UCLA/89/TEP/40

An Alternative Large-N Limit for QCD and its Implications for Low Energy Nuclear Phenomena *

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Abstract

The Corrigan-Ramond model for large-N QCD is analysed in detail. The spectrum, leading order results for interactions and an effective Lagrangian describing large-N interactions are derived. This Lagrangian, when quantized, provides an effective quantum field theory for mesons and baryons. The applicability of such a theory to low energy nuclear phenomena is studied. It is found that the model is in disagreement with standard hadron phenomenology. It is shown that the features of baryons at low energies, inspired by the standard large-N approach to QCD constitute the only large-N description compatible with phenomenology.

*This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grants PHY-85-15857, PHY-86-13210.

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1 Introduction

There is a plethora of indications by now, that Quantum Chromodynamics (QCD) is the theory that correctly describes the strong interactions. In QCD the fundamental degrees of freedom are gluons and quarks, which become free at asymptotically high energies (asymptotic freedom). However, at low energy the coupling becomes strong and perturbative techniques do not apply. Unfortunately this is the important region for Nuclear Physics. In particular, mass spectra of asymptotic states, as well as dynamics at very low energies, (~ 1 Gev), are issues to which no rigorous answer exists so far within QCD.

There are several methods that try to uncover the behaviour of QCD at low energy (strong coupling). A standard one is lattice QCD which consists of evaluating the path integral of QCD by brute force, converting it to a finite dimensional integral by taking spacetime to be a finite lattice. Such an approach so far had a restricted scope due to limitations in computing power*.

Characteristically, the biggest lattice sizes that can be used today have a physical scale that can barely reach the nucleon size. Nonetheless, lattice techniques gave some reasonable estimates of the hadron spectrum.

As already mentioned, perturbation theory in the standard coupling of QCD gives reliable results only at high energy. If one is interested in nuclear phenomena, this kind of perturbation theory is of no practical use. There is however another expansion parameter in QCD. Strictly speaking this expansion parameter exists in a generalization of QCD where the gauge group is $SU(N)$ instead of $SU(3)$, that is the number of colors is N. From now on we will use QCD_N to denote the gauge theory with N colors. The expansion parameter mentioned above is $1/N$ when N is large. The structure of $1/N$ perturbation theory is considerably different than ordinary perturbation theory, [1,2,3]. There is certainly some non-perturbative information in this alternative expansion since the leading order already contains an infinite number of diagrams, the "planar" diagrams, that receive contributions from an arbitrary number of loops. Thought this way, the leading order of the $1/N$ expansion gives a sample of the whole conventional perturbative series.

*Exact analytic results in lattice QCD do not exist, unlike some 2-d examples, due to the complexity of the 4-d problem.

The $1/N$ expansion can also be thought of as a semiclassical expansion. For N large there is a saddle point in the QCD_N path integral that becomes exactly gaussian at $N = \infty$. Thus the leading non-trivial order in the $1/N$ expansion is generated by small fluctuations around this saddle point and large-N perturbation theory can be formulated in a well defined way, [4].

In QCD_N the leading non-trivial contributions are already too hard to sum up. Despite this, we can extract non-trivial information about the theory just from the structure of the leading diagrams and the plausible assumption of color confinement for all N. For example the physical degrees of freedom, mesons glueballs and baryons, are explicit at large N. More than that, mesons and glueballs have masses of order $O(1)$ and are non-interacting at $N = \infty$, whereas baryons have masses of order $O(N)$ etc. [1].

In the nuclear energy regime, since our quantitative understanding of QCD is far from sufficient, one usually tries to describe physics using effective Lagrangians for the relevant physical degrees of freedom, which in that case are the low lying mesons and baryons†. The reasoning for writing down effective Lagrangians is based on symmetries, simplicity and ultimately, agreement with data. Various approaches have been used to describe nuclear phenomena this way, including both relativistic and non-relativistic theories, [5,6,7,8,9]. In ref. [10] we used several results from large-N QCD_N to construct effective Lagrangians for the low lying degrees of freedom.‡ These theories are obviously relativistic and of the Yukawa-type. However, nucleon propagation as well as Yukawa interactions should be non-local in order to agree with large-N QCD_N , [10].

As it was mentioned before, at large N we have a saddle-point where quantum fluctuations are suppressed. When we move away from $N = \infty$ the effects of fluctuations are not negligible any more. The question is, what is a tractable and at the same time physically reasonable way of taking such fluctuations into account (after all, we eventually care about $N = 3$). At $N = \infty$ the theory

†One ignores glueballs, taking into account the experimental fact that they have not been seen in a certain energy regime which could mean that either they are heavy and/or they are very-very broad. In both cases one can neglect their affects for all practical purposes.

‡Effective actions of similar form have been also considered in [11] for a somewhat higher energy regime.

is classical. One can easily derive an effective Lagrangian whose tree level expansion reproduces the leading order of the $1/N$ expansion. Of course, values of couplings cannot be calculated directly but this is not a big concern because after all such couplings can be fit to data. Generally, this effective Lagrangian will contain arbitrary-dimension operators. In the large-N QCD_N case there are only a few couplings that are non-local, the rest of them being local. Thus, to describe fluctuations away from $N = \infty$ the natural approach to follow is to quantize the classical effective Lagrangian found at the large-N limit, [10]. However such a quantization should be done with care, in order to be consistent with exact results obtained directly from QCD_N . For example in standard large-N QCD_N one should quantize the mesons but keep the baryons classical, (alternatively speaking, there are no baryonic loops, see also [12])[§]. Since arbitrary dimension operators appear in the large-N action, the quantum field theory defined that way is non-renormalizable. However, this is not important for two reasons. The effective theory already possesses a cut-off scale of order of the mass of the heavy classical particles (nucleons). In addition, by renormalization group arguments, contributions from high-dimension operators will be suppressed at low energy, their only effect being a renormalization of relevant coupling constants. Thus, for all practical purposes one can limit attention to renormalizable interactions if one wishes so.

Of course, such a procedure can be *in principle* implemented directly in QCD_3. One has to define appropriate composite operators, representing gauge invariant (physical) degrees of freedom in terms of the fundamental fields (quarks and gluons), and then derive the effective action of these operators, [13]. Unfortunately, despite the fact that such a procedure is well defined, it is computationaly intractable. The advantage of going through the large-N limit is that without too much computation we can derive the salient features of the effective theory.

So far we have been talking about the large-N limit of QCD_N . For a pure

[§] There is an alternative description of mesons which presumably becomes exact at large-N and this is the Skyrme model. In this model the baryons are solitons, that is, extendend, semi-classical objects. This alternative description is in aggreement with our description since here baryons are treated as point-like and thus the fact that they have non-local interactions should come as no surprise.

gauge theory the large-N generalization is uniquely defined. However if one includes quarks then the large-N extension is somewhat ambiguous. The reason is the following. One has to assign the quarks into some (in general reducible) representation of the color-group $SU(N)$, in such a way that at $N = 3$ one can recover the usual content of QCD, that is, six 3's and six $\bar{3}$'s. The simplest choice for general N would be to consider six N's and six $\bar{N}$'s. This is the standard large-N model that has been analysed extensively in the literature, [1,2,3]. In this model the mesons are $q\bar{q}$ states whereas the baryons are states containing N quarks. This feature is what makes the baryons in the standard model behave rather differently from the mesons. There are however alternative quark assignments with the same $N = 3$ limit. In fact if one wants to maintain asymptotic freedom for arbitrarily large N, then the only other representation that one can add is the two-index antisymmetric tensor representation of $SU(N)$. Thus we are led to the model proposed by Corrigan and Ramond, [14], where we have n_q N's and $6 - n_q$ antisymmetric tensor representations (of dimension $N(N-1)/2$) along with their conjugate ones. In [14] the "quarks", Q, associated with the antisymmetric tensor representation have been named "larks", name that we will also use here. It is obvious that for $N = 3$ this model is the standard QCD model since for $N = 3$ the two-index antisymmetric tensor representation is equivalent to the $\bar{3}$ of $SU(3)$.

The original motivation for proposing this alternative large-N limit was that, for any large N there exist baryons which contain three elementary fermions, two quarks and a lark, $qq\bar{Q}$. This type of baryons is qualitatively different than the baryons of the standard model which contain N quarks.

Our own motivation for studying the CR model in more detail is the following. As mentioned before from a large-N extension of QCD we can derive an effective quantum theory for mesons and baryons. The effective action coming from the large-N limit of standard QCD_N was shown in [10] to contain non-local Yukawa couplings, baryons were classical, and their propagation non-local. Such features being crucial, it is reasonable to investingate to what extend they depend on the particular large-N extension studied. The only other large-N extension of QCD which is asymptotically free, vector-like and at $N = 3$ coincides with QCD is the CR model. Thus we embarked into its analysis, to investigate the universality of certain features of the low-energy effective quantum theory of mesons and baryons.

Our results can be summarized as follows. There are three types of particles in the CR model, the standard mesons, $q\bar{q}$ or $Q\bar{Q}$, the l-baryons , $qq\bar{Q}$, and the h-baryons containing an order $O(N)$ number of quarks and/or larks. First, since at large N the larks behave combinatorialy like gluons, lark loops are unsuppressed. This fact induces a mixing of the standard mesons with multi-quark exotics, like $qQ\bar{Q}\bar{q}$ or $Q\bar{Q}Q\bar{Q}$. This is not a welcome feature since in real world QCD multi-quark exotic mesons are highly unstable. The same is true for the l-baryons which mix with exotics like $qqQ\bar{Q}\bar{Q}$. At the leading order mesons and l-baryons are stable with local interactions which have couplings that vanish as a negative power of N. In particular there is a mixing term between $q\bar{q}$ and $Q\bar{Q}$ mesons of order $N^{-\frac{1}{2}}$. The h-baryons have masses of order $O(N)$ and non-zero scattering cross-sections with the mesons and the l-baryons . One would like to argue that since the h-baryons are much heavier than the rest, by looking at low energies and integrating out the h-baryons , one could consider the effective theory of mesons and l-baryons which is a nicely behaving field theory. However as $N \to 3$ the l-baryons and h-baryons become degenerate and such a split does not make sense. Thus, despite the fact that, superficially, the CR model seems to give a smooth theory for mesons and baryons at large-N, its implications at small N are almost certain to be false.

There are other serious reasons indicating that the phenomenology of the CR model is in disagreement with the standard strong interaction phenomenology. Some of them are: Zweig's rule is violated due to the mixing, discussed above, between various meson states. Also the observed baryon widths are large unlike what is happening with l-baryons where the widths vanish at large-N. Baryons and baryonic resonanses are generally much heavier than mesons and their resonances. Their Regge trajectories behave very differently. The baryonic interaction seems to be strong, whereas l-baryons interact weakly. Nuclear phenomena would be quite different if the nuclear interaction is weak. For those and other reasons it seems that the C-R model is not a good description of nuclear pnenomena. The pattern of the realization of global symmetries of the model is also different. The model has a manifest $U(n_q)_L \otimes U(n_q)_R \otimes U(6-n_q)_L \otimes U(6-n_q)_R$ chiral symmetry when the fundamental fermions are massless. Instanton effects at large N in the standard model are

subleading so that the axial U(1) is still a symmetry. Here, on the contrary, the axial U(1) corresponding to larks is anomalous at the leading order so that the correct global symmetry is $U(n_q)_L \otimes U(n_q)_R \otimes SU(6-n_q)_L \otimes SU(6-n_q)_R \otimes U(1)_V$. Chiral symmetry breaking occurs to a vector subgroup, but we do not know if the unbroken symmetry group is maximal.

The structure of this paper is as follows. In section 2 we review the large-N results for the standard model and discuss how to use them to derive an effective quantum theory for the physical degrees of freedom. In section 3 we analyse in detail the large-N behavior of the CR model. In section 4 we derive and analyse the low-energy effective quantum theory of the physical degrees of freedom. Finally section 5 contains our concluding remarks.

2 Standard large N QCD

In this section we are going to review the results of the $1/N$ expansion in QCD_N with six quarks in the $N + \bar{N}$ of $SU(N)$. Since both the basic features as well as the consequences of this approach have been extensively discussed in the literature, [1,2,3] we will only give a brief account of the main results in order to acquaint the reader with the methods of the large-N expansion in QCD. We will also explain how one can use the results of the large-N expansion to derive the effective, relativistic quantum theory of mesons and baryons, [10].

The main idea of the large-N expansion is based on the fact that by generalizing the gauge group from $SU(3)$ to $SU(N)$ we can formulate a well-defined perturbation series in powers of $1/N$, [4]. At the level of conventional Feynman diagrams the choice of $1/N$ as the expansion parameter leads to a series of drastic simplifications, mainly due to the proliferation of gluon degrees of freedom ($\sim N^2$) over quark degrees of freedom ($\sim N$ when quarks are in the fundamental of $SU(N)$). The key observation is that since the coupling constant between quarks and gluons behaves as $N^{-\frac{1}{2}}$ at large N,¶ a Feynman diagram will survive the large-N limit, ($N \to \infty$), only if sufficiently large color combinatorial factors can compensate for the suppression coming from the coupling. As it turns out, there is a simple geometric classification of Feynman graphs that provide the

¶Necessary in order for the theory to have a regular limit.

leading contribution to a particular process.

The graphs that give subleading contributions in $1/N$ to a particular process are of two kinds.

1. $Non-planar$ diagrams, e.g. diagrams that cannot be drawn on the plain without gluon lines crossing each other at points where there are no interaction vertices.

2. Diagrams containing *quark loops*. Such diagrams are suppressed by factors of $1/N$ relative to diagrams where the quark loop is replaced by a gluon loop.

In other words, the dominant diagrams at large N are planar diagrams with no quark loops. It is important to emphasize that planar diagrams have the same leading large-N behaviour irrespective of the number of loops. Therefore the leading order of the $1/N$ expansion certainly contains non-perturbative information.

In order that useful results are obtained via large-N techniques in QCD one must also assume confinement. In particular we must assume that QCD_N confines for arbitrary N. This is certainly a reasonable assumption, known to be true for $N = 2, 3$.‖ If confinement is assumed then physical asymptotic states must be gauge singlets, e.g. mesons, baryons or glueballs.

As mentioned in the introduction, leading order results in QCD_N in four dimensions are still beyond our calculational capabilities.** However, if one assumes confinement, and uses the selection rules for Feynman diagrams mentioned above, there is a whole series of qualitative results that emerges, which turn out to be extremely interesting.

To be more specific, in the mesonic sector we have the following picture.†† Meson states are generated by composite operators of the form, $q_i\bar{q}^i$, (where

‖There are cases where the large-N limit is incompatible with confinement, [15]. In such cases it has been argued that color is not confined and the gauge symmetry is spontaneously broken.

**There are however models in 0,1,2 and 3 spacetime dimensions where leading order results are calculable, [1,16].

††From now on we will consistently ignore the glueball sector, since it is similar to the meson sector.

i is a color index summed over and the spinorial structure is suppressed) and their perturbative approximations, $q_iA^i_j\bar{q}^j$ etc. An estimate on their two-point function indicates that it is of order N providing the normalization of the one-meson states, which is $N^{-\frac{1}{2}}$. By cutting across the two-point function one can show that the propagating degrees of freedom are perturbative versions of a single one-meson state. This implies that mesons at $N = \infty$ are absolutely stable, with masses which are of order $O(1)$, and using the knowledge that the theory is asymptotically free, one can also infer that their number must be infinite. Similar analysis of the higher-point functions of mesons reveals that they can be generated as tree graphs from a *local* effective Lagrangian. This follows from the fact that the relevant leading large-N diagrams contain only single-particle poles in any possible channel. The n-point function of mesons vanishes as $N^{1-\frac{n}{2}}$ for large N, as a simple counting shows. Thus the meson interactions in the large-N limit are described by the tree diagrams of a local effective Lagrangian which is of the form,

$$L_m = \frac{1}{2}\phi(\Box - m^2)\phi + \frac{\lambda_1}{\sqrt{N}}\phi^3 + \frac{\lambda_2}{N}\phi^4 + \frac{\lambda_3}{N\sqrt{N}}\phi^5 + \cdots \qquad (2.1)$$

where ϕ denotes collectively the meson fields. The Lagrangian (2.1) is written in a schematic way. Locality of (2.1) is equivalent to the statement that the couplings are finite polynomials in derivatives. Also symmetries may force some of the terms to be absent from (2.1).

Some important consequences of the analysis above are:

1. Meson decay amplitudes are of order $N^{-\frac{1}{2}}$. Meson-meson scattering amplitudes are of order N^{-1}. Thus both are vanishing at $N = \infty$. At finite but large N, mesons have very small widths and they interact weakly.

2. Zweig's rule is exact in the large-N limit.

3. The $q\bar{q}$ sea as well as $q\bar{q}q\bar{q}$ exotic states are suppressed. Therefore mesons are approximately pure $q\bar{q}$ states in accordance with phenomenological observations.

The extension of large-N techniques in the baryonic sector is not straightforward. As mentioned before, baryons, unlike mesons that are always $q\bar{q}$ bound states irrespective of N, are formed out of N quarks, their indices coupled with the completely antisymmetric ϵ-tensor of $SU(N)$. An obvious consequence of the above is that they are bosons or fermions depending on N being even or odd.‡‡ More important, the presence of N quarks in a baryon provides additional factors of N in Feynman diagrams, obscuring the graphical analysis since it seems that lots of diagrams diverge as $N \to \infty$. However a different kind of analysis, using non-relativistic Schrödinger equations, shows how to re-organize and use diagrams in the baryon sector, [2].

Hamiltonian and path integral methods are preferable to diagrammatic techniques in this case. The baryon mass turns out to be of order $O(N)$ and both its kinetic and interaction energies are of the same order. Thus, its scattering cross-sections with mesons have finite non-zero limits as $N \to \infty$, as explained in detail in [2]. Since baryons are very heavy compared to mesons, they are practically unaffected during scattering, whereas mesons, having masses of order $O(1)$ are scattered non-trivially off baryons. In particular baryons at large N interact *strongly* unlike mesons. A direct QCD_N analysis also reveals that a baryon-antibaryon annihilation process into mesons is exponentially suppressed in N. Thus since QCD_N is crossing symmetric there should be non-trivial form-factors present in the meson-baryon interaction to restore the crossing symmetry. The momentum behavior of such form factors can be inferred from arguments like the one discussed above. This in particular implies that meson-baryon interactions as well as baryon propagation is non-local.

Another point is that QCD_N analysis implies that baryon loops are exponentially suppressed so that baryons are classical particles to all orders in $1/N$ perturbation theory. This can be easily understood. As it was mentioned above an extra quark loop in a diagram costs an extra factor of $1/N$. Therefore a baryon loop, which contains N quarks is suppressed by N^{-N}. Thus a classical Lagrangian reproducing the leading large-N results is of the form

$$L_B = \bar{\psi}(\partial\!\!\!/ A(\Box) + NB(\Box))\psi + \sqrt{N}\bar{\psi}\psi g(\Box)\phi + \cdots \qquad (2.2)$$

where we put explicit factors of N when appropriate and the form factors A, B, g should be exponentially suppressing when the momentum transfer is of order $O(N)$, [10].

It is very interesting also to see how global symmetries are realized at large-N. QCD_N , with six massless quarks has a $U(6)_L \otimes U(6)_R$ global symmetry. At leading order in $1/N$ the effects of instantons are negligible, [2] so that $U(1)_A$ is a genuine symmetry, unlike real QCD. One can argue, [17], assuming confinement, that the chiral symmetry either is unbroken or if it breaks, then it breaks to $U(6)_V$. It is crucial in this argument, that quark loops are suppressed at large N. The possibility of unbroken chiral symmetry can be excluded by a combination of t'Hooft's anomaly matching conditions and some rigorous mass-inequalities valid in QCD, [18].

Let's now discuss the implications of the above, on an effective, relativistic quantum field theory of mesons and baryons. As mentioned already, the $N = \infty$ limit is an exact saddle point of QCD_N where the theory is Gaussian (classical). This is a different ground state from the conventional QCD vacuum since this one describes mesons propagating freely, and heavy baryons interacting with mesons. An easy way to see the above is to write the Yang-Mills action for $SU(N)$ in the form,

$$S_{YM} = -\frac{1}{4g_B^2}\int Tr(F^2) \qquad (2.3)$$

where g_B is the bare Yang-Mills coupling.* We can find out how g_B renormalizes in the large-N limit. A simple way to do this is to look at the one-loop vacuum polarization diagram. This diagram is proportional to g_B^2 and has also a combinatoric factor of N due to the gluon loop. Therefore in order that the diagram is finite at $N = \infty$, $g_{ren} = g_B N^{-\frac{1}{2}}$. It can be shown that this guarantees finiteness of all correlation functions with a finite number of fields at large N. Now the action looks like

$$S_{YM} = -\frac{N}{4g_{ren}^2}\int Tr(F^2) \qquad (2.4)$$

It is obvious from (2.4) that in the quantum theory N plays the same role as $1/\hbar$, so that the usual classical limit, $\hbar \to 0$ corresponds to $N \to \infty$.

When we move away from the saddle point, fluctuations set in. To take these fluctuations into account we should allow our classical fields to fluctuate,

‡‡ This is not a very serious drawback since we can consider $SU(2N+1)$, as $N \to \infty$ and have baryons which are fermions for every N, without altering any of the large-N results.

* Terms missing from (2.3) like coupling to fermions as well as ghost and gauge fixing terms turn out to be irrelevant for the following argument.

that is, they should be quantized. Of course, such a quantization should be in accord with large-N estimates, [10]. For example, we showed before that in QCD_N , baryon loops are exponentially suppressed, thus baryons remain classical perturbatively in $1/N$. Consequently, the proper way to quantize the $N = \infty$ theory is to quantize the mesons but keep baryons classical, [10,12]. The Lagrangian of the effective quantum theory is $L_m + L_B$, which are given in (2.1), (2.2). Of course, there is a non-trivial consistency check of this procedure. Quantum effects, computed in the effective theory, should give subleading $1/N$ corrections. Otherwise stated, tree results should dominate over loop results, the latter being suppressed by extra powers of $1/N$. This is precisely true in the effective quantum theory formulated above. In fact we can easily estimate, [10], that a l-loop amplitude is suppressed relative to the respective tree amplitude by a factor N^{-l}. The Lagrangian above in general contains high dimension operators that would induce non-renormalizable interactions. This is not a problem since the theory we are describing is an effective theory and as such has a cut-off which is a few times the nucleon mass. Furthermore, such interactions would give contributions that are suppressed by powers of the cut-off and can, in most cases, safely be ignored.

Such effective relativistic theories have been extensively used in Nuclear Physics with reasonable success, [5,6,7,8,9]. What our large-N results indicate is that there is a good reason for such success. But in fact these results imply more than that. First, that meson-baryon couplings should be non-local in contrast with what is traditionally used. Second, one should not attempt to calculate baryon loop corrections. This in retrospect explains the fact that some attempts to calculate baryon loop corrections obtained results which did not make sense from the physical point of view, [19]. Therefore, in the large-N approach the instruction is clear: do *not* calculate baryon loops. Another issue that arises here concerns the non-locality of baryon-meson couplings. As a matter of fact traditional relativistic approaches to Nuclear Physics favor local meson-baryon couplings, [5]. If one had a model for QCD_N where baryons would be constructed out of three quarks for any N, then it would almost certainly follow that such baryons would have local interactions with mesons. Corrigan and Ramond introduced their model, [14], motivated as above. In the next section we will extensively analyze the CR model and try to find the effective field theory of mesons and baryons that this model suggests.

3 Large-N estimates in the CR model

In this section we are going to analyze in detail the large N limit of the CR model, [14] and the low energy effective field theory it implies. The model contains $n \geq 1$ quark fields in the fundamental of $SU(N)$, q_i, and $6-n$ lark fields in the two-index antisymmetric tensor representation of $SU(N)$ of dimension $N(N-1)/2$, Q_{ij}, as well as their conjugates,† in order to have a vector-like theory.‡

An important difference with the standard large-N model is that the number of lark degrees of freedom is $\sim N^2$, like gluons. The rules given in the previous section characterizing leading graphs are still true with one important modification: larks can propagate freely in loops, their contributions not being suppressed. This fact makes the model quite different from its standard counterpart.

An important assumption we will again make is that of confinement: All physical asymptotic states are gauge singlets. The simplest mesons come in two kinds. The first kind that we will collectively label by ϕ are of the form $\phi \sim q_i \bar{q}^i$. The other kind that we label by χ are of the form $\chi \sim Q_{ij}\bar{Q}^{ij}$. There are gauge singlets involving two quarks and a lark, $q_i q_j \bar{Q}^{ij}$, which we call l-baryons . Finally there is a whole class of heavy baryons, (baryons containing $\sim N$ quarks), which we call h-baryons . For any two non-negative integers, x, y such that $2x + y = N$, there is a h-baryon operator constructed by contracting the invariant ε-tensor of $SU(N)$ with x larks and y quarks. This h-baryon will be called of the type (x, y).

†The only indices that we will display are the color indices. Flavor or spacetime indices will be suppressed for simplicity.

‡There are large-N extensions of QCD that are chiral gauge theories. We don't consider such models since their behavior is different and in fact very singular for our purposes. Such models, being chiral, should contain massless quarks in order not to break the gauge symmetry. At the low energy limit there are solutions to the t'Hooft anomaly-matching conditions indicating the presence of *massless baryons*. This is a feature true for all $N > 3$, whereas at $N = 3$ the baryons become massive since the theory is vector-like. We believe that the existence of massless baryons is an indication that such an extension does not have much to do with QCD.

So let's first consider the two-point function of the ϕ-mesons,

$$\langle\phi\phi\rangle = \langle : q_i\bar{q}^i :: q_j\bar{q}^j : \rangle$$

. The only difference now is that, since lark loops are unsuppressed, if we cut the diagram we will find multi-meson exotics of the form,

$$\phi_n \sim q_i\bar{Q}^{ij_1}Q_{j_1i_1}\cdots\bar{Q}^{i_{n-1}j_n}Q_{j_ni_n}\bar{q}^{i_n} \tag{3.1}$$

and their perturbative approximations (dressed with gluons). Therefore, all such mesons, ϕ_k, $k = 0, 1, \cdots$, mix and the physical degrees of freedom are linear combinations of those that diagonalize the two-point function. This is already a bad feature of this model compared with the standard one.

A similar argument, as in the standard case can be made, showing that physical on-shell states propagating in the ϕ two-point function are one-particle states. As an example, consider a graph for the two-point function of a ϕ-meson with a lark loop inside the external quark loop. Cutting the graph indicates the presence of an intermediate state of the form, $q_i\bar{Q}^{ij}Q_{jk}\bar{q}^k$ propagating. This is obviously a one particle state since we cannot write it as a product of two (or more) gauge singlet pieces. Of course one can fill up the diagram with internal gluon lines without changing its order in $1/N$. The resulting states that propagate in that case are again one-particle (dressed) meson states. The whole set of ϕ_k states can be obtained from cutting diagrams which contain k internal lark loops.

We will establish now the proper normalization of the ϕ_k states. It is not difficult to show that the two-point function of un-normalized ϕ_k states scales with N as N^{2k+1}. From the contraction of each of the k $Q\bar{Q}$ pairs that exist in ϕ_k we obtain a factor of N^2 while an extra factor of N comes from contracting q with $\bar{q}$. For $k = 0$ we recover the standard result, [1,2,3]. Thus, the properly normalized states are,

$$\phi_k|0\rangle \sim N^{-k-\frac{1}{2}} : q_i\bar{Q}^{ij_1}Q_{j_1i_1}\cdots\bar{Q}^{i_{n-1}j_n}Q_{j_ni_n}\bar{q}^{i_n} : |0\rangle\ ,\ \langle\phi_k\phi_{k'}\rangle \sim O(1) \tag{3.2}$$

Armored with ϕ-states that have smooth limits for large N we can investigate correlation functions among ϕ fields. The analysis is almost parallel to the one for mesons in the standard large-N model, the only difference here being the existence of a hierarchy of ϕ-meson states labeled by k. By a direct analysis using the large-N diagrammatic rules we can establish that the general correlator of ϕ fields scales as

$$\langle\phi_{k_1}\phi_{k_2}\cdots\phi_{k_n}\rangle \sim O(N^{1-\frac{n}{2}}) \tag{3.3}$$

independent of the type k of mesons involved. This estimate is the same as for the mesons of the standard large-N model of the previous section. The rest of the properties are still there. ϕ-mesons have masses of order $O(1)$, and at $N = \infty$ they are absolutely stable. All poles in cut n-point functions are one-particle poles, showing that again we can reproduce the ϕ-meson amplitudes from tree graphs of a local Lagrangian

$$L_\phi = \frac{1}{2}\phi(\Box + m^2)\phi + \sum_{n\geq 3} N^{1-\frac{n}{2}}\lambda_n\phi^n \tag{3.4}$$

We will now focus our attention to the χ-mesons. A similar thing happens here too. Since lark loops are unsuppressed the simplest χ-mesons, $\chi_0 \sim Q_{ij}\bar{Q}^{ij}$, mix with a hierarchy of multi-meson exotics,

$$\chi_k \sim N^{-1-k} : Q_{i_0j_0}\bar{Q}^{j_0i_1}\cdots Q_{i_kj_k}\bar{Q}^{j_ki_0} :\ ,\ \langle\chi_k\chi_{k'}\rangle \sim O(1) \tag{3.5}$$

where the proper normalization was determined the same way from the appropriate two-point function. Now one can establish again the same properties that hold for the ϕ-mesons. In fact one does not even have to do any additional analysis but note that combinatorialy the χ-mesons behave exactly the same as the glueball operators in the standard model. Thus one can use the known results to establish that the mesonic sector containing ϕ and χ mesons at $N = \infty$ is a free theory with meson masses of order $O(1)$ and the leading large-N results are reproduced from the tree graphs of the following local Lagrangian,

$$L_{meson} = L_\phi + \frac{1}{2}\chi(\Box + m^2)\chi + \sum_{n\geq 3} N^{2-n}\lambda_{n,0}\chi^n + \sum_{n_1\geq 1, n_2\geq 1} N^{1-n_2-\frac{n_1}{2}}\lambda_{n_1,n_2}\phi^{n_1}\chi^{n_2} \tag{3.6}$$

We should stress again that our way of writing Lagrangians like (3.6) is very schematic. One should take into account that labels like ϕ or χ denote collectively mesons with different flavor quantum numbers, parity, and (integral) spin, couplings are a shorthand for a finite polynomial in derivatives (locality), and, depending on the symmetry properties of mesons, some of the generic couplings present in (3.6) might vanish. Some immediate consequences of (3.6) are:

1. At large but finite N the ϕ-mesons mix with the χ-mesons with strength that scales as $N^{-\frac{1}{2}}$. They are almost stable and weakly coupled.
2. When computing the N-dependence of a specific correlation function of mesons, one in general will obtain two possible large-N behaviors due to the fact that in (3.6) the coupling of χ^n is *not* a special case of the general coupling $\chi^n\phi^m$. Of course, of the two powers of N the leading power is the correct one, (the same remark applies to standard large-N QCD, in the meson-glueball sector).

We will now focus on the l-baryon sector where the simplest l-baryons have the form $\psi_0 \sim q_i q_j \bar{Q}^{ij}$. Again here a look at the two-point function of ψ_0 reveals that these l-baryons are mixing with an infinite hierarchy of other l-baryons of the form,

$$\psi_k \sim N^{-1-k} : q_{i_0} q_{j_0} \bar{Q}^{j_0 i_1} \cdots Q_{i_k j_k} \bar{Q}^{j_k i_0} : \;, \quad \langle \psi_k \psi_{k'} \rangle \sim O(1) \qquad (3.7)$$

where again we normalized the l-baryon -states appropriately. The l-baryons are obviously fermions but have similar properties as the mesons. In particular, by looking at diagrams we can establish that l-baryons have masses of order $O(1)$, they are absolutely stable at $N=\infty$, their amplitudes are the tree amplitudes of a *local* Lagrangian and their couplings can be estimated easily to be such that the total Lagrangian describing mesons and l-baryons is,

$$L_{tot} = L_{meson} + \bar{\psi}(\not{\partial} + m)\psi + \sum_{n_{1,2}\geq 0, n_3 \geq 1} N^{1-n_2-n_3-\frac{n_1}{2}} \lambda_{n_1,n_2,n_3} \phi^{n_1} \chi^{n_2} (\psi\bar{\psi})^{n_3} \qquad (3.8)$$

At this point we should say a few things about possible realizations of global symmetries in the model. When the quarks and larks are massless the model has a $U(n)_L \otimes U(n)_R \otimes U(6-n)_L \otimes U(6-n)_R$ chiral symmetry. However the $U(1)_A$ of larks is anomalous in the leading order. Thus the honest symmetry is, $U(n)_L \otimes U(n)_R \otimes SU(6-n)_L \otimes SU(6-n)_R \otimes U(1)_V$. There are two gauge invariant order parameters for the symmetry above which are bilinear in quarks or larks, $M_1^{ij} \sim \langle q^i(1+\gamma^5)\bar{q}^j \rangle$ for $U(n)$, $M_2^{ij} \sim \langle Q^i(1+\gamma^5)\bar{Q}^j \rangle$ for $SU(6-n)$. Due to lark loops being unsuppressed we cannot predict the exact way the symmetry is realized. We can however predict that it is going to break to an anomaly-free subgroup. The reason is that here, in analogy with the standard QCD_N , the theory is vector-like so that mass inequalities are applicable, [18]. Therefore, if some axial symmetry remains unbroken, that would imply the existence of massless fermions. The inequalities in that case indicate the existence of massless bosons with the right quantum numbers to be Goldstone bosons. Thus axial symmetries must also break in this model. This is all we can say about the realization of global symmetries.

4 The effective quantum theory of the CR model

In this section we are going to investigate the effective field theory of the CR model. For the moment we will assume that we can forget about the h-baryons. * In this case (3.8) contains all the dynamics governing the "classical" degrees of freedom. According to our analysis, discussed in the previous section, to take into account fluctuations that develop when we move away from the $N=\infty$ limit, we must quantize the classical degrees of freedom, ϕ, χ, ψ.

We will show explicitly that this quantization prescription is consistent with the $1/N$ expansion carried to non-leading orders, as it should.† We will first obtain large-N estimates in the effective quantum theory given by the Lagrangian in (3.8). All N-dependence in graphs is due to the explicit N-dependence of the couplings in (3.8). We can use the usual topological analysis of arbitrary graphs to estimate the large-N behavior of loop graphs in comparison with tree graphs. Let L be the number of loops in the graph, E_ϕ, E_χ, $E_{\psi\bar{\psi}}$ the number of external lines of the respective fields, I_ϕ, I_χ, I_ψ the number of the respective internal lines, W_i the number of vertices of the form χ^i and $V_{i_1 i_2 i_3}$, $(i_1, i_3) \neq (0,0)$, the number of vertices of the type $\phi^{i_1}\chi^{i_2}(\psi\bar{\psi})^{i_3}$ in (3.8), (there is an infinite number

* We will return to this point later on.

† A similar result is true for the meson sector (but not the baryon sector) of standard large-N QCD, [10]. The results of this section imply also that the same is true for the glueball sector of standard large-N QCD.

of them). Then the standard topological relations are,

$$L-1 = I_\phi + I_\chi + I_\psi - \sum_i W_i - \sum_{i_1,i_2,i_3} V_{i_1 i_2 i_3} \tag{4.1a}$$

$$\sum_{i_1,i_2,i_3} i_1 V_{i_1 i_2 i_3} = 2I_\phi + E_\phi \tag{4.1b}$$

$$\sum_{i_1,i_2,i_3} i_2 V_{i_1 i_2 i_3} + \sum_i iW_i = 2I_\chi + E_\chi \tag{4.1c}$$

$$\sum_{i_1,i_2,i_3} i_3 V_{i_1 i_2 i_3} = I_\psi + E_{\psi\bar\psi} \tag{4.1d}$$

Using (4.1) and the large-N order of elementary couplings in (3.8) we obtain that the order for a graph with L loops, and $(E_\phi, E_\chi, E_{\psi\bar\psi})$ external lines is $N^{tree} \times N^{-L-R}$ where $N^{tree} = N^{1-\frac{E_\phi}{2}-E_\chi-E_{\psi\bar\psi}}$ and $R \equiv I_\chi - \sum_i W_i$. Another way to obtain the result above is to redefine fields in (3.8) in such a way that $L = N\tilde{L}$ and inside $\tilde{L}$ the χ propagator and W vertices are proportional to N, everything else being N-independent.

We can now prove the following: for a fixed correlation function, (fixed E_ϕ, E_χ, $E_{\psi\bar\psi}$) the L-loop contribution is suppressed compared to the tree contribution by a factor N^{-L-n} where n is a non-negative integer. The above amounts to showing that $R \geq 0$ for any diagram with $E_\phi + E_{\psi\bar\psi} \geq 0$ and $R \geq -1$ for any diagram with $E_\phi = E_{\psi\bar\psi} = 0$.

Before proving the general statement above we will look at some special cases to give a feeling of the situation. Consider first, ignoring the χ fields completely. That is, they don't appear either in external or internal lines. Using again (4.1) we can now derive that an arbitrary L-loop contribution to an amplitude is suppressed compared to the tree contribution by a factor N^{-L}. In the opposite case where we completely ignore ϕ-mesons and l-baryons a similar analysis implies that the extra factor suppressing a loop contribution is N^{-2L}. The two different kinds of behavior should be attributed to the fact that the χ self interactions in (3.8) scale differently from the interactions of χ's with other particles. As we have mentioned before a similar thing happens already at tree-level.

Let's now prove our assertion. We have to distinguish two cases.

1. $E_\phi = E_{\psi\bar\psi} = 0$. In this case we would like to prove that $R \geq -1$. To find the minimum R we have to find the maximum of $W \equiv \sum_i W_i$ and the minimum of I_χ. When $W = 0, 1$ the inequality is trivially true. When $W = 2$ then $I_\chi \geq 1$ otherwise the diagram will be disconnected. Thus $R \geq -1$ here. When $W = k$ then $I_\chi \geq k-1$ in order for the diagram to be connected, QED.

2. $E_\phi + E_{\psi\bar\psi} \geq 0$. Here we want to show that $R \geq 0$. The argument goes again as above. The only difference is that since there are other external lines we would need one more χ propagator to connect the W-vertices to the rest of the diagram which accounts for the different minimum value of R by one, QED.

To recapitulate, we have proven that in the quantum theory defined by the Lagrangian (3.8) L-loop contributions to a given correlation function are suppressed relative to the tree contribution by an extra factor of N^{-L-n} where n is a non-negative integer.

We now know how diagrammatic contributions scale with N in the effective quantum theory defined by the Lagrangian (3.8). It is important to show that the $1/N$ expansion of the effective theory is consistent with the large-N expansion in the CR model. To be more precise, consider any diagram in the CR model, sub-leading in N,[‡] with external lines that are contracted in a gauge singlet. By cutting it in all possible ways and identifying the physical degrees of freedom propagating in various channels we can uniquely map it to a single diagram of the effective theory (not necessarily a tree diagram). Consistency states that the N behavior of the QCD_N diagram should be the same with the one of the effective theory diagram onto which it is mapped.

Consider for example the two-point function of ϕ. If there is a single quark loop inside the diagram, then, as mentioned before, the diagram is down by $1/N$. If we cut the diagram with one quark loop on the other hand we can verify that this corresponds in fact to a ϕ meson going around a loop. This is consisted so far with the general result for ϕ-mesons only, N^{-loops}. An iteration of the previous argument verifies the above for an arbitrary number of ϕ loops.

[‡] For leading ones this is true de-facto, by the very existence of a classical Lagrangian that reproduces the leading amplitudes.

Now χ loops have a similar effect as ϕ loops. Consider a χ loop in the two-point function for χ. In order to generate a χ loop, we have to break planarity, consequently there is a cost of a factor N^{-2}. Planarity has to be broken since we have already shown that planar graphs with an arbitrary number of lark loops when cut exhibit only one-particle states and not two as one would expect in the presence of a χ loop. Again, iteration of this argument gives the same result we obtained for the effective theory of χ, namely N^{-2L}.

Another case to consider is is that of l-baryon loops. For example to generate an l-baryon loop in the ϕ two-point function we need to create a quark and a lark loop. This would cost as a factor of $1/N$ as expected. This is in agreement with the effective theory estimate.

The general argument proving that the large-N estimates coming from the QCD diagrams are in agreement with those coming from the classical Lagrangian, (3.8) is very simple if we use the results we proved so far. The basic idea is that the equivalence is trivially true for tree graphs (almost by definition)‖ and an arbitrary loop graph can be constructed by sewing tree graphs. To make it precice, consider any QCD diagram. There is always a way to cover it with non-overlapping boxes in such a way that lines coming out of every box can be assebled into representations of physical states and any cut through the box reveals one-particle states. This representation exists since this is the explicit form of the map between QCD diagrams and effective field theory diagrams. Now, since we know that the estimates for each individual box coincide, (they are tree components where we know it to be true), the same will be true for the entire diagram.

We should pause for a moment to remind the reader what we are after. In [10] a procedure was described to obtain effective quantum field theories for mesons and baryons out of the $1/N$ expansion. Such a theory was described in [10] by using the results from the standard large-N extension of QCD. This

‖If one considers tree graphs where the external states are of the minimal form, e.g. ϕ-mesons to be represented by the operators in (3.1), then the statement is true by definition. However, the external particles can be perturbative approximations, e.g. the ϕ-meson $q_i\bar{q}^i$ can be also represented by $q_i A^i_j \bar{q}^j$. One has to make sure that replacing the simplest form of the external state with a more complicated one does get the same large-N behavior. This is indeed true when gluons are taken into account and we have shown it to be true also when larks are also involved.

effective theory had non-local Yukawa couplings and baryons remained classical. The question we are investigating is: Are such features of the theory above generic, or do they depend on the large-N extension of QCD. To answer this question we note that the CR model is the only other large-N extension of QCD which is asympotically free and vector-like.

What we have shown so far is that the effective quantum theory we have derived for the low-energy physical particles of the CR model is in fact consistent in a non-trivial way with the large-N diagrammatic expansions to all orders. It is thus a *local*, *effective*, *relativistic field theory* for mesons and l-baryons. However it seems that it has *little* if *nothing* to do with the real hadronic world. The reasons are given below¶.

The CR model, unlike QCD contains a tower of multiquark exotics that are not suppresed and they mix strongly with the usual mesons. One consequence of the above is that Zweig's rule is violated.

Baryons and baryonic resonances in general are much heavier that mesons and their resonances. They also differ in the characteristics of the their Regge trajectories. This is not true in the CR model.

The observed baryon widths are large. This is in contradiction with the widths of the l-baryons in the CR model which are of order $N^{\frac{-1}{2}}$, that is very small at large N.

The nuclear interaction is observed to be strong. Both phase-shift analyses of nucleon-nucleon scattering and data on the spin-orbit coupling in nuclear interactions confirm this result. The l-baryon self interactions in the CR model are *weak* at large N. The nuclear world would look very different if it was made out of l-baryons.

Apart from the previous objections we have neglected h-baryons and their effects on the previous picture.

Let us now turn our attention to h-baryons , to see under what circumstances we can neglect them, if any. Using the same arguments as in [2] we can immediately infer that h-baryons have masses of order $O(N)$, therefore, at large-N they are much heavier than the other physical degrees of freedom. This gives us the hope that at large-N we can integrate them out leaving behind only

¶We would like to thank R. Seki for numerous conversations on this point.

an effective theory for the light degrees of freedom. There is however a major obstacle in doing this. We are eventually interested in $N \sim 3$ and in this region it is not possible to integrate out the h-baryons since they can be pair-produced. If one includes the h-baryons in the effective theory though, then we have again the features of the previous theory, namely non-local Yukawa and four-Fermi interactions.

Independent of the above, there is another issue which is interesting: Whether h-baryons loops are suppressed exponentially in the CR model. Despite the fact that lark loops are unsuppressed, h-baryons loops scale exponentially with N. This behavior can be inferred from the fact that the amplitude for a light particle to create a heavy baryon-antibaryon pair is exponentially suppressed and this depends only on the h-baryon containing order $O(N)$ constituents. Thus again in the effective quantum theory the h-baryons are still classical objects. If we are at large enough N it makes sense to integrate out the h-baryons . If the h-baryons were not classical then the effect of integrating them out would be that the new effective quantum theory for the light degrees of freedom would become horribly non-local. However, "integrating out" classical particles simply means drop them from the Lagrangian.

Thus, at large-N, the effective quantum theory at energy scales of order $O(1)$ would just be the local-one described by (3.8). At low N such a split between light and heavy particles is no longer feasible and the theory is non-local.

Therefore the answer to our "universality" question about effective actions would be that these *are "universal"*, since the only alternative model, the CR model which would invalidate the previous statement, is found to be in disagreement with nuclear phenomenology.

5 Conclusions

In this paper we have analysed in detail the CR model for large-N QCD. At $N = \infty$ QCD is described by a classical Lagrangian, which can be determined from an analysis of large-N perturbation theory. Appropriate quantization of the physical degrees of freedom with classical dynamics governed by the $N = \infty$ Lagrangian describes effects subleading in $1/N$ which describe the finite N theory. This effective quantum theory should describe low energy interactions of physical degrees of freedom (mesons-baryons). In [10] standard large-N QCD was used to derive this theory which is of the usual relativistic Yukawa type, but with non-local Yukawa couplings and classical baryons (no baryon loops). What we have primarily investigated in this paper is if conclusions like non-locality of Yukawa couplings and baryons being classical depend on the particular large-N extension of QCD or if they are universal.

The only other model for large-N QCD which is asympotically free, vector-like and at $N = 3$ coincides with standard QCD is the CR model [14]. Thus we embarked in an effort to analyse in detail the spectrum and other large-N estimates in this model. The spectrum consists of two kinds of mesons and light baryons, all of which have masses of order $O(1)$ and heavy baryons having masses of order $O(N)$. At $N \to \infty$ the classical effective theory of the light sector (mesons, light baryons) is a *local* field theory. Away from $N = \infty$ the effective quantum field theory is obtained by quantizing the light degrees of freedom but leaving thew heavy baryons classical since is directly implied by a diagrammatic analysis.

At finite but large N, it is physically consistent to integrate out the heavy baryons, which in that case means simply to ignore them. Then at low energy the effective quantum theory for the light degrees of freedom is a *local* quantum field theory with baryons that can go around loops. However in the case of interest, $N \to 3$, the heavy baryons are close in mass with the light degrees of freedom and cannot be integrated out. In this case the effective theory contains non-local Yukawa couplings between heavy baryons and light particles, heavy baryons are classical, and their propagators are non-local.

Even if we remain at large N there are reasons which indicate that the CR model is at odds with nuclear phenomenology. In the CR model Zweig's rule is not valid, the multiquark exotic mesons mix strongly with the usual ones, the baryon widths are small and baryonic forces are weak, all the above in contradiction with experiment.

Thus our basic conclusion is that *the only good large N limit for QCD is the standard one*. Consequently, the results of [10], namely non-local meson-baryon interactions and absence of baryon loops *are universal*.

Acknowledgement

One of us, (E.K.), would like to thank R. Seki for numerous discussions.

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