Gauge Invariance and Unstable Particles

Joannis Papavassiliou[a] and Apostolos Pilaftsis[b*]

[a]*Department of Physics, New York University, 4 Washington Place, New York,*

NY 10003, USA

[b]*Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, UK*

ABSTRACT

A gauge-independent approach to resonant transition amplitudes with nonconserved external currents is presented, which is implemented by the pinch technique. The analytic expressions derived with this method are $U(1)_{em}$ invariant, independent of the choice of the gauge-fixing parameter, and satisfy a number of required theoretical properties, including unitarity. Although special attention is paid to resonant scatterings involving the γWW and ZWW vertices in the minimal Standard Model, our approach can be extended to the top quark or other unstable particles appearing in renormalizable models of new physics.

PACS nos.: 14.70.Fm, 11.15.Bt, 11.15.Ex

---

[*]E-mail address: pilaftsis@v2.rl.ac.uk

arXiv:hep-ph/9506417v2 26 Jun 1995

Three decades after Veltman's pioneering work [1], the correct treatment of unstable particles in the context of renormalizable gauge field theories is still an open question. The interest in the problem resurfaced in recent years [2], mainly motivated by a plethora of phenomenological applications linked to machines, such as the CERN Large Electron Positron collider (LEP), the LEP2, planned to operate at centre of mass system (c.m.s.) energy $s = 200$ GeV, the TEVATRON at Fermilab, and the CERN Large Hadron Collider (LHC).

Even though the need for a resummed propagator is evident when dealing with unstable particles within the framework of the S-matrix perturbation theory, its incorporation to the amplitude of a resonant process is non-trivial. When this incorporation is done naively, *e.g.* by simply replacing the bare propagators of a tree-level amplitude by resummed propagators, one is often unable to satisfy basic field theoretical requirements, such as the gauge-parameter independence of the resulting S-matrix element, $U(1)_{em}$ symmetry, high-energy unitarity, and the optical theorem. This fact is perhaps not so surprising, since the naive resummation of the self-energy graphs takes into account higher order corrections, for *only* certain parts of the tree-level amplitude. Even though the amplitude possesses all the desired properties, this unequal treatment of its parts distorts subtle cancellations, resulting in numerous pathologies, which are artifacts of the method used. It is therefore important to devise a self-consistent calculational scheme, which *manifestly* preserves the afore-mentioned field theoretical properties that are *intrinsic* in every S-matrix element.

In this paper, we present a new gauge-independent (g.i.) approach to resonant transition amplitudes implemented by the pinch technique (PT) [3]. The PT is an algorithm which systematically exploits all the healthy properties of the S matrix and hence has numerous applications in electro-weak physics. Operationally, it amounts to a rearrangement of the Feynman graphs contributing to a g.i. amplitude so as to define *individually* g.i. propagator-, vertex-, and box-like structures. In particular, due to technical limitations,

one often attempts to cast the higher order corrections to a given process in the form of its tree-level amplitude, by essentially omitting box diagrams. Since box graphs are however crucial for the final gauge independence (among other things) of the S matrix, their omission introduces artificial gauge-dependences into the rest of the amplitude. The way the PT circumvents such problems is by recognizing that the parts of the boxes, which are instrumental for restoring the final gauge invariance, are effectively propagator-like and vertex-like, and thus can be naturally cast in the form of the original tree-level amplitude. The remaining box contributions are a g.i. subset and can be consistently subtracted out. Even though the situation is conceptually and technically more subtle, the underlying objective remains the same when dealing with resonant amplitudes. Again, one attempts to account for resonant effects by "dressing up" the tree-level amplitude; when this is done without a concrete guiding principle, one ends up with the type of pathologies mentioned above.

The crucial novelty we introduce here is that the resummation of graphs must take place only *after* the amplitude of interest has been cast via the PT algorithm into manifestly g.i. sub-amplitudes, with distinct kinematic properties, order by order in perturbation theory. The application of the PT remarkably remedies all the afore-mentioned field-theoretical problems existing at present in the literature. In particular,

(i) The analytic results obtained within our approach are, by construction, *independent* of the gauge-fixing parameter, in *every* gauge-fixing scheme (R_ξ gauges, axial gauges, background field method, *etc.*). In addition, by virtue of the tree-level Ward identities satisfied by the PT Green's functions, the $U(1)_{em}$ invariance can be enforced, without introducing residual gauge-dependent terms of higher orders.

(ii) The PT treats bosonic and fermionic contributions to the resummed propagator of the W-, Z-boson, t quark or other unstable particles, *on equal footing.* This feature is highly desirable for applications to extensions of the SM at high energy colliders, such as the LHC.

For example, a heavy Higgs boson in the SM or new gauge bosons, such as *e.g.* Z', W', Z_R, *etc.*, predicted in models beyond the SM, can have widths predominantly originating from bosonic channels. In this way, it becomes even more obvious that prescriptions based on resumming *only* fermionic contributions as g.i. subsets of graphs, are insufficient.

(iii) The use of an expansion of the resonant matrix element in terms of a *constant* complex pole produces unavoidably space-like threshold terms to all orders, while non-resonant corrections remove such terms only up to a given order. These space-like terms, which explicitly violate unitarity, manifest themselves when the c.m.s. energy of the process does not coincide with the position of the resonant pole. On the contrary, the PT circumvents these difficulties by giving rise to an energy-dependent complex-pole regulator. For instance, possible unphysical absorptive parts originating from channels below their production threshold have already been eliminated by the corresponding kinematic θ functions.

(iv) Lastly, the amplitude obtained from our approach exhibits a good high-energy unitarity behaviour, as the c.m.s. energy $s \to \infty$. In fact, far away from the resonance, the resonant amplitude tends to the usual PT amplitude, thus displaying the correct high-energy unitarity limit of the entire tree-level process.

We will now study some characteristic examples. Within the PT framework, the transition amplitude $T(s, t, m_i)$ of a $2 \to 2$ process, such as $e^- \bar{\nu}_e \to \mu^- \bar{\nu}_\mu$ with massive external charged leptons, can be decomposed as

$$T(s, t, m_i) = \widehat{T}_1(s) + \widehat{T}_2(s, m_i) + \widehat{T}_3(s, t, m_i), \tag{1}$$

where the piece $\widehat{T}_1$ contains three *individually* g.i. quantities: The WW self-energy $\widehat{\Pi}^W_{\mu\nu}$, the WG mixing term $\widehat{\Theta}_\mu$, and the GG self-energy $\widehat{\Omega}$. Similarly, $\widehat{T}_2(s, m_i)$ consists of two pairs of g.i. vertices $We^-\bar{\nu}_e$, $Ge^-\bar{\nu}_e$ [$\widehat{\Gamma}^{(1)}_\mu$ and $\widehat{\Lambda}^{(1)}$], and $W\mu^-\bar{\nu}_\mu$ and $G\mu^-\bar{\nu}_\mu$ [$\widehat{\Gamma}^{(2)}_\mu$ and $\widehat{\Lambda}^{(2)}$]. Most importantly, in addition to being g.i., the PT self-energies and vertices satisfy the

following *tree-like* Ward identities:

$$q^\mu \widehat{\Pi}^W_{\mu\nu} - M_W \widehat{\Theta}_\nu = 0,$$

$$q^\mu \widehat{\Theta}_\mu - M_W \widehat{\Omega} = 0,$$

$$q^\mu \widehat{\Gamma}^i_\mu - M_W \widehat{\Lambda}^i = 0 \ (i = 1, 2). \tag{2}$$

These Ward identities are a direct consequence of the requirement that $\widehat{T}_1$ and $\widehat{T}_2$ are fully ξ independent [3]. If we assume that the PT decomposition in Eq. (1) holds to any order in perturbation theory (the validity of this assumption will be discussed extensively in Ref. [4]), and sum up contributions from all orders, we obtain for $\widehat{T}_1$ (suppressing contraction of Lorentz indices)

$$\widehat{T}_1 = \Gamma_0 U_W \Gamma_0 + \Gamma_0 U_W \widehat{\Pi}^W U_W \Gamma_0 + \Gamma_0 U_W \widehat{\Pi}^W \cdots \widehat{\Pi}^W U_W \Gamma_0 = \Gamma_0 \widehat{\Delta}_W \Gamma_0, \tag{3}$$

where $U_{W\mu\nu}(q) = t_{\mu\nu}(q)(q^2 - M_W^2)^{-1} + \ell_{\mu\nu}(q)M_W^{-2}$ $[t_{\mu\nu}(q) = -g_{\mu\nu} + q_\mu q_\nu/q^2$ and $\ell_{\mu\nu}(q) = q_\mu q_\nu/q^2]$, and

$$\widehat{\Delta}_{W\mu\nu}(q) = \frac{t_{\mu\nu}(q)}{q^2 - M_W^2 - \widehat{\Pi}^W_T(q^2)} - \frac{\ell_{\mu\nu}(q)}{M_W^2 - \widehat{\Pi}^W_L(q^2)}. \tag{4}$$

In Eq. (4), we have decomposed $\widehat{\Pi}^W_{\mu\nu} = t_{\mu\nu}\widehat{\Pi}^W_T + \ell_{\mu\nu}\widehat{\Pi}^W_L$.

Next we apply our formalism to the process $\gamma e^- \to \mu^- \bar{\nu}_\mu \nu_e$, in which two W gauge bosons are involved. This process is of potential interest at the LEP2. We concentrate on the part of the amplitude ($\widehat{T}_{1\mu}$) involving the γWW vertex, as given in Fig. 1. As discussed above, the PT method reorders the Feynman graphs into manifestly g.i. subsets. Resumming the PT self-energies one obtains the following resonant transition amplitude:

$$\widehat{T}_{1\mu} = \Gamma_0 \widehat{\Delta}_W (\Gamma_{0\mu}^{\gamma W^- W^+} + \widehat{\Gamma}_\mu^{\gamma W^- W^+}) \widehat{\Delta}_W \Gamma_0 + \Gamma_0 S_0^{(e)} \Gamma_{0\mu}^\gamma \widehat{\Delta}_W \Gamma_0 + \Gamma_0 \widehat{\Delta}_W \Gamma_{0\mu}^\gamma S_0^{(\mu)} \Gamma_0, \tag{5}$$

where $S^{(f)}$ is the free f-fermion propagator and $\Gamma_{0\mu}^\gamma$ is the tree γff coupling. In Eq. (5), contraction over all Lorentz indices except of the photonic one is implied. Since the action of the photonic momentum (q) on the tree-level and one-loop PT γWW vertices gives:

$\frac{1}{e}q^{\mu}\Gamma_{0\,\mu\nu\lambda}^{\gamma W^- W^+} = U_{W\nu\lambda}^{-1}(p_+) - U_{W\nu\lambda}^{-1}(p_-)$ and $\frac{1}{e}q^{\mu}\widehat{\Gamma}_{\mu\nu\lambda}^{\gamma W^- W^+} = \widehat{\Pi}_{\nu\lambda}^{W}(p_-) - \widehat{\Pi}_{\nu\lambda}^{W}(p_+)$, respectively, the $U(1)_{em}$ gauge invariance of this resonant process is restored, $i.e.$ $q^{\mu}\widehat{T}_{1\mu} = 0$. To any loop order, $U(1)_{em}$ and R_{ξ} invariance are warranted by virtue of the tree-type Ward identities that the PT vertex γWW satisfy (all momenta flow into the vertex, $i.e.$, $q + p_- + p_+ = 0$):

$$
\begin{aligned}
\frac{1}{e}q^{\mu}\widehat{\Gamma}_{\mu\nu\lambda}^{\gamma W^- W^+} &= \widehat{\Pi}_{\nu\lambda}^{W}(p_-) - \widehat{\Pi}_{\nu\lambda}^{W}(p_+), \\
\frac{1}{e}[p_-^{\nu}\widehat{\Gamma}_{\mu\nu\lambda}^{\gamma W^- W^+} - M_W\widehat{\Gamma}_{\mu\lambda}^{\gamma G^- W^+}] &= \widehat{\Pi}_{\mu\lambda}^{W}(p_+) - \widehat{\Pi}_{\mu\lambda}^{\gamma}(q) - \frac{c_w}{s_w}\widehat{\Pi}_{\mu\lambda}^{\gamma Z}(q), \\
\frac{1}{e}[p_+^{\lambda}\widehat{\Gamma}_{\mu\nu\lambda}^{\gamma W^- W^+} + M_W\widehat{\Gamma}_{\mu\nu}^{\gamma W^- G^+}] &= -\widehat{\Pi}_{\mu\nu}^{W}(p_-) + \widehat{\Pi}_{\mu\nu}^{\gamma}(q) + \frac{c_w}{s_w}\widehat{\Pi}_{\mu\nu}^{\gamma Z}(q).
\end{aligned}
\tag{6}
$$

Of particular interest in testing electroweak theory at TEVATRON is the process $QQ' \to e^-\bar{\nu}_e\mu^-\mu^+$. In addition to the γWW vertex, the ZWW coupling is now important, especially when the invariant-mass cut $m(\mu^-\mu^+) \simeq M_Z$ is imposed. Noticing that the PT self-energy of the photon and $Z\gamma$ mixing are transverse $[q^{\mu}\widehat{\Pi}_{\mu\lambda}^{\gamma Z}(q) = q^{\mu}\widehat{\Pi}_{\mu\lambda}^{\gamma}(q) = 0]$, we find that the part of $\widehat{T}_1$, in which all tree photonic interactions are absent as shown in Fig. 2, is individually g.i., having the form

$$
\begin{aligned}
\widehat{T}_1^Z &= \Gamma_0\widehat{\Delta}_W(p_-)(\Gamma_0^{ZW^- W^+} + \widehat{\Gamma}^{ZW^- W^+})\widehat{\Delta}_Z(q)\Gamma_0^Z\widehat{\Delta}_W(p_+)\Gamma_0 \\
&+ \Gamma_0 S_0^{(Q)}\Gamma_0^Z\widehat{\Delta}_Z(q)\Gamma_0^Z\widehat{\Delta}_W(p_+)\Gamma_0 + \Gamma_0^Z S_0^{(Q')}\Gamma_0\widehat{\Delta}_Z(q)\Gamma_0^Z\widehat{\Delta}_W(p_+)\Gamma_0 \\
&+ \Gamma_0\widehat{\Delta}_W(p_-)\Gamma_0^Z\widehat{\Delta}_Z(q)\Gamma_0^Z S_0^{(e)}\Gamma_0 + \Gamma_0\widehat{\Delta}_W(p_-)\Gamma_0^Z\widehat{\Delta}_Z(q)\Gamma_0 S_0^{(\nu_e)}\Gamma_0^Z \\
&+ \Gamma_0\widehat{\Delta}_W(p_-)\Gamma_0 S_0^{(\nu_\mu)}\Gamma_0\widehat{\Delta}_W(p_+)\Gamma_0,
\end{aligned}
\tag{7}
$$

where Γ_0^Z stands for the tree Zff coupling. The PT Ward identities maintaining the gauge invariance of this process have been derived in [5].

We continue with some important technical remarks. We first focus on issues of resummation, and argue that the g.i. PT self-energy may be resummed, exactly as one carries out the Dyson summation of the conventional self-energy. The crucial point is that, even though contributions from vertices and boxes are instrumental for the definition of the PT self-energies, their resummation does *not* require a corresponding resummation of

vertex or box parts. In order to construct g.i. chains of self-energy bubbles, one can borrow pinch contributions from existing graphs, which are however not sufficient to convert each $\Pi_{\mu\nu}$ in the chain into the corresponding $\widehat{\Pi}_{\mu\nu}$. If we add the missing pieces by hand, and subsequently subtract them, we notice that: **(i)** The regular string has been converted into a g.i. string, with $\Pi_{\mu\nu} \to \widehat{\Pi}_{\mu\nu}$, and **(ii)** The left-overs, due to the characteristic presence of the inverse bare propagator, $(U^{\mu\nu})^{-1}$, are effectively one-particle *irreducible*. In fact, the left-over terms have the same structure as the one-particle irreducible self-energy graphs, and together with the genuine vertex (V^P) and box pinch contributions (B^P) will convert the conventional self-energy into the g.i. PT self-energy. This procedure is generalizable to an arbitrary order. So, the transverse propagator-like pinch contributions in the Feynman gauge, to a given order n in perturbation theory, have the general form

$$\Pi_n^P(q^2) = (q^2 - m_0^2)V_n^P(q^2) + (q^2 - m_0^2)^2 B_n^P(q^2) + R_n^P(q^2)\,, \tag{8}$$

where R_n^P are the residual pieces of order n. For $n = 2$, for example, it is easy to check that the string $(\frac{1}{q^2-m_0^2})\Pi(\frac{1}{q^2-m_0^2})\Pi(\frac{1}{q^2-m_0^2})$, together with existing pinch pieces from graphs containing vertices, needs an additional amount $-R_2^P$, given by

$$- R_2^P(q^2) \;=\; \Pi V_1^P + \frac{3}{4}(q^2 - m_0^2)V_1^P V_1^P\,, \tag{9}$$

in order to be converted into the g.i. string $(\frac{1}{q^2-m_0^2})\widehat{\Pi}(\frac{1}{q^2-m_0^2})\widehat{\Pi}(\frac{1}{q^2-m_0^2})$. However, R_2^P will be absorbed by the one-particle irreducible two-loop self-energy shown in Fig. 3. In general, the R_n^P terms consist of products of lower order conventional self-energies $\Pi_k(q^2)$, and lower order pinch contributions V_ℓ^P and (or) B_ℓ^P, with $k + \ell = n$ [4].

Another issue is whether the g.i. PT complex pole is identical to the g.i. physical pole of the amplitude. Here we concentrate on the case of a stable particle, and demonstrate how its mass does not get shifted by the PT. The masses m and $\widehat{m}$ are respectively defined as the solution of the equations: $m^2 = m_0^2 + \Pi(m^2)$ and $\widehat{m}^2 = m_0^2 + \widehat{\Pi}(\widehat{m}^2)$. In perturbation theory, $m^2 = m_0^2 + \sum_1^\infty g^{2n} C_n$ and $\widehat{m}^2 = m_0^2 + \sum_1^\infty g^{2n} \widehat{C}_n$, and one has hence to show that

$C_n - \widehat{C}_n = O(g^{2n+1})$. To zeroth order $m^2 = \widehat{m}^2 = m_0^2$. Similarly, from Eq. (8), using the fact that $B_1^P = 0$ (in the Feynman gauge), and $R_1^P=0$ (in any gauge), we have that $C_1 = \widehat{C}_1$, because the pinch contribution $(q^2 - m_0^2)V_1^P$ is of $O(g^4)$. The non-trivial step in generalizing this proof to higher orders is to observe that not all pinch contributions of Eq. (8) contribute terms of higher order. To be precise, the terms of R_n^P which do not have the characteristic factor $(q^2 - m_0^2)$ in front are *not* of higher order, and are instrumental for our proof. We will illustrate this point at the two-loop order. The second order m^2 and $\widehat{m}^2$ are given by:

$$m^2 = m_0^2 + \Pi_1(m^2) + \Pi_2(m^2)$$

$$\widehat{m}^2 = m_0^2 + \Pi_1(\widehat{m}^2) + \Pi_2(\widehat{m}^2) + \Pi_1^P + \Pi_2^P$$

where the subscripts 1 and 2 denote loop order, and

$$\Pi_1^P(\widehat{m}^2) + \Pi_2^P(\widehat{m}^2) = (\widehat{m}^2 - m_0^2)[V_1^P(\widehat{m}^2) + V_2^P(\widehat{m}^2)] + (\widehat{m}^2 - m_0^2)^2[B_1^P(\widehat{m}^2) + B_2^P(\widehat{m}^2)]$$

$$+ R_2^P(\widehat{m}^2). \tag{10}$$

It is not difficult to show that $\Pi_1^P(\widehat{m}^2) + \Pi_2^P(\widehat{m}^2) = O(g^6)$. Substituting $\widehat{m}^2 - m_0^2 = \Pi_1(\widehat{m}^2) + O(g^4)$ into Eq. (10), and neglecting terms of $O(g^6)$ or higher, we find

$$\Pi_1^P(\widehat{m}^2) + \Pi_2^P(\widehat{m}^2) = R_2^P(\widehat{m}^2) + \Pi_1(\widehat{m}^2)V_1^P(\widehat{m}^2) + O(g^6) = 0 + O(g^6),$$

where we have also used Eq. (9) at $q^2 = \widehat{m}^2$. The generalization of the proof to an arbitrary order n in perturbation theory proceeds by induction and will be given in Ref. [4], together with the case of an unstable particle—both mass and width remain unshifted.

Another point, important for unitarity, is whether the PT self-energy contains any unphysical absorptive parts. In particular, the propagator-like part $\widehat{T}_1$ of a reaction should contain imaginary parts associated with physical Landau singularities only, whereas the unphysical poles related to Goldstone bosons and ghosts must vanish in the loop. Explicit

calculations (see Ref. [4]) show that, indeed, our g.i. procedure does not introduce any fixed unphysical poles. Here we offer only a qualitative argument in that vein, namely that the PT results may be obtained equally well if one works *directly* in the unitary gauge, where only physical Landau poles are present.

Although our discussion has been restricted to the W and Z gauge bosons, our considerations are also valid for the heavy top quark, and will provide a self-consistent framework for investigating the CP properties of the t quark at LHC. Moreover, our analytic g.i. approach can be straightforwardly extended to analyze possible new-physics phenomena induced by non-SM gauge bosons, such as the bosons W_R, Z', *etc.*, predicted in $SO(10)$ or E_6 unified models [6]. Since our analytic method treats bosonic and fermionic contributions equally, it can provide a consistent framework for the study of the resonant dynamics of a heavy Higgs boson and of a strong Higgs sector at the LHC.

References

[1] M. Veltman, *Physica* **29**, 186 (1963).

[2] A. Pilaftsis, *Z. Phys.* **C47**, 95 (1990); S. Willenbrock and G. Valencia, *Phys. Lett.* **B259**, 373 (1991); R.G. Stuart, *Phys. Lett.* **B262**, 113 (1991); *Phys. Lett.* **B272**, 353 (1991); *Phys. Rev. Lett.* **70**, 3193 (1993); Michigan of University report (1995), UM-TH-95-13; A. Sirlin, *Phys. Rev. Lett.* **67**, 2127 (1991); *Phys. Lett.* **B267**, 240 (1991); M. Nowakowski and A. Pilaftsis, *Z. Phys.* **C60**, 121 (1993); G. López Castro, J. L. Lucio M., and J. Pestieau, *Mod. Phys. Lett.* **6**, 3679 (1991); hep-ph/9504351; H. Veltman, *Z. Phys.* **C62**, 35 (1994); A. Aeppli, G.J. van Oldenborgh, and D. Wyler, *Nucl. Phys.* **B428**, 126 (1994); U. Baur and D. Zeppenfeld, Madison preprint (1995), MAD/PH/878; C.G. Papadopoulos, Durham preprint (1995), DTP/95/20.

[3] J.M. Cornwall, *Phys. Rev.* **D26**, 1453 (1982); J. M. Cornwall and J. Papavassiliou, *Phys. Rev.* **D40**, 3474 (1989); J. Papavassiliou, *Phys. Rev.* **D41**, 3179 (1990); G. Degrassi and A. Sirlin, *Phys. Rev.* **D46**, 3104 (1992); G. Degrassi, B. Kniehl, and A. Sirlin, *Phys. Rev.* **D48**, R3963 (1993); J. Papavassiliou and A. Sirlin, *Phys. Rev.* **D50**, 5951 (1994); J. Papavassiliou, *Phys. Rev.* **D50**, 5958 (1994); N. J. Watson, *Phys. Lett* **B349**, 155 (1995).

[4] J. Papavassiliou and A. Pilaftsis, New York University and Rutherford Lab. reports (1995), NYU-TH/95-03 and RAL-TR/95-022.

[5] J. Papavassiliou and K. Philippides, *Phys. Rev.* **D48**, 4255 (1993); J. Papavassiliou and K. Philippides, hep-ph/9503377, to appear in *Phys. Rev. D.*

[6] For reviews, see, *e.g.*, P. Langacker, *Phys. Rep.* **C72**, 185 (1981); R.N. Mohapatra, *Unification and Supersymmetry*, Springer-Verlag, New York, (1986).

Figure Captions

Fig. 1:   The process $e^-\gamma \to \mu^-\bar{\nu}_\mu\nu_e$ in our PT approach.

Fig. 2:   The reaction $QQ' \to \mu^+\mu^-e^-\bar{\nu}_e$, where photonic and $Z\gamma$-mixing graphs are not shown.

Fig. 3:   Typical two-loop self-energy graphs (a)–(d), and some of the residual pinch contributions (e)–(h) contained in R_2^P.
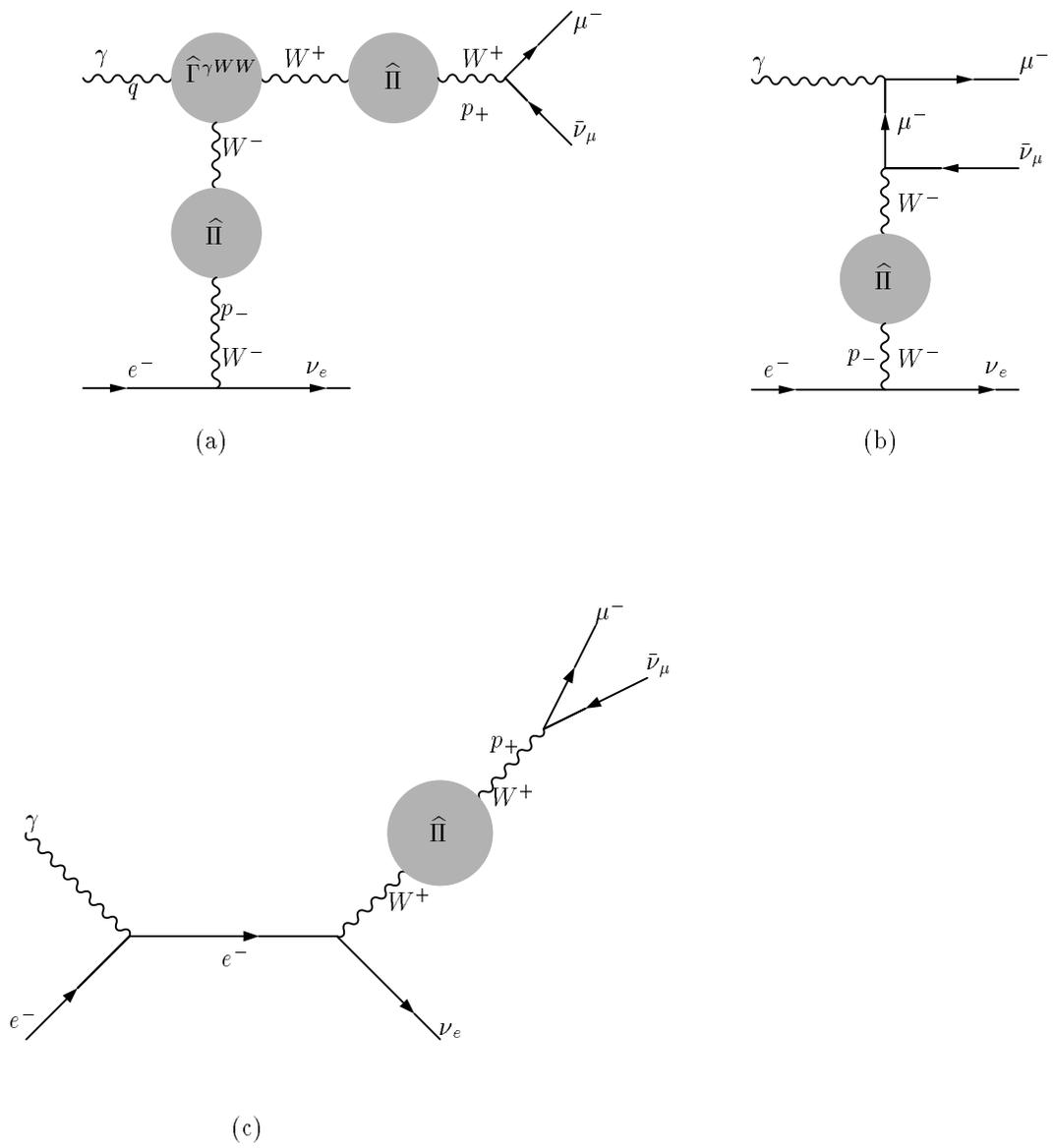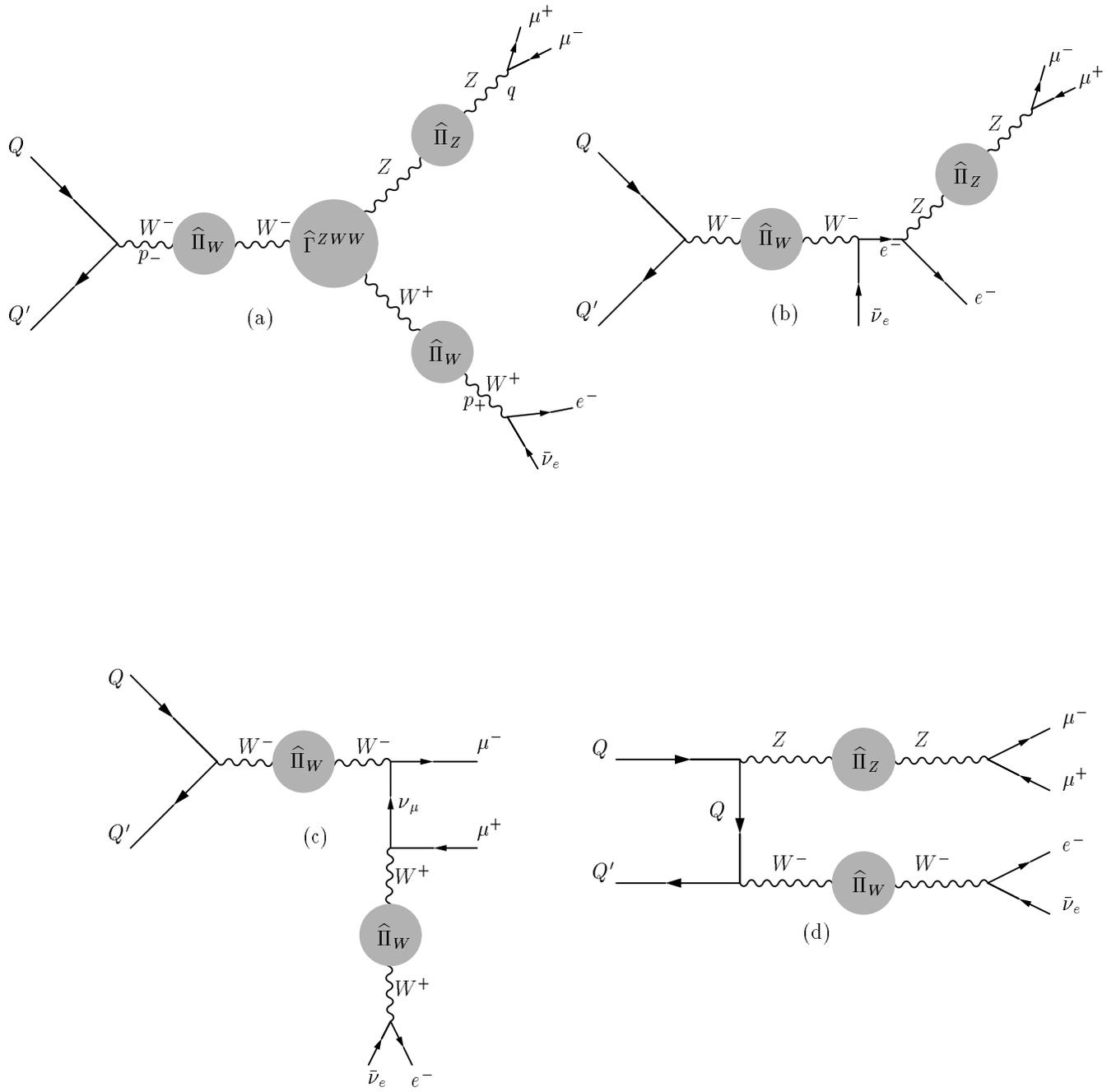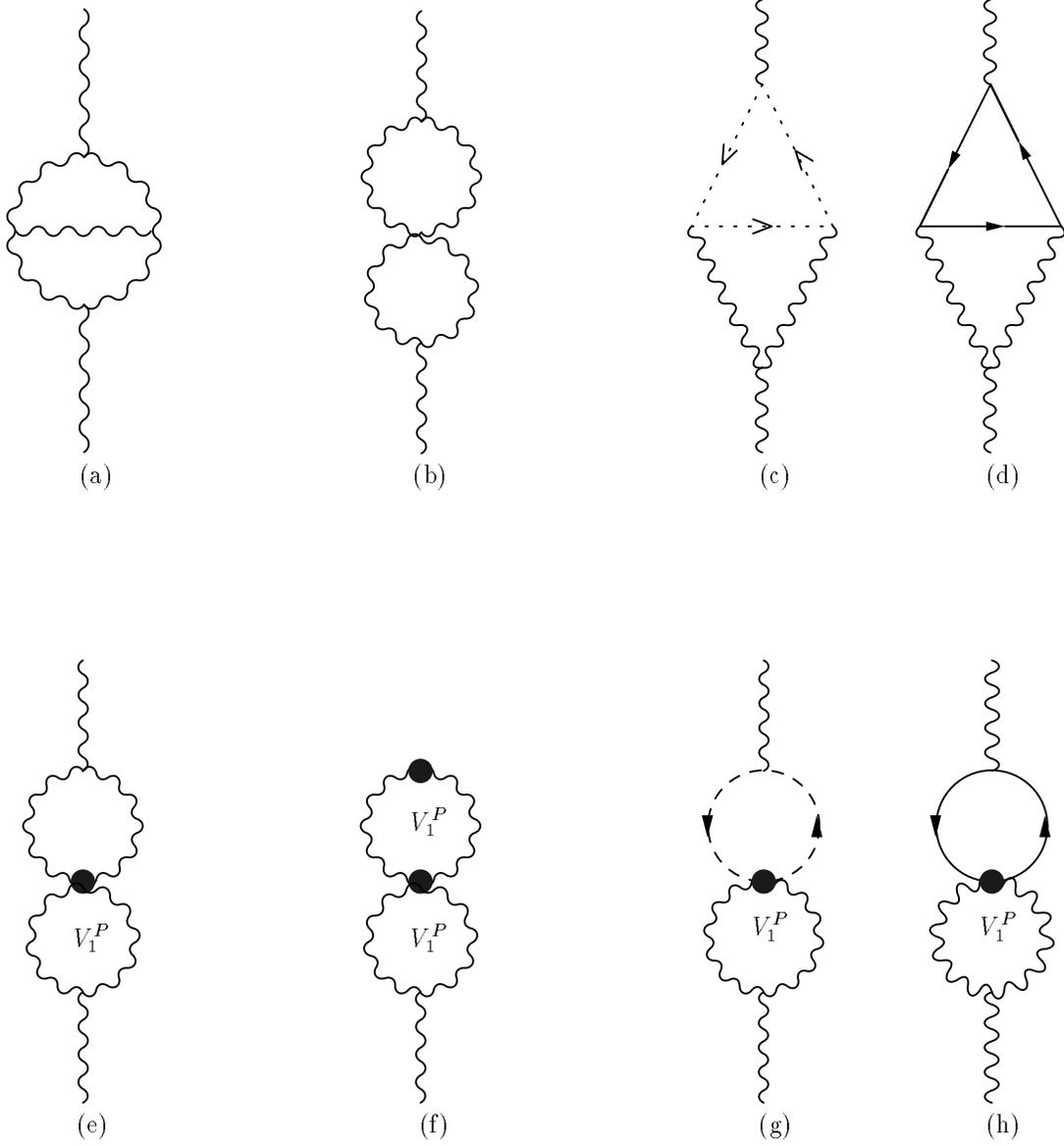
Figure 1

Figure 2

(a)   (b)   (c)   (d)

(e)   (f)   (g)   (h)

Figure 3