a quark model description which would require $4q1\bar{q}$ and/or higher Fock space terms (note that the meson and baryon of the threshold might also correspond to dynamically generated states). This argument can be extended to resonances where the meson-baryon thresholds are above the $3q$ masses if the meson-baryon interaction is sufficiently attractive as to provide the binding required by data.

As a matter of fact, meson-baryon components are present in all baryonic resonances. In some cases the contribution of these components to the masses may be properly taken into account by making use of the $3q$ description with effective parameters for the quark-quark interaction. In other cases, as explained above, this may not be possible. The intermediate situation corresponds to the case of resonances for which both approximations may reasonably reproduce their masses. In such a case the two descriptions may be at least to some extent equally valid alternatives, the values of their effective parameters taking implicitly into account the non-explicit ($3q$ or meson-baryon) component contribution.

In this article we take these considerations into account to analyze N and Δ resonances with $J^P = 1/2^+, 3/2^+, 5/2^+$ sectors. The motivation for this study comes mainly from the puzzle concerning the $\Delta(2000) F_{35} (**)$ since the nominal mass of this resonance does not correspond in fact to any experimental analysis but to an estimation based on the value of the masses (~ 1740 MeV and ~ 2200 MeV) extracted from different data analyses [2]. This makes feasible the existence of a hidden $\Delta(1740) F_{35}$ resonance which could not be reasonably accommodated within a $3q$ framework description what might be indicating its dynamically gener-

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ated character. The theoretical examination of such a possible character is the main objective of this article. For this purpose we shall follow a procedure based on the combination of chiral Lagrangians with nonperturbative unitary techniques in coupled channels of baryons and/or pseudoscalar and/or vector mesons. This scheme has been very fruitful in the description of other baryonic resonances through the analysis of the poles of the meson-baryon or meson-meson-baryon scattering amplitudes (see for instance Ref. [7] and references therein).

If existing, the $\Delta(1740) F_{35}$ resonance could be generated from $\pi - (N(1675) D_{15})$ as suggested in Ref. [6]. Since the $N_{5/2-}(1675)$ has been dynamically generated as a bound state of $\rho\Delta(1232)$ in the $I = 1/2$ sector (the same interaction generating the $\Delta_{5/2-}(1930)$ for $I = 3/2$) [5], we shall investigate the three-body π - ρ - Δ system but keeping the strong correlations of the $\rho\Delta$ system which generate the $N_{5/2-}(1675)$. In such a situation the use of the Fixed Center Approximation (FCA) to the Faddeev equations is justified [8]. For the sake of consistency, N and Δ resonances which can be dynamically generated altogether with $\Delta_{5/2+}$ will be also analyzed.

The contents of the article are organized as follows. In Section II, we revisit the cataloged $\Delta_{5/2+}$ resonances and comment on their $3q$ description. In Section III, we present the FCA formalism to analyze the $\pi - (\Delta\rho)_{N_{5/2-}(1675)}$ system. The analysis of the $\pi - (\Delta\rho)_{N_{5/2-}(1675)}$ scattering amplitude is extracted in Section IV and a tentative assignment peaks in the amplitudes to baryonic resonances is proposed. Finally, in Section V we summarize our approach and main findings.

II. THE $\Delta(2000) F_{35}$ PUZZLE

In the PDG [2] there is only a well established $\Delta_{5/2+}$ resonance, $\Delta(1905) F_{35}$ (* **), and fair evidence of the existence of another one, $\Delta(2000) F_{35}$ (**). However, a careful look at this last resonance shows that its nominal mass is in fact estimated from $\Delta(1752 \pm 32)$, $\Delta(1724 \pm 61)$ and $\Delta(2200 \pm 125)$, respectively, extracted from three independent analyses [9–11] of different character: $\pi N \rightarrow \pi N, \pi\pi N$ in Ref. [9], multichannel in Ref. [10] and $\pi N \rightarrow \pi N$ in Ref. [11]. Moreover a recent new data analysis has reported a $\Delta_{5/2+}$ with a pole position at 1738 MeV [12]. In this last analysis, incorporating $\pi N, \gamma N \rightarrow \pi N, \eta N, \pi\pi N$ data, the resonance is obtained from a bare state at 2162 MeV through its coupling to meson-baryon channels. This bare state represents the quark core component of the resonance within this calculation framework.

It is important to remark that i) all the analyses extract the $\Delta_{5/2+}(1905)$ and ii) the non extraction of $\Delta_{5/2+}(2200)$ in most of the mentioned analyses may be related to the restricted range of energy examined (typically below 2200 MeV).

From a $3q$ description the $\Delta_{5/2+}(1905)$ is naturally accommodated as the lowest $\Delta_{5/2+}$ state in the second

energy band of a double harmonic oscillator potential (one oscillator for each Jacobi coordinate of the $3q$ system) that provides (up to perturbative terms) a reasonable overall description of the whole baryon spectrum [13]. Actually quark models predict two states close in energy for the lowest symmetric and mixed symmetric orbital configurations in the second energy band. The $\Delta_{5/2+}(1905)$ is then assigned to the orbitally symmetric state. Experimental evidence for the mixed symmetric one has also been reported near 2000 MeV [14]. Similarly, the reported $\Delta_{5/2+}(2200)$, with a more uncertain mass (2200 ± 125 MeV), may be reasonably located in the fourth energy band. On the contrary, $\Delta_{5/2+}(1740)$ lying far below the energy of the lowest $\Delta_{5/2+}$ state in the second energy band (the first available band by symmetry to a $\Delta_{5/2+}$ state) could not be accommodated as a $3q$ state without seriously spoiling the overall spectral description.

The same kind of problem was tackled in Ref. [5] regarding the description of $\Delta_{5/2-}(1930)$ with a mass much lower than the corresponding to the third energy band, the first available band for such a state. There the consideration of the $\rho\Delta$ channel whose threshold (2002 MeV) lies close above the experimental mass of the resonance and far below the $3q$ mass (~ 2150 MeV) allowed for an explanation of $\Delta_{5/2-}(1930)$ and its partners, $\Delta_{3/2-}(1940)$ and $\Delta_{1/2-}(1900)$, as $\rho\Delta$ bound states in the $I = 3/2$ sector. In addition $N_{1/2-}(1650)$, $N_{3/2-}(1700)$ and $N_{5/2-}(1675)$ were also well described as $\rho\Delta$ bound states in the $I = 1/2$ sector, although the bigger sensitivity in this case to the cutoff parameter employed left some room for alternative assignments of these $\rho\Delta$ bound states to nucleonic resonances [15]. It should be pointed out that, contrary to the $\Delta_{5/2-}(1930)$ and its partners, these nucleon resonances around 1700 MeV can also be reasonably described as $3q$ states in the first energy band [13]. Therefore a more reliable explanation of data should include the contribution of the $3q$ states as well as of the possible $\rho\Delta$ bound states.

Back to $\Delta_{5/2+}(1740)$ one can easily identify a meson-baryon threshold, $[\pi N_{5/2-}(1675)]_{\text{threshold}} = 1814$ MeV, in between the $3q$ mass (~ 1910 MeV) and the data. Then one can wonder about the possibility that the $\pi N_{5/2-}(1675)$ system may give rise to a bound state which could provide theoretical support to the fair evidence of the existence of $\Delta_{5/2+}(1740)$. Actually this bound state nature could explain why this resonance is extracted in some data analyses but not in others. It turns out that only analyses reproducing the $\pi\pi N$ production cross section data extract it. Let us note that this would be a necessary condition to extract $\Delta_{5/2+}(1740)$ if corresponding to a $\pi N_{5/2-}(1675)$ state (let us recall that $N_{5/2-}(1675)$ decays to πN and to $\pi\pi N$ with branching fractions of 40% and 55% respectively).

To examine this possibility we perform next an analysis of the $\pi N_{5/2-}(1675)$ system by assuming that $N_{5/2-}(1675)$ is a $\rho\Delta$ bound state. Although according to our discussion above, a combined ($3q + \rho\Delta$) descrip-

tion of $N_{5/2-}(1675)$ would be more appropriate we shall consider only the $\rho\Delta$ bound state option (adequate to our formalism) and assume that the value of the parameter (cutoff or subtraction constant) involved in the dynamical generation of $N_{5/2-}(1675)$ from $\rho\Delta$ takes implicitly into account the $3q$ component. Furthermore, the same consideration is extended to the dynamical generation of resonances from $\pi N_{5/2-}(1675)$.

We should finally notice that $\pi N_{5/2-}(1675)$ may couple to other s -wave meson-baryon channel, like $\pi\Delta_{5/2-}(1930)$. We do not expect this channel to play any relevant role in the generation of $\Delta_{5/2+}(1740)$ since its threshold is far above in energy. However, the $\pi\Delta_{5/2-}(1930)$ channel could influence the possible generation of higher mass resonances. For the sake of simplicity we shall not include it in our calculation. We should then keep in mind that the calculated masses for the higher resonances have a higher degree of uncertainty.

III. FORMALISM

The interaction of a particle with a bound state of a pair of particles at very low energies or below threshold can be efficiently and accurately studied by means of the fixed center approximation (FCA) to the Faddeev equations for the three-particle system [16]. We shall extend this formalism to include states above threshold and apply it to $\pi - (\Delta\rho)_{N_{5/2-}(1675)}$. The analysis of the $\pi - (\Delta\rho)_{N_{5/2-}(1675)}$ scattering amplitude will allow us to identify dynamically generated resonances with states listed in the PDG.

The FCA to the Faddeev equations has been used with success recently in similar problems of bound three-body systems and contrasted with full Faddeev or variational calculations. In this sense, the $N\bar{K}K$ system has been studied with the FCA in Ref. [17], with very similar results as found in the full Faddeev calculations in Refs. [18] and in the variational estimate in Ref. [19]. Similarly, the study [20] of the $\bar{K}NN$ system within the FCA has led to very similar results as the variational calculations of [21] when the $\bar{K}N$ chiral amplitudes are used, or the Faddeev calculation of [22], when the energy dependence of the $\bar{K}N$ amplitude is used in agreement with chiral dynamics.

The important ingredients in the calculation of the total scattering amplitude for the Δ - ρ - π system using the FCA are the two-body Δ - ρ , Δ - π and ρ - π unitarized s -wave interactions from the chiral unitary approach. Although the form of these interactions have been detailed elsewhere [5, 23, 24], we shall briefly revisit in Subsection A the Δ - ρ case. This will allow us to remind the general procedure of calculating the two-body amplitudes entering the FCA equations. Then in Subsections B and C the calculation of the π - $(\Delta\rho)$ amplitude will be detailed.

A. Unitarized $\Delta\rho$ interaction

The $\Delta\rho$ interaction has been analyzed in the framework of the hidden gauge formalism [25] in terms of the exchange of a ρ meson in the t -channel between the Δ and the ρ [5, 15]. Under the low energy approximation of neglecting q^2/M_V^2 in the propagator of the exchanged vector meson, where q is the momentum transfer, and also the three momentum \vec{k} of the vector meson, one obtains for the $\Delta\rho \rightarrow \Delta\rho$ potential the form

$$V_{pol} = -\frac{1}{4f^2}C_I(k^0 + k'^0)\vec{\epsilon} \cdot \vec{\epsilon}', \quad (1)$$

where $f = 93$ MeV is the pion decay constant, $k^0(\vec{\epsilon})$ and $k'^0(\vec{\epsilon}')$ is the energy (polarization) of the incoming/outgoing rho meson and C_I an isospin dependent coefficient with values

$$I_{\Delta\rho} = \frac{1}{2}, C_{1/2} = 5, \quad (2)$$

$$I_{\Delta\rho} = \frac{3}{2}, C_{3/2} = 2, \quad (3)$$

$$I_{\Delta\rho} = \frac{5}{2}, C_{5/2} = -3. \quad (4)$$

Then one can solve the Bethe-Salpeter equation with the on-shell factorized potential and, thus, the T -matrix will be given by

$$T = \frac{V}{1 - VG}, \quad (5)$$

with V the potential of Eq. (1) in the isospin basis without the polarization factor $\vec{\epsilon} \cdot \vec{\epsilon}'$. G is the loop function for intermediate $\Delta\rho$ states that can be regularized both with a cutoff prescription as done in Ref. [5], or with dimensional regularization in terms of a subtraction constant as done in Ref. [15]. Here we shall make use of the dimensional regularization scheme better suited to analyze the sensitivity of our results against variations of the parameter (small changes of the subtraction constant translate into significant changes in the values of the cutoff [26]). The expression for G is then

$$\begin{aligned} G(s_{\Delta\rho}) &= i \int \frac{d^4q}{(2\pi)^4} \frac{2M_\Delta}{[(P-q)^2 - M_\Delta^2 + i\epsilon](q^2 - m_\rho^2 + i\epsilon)} \\ &= \frac{2M_\Delta}{16\pi^2} \left\{ a(\mu) + \ln \frac{M_\Delta^2}{\mu^2} + \frac{m_\rho^2 - M_\Delta^2 + s_{\Delta\rho}}{2s_{\Delta\rho}} \ln \frac{m_\rho^2}{M_\Delta^2} \right. \\ &\quad \left. + \frac{\bar{q}}{\sqrt{s_{\Delta\rho}}} [\ln(s_{\Delta\rho} - (M_\Delta^2 - m_\rho^2) + 2\bar{q}\sqrt{s_{\Delta\rho}}) + \right. \\ &\quad \ln(s_{\Delta\rho} + (M_\Delta^2 - m_\rho^2) + 2\bar{q}\sqrt{s_{\Delta\rho}}) - \\ &\quad \ln(-s_{\Delta\rho} + (M_\Delta^2 - m_\rho^2) + 2\bar{q}\sqrt{s_{\Delta\rho}}) - \\ &\quad \left. \left. \ln(-s_{\Delta\rho} - (M_\Delta^2 - m_\rho^2) + 2\bar{q}\sqrt{s_{\Delta\rho}}) \right] \right\}, \quad (6) \end{aligned}$$

where P is the total incident momentum, which in the center of mass frame is $(\sqrt{s_{\Delta\rho}}, 0, 0, 0)$ being $\sqrt{s_{\Delta\rho}}$ the

invariant mass of the $\Delta\rho$ system. In Eq. (6), μ is the scale of dimensional regularization and $a(\mu)$ the subtraction constant. Note that the only parameter dependent part of G is $a(\mu)+\ln\frac{M_\Delta^2}{\mu^2}$. Due to renormalization group invariance any change in μ is reabsorbed by a change in $a(\mu)$ through $a(\mu')-a(\mu)=\ln\frac{\mu'^2}{\mu^2}$ so that the amplitude is scale-independent. In Eq. (6), \vec{q} is the momentum of the Δ or the ρ in the $\Delta\rho$ center of mass frame, which is given by

$$\vec{q} = \frac{\sqrt{(s_{\Delta\rho} - (M_\Delta + m_\rho)^2)(s_{\Delta\rho} - (M_\Delta - m_\rho)^2)}}{2\sqrt{s_{\Delta\rho}}}. \quad (7)$$

However, since the Δ baryon and ρ meson have large total decay widths Γ_Δ and Γ_ρ , they should be taken into account. For this purpose we replace the G function in Eq. (5) by \tilde{G} :

$$\begin{aligned} \tilde{G}(s_{\Delta\rho}) &= \frac{1}{N_\Delta N_\rho} \int_{M_\Delta-2\Gamma_\Delta}^{M_\Delta+2\Gamma_\Delta} d\tilde{M} \left(-\frac{1}{\pi}\right) \times \\ &\quad \mathcal{I}m \frac{1}{\tilde{M} - M_\Delta + i\frac{\Gamma_1(\tilde{M})}{2}} \int_{(m_\rho-2\Gamma_\rho)^2}^{(m_\rho+2\Gamma_\rho)^2} d\tilde{m}^2 \\ &\quad \times \left(-\frac{1}{\pi}\right) \mathcal{I}m \frac{1}{\tilde{m}^2 - m_\rho^2 + i\tilde{m}\Gamma_2(\tilde{m})} \\ &\quad \times G(s_{\Delta\rho}, \tilde{M}, \tilde{m}), \end{aligned} \quad (8)$$

with

$$\begin{aligned} N_\Delta &= \int_{M_\Delta-2\Gamma_\Delta}^{M_\Delta+2\Gamma_\Delta} d\tilde{M} \left(-\frac{1}{\pi}\right) \mathcal{I}m \frac{1}{\tilde{M} - M_\Delta + i\frac{\Gamma_1(\tilde{M})}{2}}, \\ N_\rho &= \int_{(m_\rho-2\Gamma_\rho)^2}^{(m_\rho+2\Gamma_\rho)^2} d\tilde{m}^2 \left(-\frac{1}{\pi}\right) \mathcal{I}m \frac{1}{\tilde{m}^2 - m_\rho^2 + i\tilde{m}\Gamma_2(\tilde{m})}, \end{aligned}$$

where

$$\begin{aligned} \Gamma_1(\tilde{M}) &= \Gamma_\Delta \left(\frac{\lambda^{1/2}(\tilde{M}^2, M_N^2, m_\pi^2) 2M_\Delta}{\lambda^{1/2}(M_\Delta^2, M_N^2, m_\pi^2) 2\tilde{M}} \right)^3 \\ &\quad \times \theta(\tilde{M} - M_N - m_\pi), \\ \Gamma_2(\tilde{m}) &= \Gamma_\rho \left(\frac{\tilde{m}^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2} \right)^{3/2} \theta(\tilde{m} - 2m_\pi), \end{aligned}$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$ the triangle function. We shall take $\Gamma_\Delta = 120$ MeV and $\Gamma_\rho = 150$ MeV.

In addition, one issue worth mentioning is that the spin dependence comes from the $\vec{\epsilon} \cdot \vec{\epsilon}$ factor of the ρ meson. The spin of the Δ baryon does not appear in the present formalism due to the approximations done. The $\vec{\epsilon} \cdot \vec{\epsilon}$ scalar structure indicate s -wave interaction of $\Delta\rho$, therefore, one has degeneracy for the $J^P = 1/2^-, 3/2^-, 5/2^-$ states for both $I_{\Delta\rho} = 1/2$ and $I_{\Delta\rho} = 3/2$.

In order to evaluate the value of the scattering amplitude we have to fix the parameter $a(\mu)+\ln\frac{M_\Delta^2}{\mu^2}$. As explained above the choice of μ is rather arbitrary since a

change in it is reabsorbed by a change in $a(\mu)$. Values of μ from 630 MeV to 1000 MeV have been employed in the literature. We choose $\mu = 800$ MeV, a value rather close to the cutoff employed in Ref. [5] ($q_{max} = 770$ MeV), and fix $a(\mu)$, according to our comment at the end of the Section II, to get the $(\Delta\rho)_{I=1/2}$ bound state at 1675 MeV as corresponding to the estimated mass of $N_{5/2^-}(1675)$ in Ref. [2]. We get $a_{\Delta\rho} = -2.28$ (if instead we had used $\mu = 630, 1000$ MeV we would have obtained $a_{\Delta\rho} = -2.76, -1.83$).

In Fig. 1 the modulus squared of the scattering amplitude as a function of the invariant mass of the $\Delta\rho$ system for $I_{\Delta\rho} = 1/2$ is shown. Note that in the $I_{\Delta\rho} = 3/2$ sector the bound state is located at $\sqrt{s_{\Delta\rho}} = 1887$ MeV, a little bit lower than its location in our previous study [5] as a consequence of the fine tuning of the parameter to get the $I_{\Delta\rho} = 1/2$ bound state at $\sqrt{s_{\Delta\rho}} = 1675$ MeV. Notice anyhow that the assignment of the $I_{\Delta\rho} = 3/2$ states at 1887 MeV to $\Delta(1900)S_{31}(**)$, $\Delta(1940)D_{33}(*)$ and $\Delta(1930)D_{35}(**)$ remains unambiguous.

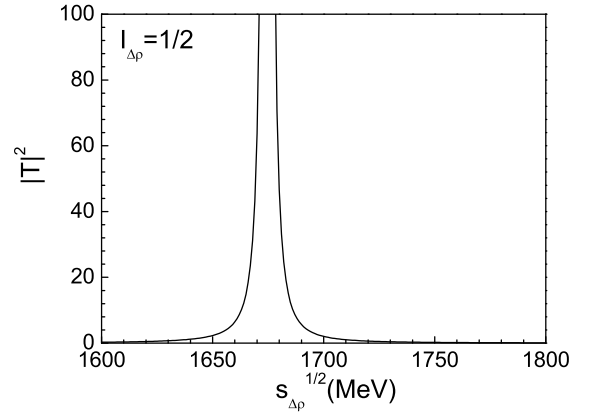


FIG. 1: Modulus squared of the $\Delta\rho$ scattering amplitude for $I_{\Delta\rho} = 1/2$.

B. Single-scattering contribution for the π interaction with the $\Delta\rho$ system

The FCA to the Faddeev equations for the three body $\Delta\rho\pi$ system is depicted diagrammatically in Fig. 2. The external π meson interacts successively with the Δ baryon and ρ meson which form the $N_{5/2^-}(1675)$ ($\equiv N^*$). In terms of two partition functions T_1 and T_2 , the FCA equations are

$$T_1 = t_1 + t_1 G_0 T_2, \quad (9)$$

$$T_2 = t_2 + t_2 G_0 T_1, \quad (10)$$

$$T = T_1 + T_2, \quad (11)$$

where T is the total three-body scattering amplitude and T_i ($i = 1, 2$)¹ account for the diagrams starting with the interaction of the external particle with particle i of the compound system. Hence, t_i represent the $\Delta\pi$ and $\rho\pi$ unitarized scattering amplitudes whose forms were derived in Refs. [23] and [24] respectively to which we refer for details. In the above equations, G_0 is the loop function for the π meson propagating inside the $N_{5/2^-}$ (1675) resonance which will be discussed later on.

More specifically t_1 is the appropriate combination of the $I = 1/2, 3/2$ and $5/2$ unitarized two-body $\Delta\pi$ scattering amplitudes ($t_{\Delta\pi}^{1/2}$, $t_{\Delta\pi}^{3/2}$, and $t_{\Delta\pi}^{5/2}$) whereas t_2 stands for the corresponding combination of the $I = 0, 1, 2$ two-body $\rho\pi$ scattering amplitudes ($t_{\rho\pi}^0$, $t_{\rho\pi}^1$, $t_{\rho\pi}^2$). For example, let us consider a cluster of $\Delta\rho$ in isospin $I = 1/2$, the constituents of which we call 1 and 2, and the external

¹ In the present work, the label 1 represents Δ baryon of the compound system, while 2 represents the ρ meson.

π meson we call number 3. The $\Delta\rho$ isospin states are written as

$$|\Delta\rho \rangle_{I=1/2, I_z=1/2} = \sqrt{\frac{1}{2}} |(\frac{3}{2}, -1) \rangle - \sqrt{\frac{1}{3}} |(\frac{1}{2}, 0) \rangle + \sqrt{\frac{1}{6}} |(-\frac{1}{2}, 1) \rangle, \quad (12)$$

$$|\Delta\rho \rangle_{I=1/2, I_z=-1/2} = \sqrt{\frac{1}{6}} |(\frac{1}{2}, -1) \rangle - \sqrt{\frac{1}{3}} |(-\frac{1}{2}, 0) \rangle + \sqrt{\frac{1}{2}} |(-\frac{3}{2}, 1) \rangle, \quad (13)$$

where the kets on the right hand sides indicate the I_z components of the particles 1 and 2, $|I_z^{(1)}, I_z^{(2)} \rangle$.

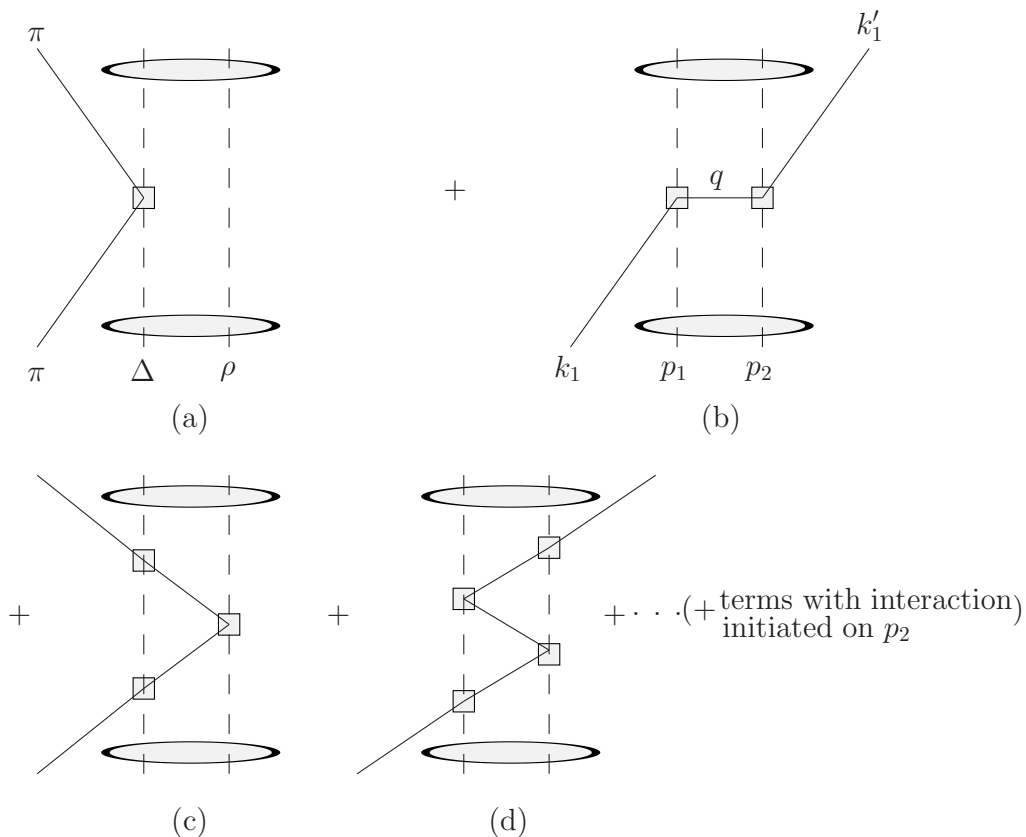


FIG. 2: Diagrammatic representation of the fixed center approximation to the Faddeev equations. Diagrams (a) and (b) represent the first contributions to the Faddeev equations from single scattering and double scattering respectively. Diagrams (c) and (d) represent iterations of the interaction.

The scattering potential $\langle \Delta\rho\pi|V|\Delta\rho\pi \rangle$ for the sin-

gle scattering contribution (Fig. 2 (a) + term with inter-

action initiated on p_2) can be easily obtained in terms of the two body potentials V_{31} and V_{32} derived in Refs. [23] and [24].

Here we write explicitly the case of $I_{\Delta\rho} = 1/2$ and total isospin $I_{\Delta\rho\pi} = 1/2$,

$$\begin{aligned}
& \langle \Delta\rho\pi | V | \Delta\rho\pi \rangle = \left(\sqrt{\frac{1}{3}} \langle (\Delta\rho) |_{I_z=1/2} \otimes \langle \pi |_{I_z=0} - \sqrt{\frac{2}{3}} \langle (\Delta\rho) |_{I_z=-1/2} \otimes \langle \pi |_{I_z=1} \right) (V_{31} + V_{32}) \\
& \left(\sqrt{\frac{1}{3}} |(\Delta\rho) \rangle_{I_z=1/2} \otimes |\pi \rangle_{I_z=0} - \sqrt{\frac{2}{3}} |(\Delta\rho) \rangle_{I_z=-1/2} \otimes |\pi \rangle_{I_z=1} \right) \\
& = \left(\sqrt{\frac{1}{3}} \left(\sqrt{\frac{1}{2}} \langle \left(\frac{3}{2}, -1\right) | - \sqrt{\frac{1}{3}} \langle \left(\frac{1}{2}, 0\right) | + \sqrt{\frac{1}{6}} \langle \left(-\frac{1}{2}, 1\right) | \right) \otimes \langle 0 | - \right. \\
& \left. \sqrt{\frac{2}{3}} \left(\sqrt{\frac{1}{6}} \langle \left(\frac{1}{2}, -1\right) | - \sqrt{\frac{1}{3}} \langle \left(-\frac{1}{2}, 0\right) | + \sqrt{\frac{1}{2}} \langle \left(-\frac{3}{2}, 1\right) | \right) \otimes \langle 1 | \right) (V_{31} + V_{32}) \\
& \left(\sqrt{\frac{1}{3}} \left(\sqrt{\frac{1}{2}} \left(\frac{3}{2}, -1\right) \rangle - \sqrt{\frac{1}{3}} \left(\frac{1}{2}, 0\right) \rangle + \sqrt{\frac{1}{6}} \left(-\frac{1}{2}, 1\right) \rangle \right) \otimes |0 \rangle - \right. \\
& \left. \sqrt{\frac{2}{3}} \left(\sqrt{\frac{1}{6}} \left(\frac{1}{2}, -1\right) \rangle - \sqrt{\frac{1}{3}} \left(-\frac{1}{2}, 0\right) \rangle + \sqrt{\frac{1}{2}} \left(-\frac{3}{2}, 1\right) \rangle \right) \otimes |1 \rangle \right) \\
& = \langle \frac{\sqrt{10}}{6} \left(\left(\frac{3}{2}, \frac{3}{2}\right), -1 \right) - \left(\frac{\sqrt{15}}{9} \left(\frac{3}{2}, \frac{1}{2}\right) - \frac{2\sqrt{3}}{9} \left(\frac{1}{2}, \frac{1}{2}\right) \right), 0 \rangle + \left(\frac{\sqrt{30}}{18} \left(\frac{3}{2} - \frac{1}{2}\right) - \frac{2\sqrt{6}}{9} \left(\frac{1}{2} - \frac{1}{2}\right) \right), 1 \rangle |V_{31} \\
& | \frac{\sqrt{10}}{6} \left(\left(\frac{3}{2}, \frac{3}{2}\right), -1 \right) - \left(\frac{\sqrt{15}}{9} \left(\frac{3}{2}, \frac{1}{2}\right) - \frac{2\sqrt{3}}{9} \left(\frac{1}{2}, \frac{1}{2}\right) \right), 0 \rangle + \left(\frac{\sqrt{30}}{18} \left(\frac{3}{2} - \frac{1}{2}\right) - \frac{2\sqrt{6}}{9} \left(\frac{1}{2} - \frac{1}{2}\right) \right), 1 \rangle + \\
& \langle \frac{\sqrt{3}}{6} \left((2-1) - (1-1) \right), \frac{3}{2} \rangle - \left(\frac{\sqrt{6}}{6} (20) - \frac{\sqrt{2}}{6} (10) \right), \frac{1}{2} \rangle + \left(\frac{1}{2} (21) - \frac{1}{6} (11) \right), -\frac{1}{2} \rangle - \frac{\sqrt{3}}{3} \left((22), -\frac{3}{2} \right) |V_{32} \\
& | \frac{\sqrt{3}}{6} \left((2-1) - (1-1) \right), \frac{3}{2} \rangle - \left(\frac{\sqrt{6}}{6} (20) - \frac{\sqrt{2}}{6} (10) \right), \frac{1}{2} \rangle + \left(\frac{1}{2} (21) - \frac{1}{6} (11) \right), -\frac{1}{2} \rangle - \frac{\sqrt{3}}{3} \left((22), -\frac{3}{2} \right) \rangle, \quad (14)
\end{aligned}$$

where the notation for the states in the third equality is $((I_{\Delta\pi} I_{\Delta\pi}^z), I_{\rho}^z)$ for the V_{31} matrix element, and $((I_{\rho\pi} I_{\rho\pi}^z), I_{\Delta}^z)$ for the V_{32} one. This leads to the following amplitudes for the single scattering contribution,

$$t_1 = \frac{5}{9} t_{\Delta\pi}^{I=3/2} + \frac{4}{9} t_{\Delta\pi}^{I=1/2} \equiv \frac{5}{9} t_{\Delta\pi}^{3/2} + \frac{4}{9} t_{\Delta\pi}^{1/2}, \quad (15)$$

$$t_2 = \frac{5}{6} t_{\rho\pi}^{I=2} + \frac{1}{6} t_{\rho\pi}^{I=1} \equiv \frac{5}{6} t_{\rho\pi}^2 + \frac{1}{6} t_{\rho\pi}^1. \quad (16)$$

Proceeding in a similar way, we can get all the amplitudes for the single scattering contribution required in the present calculation which are shown in Table I.

TABLE I: Unitarized two-body scattering amplitudes for the single scattering contribution.

$I_{\Delta\rho}$	I_{total}	t_1	t_2
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{4}{9} t_{\Delta\pi}^{1/2} + \frac{5}{9} t_{\Delta\pi}^{3/2}$	$\frac{1}{6} t_{\rho\pi}^1 + \frac{5}{6} t_{\rho\pi}^2$
$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{36} t_{\Delta\pi}^{1/2} + \frac{2}{9} t_{\Delta\pi}^{3/2} + \frac{3}{4} t_{\Delta\pi}^{5/2}$	$\frac{1}{6} t_{\rho\pi}^0 + \frac{5}{12} t_{\rho\pi}^1 + \frac{5}{12} t_{\rho\pi}^2$

It is worth noting that the argument of the total scattering amplitude T is a function of the total invariant

mass squared s , while the argument in t_1 is s'_1 and in t_2 is s'_2 , where s'_1 and s'_2 are the invariant masses squared of the external π meson with momentum k_1 and $\Delta(\rho)$ inside the N^* with momentum $p_1(p_2)$, which are given by

$$s'_1 = m_{\pi}^2 + M_{\Delta}^2 + \frac{(M_{N^*}^2 + M_{\Delta}^2 - m_{\rho}^2)(s - m_{\pi}^2 - M_{N^*}^2)}{2M_{N^*}^2}, \quad (17)$$

$$s'_2 = m_{\pi}^2 + m_{\rho}^2 + \frac{(M_{N^*}^2 + m_{\rho}^2 - M_{\Delta}^2)(s - m_{\pi}^2 - M_{N^*}^2)}{2M_{N^*}^2}. \quad (18)$$

Following the approach developed in Ref. [27], we can easily write down the S -matrix for the single scattering term (Fig. 2 (a) + term with interaction initiated on p_2) as,

$$\begin{aligned}
S^{(1)} &= S_1^{(1)} + S_2^{(1)} \\
&= ((-it_1) \frac{1}{\mathcal{V}^2} \sqrt{\frac{M_\Delta}{E_\Delta}} \sqrt{\frac{M_\Delta}{E'_\Delta}} \frac{1}{\sqrt{2\omega_\pi}} \frac{1}{\sqrt{2\omega'_\pi}} + \\
&\quad (-it_2) \frac{1}{\mathcal{V}^2} \frac{1}{\sqrt{2\omega_\rho}} \frac{1}{\sqrt{2\omega'_\rho}} \frac{1}{\sqrt{2\omega_\pi}} \frac{1}{\sqrt{2\omega'_\pi}}) \\
&\quad F_{N^*}(\frac{\vec{k}_1 - \vec{k}'_1}{2}) (2\pi)^4 \delta^4(k_1 + K_{N^*} - k'_1 - K'_{N^*}), \quad (19)
\end{aligned}$$

where \mathcal{V} stands for the volume of a box where we normalize to unity our plane wave states. In Eq. (19), $F_{N^*}(\frac{\vec{k}_1 - \vec{k}'_1}{2})$ is the form factor of the $N_{5/2^-}(1675)$ as a bound state of $\Delta\rho$. This form factor was taken to be unity neglecting the \vec{k}, \vec{k}' momentum in Ref. [27] since only states below threshold were considered. To consider states above threshold, we project the form factor into s-wave, the only one that we consider. Thus

$$F_{N^*}(\frac{\vec{k}_1 - \vec{k}'_1}{2}) \Rightarrow FFS(s) = \frac{1}{2} \int_{-1}^1 F_{N^*}(k) d(\cos\theta), \quad (20)$$

with

$$k = k_1 \sqrt{\frac{1 - \cos\theta}{2}}, \quad (21)$$

and

$$k_1 = \frac{\sqrt{(s - (M_{N^*} + m_\pi)^2)(s - (M_{N^*} - m_\pi)^2)}}{2\sqrt{s}}, \quad (22)$$

is the module of the momentum of π meson in the $\pi N_{5/2^-}(1675)$ center of mass frame when \sqrt{s} is above the threshold of the $\pi N_{5/2^-}(1675)$ system, otherwise, k_1 equals zero. The expression of $F_{N^*}(k)$ is given in the next section ².

In Fig. 3, we show the projection over s-wave of the form factor for the single scattering contribution as a function of the total invariant mass of $\Delta\rho\pi$ system.

C. Double-scattering and resummation contribution

In order to obtain the amplitude of the double-scattering contribution (Fig. 2 (b) + term with interaction initiated on p_2) one can proceed in the same way as in the case of the multi-rho meson interaction in Ref. [27].

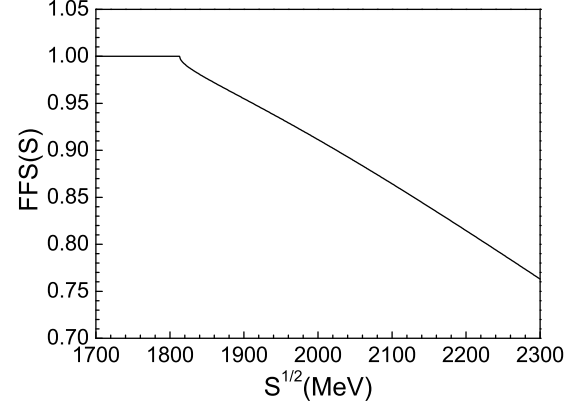


FIG. 3: Form factor for the single scattering contribution.

The expression for the S -matrix for the double scattering is ($S_2^{(2)} = S_1^{(2)}$)

$$\begin{aligned}
S_1^{(2)} &= (-it_1 t_2) (2\pi)^4 \delta^4(k_1 + K_{N^*} - k'_1 - K'_{N^*}) \times \\
&\quad \frac{1}{\mathcal{V}^2} \sqrt{\frac{M_\Delta}{E_\Delta}} \sqrt{\frac{M_\Delta}{E'_\Delta}} \frac{1}{\sqrt{2\omega_\rho}} \frac{1}{\sqrt{2\omega'_\rho}} \frac{1}{\sqrt{2\omega_\pi}} \frac{1}{\sqrt{2\omega'_\pi}} \\
&\quad \times \int \frac{d^3\vec{q}}{(2\pi)^3} F_{N^*}(q) \frac{1}{q^0^2 - \vec{q}^2 - m_\pi^2 + i\epsilon}, \quad (23)
\end{aligned}$$

where F_{N^*} is the $N_{5/2^-}(1675)$ form factor, and we will take q^0 in the $\pi N_{5/2^-}(1675)$ center of mass frame, $q^0 = (s + m_\pi^2 - M_{N^*}^2)/(2\sqrt{s})$.

Following the approach of Ref. [29], we can get the expression for the form factor $F_{N^*}(q)$,

$$\begin{aligned}
F_{N^*}(q) &= \frac{1}{\mathcal{N}} \int_{|\vec{p}| < \Lambda, |\vec{p} - \vec{q}| < \Lambda} d^3\vec{p} \frac{M_\Delta}{E_\Delta(\vec{p})} \frac{1}{2\omega_\rho(\vec{p})} \times \\
&\quad \frac{1}{M_{N^*} - E_\Delta(\vec{p}) - \omega_\rho(\vec{p}) + i(\frac{\Gamma_\Delta + \Gamma_\rho}{2})} \times \\
&\quad \frac{M_\Delta}{E_\Delta(\vec{p} - \vec{q})} \frac{1}{2\omega_\rho(\vec{p} - \vec{q})} \times \\
&\quad \frac{1}{M_{N^*} - E_\Delta(\vec{p} - \vec{q}) - \omega_\rho(\vec{p} - \vec{q}) + i(\frac{\Gamma_\Delta + \Gamma_\rho}{2})}, \quad (24)
\end{aligned}$$

where the normalization factor \mathcal{N} is

$$\begin{aligned}
\mathcal{N} &= \int_{|\vec{p}| < \Lambda} d^3\vec{p} \left(\frac{M_\Delta}{E_\Delta(\vec{p})} \frac{1}{2\omega_\rho(\vec{p})} \right)^2 \times \\
&\quad \frac{1}{(M_{N^*} - E_\Delta(\vec{p}) - \omega_\rho(\vec{p}) + i(\frac{\Gamma_\Delta + \Gamma_\rho}{2}))^2}, \quad (25)
\end{aligned}$$

with Γ_Δ and Γ_ρ the total decay width of the Δ baryon and the ρ meson, respectively, taken as in Subsection A equal to 120 MeV and 150 MeV. Since $M_{N^*} < M_\Delta + m_\rho$

² The form factor that we use is suited to a molecule with two components with equal masses. Some different recoil corrections are needed when the two masses are different [28], but the results only affect moderately the peak around 2200 MeV.

the effect of the widths of Δ baryon and ρ meson is not very important.

To connect with the dimensional regularization procedure we choose the cutoff Λ such that the value of the G function of Eq. (6) at threshold coincides in both methods. Thus for $\Lambda = 820$ MeV we get $M_{N_{5/2^-}} = 1675$ MeV as required.

We show the form factor $F_{N^*}(q)$ in Fig. 4 with $\Lambda = 820$ MeV. The condition $|\vec{p} - \vec{q}| < \Lambda$ implies that the form factor is exactly zero for $q > 2\Lambda$. Therefore the integration in Eq. (23) has an upper limit of 2Λ .

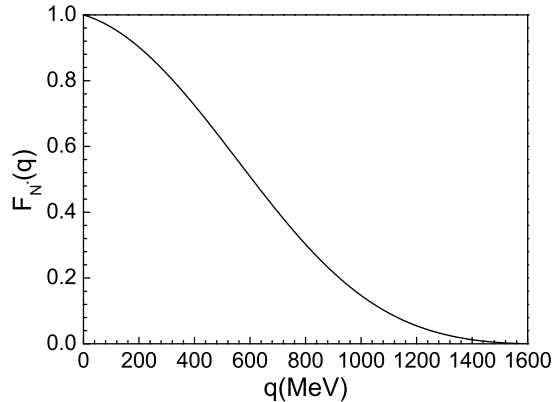


FIG. 4: Form factor of the $N_{5/2^-}(1675)$ as a $\Delta\rho$ bound state.

Before proceeding further, we examine the normalization for the S matrix. We follow Mandl-Shaw [30] normalization for the fields of baryons and mesons, then the S -matrix for πN^* scattering is written as

$$S = -iT_{\pi N^*} \frac{1}{\mathcal{V}^2} \sqrt{\frac{M_{N^*}}{E_{N^*}}} \sqrt{\frac{M_{N^*}}{E'_{N^*}}} \frac{1}{\sqrt{2\omega_\pi}} \frac{1}{\sqrt{2\omega'_\pi}} \times (2\pi)^4 \delta^4(k_1 + K_{N^*} - k'_1 - K'_{N^*}). \quad (26)$$

By comparing Eq. (26) with Eq. (19) for the single-scattering and Eq. (23) for the double-scattering, we see we have to give a weight to t_1 and t_2 such that Eqs. (19) and (23) get the weight factors that appear in the general formula of Eq. (26). This is achieved by replacing

$$t_1(t_{\Delta\pi}) \rightarrow \tilde{t}_1(\tilde{t}_{\Delta\pi}) \equiv t_1 \sqrt{\frac{M_\Delta}{E_\Delta}} \sqrt{\frac{M_\Delta}{E'_\Delta}} \sqrt{\frac{E_{N^*}}{M_{N^*}}} \sqrt{\frac{E_{N^*}}{M'_{N^*}}},$$

$$t_2(t_{\rho\pi}) \rightarrow \tilde{t}_2(\tilde{t}_{\rho\pi}) \equiv t_2 \frac{1}{\sqrt{2\omega_\rho}} \frac{1}{\sqrt{2\omega'_\rho}} \sqrt{\frac{E_{N^*}}{M_{N^*}}} \sqrt{\frac{E_{N^*}}{M'_{N^*}}}.$$

By solving Eqs. (9), (10) and summing the two partitions T_1 and T_2 , we find that

$$T_{\pi N^*} = \frac{\tilde{t}_1 + \tilde{t}_2 + 2\tilde{t}_1\tilde{t}_2 G_0(s)}{1 - \tilde{t}_1\tilde{t}_2 G_0^2(s)} + (\tilde{t}_1 + \tilde{t}_2)(FFS(s) - 1), \quad (27)$$

where $G_0(s)$ is given by

$$G_0(s) = \frac{\sqrt{\frac{M_{N^*}}{E_{N^*}}} \sqrt{\frac{M_{N^*}}{E'_{N^*}}} \int \frac{d^3\vec{q}}{(2\pi)^3} F_{N^*}(q) \times \frac{1}{q^{0^2} - \vec{q}^2 - m_\pi^2 + i\epsilon}. \quad (28)$$

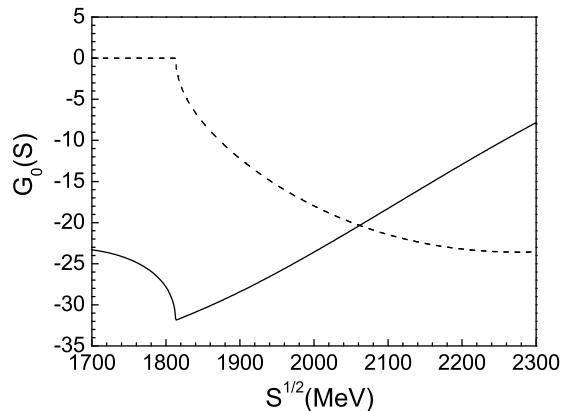


FIG. 5: Real(solid line) and imaginary(dashed line) parts of the G_0 function for the $I_{\Delta\rho} = 1/2$.

In Fig. 5, we show the real and imaginary parts of the G_0 function for $I_{\Delta\rho} = 1/2$ as a function of the total Δ - ρ - π system invariant mass.

IV. RESULTS AND DISCUSSION

The dynamical generation of resonances from our formalism depends on three subtraction constants, $a_{\Delta\rho}$, $a_{\Delta\pi}$ and $a_{\rho\pi}$, respectively associated to the two-body Δ - ρ , Δ - π and ρ - π unitarized s -wave interactions entering our calculation. According to our comments in Section II we assume they are effective parameters, their values implicitly taking into account the effects of the $3q$ component of $N_{5/2^-}(1675)$. Following this assumption we have fixed $a_{\Delta\rho} = -2.28$ in Subsection III.A to get the mass of $N(1675)$ at its estimated value. Concerning the values of $a_{\Delta\pi}$ and $a_{\rho\pi}$ they should implement the effect of the $\pi - (N(1675))_{3q}$ interaction. Therefore they could differ significantly from the values (around -2 for $\mu \sim 800$ MeV) used in the $\Delta\pi$ and $\rho\pi$ calculations of references [23, 24]. In this regard our study has an exploratory character. We examine first the interval of values of $a_{\Delta\pi}$ and $a_{\rho\pi}$ around -2 allowing for the dynamic generation of $\pi N(1675)$ bound states in the $I = 3/2$ sector. Then we analyze within these intervals the possible generation of a $\Delta_{5/2^+}(1740)$.

The general results of this study can be summarized as:

i) the dynamic generation of $I = 3/2$ bound states depends essentially on the value of $a_{\Delta\pi}$. Only values in the interval $a_{\Delta\pi} < -2.5$ give rise to bound states independently of the value of $a_{\rho\pi}$ (we have checked this for $-3.0 < a_{\rho\pi} < -1.0$ and $-4.5 < a_{\Delta\pi} < -2.5$).

ii) if a $I = 3/2$ bound state is generated, then two other $I = 3/2$ resonances lying above threshold (1815 MeV) and below 2300 MeV are also generated.

Examples of these results are graphically shown in Fig. 6. In Fig. 6a (6b) the value of $a_{\rho\pi}$ ($a_{\Delta\pi}$) is fixed whereas $a_{\Delta\pi}$ ($a_{\rho\pi}$) is varied within the selected interval.

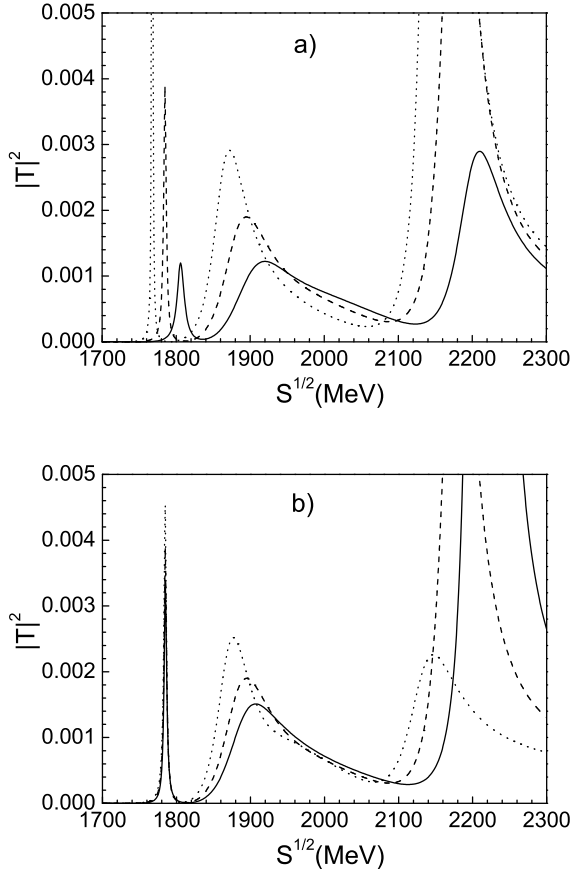


FIG. 6: Modulus squared of the three-body scattering amplitude for $I = 3/2$. a): results obtained with $a_{\rho\pi} = -2.0$ and $a_{\Delta\pi} = -2.6$ (solid line), -3.0 (dash line), -3.4 (dotted line). (b): results obtained with $a_{\Delta\pi} = -3.0$ and $a_{\rho\pi} = -1.4$ (solid line), -2.0 (dash line), -2.6 (dotted line).

As the positions of the three peaks in the figures are quite stable (within 60 MeV) against variation of the parameters in the ranges of values considered, they may be unambiguously assigned to $\Delta_{5/2^+}(1740)$, $\Delta_{5/2^+}(1905)$ and $\Delta_{5/2^+}(2200)$. Let us realize that in the region of the second peak around 2000 MeV there might be a second resonance, as reported in Ref. [14].

Note that the location of the first peak varies in Fig. 6a from 1770 MeV to 1800 MeV whereas the estimated

masses of the existing candidates in Ref. [2], $\Delta(1752 \pm 32)$ and $\Delta(1724 \pm 61)$, have their upper limits at 1785 MeV. This indicates that values $a_{\Delta\pi} \leq -3.0$ can reproduce the experimental mass. Indeed, we could force $a_{\Delta\pi} = -4.3$ to get an average mass of 1740 MeV. Then the second peak appears at 1830 MeV and would lie below the estimated mass interval (1865 – 1915 MeV) of $\Delta_{5/2^+}(1905)$. However, we should not forget that the additional consideration of the coupling to $\pi\Delta_{5/2^-}(1930)$ could have more effect on this state as well as on $\Delta_{5/2^+}(2200)$. Concerning the needed values of $a_{\Delta\pi}$ to get the $\Delta_{5/2^+}(1740)$ the important difference with the reference value -2 seems to indicate the relevant role played by the $3q$ component of $N(1675)$ in the binding process.

Regarding Fig. 6b, a lack of dependence of the bound state $\Delta_{5/2^+}(1740)$ on $a_{\rho\pi}$ is observed. This means that all the effect of the $\pi-(N(1675))_{3q}$ interaction in $\pi-(\Delta\rho)$ is incorporated through $a_{\Delta\pi}$. The ρ - π interaction seems to play a marginal role.

Although encouraging, our results should mainly be interpreted as a fit to fix the parameters in our formalism. In order to gain confidence about the possible existence of $\Delta_{5/2^+}(1740)$ it becomes essential that further predictions from our formalism (with no free parameters) are successful in the interpretation of data. Let us examine the situation with more detail in the $I = 3/2, 1/2$ sectors.

A. $I=3/2$

Δ resonances generated from $\pi N_{3/2^-}(1700)$ and $\pi N_{1/2^-}(1650)$ are of particular interest since $N_{3/2^-}(1700)$ and $N_{1/2^-}(1650)$ are dynamically generated from $\Delta\rho$ as degenerate states to $N_{5/2^-}(1675)$. As the small mass difference among these nucleon states (N^*) does not give rise to important mass differences in the πN^* resonances, we predict $J^P = 1/2^+, 3/2^+$ experimental Δ states almost degenerate to $\Delta_{5/2^+}(1740)$, $\Delta_{5/2^+}(1905)$ and $\Delta_{5/2^+}(2200)$. Regarding their experimental assignment we shall centre on possible candidates to be $\Delta_{3/2^+, 1/2^+}(\sim 1740)$ and $\Delta_{3/2^+, 1/2^+}(\sim 1905)$ since the extensive set of data available in the energy region below 2.0 GeV makes us confident that all resonances may have been identified. We shall pay particular attention to the data analyses of references [9] and [10] extracting both $\Delta_{5/2^+}(1740)$ and $\Delta_{5/2^+}(1905)$. Concerning $\Delta_{3/2^+, 1/2^+}(\sim 2200)$ they should be considered as predicted resonances to be extracted when a more complete data set allows for thorough analyses in the corresponding energy region.

In Table II we list our findings taking for comparison to data the values we obtain with $a_{\Delta\pi} = -3.4$ and $a_{\rho\pi} = -1.4$.

As can be checked all predicted states can be unambiguously assigned to experimental resonances.

Particularly interesting is the generation of $\Delta_{1/2^+}(1750)$. This resonance is forced by symmetry to belong to the second energy band and the quark model overpredicts its mass by about 90 MeV [13] (we

do not know any other quark model based on two-quark interactions that does better). In reference [6] it was argued that it could be generated from $\pi N_{1/2-}$ (1650) as it is done here (alternatively $\pi\Delta_{1/2-}$ (1620) might be also generating it). It should be remarked that only analyses reproducing the $\pi\pi N$ production cross section data extract it as it was the case for $\Delta_{5/2+}$ (1740). Therefore the mere existence of $\Delta_{1/2+}$ (1750)(*) could be considered within our calculation framework as an argument in favor of the existence of $\Delta_{5/2+}$ (1740).

In what respects $\Delta_{3/2+}$ (~ 1770), it is assigned to the

$\Delta_{3/2+}$ (1600), which appears with masses around 1700 MeV in the analyses of Refs. [9, 10]. It should be noted that its mass is largely overpredicted by $3q$ models as the first radial excitation of the Δ (1232). However, other channels as $\sigma\Delta$ (1232) and $\pi N_{3/2-}$ (1520) could be playing a more important role in the generation of this resonance.

For the states around 1900 MeV we should recall that all of them admit a good $3q$ description. Hence our assignments points out that an approximately equivalent alternative meson-baryon description is feasible.

TABLE II: Assignment of $I = 3/2$ predicted states to $J^P = 1/2^+, 3/2^+, 5/2^+$ resonances. Estimated PDG masses for these resonances as well as their extracted values from references [9] and [10] (in brackets) are shown for comparison. N. C. stands for a non cataloged resonance in the PDG review

Predicted		PDG data			
Mass (MeV)	Name J^P	Estimated Mass (MeV)	Extracted Mass (MeV)	Status	
1770	Δ (1740) $5/2^+$		1752 ± 32 (1724 ± 61)	N.C.	
	Δ (1600) $3/2^+$	1550 – 1700	1706 ± 10 (1687 ± 44)	***	
	Δ (1750) $1/2^+$	≈ 1750	1744 ± 36 (1721 ± 61)	*	
1875	Δ (1905) $5/2^+$	1865 – 1915	1881 ± 18 (1873 ± 77)	****	
	Δ (1920) $3/2^+$	1900 – 1970	2014 ± 16 (1889 ± 100)	***	
	Δ (1910) $1/2^+$	1870 – 1920	1882 ± 10 (1995 ± 12)	****	

B. $I=1/2$

N resonances are also generated from $\pi N_{5/2-}$ (1675) and their partners $\pi N_{3/2-}$ (1700) and $\pi N_{1/2-}$ (1650). In Fig. 7, we show the results we get for them with $a_{\Delta\pi} = -3.4$ and $a_{\rho\pi} = -1.4$ where, as is the general case in the parameter region explored, there appears two well defined peaks.

The first peak corresponds to a nucleon resonance almost degenerate with $\Delta_{5/2+}$ (1740). The mass difference with the second peak is always about 55 MeV bigger than that between $\Delta_{5/2+}$ (~ 1740) and $\Delta_{5/2+}$ (1905).

In Table III we show the assignment to experimental states. Again an unambiguous assignment of predicted states to experimental candidates can be done. This provides additional support to our previous predictions. When comparing our results to data we should be aware, though, that the values used for the param-

eters have been fixed from the fitting to Δ resonances whereas a fitting to N resonances could give rise to different values of these parameters. This would be a reflection of the different character of the $\pi - (N(1675))_{3q}$ and $\pi - (N(1675))_{\rho\Delta}$ interactions. Thus any appreciable deviation of our results from data could be indicating such a circumstance. This could be indeed the case in Table III since our predicted masses seem to be systematically higher than data.

It is worthwhile to recall that $N_{5/2+}$ (1680), $N_{3/2+}$ (1720) and $N_{1/2+}$ (1710) are not well described by $3q$ models, usually overpredicting their masses by about 70 MeV. On the other hand other meson-baryon and meson-meson-baryon channels may be contributing as well to the explanation of these resonances. For instance $\pi\Delta_{3/2-}$ (1700) may contribute to $N_{3/2+}$ (1720) and $\pi\Delta_{1/2-}$ (1620) as well as σN to $N_{1/2+}$ (1710). Indeed in this latter case the resonance has been dynamically

generated as a σN state [31].

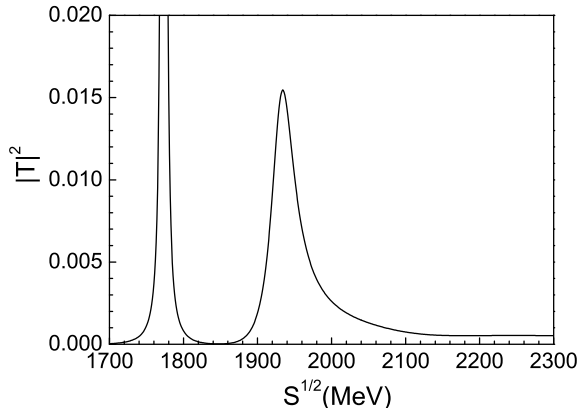


FIG. 7: Modulus squared of the three-body scattering amplitude for $I = 1/2$ with $a_{\Delta\pi} = -3.4$ and $a_{\rho\pi} = -1.4$.

TABLE III: Assignment of $I = 1/2$ predicted states to $J^P = 1/2^+, 3/2^+, 5/2^+$ resonances. Estimated PDG masses for these resonances as well as their extracted values from references [9] and [10] (in brackets) are shown for comparison. N. C. stands for a non cataloged resonance in the PDG review. In this N. C. case the quoted mass corresponds to reference [9].

Predicted		PDG data			
Mass (MeV)	Name J^P	Estimated Mass (MeV)	Mass (MeV)	Status	
1770	$N(1680) 5/2^+$	1680 – 1690	1684 ± 4 (1679 ± 3)	****	
	$N(1720) 3/2^+$	1700 – 1750	1717 ± 31 (1716 ± 112)	****	
	$N(1710) 1/2^+$	1680 – 1740	1717 ± 28 (1699 ± 65)	***	
1930	$N(2000) 5/2^+$	≈ 2000	1903 ± 87	**	
	$N(1900) 3/2^+$	≈ 1900	1879 ± 17	**	
	$N(1900) 1/2^+$		1885 ± 30	N. C.	

V. SUMMARY

We have performed a Faddeev calculation for the $\pi - N_{5/2^-}(1675)$ system treating the $N_{5/2^-}(1675)$ as a $(\Delta\rho)$ bound state as found in a previous study of the $\Delta-\rho$ system. We have used the fixed center approximation (FCA) to describe the $\pi - (\Delta\rho)_{N_{5/2^-}(1675)}$ system in terms of the two-body interactions, $\Delta-\rho$, $\Delta-\pi$, $\rho-\pi$, provided by the chiral unitary approach. In order to get a more complete description of $N_{5/2^-}(1675)$ the cutoffs or the subtraction constants a needed to calculate the two-body amplitudes are considered as effective parameters whose values may implicitly take into account the effect of the missing $3q$ component of $N_{5/2^-}(1675)$. Thus

$a_{\Delta\rho}$ has been fitted to reproduce the nominal mass of $N_{5/2^-}(1675)$ whereas $a_{\Delta\pi}$ and $a_{\rho\pi}$ are assumed to incorporate the effects of the $\pi - (N_{5/2^-}(1675))_{3q}$ interaction. The variation of the parameters around some values of reference employed in previous studies of the free $\Delta-\pi$ and $\rho-\pi$ interactions shows that a quite stable (against variation of the parameters) bound state is found for $a_{\Delta\pi} < -2.5$ independently of the value of $a_{\rho\pi}$ what suggests that all the effect of the $3q$ component interaction can be then absorbed in $a_{\Delta\pi}$. The significant difference of the resulting values of $a_{\Delta\pi}$ with respect to the value of reference seems to indicate the relevance of $3q$ effects. Indeed, the departure of the subtraction constants $a(\mu)$ from their natural value is interpreted in Ref. [32] as a

measure of the relevance of genuine component in the wave function beyond the meson-baryon ones.

The bound state is always accompanied by the presence of two other resonances so that a quite precise correspondence to experimental states can be achieved when the existence of a $\Delta_{5/2^+}(1740)$, extracted by two independent data analyses but non cataloged in the Particle Data Book Review, is taken for granted. Actually the possibility of providing a theoretical explanation of such resonance was the main motivation for our study since its description is clearly out of the scope of the $3q$ model.

Once the parameters are restricted to the bound state region we can generate a set of definite predictions for $I = 3/2, 1/2$ and $J^P = 3/2, 1/2$. All the generated resonances can be unambiguously assigned to experimental states. It should be emphasized that this assignment provides a natural explanation to all the degeneracies observed in the baryonic sectors studied. In particular it provides theoretical support to the currently poor existence of $\Delta_{1/2^+}(1750)$ (*) as an almost degenerate state to $\Delta_{5/2^+}(1740)$. It also points out, confirming previous proposals, the relevance that meson-baryon components may have in a detailed explanation of nucleon states as $N_{5/2^+}(1680)$, $N_{3/2^+}(1720)$ and $N_{1/2^+}(1710)$ with a deficient $3q$ description.

The consistency of the whole scheme and the good agreement with data makes us confident that the approximations followed draw the essential dynamics. From our results we may conclude that there is a sound theoretical basis to support the data analyses extracting two distinctive resonances, $\Delta_{5/2^+}(1740)$ and $\Delta_{5/2^+}(2200)$, cataloged altogether as $\Delta_{5/2^+}(2000)$ in the Particle Data Book Review. Besides we predict the existence of $\Delta_{1/2^+, 3/2^+}$ resonances about 2200 MeV, partners of $\Delta_{5/2^+}(2200)$, which may deserve additional theoretical and experimental analysis.

Concerning $\Delta_{5/2^+}(1740)$ (equivalently for $\Delta_{1/2^+}(1750)$) our derivation makes clear its domi-

nant meson-baryon character so that experimental analyses looking in detail into specific decay channels would be most welcome. Some of them ($N\rho$ and $\Delta\pi$ [10]) are already available in the PDG book. From our theoretical model, a $\Delta\rho$ state ($N_{5/2^-}(1675)$) decays into πN and $\pi\Delta$, through a diagram involving $\rho \rightarrow \pi\pi$ with one π exchange in the t -channel and the other π in the final state [33]. This is in agreement with data. Since our claim for the $\Delta_{5/2^+}(1740)$ is a molecular state of $\pi N_{5/2^-}(1675)$, the natural decay modes would be $\pi\pi N$ and $\pi\pi\Delta$. Current data in Ref. [10] suggest that the $\pi\pi N$ would be the dominant mode.

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- [1] E. Klempt and J. M. Richard, *Rev. Mod. Phys.* **82**, 1095 (2010).
 - [2] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
 - [3] M. Jones, R. H. Dalitz and R. R. Horgan, *Nucl. Phys. B* **29**, 45 (1977).
 - [4] N. Kaiser, P. B. Siegel and W. Weise, *Phys. Lett. B* **362**, 23 (1995); E. Oset and A. Ramos, *Nucl. Phys. A* **635**, 99 (1998); C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, *Phys. Rev. D* **67**, 076009 (2003); T. Hyodo, S. I. Nam, D. Jido and A. Hosaka, *Phys. Rev. C* **68**, 018201 (2003); V. K. Magas, E. Oset and A. Ramos, *Phys. Rev. Lett.* **95**, 052301 (2005); E. Oset, A. Ramos and C. Bennhold, *Phys. Lett. B* **527**, 99 (2002) [Erratum-*ibid.* **B 530**, 260 (2003)].
 - [5] P. González, E. Oset and J. Vijande, *Phys. Rev. C* **79**, 025209 (2009).
 - [6] P. González, J. Vijande and A. Valcarce, *Phys. Rev. C* **77**, 065213 (2008).
 - [7] E. Oset *et al.*, *Nucl. Phys. A* **835**, 271 (2010).
 - [8] A. Gal, *Int. J. Mod. Phys. A* **22**, 226 (2007).
 - [9] D.M. Manley and E.M. Saleski, *Phys. Rev. D* **45**, 4002 (1992).
 - [10] T. P. Vrana, S. A. Dytman and T.-S. H. Lee, *Phys. Rep.* **328**, 181 (2000).
 - [11] R.E. Cutkosky, C.P. Forsyth, J.B. Babcock, R.L. Kelly, and R.E. Hendrick, *Proceedings of the IV International Conference on Baryon Resonances (Baryon 1980)*, edited by N. Isgur, Toronto 1980.
 - [12] N. Suzuki *et al.* (EBAC), *Phys. Rev. Lett.* **104**, 042302 (2010).
 - [13] S. Capstick and N. Isgur, *Phys. Rev. D* **34**, 2809 (1986); S. Capstick and W. Roberts, *Prog. Part. Nucl. Phys.* **45** S241 (2000).
 - [14] D. Mark Manley, *Phys. Rev. Lett.* **52**, 2122 (1984).
 - [15] Sourav Sarkar, Bao-Xi Sun, E. Oset, and M. J. Vicente

- Vacas, *Eur. Phys. A* **44**, 431 (2010).
- [16] R. Chand and R. H. Dalitz, *Annals Phys.* **20**, 1 (1962); R. C. Barrett and A. Deloff, *Phys. Rev. C* **60**, 025201 (1999); R. C. Barrett and A. Deloff, *Phys. Rev. C* **60**, 025201 (1999); S. S. Kamalov, E. Oset and A. Ramos, *Nucl. Phys. A* **690**, 494 (2001).
- [17] J. J. Xie, A. Martínez Torres and E. Oset, arXiv:1010.6164 [nucl-th].
- [18] A. Martínez Torres, K. P. Khemchandani, and E. Oset, *Phys. Rev. C* **79**, 065207 (2009); A. Martínez Torres and D. Jido, *Phys. Rev. C* **82**, 038202 (2010).
- [19] D. Jido and Y. Kanada-En'yo, *Phys. Rev. C* **78**, 035203 (2008).
- [20] M. Bayar, J. Yamagata-Sekihara and E. Oset, arXiv:1102.2854 [hep-ph].
- [21] A. Dote, T. Hyodo and W. Weise, *Phys. Rev. C* **79**, 014003 (2009).
- [22] Y. Ikeda, H. Kamano and T. Sato, *Prog. Theor. Phys.* **124**, 533 (2010).
- [23] Sourav Sarkar, E. Oset, and M. J. Vicente Vacas, *Nucl. Phys. A* **750**, 294 (2005), Erratum-ibid. A **780**, 90 (2006), and references therein.
- [24] L. Roca, E. Oset, and J. Singh, *Phys. Rev. D* **72**, 014002 (2005) and references therein; A. Martínez Torres, K. P. Khemchandani, L. S. Geng, M. Napsuciale and E. Oset, *Phys. Rev. D* **78**, 074031 (2008).
- [25] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, *Phys. Rev. Lett.* **54**, 1215 (1985); U. G. Meissner, *Phys. Rept.* **161**, 213 (1988); M. Bando, T. Kugo, and K. Yamawaki, *Phys. Rept.* **164**, 217 (1988); M. Harada and K. Yamawaki, *Phys. Rept.* **381**, 1 (2003).
- [26] J. A. Oller, and U. G. Meissner, *Phys. Lett. B* **500**, 263 (2001).
- [27] L. Roca and E. Oset, *Phys. Rev. D* **82**, 054013 (2010); J. Yamagata-Sekihara, L. Roca and E. Oset, *Phys. Rev. D* **82**, 094017 (2010).
- [28] J. Yamagata-Sekihara, J. Nieves and E. Oset, *Phys. Rev. D* **83**, 014003 (2010).
- [29] D. Gamermann, J. Nieves, E. Oset, and E. Ruiz Arriola, *Phys. Rev. D* **81**, 014029 (2010).
- [30] F. Mandl and G. Shaw, *Quantum Field Theory*, Wiley-Interscience, (1984).
- [31] K. P. Khemchandani, A. Martínez Torres and E. Oset, *Eur. Phys. J. A* **37**, 233 (2008).
- [32] T. Hyodo, D. Jido, A. Hosaka, *Phys. Rev. C* **78**, 025203 (2008).
- [33] E. Oset, E. J. Garzon, Ju-Jun Xie, P. González, A. Ramos, A. Martínez Torres, [arXiv:1103.0807 [nucl-th]].