

Effective Charge of the Higgs Boson

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Abstract

The Higgs-boson lineshape is studied within the pinch technique resummation formalism. It is shown that any resonant Higgs-boson amplitude contains a universal part which is gauge independent, renormalization-group invariant, satisfies the optical and equivalence theorems, and constitutes the natural extension of the QED effective charge to the case of the Higgs scalar.

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The production of the Standard Model (SM) Higgs boson [1] and the detailed study of its lineshape, mass and width, are expected to dominate the particle physics scene for the next two decades. A Higgs boson with mass M_H less than 100 GeV can be discovered at the CERN Large Electron Positron collider LEP2 through the Bjorken process $e^+e^- \rightarrow ZH$. If the Higgs boson turns out to be heavier, its discovery will become again possible at the CERN Large Hadron Collider through a variety of sub-processes, such as $X \rightarrow H^* \rightarrow X'$, where $X, X' = t\bar{t}, ZZ, W^+W^-$. Depending on the value of M_H and the specific kinematic circumstances, any of the above transitions may be resonant. The phenomenological importance of the above processes makes the need for solving a subtle theoretical problem [2], namely the self-consistent treatment of the Higgs boson resonance in the framework of S-matrix perturbation theory, all the more pressing. In particular, a resummation formalism needs be devised which complies with a set of very stringent and tightly interlocked physical requirements. To any finite order in perturbation theory, physical amplitudes reflect the local gauge symmetry, respect unitarity, are invariant under the renormalization group, and satisfy the equivalence theorem [3,4]. All of the above properties should be also present after resummation; unfortunately, resummation methods often end up violating one or more of them, essentially because subtle cancellations are distorted when certain parts of the amplitude are resummed to all orders in perturbation theory, whereas others, carrying important physical information, are only considered to a finite order.

Recently however [5], a formalism based on the pinch technique (PT) [6] has been developed, which manifestly preserves the crucial physical properties during all intermediate steps of the resummation procedure. The PT algorithm rearranges systematically a given amplitude into *physically meaningful* sub-amplitudes, which have the same kinematic properties as their conventional counterparts, but none of their individual pathologies. In this Letter, the above formalism is extended to the case of resonant transitions involving the SM Higgs boson. The main novel results of our study are: (i) The PT gives rise to a Higgs boson self-energy which is *independent* of the gauge-fixing parameter (GFP) in every gauge fixing scheme, is *universal* in the sense that it is *process-independent*, it may be *resummed*

following the method presented in Ref. [5], it displays only *physical* fermionic and bosonic thresholds, and satisfies *individually* the optical theorem for *both* fermionic as well as bosonic contributions. (ii) When the resummed Higgs boson propagator is multiplied by the universal quantity g_w^2/M_W^2 , or, equivalently, by the inverse square of the vacuum expectation value (VEV) of the Higgs field, it gives rise to a *renormalization group invariant* quantity, in direct analogy to the *effective charge* of the photon, or the W and Z bosons [7], which constitutes a common component in every Higgs-boson mediated process, and can be viewed as a physical entity intrinsic to the Higgs boson. (iii) Any amplitude involving longitudinally polarized gauge bosons satisfies the equivalence theorem, but its individual s -channel and t -channel contributions do not. Instead, the PT rearrangement of such an amplitude gives rise to two kinematically distinct pieces, a genuine s -channel and a genuine t -channel, which satisfy the equivalence theorem *individually*. In particular, the above property *persists* even *after* the s -channel Higgs boson self-energy has been resummed, thus solving a long-standing problem.

We shall now analyze the above points in the context of specific examples. When the center-of-mass (c.m.) energy \sqrt{s} approaches M_H , amplitudes containing an s -channel Higgs boson become singular, and must be regulated. The naive extension of the standard Breit-Wigner procedure to this case would consist of replacing the free Higgs boson propagator $\Delta_H(s) = (s - M_H^2)^{-1}$ by a resummed propagator of the form $[s - M_H^2 + \Pi^{HH}(s)]^{-1}$, where $\Pi^{HH}(s)$ is the one-loop Higgs boson self-energy. However, bosonic radiative corrections induce an additional dependence on the GFP, as one can verify by explicit calculations in a variety of conventional gauges, such as the renormalizable (R_ξ), or axial gauges. Turning to more elaborate gauge fixing schemes does not improve the situation. For example, within the background field gauges (BFG's) and with irrelevant tadpole graphs omitted, the contribution of the Z boson-loop reads: [8]

$$\begin{aligned} \Pi_{(ZZ)}^{\widehat{H}\widehat{H}}(s, \xi_Q) = & \frac{\alpha_w}{32\pi} \frac{s^2}{M_W^2} \left\{ \left(1 - 4 \frac{M_Z^2}{s} + 12 \frac{M_Z^4}{s^2} \right) B_0(s, M_Z^2, M_Z^2) \right. \\ & \left. - \left[1 + 4\xi_Q \frac{M_Z^2}{s} - (M_H^2 + 4\xi_Q M_Z^2) \frac{M_H^2}{s^2} \right] B_0(s, \xi_Q M_Z^2, \xi_Q M_Z^2) \right\}, \quad (1) \end{aligned}$$

where $\alpha_w = g_w^2/(4\pi)$ is the weak fine structure constant and B_0 is the usual Passarino-Veltman function. The presence of the GFP ξ_Q results in bad high energy behavior and the appearance of unphysical thresholds, as can be verified directly using $\Im m B_0(s, M^2, M^2) = \theta(s-4M^2)\pi(1-4M^2/s)^{1/2}$. Even though to any order in perturbation theory physical amplitudes are GFP-independent, and display only physical thresholds, resumming $\hat{\Pi}_{(ZZ)}^{\hat{H}\hat{H}}(s, \xi_Q)$ will introduce artifacts to the resonant amplitude. Even in the unitary gauge ($\xi_Q \rightarrow \infty$), where only physical thresholds survive, the s^2 -growth in Eq. (1) grossly contradicts the equivalence theorem.

In the PT framework however, a modified one-loop self-energy for the Higgs boson can be constructed, by appending to the conventional self-energy additional propagator-like contributions concealed inside vertices and boxes. These contributions can be identified systematically, by resorting exclusively to elementary Ward identities of the form $\not{k}(v + a\gamma_5) = (\not{k} + \not{p} - m)(v + a\gamma_5) - (v - a\gamma_5)(\not{p} - m) + 2am\gamma_5$, triggered by the longitudinal virtual momenta k_μ . Following this procedure, we find the PT Higgs-boson self-energy [8]

$$\hat{\Pi}_{(ZZ)}^{HH}(s) = \frac{\alpha_w}{32\pi} \frac{M_H^4}{M_W^2} \left[1 + 4 \frac{M_Z^2}{M_H^2} - 4 \frac{M_Z^2}{M_H^4} (2s - 3M_Z^2) \right] B_0(s, M_Z^2, M_Z^2), \quad (2)$$

which is GFP-independent in any gauge fixing scheme, universal [9], grows linearly with s , and displays physical thresholds only. For illustration, in Fig. 1, we plot the dependence of the running width, $\Im m \hat{\Pi}_{(ZZ)}^{HH}(s)$, on \sqrt{s} within the PT resummation formalism, the BFG with $\xi_Q = 0$, and the unitary gauge. The difference in the phenomenological predictions between the three approaches is rather striking, in accordance with the discussion given above.

The PT self-energies satisfy the optical theorem *individually*, as explained in [5,7]. To verify that $\hat{\Pi}_{(ZZ)}^{HH}(s)$ has this property, consider the tree-level transition amplitude $\mathcal{T}(ZZ)$ for the process $f(p_1)\bar{f}(p_2) \rightarrow Z(k_1)Z(k_2)$; it is the sum of an s - and a t - channel contribution, denoted by $\mathcal{T}_s^H(ZZ)$ and $\mathcal{T}_t(ZZ)$, respectively, given by

$$\mathcal{T}_{s\mu\nu}^H(ZZ) = \Gamma_{0\mu\nu}^{HZZ} \Delta_H(s) \bar{v}(p_2) \Gamma_0^{Hf\bar{f}} u(p_1), \quad (3)$$

$$\mathcal{T}_{t\mu\nu}(ZZ) = \bar{v}(p_2) \left(\Gamma_{0\nu}^{Zf\bar{f}} \frac{1}{\not{p}_1 + \not{k}_1 - m_f} \Gamma_{0\mu}^{Zf\bar{f}} + \Gamma_{0\mu}^{Zf\bar{f}} \frac{1}{\not{p}_1 + \not{k}_2 - m_f} \Gamma_{0\nu}^{Zf\bar{f}} \right) u(p_1). \quad (4)$$

Here, $s = (p_1 + p_2)^2 = (k_1 + k_2)^2$ is the c.m. energy squared, $\Gamma_{0\mu\nu}^{HZZ} = ig_w M_Z^2 / M_W g_{\mu\nu}$, $\Gamma_0^{Hf\bar{f}} = -ig_w m_f / (2M_W)$ and $\Gamma_{0\mu}^{Zf\bar{f}} = -ig_w / (2c_w) \gamma_\mu [T_z^f (1 - \gamma_5) - 2Q_f s_w^2]$, with $c_w = \sqrt{1 - s_w^2} = M_W / M_Z$, are the tree-level HZZ , $Hf\bar{f}$ and $Zf\bar{f}$ couplings, respectively, and Q_f is the electric charge of the fermion f , and T_z^f its z -component of the weak isospin. We then calculate the expression $[\mathcal{T}_s^H(ZZ) + \mathcal{T}_{t\mu\nu}(ZZ)] Q^{\mu\rho}(k_1) Q^{\nu\sigma}(k_2) [\mathcal{T}_s^H(ZZ) + \mathcal{T}_{t\rho\sigma}(ZZ)]^*$, where $Q^{\mu\nu}(k) = -g^{\mu\nu} + k^\mu k^\nu / M_Z^2$ denotes the usual polarization tensor, and isolate its Higgs-boson mediated part. To accomplish this, one must first use the longitudinal momenta coming from $Q^{\mu\rho}(k_1)$ and $Q^{\nu\sigma}(k_2)$ in order to extract the Higgs-boson part of $\mathcal{T}_t^{\mu\nu}(ZZ)$, *i.e.*,

$$\frac{k_1^\mu k_2^\nu}{M_Z^2} \mathcal{T}_{t\mu\nu}(ZZ) = \mathcal{T}_P^H + \dots = -\frac{ig_w}{2M_W} \bar{v}(p_2) \Gamma_0^{Hf\bar{f}} u(p_1) + \dots, \quad (5)$$

where the ellipses denote genuine t -channel (not Higgs-boson related) contributions. Then, one must append the piece $\mathcal{T}_P^H \mathcal{T}_P^{H*}$ to the “naive” Higgs-dependent part $\mathcal{T}_s^H(ZZ) Q^{\mu\rho}(k_1) Q^{\nu\sigma}(k_2) \mathcal{T}_s^{H*}(ZZ)$. Integrating the expression so obtained over the two-body phase space, we finally arrive at the imaginary part of Eq. (2), which is the announced result.

The gauge-invariance of the S matrix imposes tree-level Ward identities on the unrenormalized one-loop PT Green’s functions [6,10]. The requirement that the same Ward identities should be maintained after renormalization leads to important QED-type relations for the renormalization constants of the theory. Specifically, we find

$$\hat{Z}_W = \hat{Z}_{g_w}^{-2}, \quad \hat{Z}_Z = \hat{Z}_W \hat{Z}_{c_w}^2, \quad \hat{Z}_H = \hat{Z}_W (1 + \delta M_W^2 / M_W^2), \quad (6)$$

where \hat{Z}_W , \hat{Z}_Z , and \hat{Z}_H are the wave-function renormalizations of the W , Z and H fields, respectively, \hat{Z}_{g_w} is the coupling renormalization, and $\hat{Z}_{c_w} = (1 + \delta M_W^2 / M_W^2)^{1/2} (1 + \delta M_Z^2 / M_Z^2)^{-1/2}$. The renormalization of the bare resummed Higgs-boson propagator $\hat{\Delta}^{H,0}(s)$ proceeds as follows:

$$\hat{\Delta}^{H,0}(s) = [s - (M_H^0)^2 + \hat{\Pi}^{HH,0}(s)]^{-1} = \hat{Z}_H [s - M_H^2 + \hat{\Pi}^{HH}(s)]^{-1} = \hat{Z}_H \hat{\Delta}^H(s), \quad (7)$$

with $(M_H^0)^2 = M_H^2 + \delta M_H^2$. The renormalized Higgs-boson mass M_H^2 may be defined as the real part of the complex pole position of $\hat{\Delta}^H(s)$. Notice that within the PT resummation formalism the gauge-independent pole [11] of a resonant transition amplitude does not get shifted [5], and the HZ mixing is absent up to two loops [12]. Employing the relations in Eq. (6), we observe that the universal quantity

$$\hat{R}^{H,0}(s) = \frac{(g_w^0)^2}{(M_W^0)^2} \hat{\Delta}^{H,0}(s) = \frac{g_w^2}{M_W^2} \hat{\Delta}^H(s) = \hat{R}^H(s) \quad (8)$$

is invariant under the renormalization group. This important universal property of the Higgs boson is true for non-Abelian gauge theories with spontaneous symmetry breaking (SSB), but does not hold in general. For example, in pure scalar theories any attempt to construct quantities analogous to $\hat{R}^H(s)$ fails to be process independent [8]. In that sense, $\hat{R}^H(s)$ provides a natural extension of the notion of the QED effective charge for the SM Higgs boson, *i.e.*, H couples universally to matter with an effective “charge” inversely proportional to its VEV. In the high-energy limit, $s \gg M_H^2$, the dispersive part of Higgs self-energy behaves as $\Re e \hat{\Pi}^{HH}(s) \sim -\alpha_w s \ln(s/M_H^2)(3m_t^2 - 4M_W^2 - 2M_Z^2)/(8\pi M_W^2)$. If the heavy top quark were assumed to be absent, the coefficient accompanying the leading logarithm in $\Re e \hat{\Pi}^{HH}(s)$ would be positive. This feature is reminiscent of the PT self-energy in pure Yang-Mills theories [6,5,7], whose leading logarithm is proportional to $b_1 = 11c_A/3 > 0$, where c_A is the Casimir eigenvalue of the adjoint representation, thus reflecting the asymptotic freedom of the theory. On similar theoretical grounds, $\Im m \hat{\Pi}_{(ZZ)}^{HH}(s)$ turns negative for c.m. energies much higher than M_H , *viz.*, the Higgs self-energy cannot be spectrally represented.

An additional, highly non-trivial constraint, must be imposed on resummed amplitudes; they have to obey the (generalized) equivalence theorem (GET), which is known to be satisfied before resummation, order by order in perturbation theory. For the specific example of the amplitude $\mathcal{T}(ZZ) = \mathcal{T}_s^H + \mathcal{T}_t$, the GET states that

$$\mathcal{T}(Z_L Z_L) = -\mathcal{T}(G^0 G^0) - i\mathcal{T}(G^0 z) - i\mathcal{T}(z G^0) + \mathcal{T}(zz), \quad (9)$$

where Z_L is the longitudinal component of the Z boson, G^0 is its associated would-be Goldstone boson, and $z^\mu(k) = \varepsilon_L^\mu(k) - k^\mu/M_W$ is the energetically suppressed part of the

longitudinal polarization vector ε_L^μ . It is crucial to observe, however, that already at the tree level, the conventional s - and t - channel sub-amplitudes \mathcal{T}_s^H and \mathcal{T}_t fail to satisfy the GET individually. To verify that, one has to calculate $\mathcal{T}_s^H(Z_L Z_L)$, using explicit expressions for the longitudinal polarization vectors, and check if the answer obtained is equal to the Higgs-boson mediated s -channel part of the LHS of Eq. (9). In particular, in the c.m. system, we have $z^\mu(k_1) = \varepsilon_L^\mu(k_1) - k_1^\mu/M_Z = -2M_Z k_2^\mu/s + \mathcal{O}(M_Z^4/s^2)$, and exactly analogous expressions for $z^\mu(k_2)$. The residual vector $z^\mu(k)$ has the properties $z_\mu k^\mu = -M_Z$ and $z^2 = 0$. After a straightforward calculation, we obtain $\mathcal{T}_s^H(Z_L Z_L) = -\mathcal{T}_s^H(G^0 G^0) - i\mathcal{T}_s^H(zG^0) - i\mathcal{T}_s^H(G^0 z) + \mathcal{T}_s^H(zz) - \mathcal{T}_P^H$, where

$$\mathcal{T}_s^H(G^0 G^0) = \Gamma_0^{HG^0 G^0} \Delta_H(s) \bar{v}(p_2) \Gamma_0^{Hf\bar{f}} u(p_1), \quad (10)$$

$$\mathcal{T}_s^H(zG^0) + \mathcal{T}_s^H(G^0 z) = [z^\mu(k_1) \Gamma_{0\mu}^{HZZG^0} + z^\nu(k_2) \Gamma_{0\nu}^{HG^0 Z}] \Delta_H(s) \bar{v}(p_2) \Gamma_0^{Hf\bar{f}} u(p_1),$$

and $\mathcal{T}_s^H(zz) = z^\mu(k_1) z^\nu(k_2) \mathcal{T}_{s\mu\nu}^H(ZZ)$, with $\Gamma_0^{HG^0 G^0} = -ig_w M_H^2/(2M_W)$ and $\Gamma_{0\mu}^{HZZG^0} = -g_w(k_1 + 2k_2)_\mu/(2c_w)$. Evidently, the presence of the term \mathcal{T}_P^H prevents $\mathcal{T}_s^H(Z_L Z_L)$ from satisfying the GET. This is not surprising however, since an important Higgs-boson mediated s -channel part has been omitted. Specifically, the momenta k_1^μ and k_2^ν stemming from the leading parts of the longitudinal polarization vectors $\varepsilon_L^\mu(k_1)$ and $\varepsilon_L^\nu(k_2)$ extract such a term from $\mathcal{T}_t(Z_L Z_L)$. Just as happens in Eq. (5), this term is precisely \mathcal{T}_P^H , and must be added to $\mathcal{T}_s^H(Z_L Z_L)$, in order to form a well-behaved amplitude at very high energies. In other words, the amplitude $\hat{\mathcal{T}}_s^H(Z_L Z_L) = \mathcal{T}_s^H(Z_L Z_L) + \mathcal{T}_P^H$ satisfies the GET independently (*cf.* Eq. (9)). In fact, this crucial property persists after resummation. Indeed, as shown in Fig. 2, the resummed amplitude $\bar{\mathcal{T}}_s^H(Z_L Z_L)$ may be constructed from $\mathcal{T}_s^H(Z_L Z_L)$ in Eq. (3), if $\Delta_H(s)$ is replaced by the resummed Higgs-boson propagator $\hat{\Delta}^H(s)$, and $\Gamma_{0\mu\nu}^{HZZ}$ by the expression $\Gamma_{0\mu\nu}^{HZZ} + \hat{\Gamma}_{\mu\nu}^{HZZ}$, where $\hat{\Gamma}_{\mu\nu}^{HZZ}$ is the one-loop HZZ vertex calculated within the PT [8]. It is then straightforward to show that the Higgs-mediated amplitude $\tilde{\mathcal{T}}_s^H(Z_L Z_L) = \bar{\mathcal{T}}_s^H(Z_L Z_L) + \mathcal{T}_P^H$ respects the GET *individually*; to that end we only need to employ the following tree-level-type PT WI's:

$$\begin{aligned}
k_2^\nu \widehat{\Gamma}_{\mu\nu}^{HZZ}(q, k_1, k_2) + iM_Z \widehat{\Gamma}_\mu^{HZG^0}(q, k_1, k_2) &= -\frac{g_w}{2c_w} \widehat{\Pi}_\mu^{ZG^0}(k_1), \\
k_1^\mu \widehat{\Gamma}_\mu^{HZG^0}(q, k_1, k_2) + iM_Z \widehat{\Gamma}^{HG^0G^0}(q, k_1, k_2) &= -\frac{g_w}{2c_w} \left[\widehat{\Pi}^{HH}(q^2) + \widehat{\Pi}^{G^0G^0}(k_2^2) \right], \\
k_1^\mu k_2^\nu \widehat{\Gamma}_{\mu\nu}^{HZZ}(q, k_1, k_2) + M_Z^2 \widehat{\Gamma}^{HG^0G^0}(q, k_1, k_2) &= \frac{ig_w M_Z}{2c_w} \left[\widehat{\Pi}^{HH}(q^2) + \widehat{\Pi}^{G^0G^0}(k_1^2) + \widehat{\Pi}^{G^0G^0}(k_2^2) \right],
\end{aligned} \tag{11}$$

where $\widehat{\Gamma}_\mu^{HZG^0}$ and $\widehat{\Gamma}^{HG^0G^0}$ are the one-loop PT HZG^0 and HG^0G^0 vertices, respectively. In this derivation, one should also make use of the PT WI involving the ZG^0 - and G^0G^0 -self-energies: $\widehat{\Pi}_\mu^{ZG^0}(k) = -iM_Z k_\mu \widehat{\Pi}^{G^0G^0}(k^2)/k^2$.

In conclusion, we have explicitly demonstrated that within the PT resummation approach, any resonant Higgs-mediated amplitude contains a gauge-independent universal part, which is invariant under the renormalization group and satisfies the optical and equivalence theorems individually. It would be of great phenomenological interest to confront the theoretical predictions for this universal quantity against data obtained from future Higgs-boson experiments.

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FIGURES

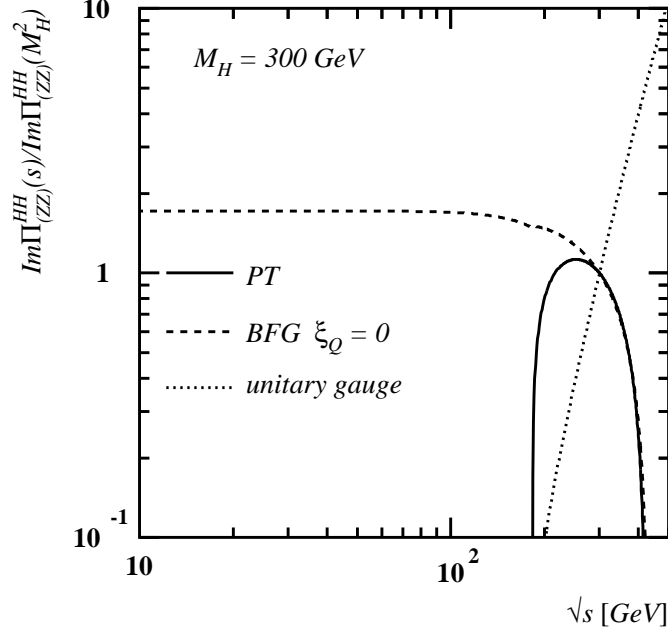


FIG. 1. Dependence of $\Im m\Pi_{(ZZ)}^{HH}(s)/\Im m\Pi_{(ZZ)}^{HH}(M_H^2)$ on $s^{1/2}$ in the PT, the BFM with $\xi_Q = 0$, and the unitary gauge.

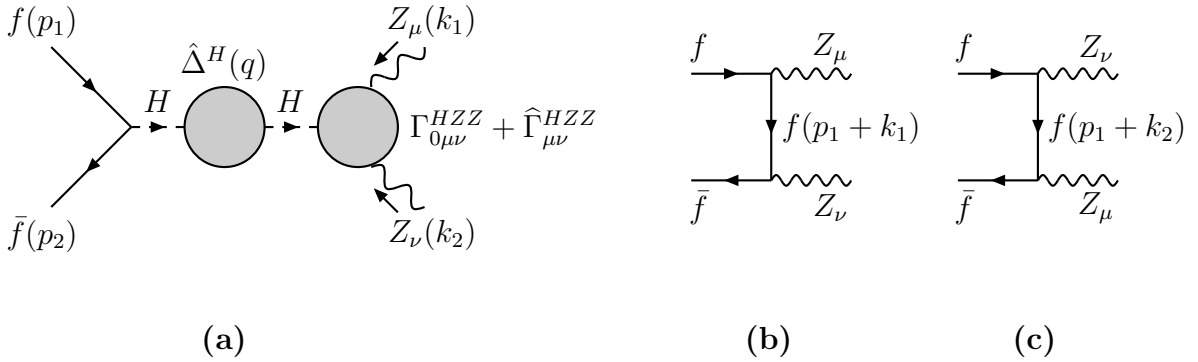


FIG. 2. Resummation of the Higgs-mediated amplitude pertinent to $f\bar{f} \rightarrow ZZ$.