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Standard Model Higher Order Corrections to the $WW\gamma/WWZ$ Vertex

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Using the S–matrix pinch technique we obtain to one loop order gauge independent $\gamma W^- W^+$ and $ZW^- W^+$ vertices in the context of the standard model, with all incoming momenta off–shell. We show that the vertices so constructed satisfy simple QED–like Ward identities. These gauge invariant vertices give rise to expressions for the magnetic dipole and electric quadrupole form factors of the W gauge boson, which, unlike previous treatments, satisfy the crucial properties of infrared finiteness and perturbative unitarity. [1]

INTRODUCTION

A new and largely unexplored frontier on which the ongoing search for new physics will soon focus is the study of the structure of the three-boson couplings (1). A general parametrization of the trilinear gauge boson vertex for two on–shell Ws and one off–shell $V = \gamma,\ Z$ is (2)

$$\begin{aligned}\Gamma^V_{\mu\alpha\beta} = \quad & -ig_V \Big[\ f\,[\ 2g_{\alpha\beta}\Delta_\mu + 4(g_{\alpha\mu}Q_\beta - g_{\beta\mu}Q_\alpha)\ ] \\ & +\ 2\Delta\kappa_V\ (g_{\alpha\mu}Q_\beta - g_{\beta\mu}Q_\alpha) \\ & +\ 4\tfrac{\Delta Q_V}{M_W^2}(\Delta_\mu Q_\alpha Q_\beta - \tfrac{1}{2}Q^2 g_{\alpha\beta}\Delta_\mu)\ \Big]\ , \end{aligned} \tag{1}$$

with $g_\gamma = gs$, $g_Z = gc$, where g is the $SU(2)$ gauge coupling, $s \equiv sin\theta_W$ and $c \equiv cos\theta_W$. In the above formula terms which are odd under the individual discrete symmetries of C, P, or T have been omitted. The four-momenta Q and Δ are related to the incoming momenta q, p_1 and p_2 of the gauge bosons V, $W^-$and W^+ respectively, by $q = 2Q$, $p_1 = \Delta - Q$ and $p_2 = -\Delta - Q$. The form factors $\Delta\kappa_V$ and ΔQ_V, also defined as

$$\Delta\kappa_V = \kappa_V + \lambda_V - 1\ , \tag{2}$$

and

$$\Delta Q_V = -2\lambda_V\ , \tag{3}$$

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are compatible with C, P, and T invariance, and are related to the magnetic dipole moment μ_W and the electric quadrupole moment Q_W, by the following expressions (3), (4), (5), (6):

$$\mu_W = \frac{e}{2M_W}(2 + \Delta\kappa_\gamma) \ , \tag{4}$$

and

$$Q_W = -\frac{e}{M_W^2}(1 + \Delta\kappa_\gamma + \Delta Q_\gamma) \ . \tag{5}$$

In the context of the standard model, their canonical, tree level values, are $f = 1$ and $\Delta\kappa_V = \Delta Q_V = 0$. To determine the radiative corrections to these quantities one must cast the resulting one–loop expressions in the following form:

$$\Gamma^V_{\mu\alpha\beta} = -ig_V[a_1^V g_{\alpha\beta}\Delta_\mu + a_2^V(g_{\alpha\mu}Q_\beta - g_{\beta\mu}Q_\alpha) + a_3^V \Delta_\mu Q_\alpha Q_\beta] \ , \tag{6}$$

where a_1^V, a_2^V, and a_3^V are complicated functions of the momentum transfer Q^2, and the masses of the particles appearing in the loops. It then follows that $\Delta\kappa_V$ and ΔQ_V are given by the following expressions:

$$\Delta\kappa_V = \frac{1}{2}(a_2^V - 2a_1^V - Q^2 a_3^V) \tag{7}$$

and

$$\Delta Q_V = \frac{M_W^2}{4} a_3^V \ . \tag{8}$$

Calculating the one-loop expressions for $\Delta\kappa_V$ and ΔQ_V is a non-trivial task, both from the technical and the conceptual point of view. If one calculates just the Feynman diagrams contributing to the γW^+W^- vertex and then extracts from them the contributions to $\Delta\kappa_\gamma$ and ΔQ_γ, one arrives at expressions that are plagued with several pathologies, gauge-dependence being one of them. Indeed, even if the two W are considered to be on shell, since the incoming photon is not, there is no *a priori* reason why a gauge-independent answer should emerge. In the context of the renormalizable R_ξ gauges the final answer depends on the choice of the gauge fixing parameter ξ, which enters into the one-loop calculations through the gauge-boson propagators (W, Z, γ, and unphysical scalar particles). In addition, as shown by an explicit calculation performed in the Feynman gauge ($\xi = 1$), the answer for $\Delta\kappa_\gamma$ is *infrared divergent* and violates perturbative unitarity, e.g. it grows monotonically for $Q^2 \to \infty$ (7). Clearly, regardless of the measurability of quantities like $\Delta\kappa_\gamma$ and ΔQ_γ, from the theoretical point of view one should at least be able to satisfy such crucial requirements as gauge-independence and infrared finiteness, when calculating the model's prediction for them. Indeed, all the above pathologies may be circumvented if one adopts the pinch technique

(PT), first invented by Cornwall (8). The application of this method gives rise to new expressions, $\hat{\Delta}\kappa_\gamma$ and $\hat{\Delta}Q_\gamma$, which are gauge fixing parameter (ξ) independent, ultraviolet *and* infrared finite, and well behaved for large momentum transfers Q^2 (9).

I. THE PINCH TECHNIQUE

The simplest example that demonstrates how the PT works is the gluon two point function (10). Consider the S-matrix element T for the elastic scattering such as $q_1\bar{q}_2 \to q_1\bar{q}_2$, where q_1,q_2 are two on-shell test quarks with masses m_1 and m_2. To any order in perturbation theory T is independent of the gauge fixing parameter ξ. On the other hand, as an explicit calculation shows, the conventionally defined proper self-energy depends on ξ. At the one loop level this dependence is canceled by contributions from other graphs, which, at first glance, do not seem to be propagator-like. That this cancellation must occur and can be employed to define a gauge-independent self-energy, is evident from the decomposition:

$$T(s,t,m_1,m_2) = T_0(t,\xi) + \sum_{i=1}^{2} T_i(t,m_i,\xi) + T_3(s,t,m_1,m_2,\xi) \ , \qquad (9)$$

where the function $T_0(t,\xi)$ depends kinematically only on the Mandelstam variable $t = -(\hat{p}_1 - p_1)^2 = -q^2$, and not on $s = (p_1 + p_2)^2$ or on the external masses. Typically, self-energy, vertex, and box diagrams contribute to T_0, T_1, T_2, and T_3, respectively. Such contributions are ξ dependent, in general. However, as the sum $T(s,t,m_1,m_2)$ is gauge-independent, it is easy to show that Eq(9) can be recast in the form

$$T(s,t,m_1,m_2) = \hat{T}_0(t) + \hat{T}_1(t,m_1) + \hat{T}_2(t,m_2) + \hat{T}_3(s,t,m_1,m_2) \ , \qquad (10)$$

where the $\hat{T}_i$ $(i = 0,1,2,3)$ are *individually* ξ-independent. The propagator-like parts of vertex and box graphs which enforce the gauge independence of $T_0(t)$, are called pinch parts. They emerge every time a gluon propagator or an elementary three-gluon vertex contributes a longitudinal k_μ to the original graph's numerator. The action of such a term is to trigger an elementary Ward identity of the form

$$\not{k} = (\not{p} + \not{k} - m) - (\not{p} - m) \qquad (11)$$

when it gets contracted with a γ matrix. The first term removes (pinches out) the internal fermion propagator, whereas the second vanishes on shell. From the gauge-independent functions $\hat{T}_i$ $(i = 0,1,2,3)$ one may now extract a gauge-independent effective gluon (G) self-energy $\hat{\Pi}_{\mu\nu}(q)$, gauge-independent $Gq_i\bar{q}_i$ vertices $\hat{\Gamma}^{(i)}_\mu$, and a gauge-independent box $\hat{B}$, in the following way:

$$\begin{aligned}
\hat{T}_0 &= g^2\bar{u}_1\gamma^\mu u_1[(\tfrac{1}{q^2})\hat{\Pi}_{\mu\nu}(q)(\tfrac{1}{q^2})]\bar{u}_2\gamma^\nu u_2 \ , \\
\hat{T}_1 &= g^2\bar{u}_1\hat{\Gamma}^{(1)}_\nu u_1(\tfrac{1}{q^2})\bar{u}_2\gamma^\nu u_2 \ , \\
\hat{T}_2 &= g^2\bar{u}_1\gamma^\mu u_1(\tfrac{1}{q^2})\bar{u}_2\hat{\Gamma}^{(2)}_\nu u_2 \ , \\
\hat{T}_3 &= \hat{B} \ ,
\end{aligned} \tag{12}$$

where u_i are the external spinors, and g is the gauge coupling. Since all hatted quantities in the above formula are gauge-independent, their explicit form may be calculated using any value of the gauge-fixing parameter ξ, as long as one properly identifies and allots all relevant pinch contributions. The choice $\xi = 1$ simplifies the calculations significantly, since it eliminates the longitudinal part of the gluon propagator. Therefore, for $\xi = 1$ the pinch contributions originate only from momenta carried by the elementary three-gluon vertex The one-loop expression for $\hat{\Pi}_{\mu\nu}(q)$ is given by (10) :

$$\hat{\Pi}_{\mu\nu}(q) = \Pi^{(\xi=1)}_{\mu\nu}(q) + t_{\mu\nu}\Pi^P(q) \ , \tag{13}$$

where

$$t_{\mu\nu} = (g_{\mu\nu}q^2 - q_\mu q_\nu) \tag{14}$$

and

$$\Pi^P(q) = -2ic_a g^2 \int_n \frac{1}{k^2(k+q)^2} \ , \tag{15}$$

where $\int_n \equiv \int \frac{d^n k}{(2\pi)^n}$ is the dimensionally regularized loop integral, and c_a is the quadratic Casimir operator for the adjoint representation [for $SU(N)$, $c_a = N$]. After integration and renormalization we find

$$\Pi^P(q) = -2c_a(\frac{g^2}{16\pi^2})\ln(\frac{-q^2}{\mu^2})] \ . \tag{16}$$

Adding this to the Feynman-gauge proper self-energy

$$\Pi^{(\xi=1)}_{\mu\nu}(q) = -[\frac{5}{3}c_a(\frac{g^2}{16\pi^2})\ln(\frac{-q^2}{\mu^2})]t_{\mu\nu} \ , \tag{17}$$

we find for $\hat{\Pi}_{\mu\nu}(q)$

$$\hat{\Pi}_{\mu\nu}(q) = -bg^2\ln(\frac{-q^2}{\mu^2})t_{\mu\nu} \ , \tag{18}$$

where $b = \frac{11c_a}{48\pi^2}$ is the coefficient of $-g^3$ in the usual β function.

This procedure can be extended to an arbitrary n-point function; of particular physical interest are the gauge-independent three and four point functions

$\hat{\Gamma}_{\mu\nu\alpha}$ (11) and $\hat{\Gamma}_{\mu\nu\alpha\beta}$ (12), which at one-loop satisfy the following *tree-level* Ward identities:

$$\begin{aligned} q_1^\mu \hat{\Gamma}_{\mu\nu\alpha}(q_1,q_2,q_3) &= t_{\nu\alpha}(q_2)\hat{d}^{-1}(q_2) - t_{\nu\alpha}(q_3)\hat{d}^{-1}(q_3) \\ q_1^\mu \hat{\Gamma}^{abcd}_{\mu\nu\alpha\beta} &= f_{abp}\hat{\Gamma}^{cdp}_{\nu\alpha\beta}(q_1+q_2,q_3,q_4) + c.p. \ , \end{aligned} \tag{19}$$

where $\hat{d} = \left[q^2 - \hat{\Pi}(q)\right]^{-1}$ and f^{abc} the structure constants of the gauge group.

Finally, the generalization of the PT to the case of non-conserved external currents is technically more involved (13). The main reasons are the following:

(a) The charged W couples to fermions with different, non-vanishing masses $m_i, m_j \neq 0$, and consequently the elementary Ward identity of Eq.(11) gets modified to :

$$k_\mu \gamma^\mu P_L \equiv \not{k} P_L = S_i^{-1}(p+k)P_L - P_R S_j^{-1}(p) + m_i P_L - m_j P_R \tag{20}$$

where

$$P_{R,L} = \frac{1 \pm \gamma_5}{2} \tag{21}$$

are the chirality projection operators. The first two terms of Eq(20) will pinch and vanish on shell, respectively, as they did before. But in addition, a term proportional to $m_i P_L - m_j P_R$ is left over. In a general R_ξ gauge such terms give rise to extra propagator and vertex-like contributions, not present in the massless case.

(b) Additional graphs involving the "unphysical" would-be Goldstone bosons χ and ϕ, and physical Higgs H, which do not couple to massless fermions, must now be included. Such graphs give rise to new pinch contributions, even in the Feynman gauge, due to the momenta carried by interaction vertices such as $\gamma\phi^+\phi^-$, $Z\phi^+\phi^-$, $W^+\phi^-\chi$, $HW^+\phi^-$, e.g. vertices with one vector gauge boson and two scalar bosons.

II. THE CURRENT ALGEBRA FORMULATION OF THE PINCH TECHNIQUE

We now present an alternative formulation of the PT introduced in the context of the standard model (14). In this approach the interaction of gauge bosons with external fermions is expressed in terms of current correlation functions (15), i.e. matrix elements of Fourier transforms of time-ordered products of current operators. This is particularly economical because these amplitudes automatically include several closely related Feynman diagrams. When one of the current operators is contracted with the appropriate four-momentum, a Ward identity is triggered. The pinch part is then identified with the contributions involving the equal-time commutators in the Ward identities, and therefore involve amplitudes in which the number of current

operators has been decreased by one or more. A basic ingredient in this formulation are the following equal-time commutators;

$$\begin{aligned}
&\delta(x_0-y_0)[J^0_W(x),J^\mu_Z(y)] = c^2 J^\mu_W(x)\delta^4(x-y)\ ,\\
&\delta(x_0-y_0)[J^0_W(x),J^{\mu\dagger}_W(y)] = -J^\mu_3(x)\delta^4(x-y)\ ,\\
&\delta(x_0-y_0)[J^0_W(x),J^\mu_\gamma(y)] = J^\mu_W(x)\delta^4(x-y)\ ,\\
&\delta(x_0-y_0)[J^0_V(x),J^\mu_{V'}(y)] = 0\ ,
\end{aligned} \tag{22}$$

where $J^\mu_3 \equiv 2(J^\mu_Z + s^2 J^\mu_\gamma)$ and $V, V' \in \{\gamma, Z\}$. To demonstrate the method with an example, consider the vertex Γ_μ, where now the gauge particles in the loop are W instead of gluons and the incoming and outgoing fermions are massless. It can be written as follows (with $\xi = 1$):

$$\Gamma_\mu = \int \frac{d^4k}{2\pi^4}\Gamma_{\mu\alpha\beta}(q,k,-k-q)\int d^4x e^{ikx} < f|T^*[J^{\alpha\dagger}_W(x)J^\beta_W(0)]|i > \ . \tag{23}$$

When an appropriate momentum, say k_α, from the vertex is pushed into the integral over dx, it gets transformed into a covariant derivative $\frac{d}{dx_\alpha}$ acting on the time ordered product $< f|T^*[J^{\alpha\dagger}_W(x)J^\beta_W(0)]|i >$. After using current conservation and differentiating the θ-function terms, implicit in the definition of the T^* product, we end up with the left-hand side of the second of Eq(22). So, the contribution of each such term is proportional to the matrix element of a single current operator, namely $< f|J^\mu_3|i >$; this is precisely the pinch part. Calling Γ^P_μ the total pinch contribution from the Γ_μ of Eq(23), we find that

$$\Gamma^P_\mu = -g^3 c I_{WW}(Q^2) < f|J^\mu_3|i > \ , \tag{24}$$

where

$$I_{ij}(q) = i\int_n \frac{1}{(k^2-M_i^2)[(k+q)^2-M_j^2]}\ . \tag{25}$$

Obviously, the integral in Eq(25) is the generalization of the QCD expression Eq(15) to the case of massive gauge bosons.

III. GAUGE–INVARIANT GAUGE BOSON VERTICES AND THEIR WARD IDENTITIES

We consider the S-matrix element for the process

$$e^- + \nu + e^- \to e^- + e^- + \overline{\nu}\ . \tag{26}$$

and isolate the part $T(q,p_1,p_2)$ of the S–matrix which depends only on the momentum transfers q, p_1, and p_2. The tree-level vector-boson propagator $\Delta^i_{\mu\nu}(q)$ in the R_ξ gauges is given by

$$\Delta_i^{\mu\nu}(q,\xi_i) = \frac{1}{q^2 - M_i^2}[g^{\mu\nu} - (1-\xi_i)\frac{q^\mu q^\nu}{q^2 - \xi_i M_i^2}] \quad , \tag{27}$$

with $i = W, Z, \gamma$, and $M_\gamma = 0$. Its inverse $\Delta_i^{-1}(q,\xi_i)^{\mu\nu}$ is given by

$$\Delta_i^{-1}(q,\xi)^{\mu\nu} = (q^2 - M_i^2)g^{\mu\nu} - q^\mu q^\nu + \frac{1}{\xi_i} q^\mu q^\nu \quad . \tag{28}$$

The propagators $\Delta_s(q,\xi_i)$ of the unphysical (would–be) Goldstone bosons are given by

$$\Delta_s(q,\xi_i) = \frac{-1}{q^2 - \xi_i M_i^2} \quad , \tag{29}$$

with $(s,i) = (\phi, W)$ or (χ, Z) and explicitly depend on ξ_i. On the other hand, the propagators of the fermions (quarks and leptons), as well as the propagator of the physical Higgs particle are ξ_i-independent at tree-level.

Since the final result (with pinch contributions included) is gauge-independent, we choose to work in the Feynman gauge ($\xi_i = 1$); this particular gauge simplifies the calculations because it removes all longitudinal parts from the tree-level gauge boson propagators. So, pinch contributions can only originate from appropriate momenta furnished by the tree–level gauge boson vertices. Applying the pinch technique algorithm we isolate all vertex–like parts contained in the box diagrams and allot them to the usual vertex graphs. The final expressions for one loop gauge-independent trilinear gauge boson vertices are :

$$\begin{aligned}\frac{1}{g^3 s}\widehat{\Gamma}^{\gamma W^- W^+}_{\mu\alpha\beta} &= \Gamma^{\gamma W^- W^+}_{\mu\alpha\beta}|_{\xi_i=1} + q^2\ B_{\mu\alpha\beta} + U_W^{-1}(p_1)^\rho_\alpha B^+_{\mu\rho\beta} + U_W^{-1}(p_2)^\rho_\beta B^-_{\mu\alpha\rho} \\ &\quad -2\Omega\Gamma_{\mu\alpha\beta} + p_{2\beta} g_{\mu\alpha}\ \mathcal{M}^- + p_{1\alpha} g_{\mu\beta}\ \mathcal{M}^+ \quad ,\end{aligned} \tag{30}$$

$$\begin{aligned}\frac{1}{g^3 c}\widehat{\Gamma}^{Z W^- W^+}_{\mu\alpha\beta} &= \Gamma^{Z W^- W^+}_{\mu\alpha\beta}|_{\xi_i=1} + U_Z^{-1}(q)^\rho_\mu B_{\rho\alpha\beta} + U_W^{-1}(p_1)^\rho_\alpha B^+_{\mu\rho\beta} \\ &\quad + U_W^{-1}(p_2)^\rho_\beta B^-_{\mu\alpha\rho} - 2\Omega\Gamma_{\mu\alpha\beta} + q_\mu g_{\alpha\beta}\ M_Z^2\ \mathcal{M} \\ &\quad + p_{2\beta} g_{\mu\alpha}\ M_W^2\ \mathcal{M}^- + p_{1\alpha} g_{\mu\beta}\ M_W^2\ \mathcal{M}^+ \quad ,\end{aligned} \tag{31}$$

where

$$\Omega = I_{WW}(q) + s^2 I_{W\gamma}(p_1) + c^2 I_{WZ}(p_1) + s^2 I_{W\gamma}(p_2) + c^2 I_{WZ}(p_2) \ , \tag{32}$$

and

$$\mathcal{M}^-(q,p_1,p_2) = \frac{s^2}{c^2} J_{WW\gamma} + \frac{1-2s^2}{2c^2}\ J_{WWZ} + \frac{1}{2}\ J_{WWH} + \frac{1}{2c^2}\ J_{ZHW} \quad , \tag{33}$$

with

$$J_{ABC} = \int_n \frac{1}{[(k+p_1)^2 - M_A^2]\,[(k-p_2)^2 - M_B^2]\,[k^2 - M_C^2]} \ , \tag{34}$$

and the property

$$\mathcal{M}^+(q,p_1,p_2) = -\mathcal{M}^-(q,p_2,p_1) \ . \tag{35}$$

The gauge-independent vertices satisfy the following simple Ward identities (WI), relating them to the W self energy and χWW vertex constructed also via the PT :

$$q^\mu \widehat{\Gamma}^{ZW^-W^+}_{\mu\alpha\beta} + iM_Z \widehat{\Gamma}^{\chi W^-W^+}_{\alpha\beta} = gc\ \left[\widehat{\Pi}^W_{\alpha\beta}(1) - \widehat{\Pi}^W_{\alpha\beta}(2)\right] \quad , \tag{36}$$

$$q^\mu \widehat{\Gamma}^{\gamma W^-W^+}_{\mu\alpha\beta} = gs\ \left[\widehat{\Pi}^W_{\alpha\beta}(1) - \widehat{\Pi}^W_{\alpha\beta}(2)\right] \quad . \tag{37}$$

These WI are the one–loop generalizations of the respective *tree level* WI; their validity is crucial for the gauge independence of the S–matrix. It is important to emphasize that they make no reference to ghost terms, unlike the corresponding Slavnov-Taylor identities satisfied by the conventional, gauge–dependent vertices.

For the case of *on–shell* Ws one sets $p_1^2 = p_2^2 = M_W^2$ and neglects all terms proportional to $p_{1\alpha}$ and $p_{2\beta}$, as well as the left over pinch terms of the W legs. Then the γWW vertex reduces to the form

$$\frac{1}{g^3 s}\widehat{\Gamma}^{\gamma W^-W^+}_{\mu\alpha\beta} = \Gamma^{\gamma W^-W^+}_{\mu\alpha\beta}|_{\xi_i=1} + q^2\ B_{\mu\alpha\beta}(q,p_1,p_2) - 2\Gamma_{\mu\alpha\beta} I_{WW}(q) \quad . \tag{38}$$

This is of course the same answer one obtains by applying the PT *directly* to the S–matrix of $e^+e^- \to W^+W^-$. Thus for the form factors $\Delta\kappa_\gamma,\ \Delta Q_\gamma$ the only function we need is $B_{\mu\alpha\beta}$, given below

$$g^2 B_{\mu\alpha\beta} = \sum_{V=\gamma Z} g_V^2 \int_n \frac{iR_{\alpha\beta\mu}}{[(k+p_1)^2 - M_W^2]\,[(k-p_2)^2 - M_W^2]\,[k^2 - M_V^2]} \ , \tag{39}$$

with

$$R_{\alpha\beta\mu} = g_{\alpha\beta}\ (k - \frac{3}{2}(p_1 - p_2))_\mu - g_{\alpha\mu}\ (3k+2q)_\beta - g_{\beta\mu}\ (3k-2q)_\alpha \ . \tag{40}$$

IV. MAGNETIC DIPOLE AND ELECTRIC QUADRUPOLE FORM FACTORS FOR THE W

Having constructed the gauge-independent γWW vertex we proceed to extract its contributions to the magnetic dipole and electric quadrupole form

factors of the W. We use carets to denote the gauge independent one–loop contributions. Clearly,

$$\hat{\Delta}\kappa_\gamma = \Delta\kappa_\gamma^{(\xi=1)} + \Delta\kappa_\gamma^P \ , \tag{41}$$

and

$$\hat{\Delta}Q_\gamma = \Delta Q_\gamma^{(\xi=1)} + \Delta Q_\gamma^P \ , \tag{42}$$

where $\Delta Q_\gamma^{(\xi=1)}$ and $\Delta Q_\gamma^{(\xi=1)}$ are the contributions of the usual vertex diagrams in the Feynman gauge (7), whereas ΔQ_γ^P and ΔQ_γ^P the analogous contributions from the pinch parts. The task of actually calculating $\hat{\Delta}\kappa_\gamma$ and $\hat{\Delta}Q_\gamma$ is greatly facilitated by the fact that the quantities $\Delta\kappa_\gamma^{(\xi=1)}$ and $\Delta Q_\gamma^{(\xi=1)}$ have already been calculated in (7). It must be emphasized however that the expression for $\Delta\kappa_\gamma^{(\xi=1)}$ (but not $\Delta Q_\gamma^{(\xi=1)}$) is infrared *divergent* for $Q^2 \neq 0$ due to the presence of the following double integral over the Feynman parameters (t,a), given in Eq.(26) of (7):

$$\begin{aligned} R &= -(\tfrac{\alpha_\gamma}{\pi})\tfrac{Q^2}{M_W^2}\int_0^1 da \int_0^1 \tfrac{dtt}{t^2 - t^2(1-a)a(\frac{4Q^2}{M_W^2})} \\ &= -(\tfrac{\alpha}{2\pi})\tfrac{Q^2}{M_W^2}\int_0^1 \tfrac{da}{1-(1-a)a(\frac{4Q^2}{M_W^2})}) \int_0^1 \tfrac{dt}{t} \end{aligned} \tag{43}$$

By performing the momentum integration in $B_{\mu\alpha\beta}$, we find for $p_1^2 = p_2^2 = M_W^2$

$$B_{\mu\alpha\beta} = -\frac{Q^2}{8\pi^2 M_W^2}\sum_{V=\gamma,Z} g_V^2 \int_0^1 da \int_0^1 (2tdt)\frac{F_{\mu\alpha\beta}}{L_V^2} \ , \tag{44}$$

where

$$F_{\mu\alpha\beta} = 2(\frac{3}{2} + at)g_{\alpha\beta}\Delta_\mu + 2(3at+2)[g_{\alpha\mu}Q_\beta - g_{\beta\mu}Q_\alpha] \ , \tag{45}$$

and

$$L_V^2 = t^2 - t^2 a(1-a)(\frac{4Q^2}{M_W^2}) + (1-t)\frac{M_V^2}{M_W^2} \ , \tag{46}$$

from which immediately follows that

$$a_1^P(Q^2) = -\frac{1}{2}\frac{Q^2}{M_W^2}\sum_V \frac{\alpha_V}{\pi}\int_0^1 da \int_0^1 (2tdt)\frac{2(\frac{3}{2}+at)}{L_V^2} \tag{47}$$

and

$$a_2^P(Q^2) = -\frac{1}{2}\frac{Q^2}{M_W^2}\sum_V \frac{\alpha_V}{\pi}\int_0^1 da \int_0^1 (2tdt)\frac{2(2+3at)}{L_V^2} \ , \tag{48}$$

and since there is no term proportional to $\Delta_\mu Q_\alpha Q_\beta$,

$$a_3^P(Q^2) = 0 \ . \tag{49}$$

Therefore,

$$\Delta\kappa_\gamma^P = -\frac{1}{2}\frac{Q^2}{M_W^2}\sum_V \frac{\alpha_V}{\pi}\int_0^1 da \int_0^1 (2tdt)\frac{(at-1)}{L_V^2} \ , \tag{50}$$

and

$$\Delta Q_\gamma^P = 0 \quad . \tag{51}$$

It is important to notice that even though $\Delta Q_\gamma^P = 0$ *both* μ_W and Q_W will assume values different than those predicted in the $\xi = 1$ gauge. Indeed, even though the value of λ_γ does not change, the value of κ_γ changes, and this change affects both μ_W and Q_W through Eq(4) and Eq(5). In the expression given in Eq(50) the first term (for V=Z) is infrared finite (since $M_Z \neq 0$), whereas the second term (for $V = \gamma$) is infrared divergent, since $M_\gamma = 0$. Calling this second term Θ we have

$$\Theta = -\frac{1}{2}(\frac{\alpha_\gamma}{\pi})\frac{Q^2}{M_W^2}\int_0^1 da \int_0^1 dt \frac{2t(at-1)}{t^2[1-a(1-a)\frac{4Q^2}{M_W^2}]} \ , \tag{52}$$

which can be rewritten as

$$\Theta = -R - (\frac{\alpha_\gamma}{\pi})\frac{Q^2}{M_W^2}\int_0^1 da \frac{a}{1-a(1-a)\frac{4Q^2}{M_W^2}} \ , \tag{53}$$

where R is the infrared divergent integral defined in Eq(43). On the other hand, the second term in Eq(53) is infrared finite. Clearly, including the first term of Eq(53) in the value of $\hat{\Delta}\kappa_\gamma$ *exactly* cancels the infrared divergent contribution of Eq(43), thus giving rise to an infrared finite expression for $\hat{\Delta}\kappa_\gamma$. So, after the infrared divergent part of Eq(52) is cancelled, $\Delta\kappa_\gamma^P$ is given by the following expression:

$$\Delta\kappa_\gamma^P = \Theta_\gamma + \Theta_Z \ , \tag{54}$$

with Θ_γ the second term in Eq(53), and Θ_Z the second term in Eq(50), namely

$$\Theta_\gamma = -(\frac{\alpha_\gamma}{\pi})\frac{Q^2}{M_W^2}\int_0^1 da \frac{a}{1-a(1-a)\frac{4Q^2}{M_W^2}} \ , \tag{55}$$

and

$$\Theta_Z = -\frac{Q^2}{M_W^2}(\frac{\alpha_Z}{\pi})\int_0^1 da \int_0^1 dt \frac{t(at-1)}{L_Z^2} \ , \tag{56}$$

and from Eq(41)

$$\hat{\Delta}\kappa_\gamma = [\Delta\kappa_\gamma^{(\xi=1)}]_{if} + \Theta_\gamma + \Theta_Z \ , \tag{57}$$

where the subscript (if) in the first term of the R.H.S. indicates that the contribution from the $\xi = 1$ gauge is now genuinely infrared finite. Finally, the magnetic dipole moment μ_W and electric quadrupole moment Q_W are given by

$$\mu_W = \frac{e}{2M_W}(2 + \hat{\Delta}\kappa_\gamma) \tag{58}$$

and

$$Q_W = -\frac{e}{M_W^2}(1 + \hat{\Delta}\kappa_\gamma + 2\hat{\Delta}Q_\gamma) \ . \tag{59}$$

Both $\Delta Q_\gamma^{(\xi=1)}$ and $\Delta\kappa_\gamma^{(\xi=1)}$ have been computed numerically in (7). We now proceed to compute the integrals in Eq(55) and Eq(56), which determine $\Delta\kappa_\gamma^P$. It is elementary to evaluate Θ_γ. Setting $\Theta_\gamma = -(\frac{\alpha_\gamma}{\pi})\hat{\Theta}_\gamma$ we have:

$$\begin{aligned} \hat{\Theta}_\gamma &= \tfrac{2}{\Delta}[arctg(\tfrac{1}{\Delta}) - arctg(\tfrac{-1}{\Delta})], \quad Q^2 < M_W^2 \\ &= -4, \quad Q^2 = M_W^2 \\ &= \tfrac{2}{\Delta}\ln[\tfrac{|\Delta-1|}{\Delta+1}], \quad Q^2 > M_W^2 \end{aligned} \tag{60}$$

for space-like Q^2, where $\Delta = \sqrt{|\frac{M_W^2}{Q^2} - 1|}$, and

$$\hat{\Theta}_\gamma = \frac{2}{\Delta}\ln[\frac{|\Delta - 1|}{\Delta + 1}] \tag{61}$$

for time-like Q^2, where $\Delta = \sqrt{\frac{M_W^2}{|Q^2|} + 1}$.

The double integral Θ_Z can in principle be expressed in a closed form in terms of Spence functions [see for example (16)], but this is of limited usefulness for our present calculation. Instead, we evaluated this integral numerically. We used the same values for the constants appearing in our calculations as in (7), namely $\alpha_\gamma = \frac{1}{128}$, $M_W = 80.6GeV$, $M_Z = 91.1GeV$ and $s = 0.23$.

The result of the computation is very interesting. $\Delta\kappa_\gamma^P$, which originates from pinching box diagrams, furnishes exactly the contributions needed to restore the unitarity of the final answer. Indeed, as the authors of (7) emphasized, $\Delta\kappa_\gamma^{(\xi=1)}$ is by itself not a gauge invariant object in the limit $Q^2 \to \infty$, where the local $SU(2) \times U(1)$ symmetry is restored. For large values of Q^2, $\Delta\kappa_\gamma^P$ is nearly equal in magnitude and opposite in sign to $\Delta\kappa_\gamma^P$. Therefore, when according to Eq(41) and Eq(42) both contributions are added, $\hat{\Delta}\kappa_\gamma \to 0$ as $Q^2 \to \pm\infty$. Clearly, the inclusion of the pinch parts from the box graphs is *crucial* for restoring the good asymptotic behavior of the W form factors.

V. CONCLUSIONS

We presented a study of the structure of trilinear gauge boson vertices in the context of the standard model. Using the S-matrix pinch technique gauge-independent γWW and ZWW vertices were constructed to one-loop order, with all three incoming momenta off-shell. These vertices satisfy naive QED–like Ward identities, which relate them to the gauge independent W self-energy, which were also obtained via the pinch technique. The tree-level Ward identities are to be contrasted with the complicated Ward identities satisfied by the conventionally defined gauge-dependent vertices; in particular, no ghost terms need be included. Finally, when the appropriate Lorentz structures are extracted, these vertices give rise to gauge-independent, infrared finite, and asymptotically well-behaved magnetic dipole and electric quadrupole form factors for the W, which can, at least in principle, be promoted to physical observables. It would be interesting to determine how these quantities could be directly extracted from future e^+e^- experiments.

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