

# $O(\alpha_s)$ Spin-Spin Correlations for Top and Bottom Quark Production in $e^+e^-$ Annihilation

M. M. Tung<sup>a</sup>, J. Bernabéu<sup>b</sup>, J. Peñarrocha<sup>b</sup>

<sup>a</sup>*Institute for Theoretical Physics*

*State University of New York*

*Stony Brook, NY 11794-3840, U.S.A*

<sup>b</sup>*Instituto de Física Corpuscular, Departament de Física Teòrica*

*Universitat de València, 46100 Burjassot (València), Spain*

## Abstract

We present the full  $O(\alpha_s)$  longitudinal spin-spin correlations for heavy-quark pair production at  $e^+e^-$  high-energy colliders in closed analytical form. In such reactions, quark and antiquark have strongly correlated spins, and the longitudinal components are dominant. For the explicit computation of the QCD bremsstrahlung contributions, new phase-space integrals are derived. Explicit numerical estimates are given for  $t\bar{t}$  and  $b\bar{b}$  production. Around the  $Z$ -peak, QCD one-loop corrections depolarize the spin-spin asymmetry for bottom quark pairs by approximately  $-4\%$ . For top pair production, we find at 350 GeV a 0.6% increased polarization over a value of 0.4 in the longitudinal correlation. For more than 1 TeV the  $O(\alpha_s)$  corrections enhance depolarization to  $-2\%$  in the top-pair case.

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After the long-sought discovery of the top quark at the Fermilab Tevatron [1], now much effort is made to further investigate all the properties of this exceptional particle, which completes the missing SU(3) ingredient in the third family of the Standard Model. A thorough experimental analysis of the top quark in combination with theoretical predictions offers unique possibilities to explore so-far inaccessible domains of the Standard Model.

Much work has gone into the study of polarized top production from hadron colliders [2] as these machines more easily provide the necessary energies to produce the heavy particle. However, once  $e^+e^-$  colliders have reached the required production threshold and beyond, many new interesting experimental tests may be carried out and add to the wealth of existing data on the top [3, 4]. In particular, polarized top production from  $e^+e^-$  annihilation deserves special attention. Not only are the spin properties of the top in the final state fully accessible through its rapid and predominant electroweak decay mode, but additionally  $e^+e^-$  collisions provide a clean initial state from the experimental and theoretical standpoint.

Heavy-quark production from high-energy  $e^+e^-$  collisions, especially top production, will permit high-precision measurements to directly probe the respective photon and  $Z$ -boson interaction vertices. In these processes, the longitudinal polarization  $P_L$  of quark or antiquark is large, whereas the remaining transverse components contribute to a smaller extent [5, 6]. QCD one-loop corrections for the longitudinal polarization averaged over the scattering angle have been calculated before and change  $\langle P_L \rangle$  typically by 3% for the top [7]. Other  $O(\alpha_s)$  single-spin asymmetries have also been discussed [8], using the techniques developed in Ref. [7]. A recent work by Olsen and Stav [9] investigates quark-pair polarization effects in 2- and 3-jet events from polarized  $e^+e^-$  beams.

In the present paper, we shall concentrate on the massive QCD one-loop corrections for the total spin-spin asymmetry  $\langle P_{LL} \rangle$ , which measures the bilinear spin correlations among the longitudinal quark and antiquark polarizations. The relevant observable is defined as

follows

$$\langle P_{LL} \rangle = \frac{\sigma_{tot}(\uparrow\uparrow) - \sigma_{tot}(\uparrow\downarrow) - \sigma_{tot}(\downarrow\uparrow) + \sigma_{tot}(\downarrow\downarrow)}{4 \sigma_{tot}}, \quad (1)$$

where  $\sigma_{tot}$  denotes the total cross section for  $e^+e^- \rightarrow \gamma, Z \rightarrow q\bar{q}$ , and  $\uparrow, \downarrow$  refer to positive and negative helicities for quark and antiquark, respectively. It is straightforward to see that in the numerator all combinations of  $(\mathbf{I} + \mathbf{s}_1) \otimes (\mathbf{I} + \mathbf{s}_2)$  except for the bilinear spin form  $\mathbf{s}_1 \otimes \mathbf{s}_2$  drop out. Note that the normalization factor of 1/4 is an arbitrary but convenient choice in order to obtain for massless quarks at Born level  $P_{LL}^{m=0}(Born) = 1$ , due to chiral current conservation.

For the calculation of the spin-dependent cross sections, we group the hadronic tensor  $H^{\mu\nu}$  into separate parity-parity combinations  $i, j = V, A$ , so that the structure of vector and axial-vector interference terms becomes more transparent. Then, for a fixed combination  $ij$  each contraction with the lepton tensor  $L^{\mu\nu}$  has to be multiplied by the corresponding compound coupling  $g^{ij}$ , which contains products of the usual neutral-current couplings  $v_f = 2T_z^f - 4Q_f \sin^2 \theta_W$  and  $a_f = 2T_z^f$ , and the fractional fermion charge  $Q_f$ . Finally, integration over the azimuthal ( $\varphi$ ) and polar ( $\theta$ ) angles of the quark (including the flux factor and  $\gamma, Z$  propagator term) yields

$$\sigma_{tot} = \frac{N_C}{16} \left( \frac{\alpha}{q^2} \right)^2 v \sum_{i,j=V,A} g^{ij} \int d\varphi \int d\cos\theta L^{\mu\nu} H_{\mu\nu}^{ij}, \quad (2)$$

where  $N_C = 3$  accounts for the color-singlet state of the produced quark pair, and  $v = \sqrt{1 - 4m^2/q^2}$  is the common Schwinger mass parameter ( $m$  quark mass,  $q$  total energy-momentum transfer).

In order to directly determine the total cross section, it is practical to introduce the following covariant projection operator [7]

$$\Pi_T^{\mu\nu} \sim \left( g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right),$$

so that at QCD one-loop level Eq. (2) simplifies to

$$\sigma_{tot} = \frac{3}{8} \pi \left( \frac{\alpha}{q^2} \right)^2 v \sum_{i,j=V,A} g^{ij} [H_T^{(2)ij} + H_T^{(3)ij}]. \quad (3)$$

Here, it is  $H_T \equiv \Pi_T^{\mu\nu} H_{\mu\nu}$ , and the additional superscript separates the virtual (including the Born term) from the real contributions. Note that a relative factor between two- and three-particle phase-space has to be taken into account [10].

In Eq. (3), the explicit expressions for  $g^{i,j}$  are

$$g^{VV} = Q_q^2 - 2 Q_q v_e v_q \operatorname{Re} \chi_Z + (v_e^2 + a_e^2) v_q^2 |\chi_Z|^2, \quad (4)$$

$$g^{AA} = (v_e^2 + a_e^2) a_q^2 |\chi_Z|^2, \quad (5)$$

$$g^{VA} = -Q_q a_e a_q \operatorname{Re} \chi_Z + 2 v_e a_e v_q a_q |\chi_Z|^2, \quad (6)$$

and we consider the finite width of the  $Z$  boson by taking

$$\chi_Z(q^2) = \frac{g_F M_Z^2 q^2}{q^2 - M_Z^2 + i M_Z \Gamma_Z} \quad \text{with} \quad g_F = \frac{G_F}{8\sqrt{2} \pi \alpha} \approx 4.299 \cdot 10^{-5} \text{ GeV}^{-2}. \quad (7)$$

For the explicit spin-dependent computation of  $H_T^{(3)}$  in Eq. (3), we use the following kinematical framework. As usual the direction of the outgoing quark in the center-of-momentum system (cms) is given by

$$\hat{\mathbf{p}} = \mathbf{p}_1 / |\mathbf{p}_1| = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta). \quad (8)$$

Then, the vector normal to the production plane is

$$\hat{\mathbf{n}} = (-\cos \theta \cos \varphi, -\cos \theta \sin \varphi, \sin \theta), \quad (9)$$

and all outgoing momenta in the reaction  $e^+ e^- \rightarrow q(p_1) \bar{q}(p_2) g(p_3)$  are confined to within this plane. Moreover, given also the direction of the antiquark ( $\hat{\mathbf{p}}_2$ ) the gluon direction ( $\hat{\mathbf{p}}_3$ ) is fully determined by energy-momentum conservation. Thus, for the parametrization of the three-particle phase space the following two dimensionless energy scales suffice

$$y = 1 - \frac{2 p_1 \cdot q}{q^2}, \quad z = 1 - \frac{2 p_2 \cdot q}{q^2}. \quad (10)$$

In fact, it is this choice of phase-space parametrization in combination with sophisticated integration techniques that allows for a complete analytical evaluation of  $H_T^{(3)}$ . The method was first devised in Refs. [7].

The longitudinal spin components of quark and antiquark are now given respectively by<sup>1</sup>

$$s_1 = 2\lambda_1 \xi^{-\frac{1}{2}} \left( \sqrt{(1-y)^2 - \xi}; \hat{\mathbf{p}}_1(\mathbf{1} - \mathbf{y}) \right), \quad (11)$$

$$s_2 = 2\lambda_2 \xi^{-\frac{1}{2}} \left( \sqrt{(1-z)^2 - \xi}; \hat{\mathbf{p}}_2(\mathbf{1} - \mathbf{z}) \right), \quad (12)$$

where  $\lambda_1, \lambda_2 = \pm \frac{1}{2}$  are the corresponding helicities, and  $\xi = 1 - v^2 = 4m^2/q^2$ . It is now straightforward to make the appropriate substitutions in the gamma traces  $H_T^{(3)}$ , and use

$$u(p_1, s_1) \bar{u}(p_1, s) \longrightarrow \frac{1}{2} (1 + \gamma_5 \not{s}_1) (\not{p}_1 + m), \quad (13)$$

$$v(p_1, s_1) \bar{v}(p_1, s) \longrightarrow \frac{1}{2} (1 + \gamma_5 \not{s}_2) (\not{p}_2 - m). \quad (14)$$

Here, there are no technical difficulties in the proper treatment of the axial-vector current within any renormalization scheme, since the product  $\mathbf{s}_1 \otimes \mathbf{s}_2$  implies an even number of  $\gamma_5$ -matrices. Dimensional reduction, *i.e.* the Clifford algebra of the Dirac matrices is performed in four dimensions, is the simplest renormalization procedure to control the UV sector [12, 13], and we shall adopt it in the subsequent calculations.

Note that in the previous discussion, two-particle kinematics is fully recovered by putting  $y = z = 0$ , that is, both quark and antiquark share an equal amount of the total available cms Energy  $E_{cms}^2 = q^2 = (p_1 + p_2)^2$ .

The individual  $O(\alpha_s)$  virtual corrections to the Born approximation in Eq. (3) are now derived in terms of the charge and magnetic moment form factors  $A$ ,  $B$ , and  $C = A - 2B$  (see *e.g.* Ref. [10]), where the last identity is an immediate consequence of axial-vector current conservation in the massless limit. Our explicit results are

$$H_T^{(2)VV} = \frac{16}{3} q^2 \left[ \left\{ 2(1 - 4\lambda_1 \lambda_2) + \xi(1 + 4\lambda_1 \lambda_2) \right\} (1 + 2A) + 2(1 + 4\lambda_1 \lambda_2) v^2 B \right], \quad (15)$$

$$H_T^{(2)AA} = \frac{32}{3} q^2 v^2 (1 - 4\lambda_1 \lambda_2) (1 + 2C), \quad (16)$$

$$\frac{1}{2} (H_T^{(2)VA} + H_T^{(2)AV}) = \frac{64}{3} q^2 v (\lambda_1 - \lambda_2) (1 + A + C). \quad (17)$$

It is important to notice that the form factors contain soft divergences, which have their origin in the massless nature of the gluon fields. In these expressions, the dimensional

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<sup>1</sup>The choice of metric is standard:  $g^{\mu\nu} = \text{diag}(1; -1, -1, -1)$ .

regulator  $\varepsilon = 4 - N$  is equivalent to an infinitesimal gluon mass cut-off  $\Lambda = m_g^2/q^2$ . This equivalence is readily established by the correspondence rule [14, 10]

$$\ln \Lambda \longleftrightarrow \frac{2}{\varepsilon} - \gamma_E + \ln \left( \frac{4\pi\mu^2}{q^2} \right). \quad (18)$$

These at most logarithmic divergences in  $\Lambda$  also emerge after integrating out the following differential distributions of the  $O(\alpha_s)$  tree-graph contributions in Eq. (3):

$$\begin{aligned} \frac{dH_T^{(3)VV}}{dy dz} &= \frac{\alpha_s}{\pi} C_F \frac{8q^2}{3v} \left[ -2(2+\xi) \left( \frac{1}{y} + \frac{1}{z} \right) - \frac{1}{2}\xi(2+\xi) \left( \frac{1}{y^2} + \frac{1}{z^2} \right) + 2\frac{y}{z} + 2\frac{z}{y} \right. \\ &\quad + (4-\xi^2) \frac{1}{yz} + \frac{2\lambda_1\lambda_2}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} \left\{ -4(2-\xi)(3-\xi) \right. \\ &\quad + (2-\xi)(8-7\xi) \left( \frac{1}{y} + \frac{1}{z} \right) + \xi(1-\xi)(2-\xi) \left( \frac{1}{y^2} + \frac{1}{z^2} \right) \\ &\quad - 2(1-\xi)(2-\xi)^2 \frac{1}{yz} - 4(y^2+z^2) + 4(1-\xi) \left( \frac{y^2}{z} + \frac{z^2}{y} \right) \\ &\quad + \xi(2-\xi) \left( \frac{y^2}{z^2} + \frac{z^2}{y^2} \right) + 4(3-2\xi)(y+z) + (-12+10\xi-3\xi^2) \left( \frac{y}{z} + \frac{z}{y} \right) \\ &\quad \left. \left. - \xi(2-\xi) \left( \frac{y}{z^2} + \frac{z}{y^2} \right) \right\} \right], \quad (19) \end{aligned}$$

$$\begin{aligned} \frac{dH_T^{(3)AA}}{dy dz} &= \frac{\alpha_s}{\pi} C_F \frac{8q^2}{3v} \left[ 2\xi - 4(1-\xi) \left( \frac{1}{y} + \frac{1}{z} \right) - \xi(1-\xi) \left( \frac{1}{y^2} + \frac{1}{z^2} \right) \right. \\ &\quad + (2+\xi) \left( \frac{y}{z} + \frac{z}{y} \right) + 2(1-\xi)(2-\xi) \frac{1}{yz} \\ &\quad + \frac{2\lambda_1\lambda_2}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} \left\{ -4(6-5\xi) + 2(1-\xi)(8-7\xi) \left( \frac{1}{y} + \frac{1}{z} \right) \right. \\ &\quad + 2\xi(1-\xi)^2 \left( \frac{1}{y^2} + \frac{1}{z^2} \right) - 4(1-\xi)^2(2-\xi) \frac{1}{yz} - 2(2-\xi)(y^2+z^2) \\ &\quad + 2(2-3\xi) \left( \frac{y^2}{z} + \frac{z^2}{y} \right) + 2\xi(1-\xi) \left( \frac{y^2}{z^2} + \frac{z^2}{y^2} \right) + 2(6-\xi)(y+z) \\ &\quad \left. \left. - 2(6-8\xi+\xi^2) \left( \frac{y}{z} + \frac{z}{y} \right) - 2\xi(1-\xi) \left( \frac{y}{z^2} + \frac{z}{y^2} \right) + 4\xi yz \right\} \right], \quad (20) \end{aligned}$$

$$\frac{d(H_T^{(3)VA} + H_T^{(3)AV})}{2 dy dz} = \frac{\alpha_s}{\pi} C_F \frac{16q^2}{3v} \frac{\lambda_1}{\sqrt{(1-y)^2-\xi}} \left[ 4-\xi - (4-5\xi)\frac{1}{y} - 2(4-3\xi)\frac{1}{z} \right]$$

$$\begin{aligned}
& -\xi(1-\xi)\left(\frac{1}{y^2} + \frac{1}{z^2}\right) + \xi\left(\frac{y}{z^2} - \frac{y^2}{z^2}\right) - 2z + (2-\xi)\frac{z}{y} \quad (21) \\
& + (6-\xi)\frac{y}{z} - 2\frac{y^2}{z} + 2(1-\xi)(2-\xi)\frac{1}{yz} \Big] - (\lambda_1 \rightarrow \lambda_2, y \leftrightarrow z).
\end{aligned}$$

In the last expression  $(\lambda_1 \rightarrow \lambda_2, y \leftrightarrow z)$  denotes an additional term linear in the antiquark helicity  $\lambda_2$ , and  $y$  and  $z$  are interchanged. For completeness, we have presented the full differential distributions displaying all possible spin contributions. The first term of Eq. (21) agrees with the result previously derived for longitudinal quark polarization [7]. Including the second term, Eq. (21) becomes  $C$ -odd under the exchange of quark and antiquark in the final state, so that bilinear spin terms can not occur. On the other hand, Eqs. (19) and (20) are  $C$ -even, and thus contain only unpolarized and bilinear spin terms. Note that angular extensions of these distributions can be found in Ref. [9] in different form.

We perform the integration of Eqs. (19)–(21) over the three-body phase in fully analytical form. Apart from the unpolarized and linear spin terms (which comprise known integrals of the  $I$ - and  $S$ -type, respectively [7]), entirely new phase-space integrals have to be solved for the bilinear spin contributions. In this case, phase-space symmetry is restored, and this essentially reduces the amount of integrals to be tackled by half. However, the analytical evaluation is considerably more complicated due to the additional spin-spin weight  $[\sqrt{(1-y)^2 - \xi}\sqrt{(1-z)^2 - \xi}]^{-1}$ . These new integrals are classified in the Appendix along with a listing of their explicit solutions. We shall henceforth refer to them as type  $K$ -integrals. Upon integration, one then obtains

$$\begin{aligned}
H_T^{(3)VV} = & \frac{\alpha_s}{\pi} C_F \frac{8q^2}{3v} \Big[ -4(2+\xi)I_2 - \xi(2+\xi)I_3 + 4I_4 + (4-\xi^2)I_5 \\
& + 4\lambda_1\lambda_2 \{ -2(2-\xi)(3-\xi)K_1 + (2-\xi)(8-7\xi)K_2 \\
& + \xi(1-\xi)(2-\xi)K_3 - (1-\xi)(2-\xi)^2K_4 - 4K_5 + 4(1-\xi)K_6 \\
& + \xi(2-\xi)K_7 + 4(3-2\xi)K_8 + (-12+10\xi-3\xi^2)K_9 \\
& - \xi(2-\xi)K_{10} \} \Big], \quad (22)
\end{aligned}$$

$$\begin{aligned}
H_T^{(3)AA} = & \frac{\alpha_s}{\pi} C_F \frac{8q^2}{3v} \left[ 2\xi I_1 - 8(1-\xi)I_2 - 2\xi(1-\xi)I_3 + 2(2+\xi)I_4 \right. \\
& + 2(1-\xi)(2-\xi)I_5 + 4\lambda_1\lambda_2 \left\{ -2(6-5\xi)K_1 + 2(1-\xi)(8-7\xi)K_2 \right. \\
& + 2\xi(1-\xi)^2 K_3 - 2(1-\xi)^2(2-\xi)K_4 - 2(2-\xi)K_5 + 2(2-3\xi)K_6 \\
& + 2\xi(1-\xi)K_7 + 2(6-\xi)K_8 - 2(6-8\xi+\xi^2)K_9 - 2\xi(1-\xi)K_{10} \\
& \left. \left. + 2\xi K_{11} \right\} \right], \tag{23}
\end{aligned}$$

$$\begin{aligned}
\frac{1}{2}(H_T^{(3)VA} + H_T^{(3)AV}) = & \frac{\alpha_s}{\pi} C_F \frac{16q^2}{3v} (\lambda_1 - \lambda_2) \left[ (4-\xi)S_1 - (4-5\xi)S_2 - 2(4-3\xi)S_4 \right. \\
& - \xi(1-\xi)(S_3 + S_5) + \xi(S_6 - S_7) - 2S_8 + (2-\xi)S_9 \\
& \left. + (6-\xi)S_{10} - 2S_{11} + 2(1-\xi)(2-\xi)S_{12} \right]. \tag{24}
\end{aligned}$$

We are now in the position to obtain the complete  $O(\alpha_s)$  results for each individual parity-parity combination by adding the corresponding virtual and real parts, Eqs. (15)–(17) and Eqs. (22)–(24), respectively. All logarithmic soft divergences cancel in the total sum, as we expect. Finally, Eq. (3) gives the correct normalization and flux factors, and in the total spin correlation  $\langle P_{LL} \rangle$ , *viz.* Eq. (1), only  $\lambda_1\lambda_2$  terms originating from  $H_T^{VV}$  and  $H_T^{AA}$  survive.

In Figure 1(a), we have plotted numerical estimates of  $\langle P_{LL} \rangle$  for top pair production depending on the cms energy  $E_{cms} = \sqrt{q^2}$ . In all plots the QCD one-loop corrections are indicated by dashed lines, the Born approximations by solid lines. Figure 1(b) gives the denominator term of  $\langle P_{LL} \rangle$ , that is the total unpolarized cross section. Figure 1(c) displays the impact that the  $O(\alpha_s)$  corrections have on the Born approximation of  $\langle P_{LL} \rangle$ . We give  $\langle P_{LL}(O(\alpha_s)) \rangle / \langle P_{LL}(Born) \rangle - 1$  in percent, where  $P_{LL}(O(\alpha_s))$  comprises the QCD one-loop corrections as well as the Born contribution. Figures 2 present the results for  $e^+e^- \rightarrow \gamma, Z \rightarrow b\bar{b}(g)$  in a similar fashion. The initial boundary condition for the running of the strong coupling constant is chosen to be  $\alpha_s^{(5)}(M_Z) = 0.123$  for five active quark flavors.

Close to threshold, the QCD one-loop corrections significantly enhance the unpolarized and polarized cross sections of the top quark pairs by 40% to 45%. We wish to emphasize



that in the region where  $t$  and  $\bar{t}$  are at sufficiently small distances together, a QCD Coulomb interaction is required to describe a correct threshold behavior [15]. For polarized top production the Green function method can be used [3, 16].

In Figure 1(a), the  $O(\alpha_s)$   $t\bar{t}$  spin correlations range between  $\langle P_{LL} \rangle = 0.4$  at 350 GeV and  $\langle P_{LL} \rangle = 0.92$  at 1 TeV, which include  $O(\alpha_s)$  mass effects of 0.6% and  $-0.7\%$ , respectively. Above the 1 TeV limit, the total impact of the  $O(\alpha_s)$  corrections to  $\langle P_{LL} \rangle$  amounts to  $-1.5\%$  to  $-2\%$ .

In the bottom case, the QCD corrections produce a significant depolarization around the  $Z$ -peak (Fig. 2(a)), opposite to the effect in the total rate (Fig. 2(b)). In the 80–100 GeV range, the  $O(\alpha_s)$  corrections for  $\langle P_{LL} \rangle$  vary only slightly around  $-4\%$  (Fig. 2(c)).

For very high cms energies, *i.e.*  $\xi \rightarrow 0$ , the value for  $\langle P_{LL} \rangle$  at  $O(\alpha_s)$  stabilizes around 0.96, which differs from the expected Born value of 1 by roughly  $-4\%$ .

The same effect is observable for up-type quarks, where this correction equally amounts to approximately  $-4\%$ . The origin of this effect are collinear helicity-flip terms of the order  $m^2/\xi$ . They do not emerge in the naive computation, where the quark mass is set to zero in the very beginning. Specifically, integrals  $K_3$  and  $K_7$  contain these IR/M singularities and get multiplied by  $\xi$  in Eqs. (22) and (23). Similar finite corrections emerge in the massless limit for the longitudinal single-spin polarization [7]. For a general discussion of this phenomenon we refer to Ref. [17].

In summary, we have calculated the  $O(\alpha_s)$  radiative mass corrections to the total longitudinal spin correlations of heavy quark pairs produced in  $e^+e^-$  annihilation. Fully analytical expressions have been obtained and used for numerical estimates for top and bottom production. The impact of these QCD one-loop corrections on the lowest Born approximation is considerable, and we believe that future precision measurements of such spin correlations at high-energy  $e^+e^-$  colliders help to reveal interesting new aspects of the Standard Model, or beyond.

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## Appendix: Analytical spin-spin phase-space integrals

In terms of the kinematical variables  $y = 1 - 2p_1 \cdot q/q^2$  and  $z = 1 - 2p_2 \cdot q/q^2$  the three-body phase-space is confined to a region within the limits [7]

$$y_- = \Lambda^{\frac{1}{2}}\sqrt{\xi} + \Lambda, \quad y_+ = 1 - \sqrt{\xi}, \quad (25)$$

$$z_{\pm} = \frac{2y}{4y + \xi} \left[ 1 - y - \frac{1}{2}\xi + \Lambda + \frac{\Lambda}{y} \pm \frac{1}{y} \sqrt{(1-y)^2 - \xi} \sqrt{(y-\Lambda)^2 - \Lambda\xi} \right], \quad (26)$$

where  $\Lambda = m_g^2/q^2$  relates to a spurious gluon mass that regulates the soft IR divergences in the intermediate results for the individual real and virtual contributions, *viz.* Eq. (18).

Apart from the integral classes presented in previous  $O(\alpha_s)$  calculations (for a full listing see Refs. [10] and [11]), new integral types emerge here in the bilinear spin terms. The additional spin-spin weight  $[\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}]^{-1}$  complicates the computation of the explicit analytic integral solutions. However, full phase-space symmetry ( $y \leftrightarrow z$ ) reduces the number of integrals to be tackled.

In the calculation of the longitudinal spin-spin correlations for  $e^+e^- \rightarrow q(\uparrow)\bar{q}(\uparrow)g$  the following new class of analytic integral expressions had to be derived:

$$\begin{aligned} K_1 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}} \\ &= \text{Li}_2\left(\frac{1-v}{2}\right) - \text{Li}_2\left(\frac{1+v}{2}\right) - 4\text{Li}_2\left(\frac{1}{2}\sqrt{\xi}\right) \\ &\quad + \left(-\frac{1}{2}\ln\xi + \ln 2\right) \ln\left(\frac{1+v}{1-v}\right) \\ &\quad - \frac{1}{2}\left(\ln\xi - 4\ln 2\right) \ln\xi + \frac{1}{3}\pi^2 - 2\ln^2 2 \end{aligned}$$

$$\begin{aligned} v K_2 &= v \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}} \frac{1}{y} \\ &= 2\text{Li}_2\left(\frac{2-\sqrt{\xi}}{1+v}\right) + 2\text{Li}_2\left(\frac{1-v}{2-\sqrt{\xi}}\right) - 4\text{Li}_2\left(\sqrt{\frac{1-v}{1+v}}\right) \end{aligned}$$

$$\begin{aligned}
& + \ln \left( \frac{1+v}{1-v} \right) \left[ \ln \xi - 2 \ln(2 - \sqrt{\xi}) + \ln \left( \frac{1+v}{1-v} \right) \right] \\
& + \ln \xi \left[ \frac{1}{4} \ln \xi - \ln(2 - \sqrt{\xi}) \right] + \ln^2(2 - \sqrt{\xi})
\end{aligned}$$

$$\begin{aligned}
K_3 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}} \frac{1}{y^2} \\
&= \frac{4}{v\xi} \left[ -\ln \Lambda^{\frac{1}{2}} + \frac{1}{2} \ln \xi - 2 \ln(2 - \sqrt{\xi}) + 2 \ln v + 2 \ln 2 \right] \\
&+ \frac{2}{v^3} \left[ \text{Li}_2 \left( \frac{2 - \sqrt{\xi}}{1+v} \right) + \text{Li}_2 \left( \frac{1-v}{2 - \sqrt{\xi}} \right) - \text{Li}_2 \left( \sqrt{\frac{1-v}{1+v}} \right) - \text{Li}_2 \left( \frac{\sqrt{\xi}}{1+v} \right) \right] \\
&+ \frac{1}{v^3} \ln \left( \frac{1+v}{1-v} \right) \left[ \ln \xi - 2 \ln(2 - \sqrt{\xi}) - \frac{2v}{\xi} \{ (1+v)^2 + 1 \} + \ln \left( \frac{1+v}{1-v} \right) \right] \\
&+ \frac{1}{v^3} \ln \xi \left[ \frac{1}{4} \ln \xi - \frac{v(4-\xi) + 6(1-\xi)}{\xi} \right] \\
&+ \frac{1}{v^3} \ln(2 - \sqrt{\xi}) \left[ -\ln \xi + \ln(2 - \sqrt{\xi}) + 2 \frac{v(4-\xi) + 4(1-\xi)}{\xi} \right]
\end{aligned}$$

$$\begin{aligned}
K_4 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}} \frac{1}{yz} \\
&= \frac{1}{v} S_{12} - \frac{1}{v^2} \left\{ 2 \left[ -\text{Li}_2 \left( \frac{1-v}{1+v} \right) + \text{Li}_2 \left( \frac{2 - \sqrt{\xi}}{1+v} \right) \right. \right. \\
&+ \text{Li}_2 \left( \frac{1-v}{2 - \sqrt{\xi}} \right) - \text{Li}_2 \left( \frac{1+v - \sqrt{\xi}}{2v} \right) + \text{Li}_2 \left( -\frac{1-v - \sqrt{\xi}}{2v} \right) \left. \right] \\
&+ \ln \left( \frac{1+v}{1-v} \right) \left[ \ln \xi - 2 \ln(2 - \sqrt{\xi}) - \ln v + \frac{3}{4} \ln \left( \frac{1+v}{1-v} \right) - \ln 2 \right] \\
&+ \ln^2(1+v - \sqrt{\xi}) - \ln^2(-(1-v - \sqrt{\xi})) + \ln^2(2 - \sqrt{\xi}) \\
&+ \left. \ln \xi \left[ \frac{1}{4} \ln \xi - \ln(2 - \sqrt{\xi}) \right] - \frac{1}{3} \pi^2 \right\}
\end{aligned}$$

$$\begin{aligned}
K_5 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}} y^2 \\
&= 2K_8 - K_1 \\
&+ \frac{1}{2}\xi \left[ \text{Li}_2 \left( \frac{1-v}{2} \right) - \text{Li}_2 \left( \frac{1+v}{2} \right) + 2\text{Li}_2 \left( 1 - \frac{1}{2}\sqrt{\xi} \right) - 2\text{Li}_2 \left( \frac{1}{2}\sqrt{\xi} \right) \right] \\
&+ \ln(2 - \sqrt{\xi}) \left[ \frac{1}{8}\xi^2 - \frac{1}{2}\xi(2 \ln 2 - \ln \xi) - 1 \right]
\end{aligned}$$

$$\begin{aligned}
& +\frac{1}{4}\ln\xi\left[\frac{1}{4}(1-\xi^2)+\xi(2\ln 2-\ln\xi)+\frac{7}{4}\right] \\
& +\frac{1}{2}\ln\left(\frac{1+v}{1-v}\right)\left[-\frac{1}{8}\xi^2+\frac{1}{2}\xi(2\ln 2-\ln\xi)+1\right] \\
& +\frac{1}{8}\xi^{\frac{3}{2}}-\frac{\xi^2}{8(2-\sqrt{\xi})}+\frac{3}{2}(1-\sqrt{\xi})-\frac{1}{8}v(7-v^2)
\end{aligned}$$

$$\begin{aligned}
K_6 &= \int \frac{dy dz}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} \frac{y^2}{z} \\
&= \left(1+\frac{1}{2}\xi\right)K_2-3\ln\left(\frac{1+v}{1-v}\right)-\frac{3}{2}\ln\xi+3\ln(2-\sqrt{\xi}) \\
&+ \frac{\xi}{2(2-\sqrt{\xi})}-\frac{3}{2}\sqrt{\xi}+1
\end{aligned}$$

$$\begin{aligned}
K_7 &= \int \frac{dy dz}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} \frac{y^2}{z^2} \\
&= \left(1+\frac{1}{2}\xi\right)\tilde{K}_3+\frac{2v}{\xi}\left(1-\frac{3}{v^2}\right)\left[-\frac{1}{2}\ln\xi+\ln 2-1\right] \\
&+ \left(\frac{8+\xi}{2\xi}+\frac{2+\xi}{v\xi}\right)\ln\xi+\left(\frac{6+\xi}{\xi}+2\frac{2+\xi}{v\xi}\right)\ln\left(\frac{1+v}{1-v}\right) \\
&- \frac{8+\xi}{\xi}\ln(2-\sqrt{\xi})-2\frac{2+\xi}{v\xi}[2\ln v+\ln 2]-\frac{4}{\sqrt{\xi}}-\frac{2v}{\xi}-\frac{2}{2-\sqrt{\xi}}
\end{aligned}$$

$$\begin{aligned}
K_8 &= \int \frac{dy dz}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} y \\
&= K_1+\left(1-\frac{1}{2}\xi\right)\left[-\ln\xi+2\ln(2-\sqrt{\xi})-\ln\left(\frac{1+v}{1-v}\right)\right] \\
&- 2(1-\sqrt{\xi})+v
\end{aligned}$$

$$\begin{aligned}
K_9 &= \int \frac{dy dz}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} \frac{y}{z} \\
&= K_2-\ln\xi+2\ln(2-\sqrt{\xi})-2\ln\left(\frac{1+v}{1-v}\right)
\end{aligned}$$

$$v^3 K_{10} = \int \frac{dy dz}{\sqrt{(1-y)^2-\xi}\sqrt{(1-z)^2-\xi}} \frac{y}{z^2}$$

$$\begin{aligned}
&= 2 \left[ \text{Li}_2 \left( \frac{1-v}{2-\sqrt{\xi}} \right) + \text{Li}_2 \left( \frac{2-\sqrt{\xi}}{1+v} \right) - 2\text{Li}_2 \left( \sqrt{\frac{1-v}{1+v}} \right) \right] \\
&\quad + \ln \left( \frac{1+v}{1-v} \right) \left[ \ln \left( \frac{1+v}{1-v} \right) - 2 \ln(2-\sqrt{\xi}) + \ln \xi - 4v \right] \\
&\quad + \ln(2-\sqrt{\xi}) \left[ \ln(2-\sqrt{\xi}) - \ln \xi + 6v \right] \\
&\quad + \ln \xi \left( \frac{1}{4} \ln \xi - 3v \right) + 4 \left( \frac{1}{\xi} - \frac{v^3}{\sqrt{\xi}} - 1 \right) \\
K_{11} &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi} \sqrt{(1-z)^2 - \xi}} yz \\
&= K_8 - \frac{1}{4} \left[ 10 - 9v + 2v^2 - v^3 - 2\sqrt{\xi}(4+\xi) \right] \\
&\quad - \left( 1 + \frac{1}{8}\xi^2 \right) \left[ \ln \left( \frac{1+v}{1-v} \right) + \ln \xi - 2 \ln(2-\sqrt{\xi}) \right]
\end{aligned}$$

The dilogarithm is defined by

$$\text{Li}_2(x) = - \int_0^x \frac{\ln(1-t)}{t},$$

and most of its known properties can be found in the standard reference Ref. [18].

Note that the soft divergences are at most logarithmic in  $\Lambda$ . In  $K_7$  all singularities cancel internally, and the finite part of  $K_3$  is denoted by  $\tilde{K}_3$ . The integral  $S_{12}$  has been calculated in Ref. [7].

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## Figure Captions

**Fig. 1:** (a) The longitudinal spin-spin correlation  $\langle P_{LL} \rangle$  at Born level and  $O(\alpha_s)$  for top pair production in  $e^+e^- \rightarrow \gamma, Z \rightarrow t\bar{t}$  as a function of  $E_{cms} = \sqrt{q^2}$ . (b) Total  $O(\alpha_s)$  cross section for top-quark production as a function of the cms energy  $E_{cms} = \sqrt{q^2}$ . (c) The impact of  $O(\alpha_s)$  corrections for the longitudinal top asymmetry  $\langle P_{LL} \rangle$ . Shown are values for  $\langle P_{LL}(O(\alpha_s)) \rangle / \langle P_{LL}(Born) \rangle - 1$  in percent.

**Fig. 2:** (a) QCD one-loop corrections of the longitudinal spin correlations  $\langle P_{LL} \rangle$  for  $\sigma_{tot}(e^+e^- \rightarrow \gamma, Z \rightarrow b\bar{b})$ . (b)  $O(\alpha_s)$  corrections for bottom pair production as a function of the cms energy  $E_{cms} = \sqrt{q^2}$ . (c)  $O(\alpha_s)$  corrections compared to the Born contribution for the longitudinal spin correlation  $\langle P_{LL} \rangle(e^+e^- \rightarrow \gamma, Z \rightarrow b\bar{b})$  in percent.

Figure 1a

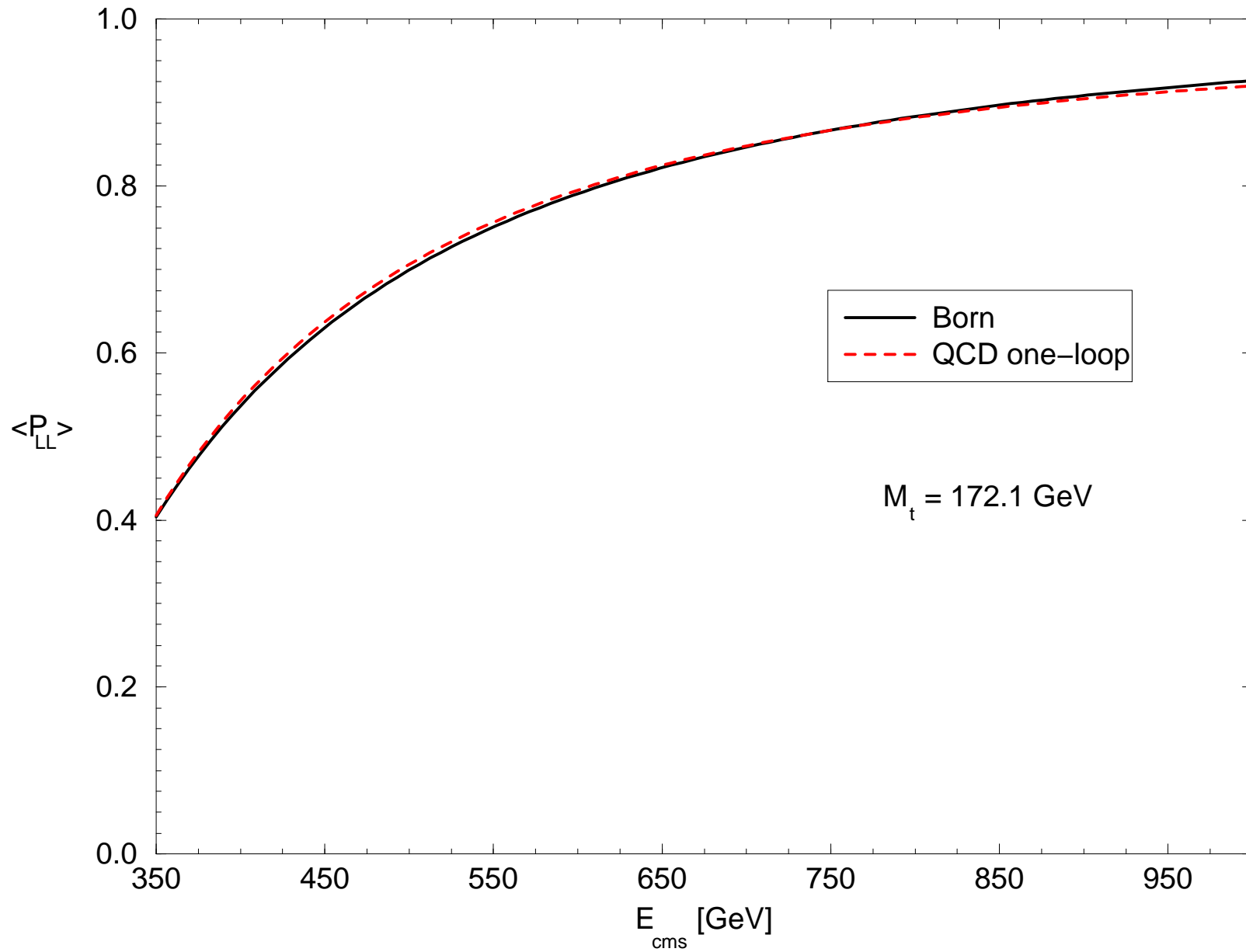


Figure 1b

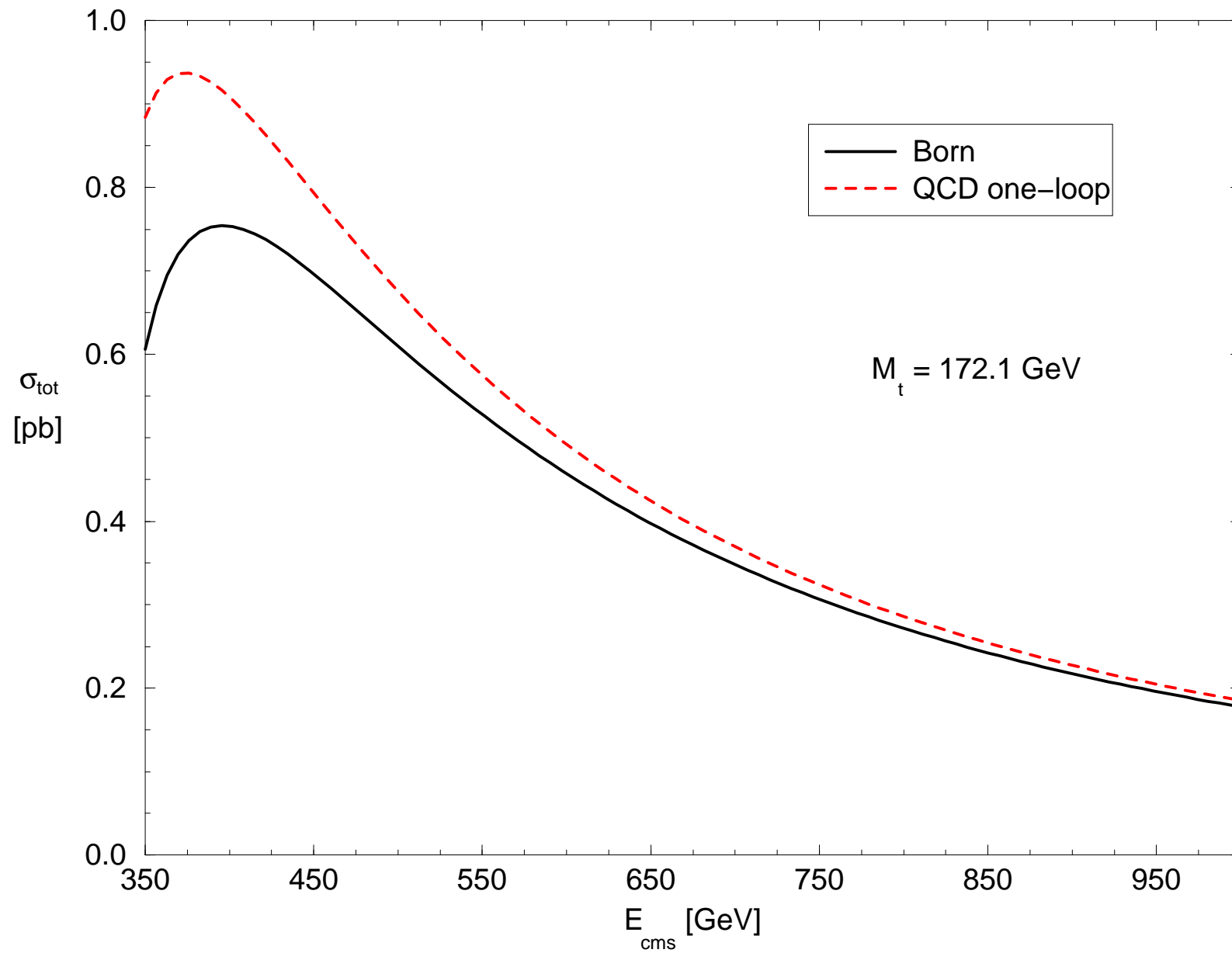


Figure 1c

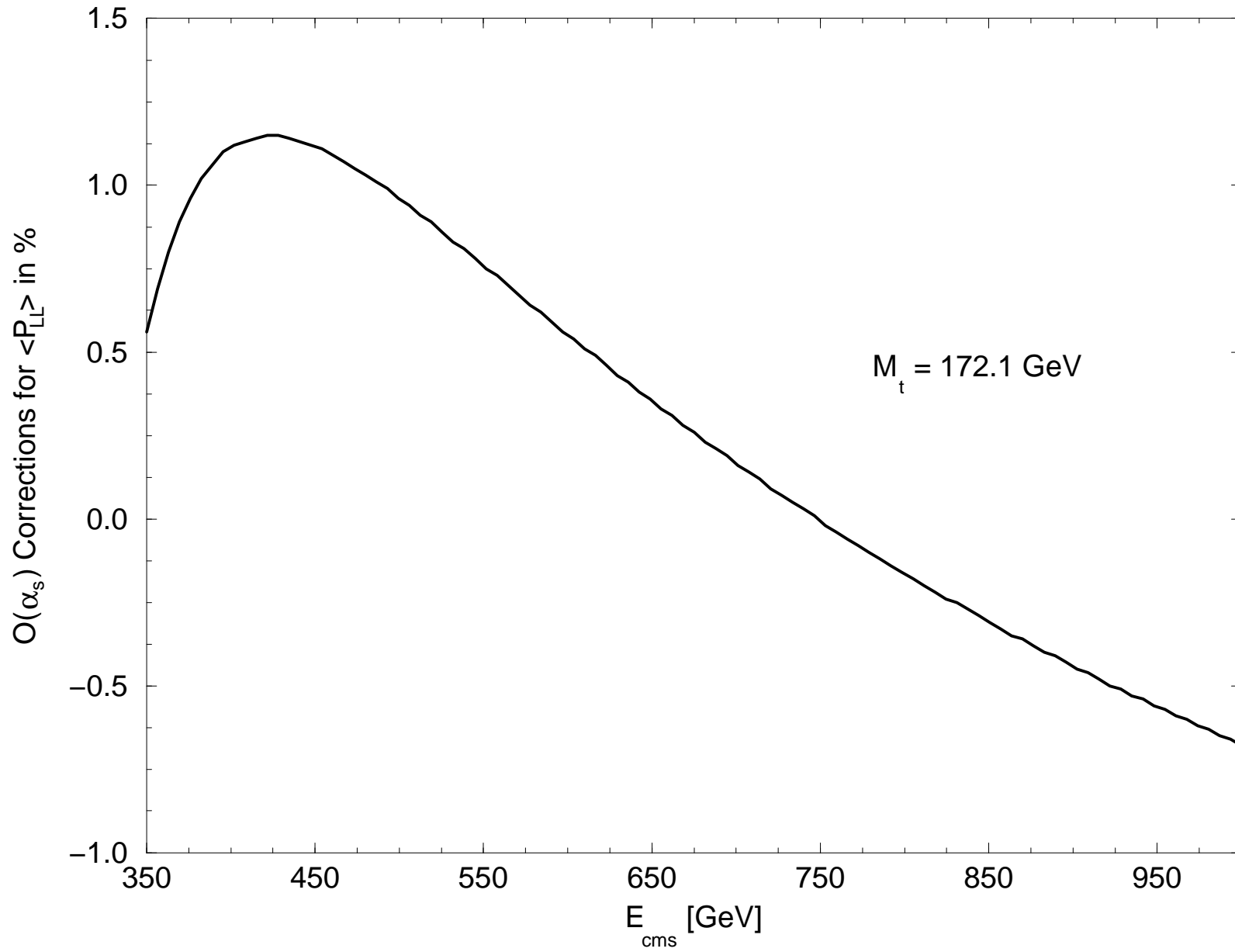


Figure 2a

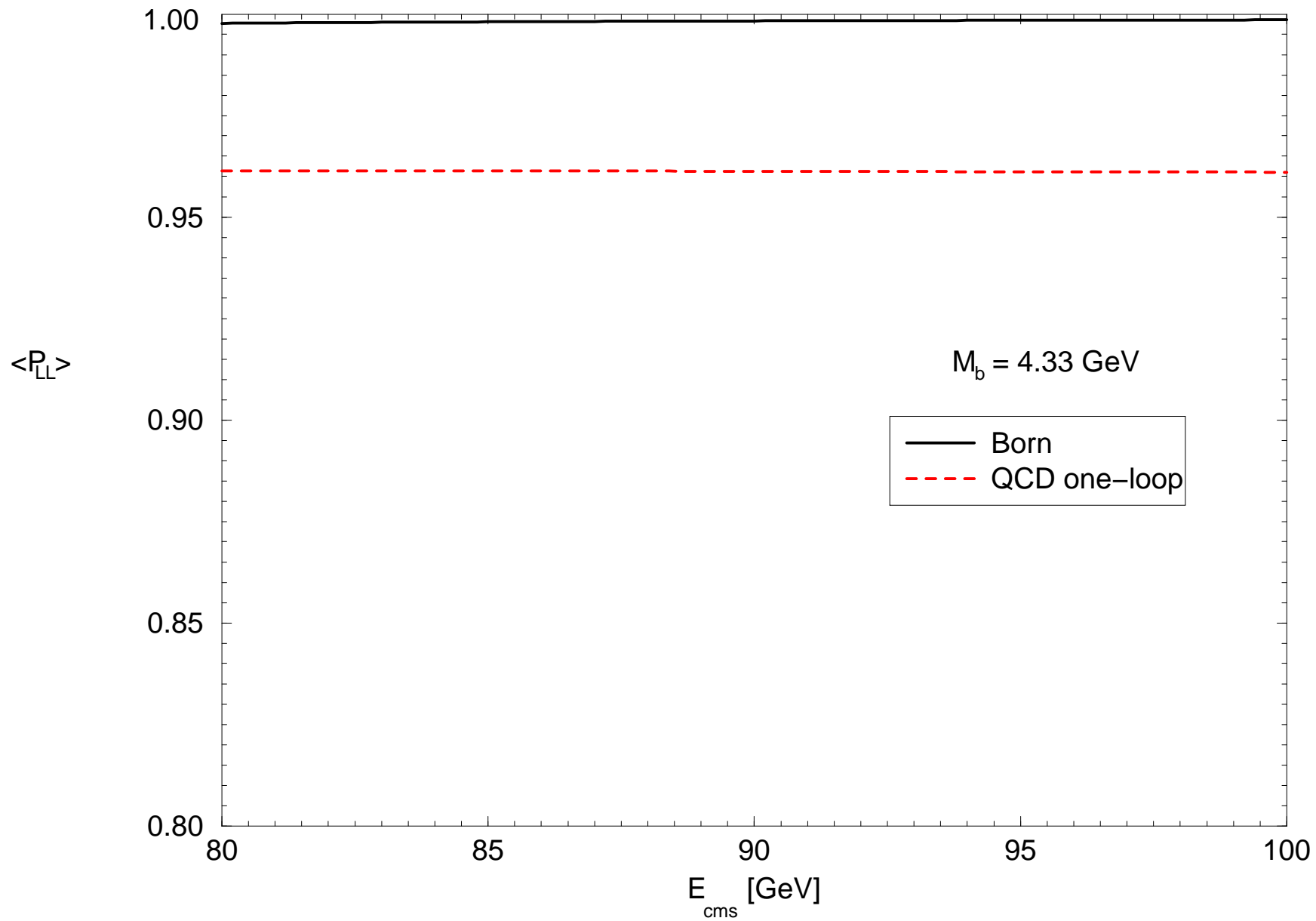


Figure 2b

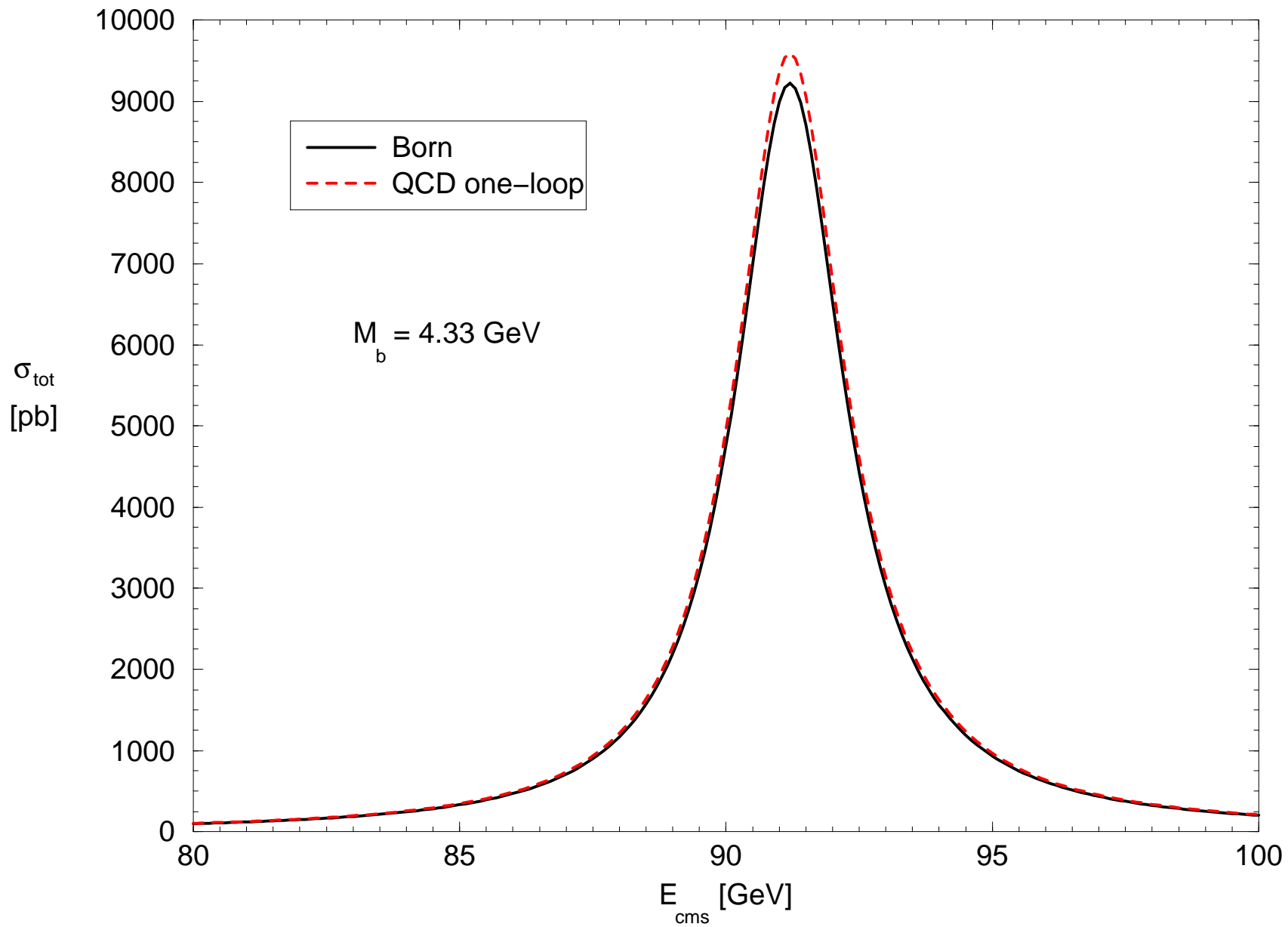


Figure 2c

