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The Pinch Technique to All Orders

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Abstract

The generalization of the pinch technique to all orders in perturbation theory is presented. The effective Green's functions constructed with this procedure are singled out in a unique way through the full exploitation of the underlying Becchi-Rouet-Stora-Tyutin symmetry. A simple all-order correspondence between the pinch technique and the background field method in the Feynman gauge is established.

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It is well-known that, to any finite order, the conventional perturbative expansion gives rise to expressions for physical amplitudes which are endowed with crucial properties. S-matrix elements, for example, are independent of the gauge-fixing scheme and parameters chosen to quantize the theory, they are gauge-invariant (current conservation), they are unitary (conservation of probability), and well behaved at high energies. However, the above properties are in general not reflected by the individual off-shell Green's functions, which are the building blocks of the aforementioned perturbative expansion. The latter depend on the gauge-fixing parameters in a complicated way , grow much faster than physical amplitudes at high energies, and display unphysical thresholds. Evidently, when combining unphysical Green's functions to form a physical amplitude, subtle field-theoretical mechanisms are at work, which enforce non-trivial cancellations among them at any given order.

There are considerable conceptual and phenomenological advantages in reformulating the perturbative expansion in terms of off-shell Green's functions which display manifestly the same properties as the physical amplitudes. To begin with, the sharp difference between observables and Green's functions suggests a great deal of redundancy in the conventional diagrammatic formulation of gauge theories, in the sense that extensive underlying cancellations beg to be made manifest and be explicitly exploited as early within a calculation as possible. Implementing these cancellations at an early stage not only renders the book-keeping aspects more tractable [1], but allows for theoretically safe reorganizations or resummations of the perturbative series. For example, identifying and Dyson-resuming the correct sub-set of propagator-like corrections gives rise to physically meaningful Born-improved amplitudes [2]. In addition, the generalization into a non-Abelian context of the characteristic properties of the QED effective charge, has a wide range of phenomenological applications [3]. Finally, n-point functions free of unphysical artifacts could serve, at least in principle, as the new building blocks of manifestly gauge-invariant Schwinger-Dyson equations [4].

It would clearly be preferable to enforce the relevant cancellations already at the level of the functional path-integral defining the theory, and obtain directly from it the desired Green's functions; this is however beyond our powers at the moment. On the other hand, there exists a *diagrammatic* method, called the pinch technique (PT) [4, 5], which systematically exploits the symmetries built into physical observables, such as S-matrix elements, in order to construct off-shell sub-amplitudes that are kinematically akin to conventional Green's functions, but, unlike the latter, are also endowed with desirable properties. The

basic observation, which essentially defines the PT, is that there exists a fundamental cancellation between sets of diagrams with different kinematic properties, such as self-energies, vertices, and boxes. This cancellation is driven by the underlying Becchi-Rouet-Stora-Tyutin symmetry [6], and is triggered when longitudinal momenta circulating inside vertex and box diagrams generate (by "pinching" out internal fermion lines) propagator-like terms. The latter are reassigned to conventional self-energy graphs in order to give rise to the aforementioned gauge-invariant effective Green's functions. In its original one- [4, 5] and two-loop [7] application, the PT boils down to the study of the kinematic rearrangements produced into *individual* Feynman diagrams when elementary tree-level Ward identities (WIs) are triggered.

One of the most pressing questions in this context is whether one can extend the PT algorithm to all orders in perturbation theory, thus achieving the systematic construction of effective n-point functions displaying the aforementioned characteristic features. To accomplish this it is clear that one needs to abandon algebraic operations inside individual Feynman graphs, and resort to a more formal procedure. In this Letter we will show that the PT algorithm can be successfully generalized to *all orders* in perturbation theory, through the collective treatment of entire sets of diagrams. This is accomplished through the judicious use of the Slavnov-Taylor identity (STI) [8] satisfied by a special Green's function, which serves as a common kernel to all higher order self-energy and vertex diagrams.

We will consider for concreteness the S-matrix element for the quark–anti-quark elastic scattering process $q(r_1)\bar{q}(r_2) \to q(p_1)\bar{q}(p_2)$ in QCD. We set $q = r_2 - r_1 = p_2 - p_1$, with $s = q^2$ the square of the momentum transfer. The longitudinal momenta responsible for the aforementioned kinematical rearrangements stem either from the bare gluon propagators or from the pinching part $\Gamma^{\rm P}_{\alpha\mu\nu}(q,k_1,k_2)$ appearing in the characteristic decomposition of the elementary tree-level three-gluon vertex $\Gamma^{eab,[0]}_{\alpha\mu\nu} = gf^{eab}\Gamma^{[0]}_{\alpha\mu\nu}$ into [4]

$$\begin{aligned}
\Gamma^{[0]}_{\alpha\mu\nu}(q,k_1,k_2) &= \Gamma^{\rm F}_{\alpha\mu\nu}(q,k_1,k_2) + \Gamma^{\rm P}_{\alpha\mu\nu}(q,k_1,k_2),\\
\Gamma^{\rm F}_{\alpha\mu\nu}(q,k_1,k_2) &= (k_1-k_2)_\alpha g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu},\\
\Gamma^{\rm P}_{\alpha\mu\nu}(q,k_1,k_2) &= k_{2\nu} g_{\alpha\mu} - k_{1\mu} g_{\alpha\nu}. \qquad (1)
\end{aligned}$$

The above decomposition is to be carried out to "external" three-gluon vertices only, *i.e.*, the vertices where the physical momentum transfer q is entering [7]. In what follows we work in the renormalizable Feynman gauge (RFG); this choice eliminates the longitudinal momenta

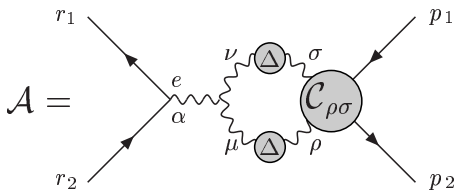

FIG. 1: The subset of the graphs of the quark–anti-quark elastic scattering process which will receive the action of the longitudinal momenta stemming from $\Gamma^{\rm P}$. Here Δ represents the full gluon propagator.

from the bare propagators, and allows us to focus our attention on the all-order study of the longitudinal momenta originating from $\Gamma^{\rm P}_{\alpha\mu\nu}$. We will denote by $\mathcal{A}$ the subset of the graphs which will receive the action of the longitudinal momenta stemming from $\Gamma^{\rm P}_{\alpha\mu\nu}(q,k_1,k_2)$ (see Fig.1). We have that

$$\mathcal{A} = ig^2\bar{u}(r_1)\frac{\lambda^e}{2}\gamma_\alpha v(r_2) f^{eab}\Gamma^{{\rm P},\,\alpha\mu\nu}(q,k_1,k_2)\mathcal{T}^{ab}_{\mu\nu}(k_1,k_2,p_1,p_2), \tag{2}$$

where λ^e are the Gell-Mann matrices, and $\mathcal{T}^{ab}_{\mu\nu}$ is the sub-amplitude $g^a_\mu(k_1)g^b_\nu(k_2) \to q(p_1)\bar{q}(p_2)$, with the gluons *off-shell* and the fermions on-shell; for the latter $\bar{v}(p_2)S^{-1}(p_2)|_{\not{p}_2=m} = S^{-1}(p_1)u(p_1)|_{\not{p}_1=m} = 0$, where $S(p)$ is the (full) quark propagator. In terms of Green's functions we have

$$\mathcal{T}^{ab}_{\mu\nu} = \bar{v}(p_2)\big[\mathcal{C}^{ab}_{\rho\sigma}(k_1,k_2,p_1,p_2)\Delta^\rho_\mu(k_1)\Delta^\sigma_\nu(k_2)\big]u(p_1). \tag{3}$$

Clearly, there is an equal contribution from the $\Gamma^{\rm P}$ situated on the right hand-side of $\mathcal{T}$.

Let us focus on the STI satisfied by the amplitude $\mathcal{T}^{ab}_{\mu\nu}$; it reads

$$k_1^\mu C^{ab}_{\mu\nu} + k_{2\nu}G^{ab}_1 - igf^{bcd}Q^{acd}_{1\nu} - gX^{ab}_{1\nu} + g\bar{X}^{ab}_{1\nu} = 0, \tag{4}$$

where the Green's function appearing in it are defined in Fig.2. The terms $X_{1\nu}$ and $\bar{X}_{1\nu}$ die on-shell, since they are missing one fermion propagator. Thus, we arrive at the on-shell STI for $\mathcal{T}^{ab}_{\mu\nu}$

$$k_1^\mu\mathcal{T}^{ab}_{\mu\nu} = \mathcal{S}^{ab}_{1\nu}, \tag{5}$$

with

$$\begin{aligned}\mathcal{S}^{ab}_{1\nu} = {} & \bar{v}(p_2)\,\big[igf^{bcd}\mathcal{Q}^{acd}_{1\nu}(k_1,k_2,p_1,p_2)D(k_1) \\ & - \; k_{2\nu}\mathcal{G}^{ab}_1(k_1,k_2,p_1,p_2)D(k_1)D(k_2)\big]\,u(p_1),\end{aligned} \tag{6}$$

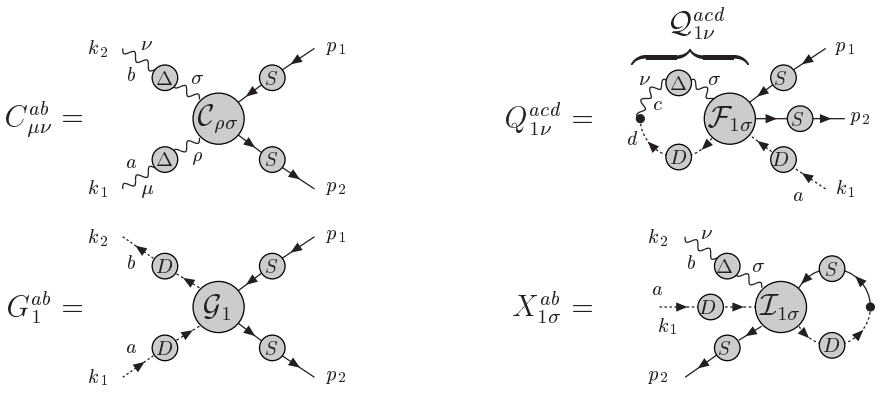

FIG. 2: Diagrammatic representation of the Green's function appearing in the STI of Eq.BasSTI. Here D and S represent the full ghost and fermion propagators respectively.

where we have defined $Q^{acd}_{1\nu}(k_1, k_2, p_1, p_2) = \mathcal{Q}^{acd}_{1\nu}(k_1, k_2, p_1, p_2)D(k_1)S(p_1)S(p_2)$.

In perturbation theory both $\mathcal{T}^{ab}_{\mu\nu}$ and $\mathcal{S}^{ab}_{1\nu}$ are given by Feynman diagrams, which can be separated into distinct classes, depending on their kinematic dependence and their geometrical properties. Graphs which do not contain information about the kinematical details of the incoming test-quarks are self-energy graphs, whereas those which display a dependence on the test quarks are vertex graphs. The former depend only on the variable s, whereas the latter on both s and the mass m of the test quarks; equivalently, we will refer to them as s-channel or t-channel graphs, respectively. In addition to the s-t decomposition, Feynman diagrams can be separated into one-particle irreducible (1PI) and one-particle reducible (1PR) ones. The crucial point is that the action of the momenta k_1^μ or k_2^ν on $\mathcal{T}^{ab}_{\mu\nu}$ does *not* respect, in general, the original s-t and 1PI-1PR separation furnished by the Feynman diagrams (see third paper of [2]). In other words, even though Eq.(5) holds for the entire amplitude, it is not true for the individual sub-amplitudes, *i.e.*,

$$k_1^\mu \left[\mathcal{T}^{ab}_{\mu\nu}\right]_{x,\mathrm{Y}} \neq \left[\mathcal{S}^{ab}_{1\nu}\right]_{x,\mathrm{Y}}, \qquad x = s, t; \quad \mathrm{Y} = \mathrm{I}, \mathrm{R}, \tag{7}$$

where I (respectively R) indicates the one-particle *irreducible* (respectively *reducible*) parts of the amplitude involved. Evidently, whereas the characterization of graphs as propagator- and vertex-like is unambiguous in the absence of longitudinal momenta (*e.g.*, in a scalar theory), their presence tends to mix propagator- and vertex-like graphs. Similarly, 1PR graphs are effectively converted into 1PI ones (the opposite cannot happen). The reason for the inequality of Eq.(7) are precisely the propagator-like terms, such as those encountered in the one- and two-loop calculations; they have the characteristic feature that, when depicted

by means of Feynman diagrams contain unphysical vertices, *i.e.*, vertices which do not exist in the original Lagrangian (Fig.3). All such diagrams cancel *diagrammatically* against each other. Thus, after the aforementioned rearrangements have taken place, for the t-channel irreducible part of the amplitude we will have the equality

$$\left[k_1^\mu \mathcal{T}^{ab}_{\mu\nu}\right]^{\rm PT}_{t,{\rm I}} \equiv \left[\mathcal{S}^{ab}_{1\nu}\right]_{t,{\rm I}}\,. \tag{8}$$

Eq.(8) merits particular attention, because it is of central importance for the generalization of the PT to all orders. The superscript "PT" on the left hand-side denotes that the corresponding amplitude must be rearranged following the well-defined PT algorithm, as it has been explained in the literature [7]. In particular, one tracks down the rearrangments induced when the action of (virtual) longitudinal momenta (k) on the bare vertices of diagrams trigger elementary WIs. Eventually a WI of the form $k_\mu\gamma^\mu = S^{-1}(\not{k}+\not{p}) - S^{-1}(\not{p})$ will give rise to propagator-like parts, by removing (pinching out) the internal bare fermion propagator $S(\not{k}+\not{p})$. Depending on the topology of the diagram under consideration this last WI may be activated immediately, or as the final outcome of a sequential triggering of intermediate WIs. We emphasize that, in order to preserve the special unitarity and analyticity properties of the PT Green's functions, "internal" three-gluon vertices should not pinch, nor should one carry out sub-integrations [7].

The non-trivial step for generalizing the PT to all orders is then the following: Instead of going through the arduous task of manipulating the left hand-side of Eq.(8), following the aforementioned rules, in order to determine the pinching parts and explicitly enforce their cancellation, use directly the right-hand side, which already contains the answer! Indeed, the right-hand side involves only conventional (ghost) Green's functions, expressed in terms of normal Feynman rules, with no reference to unphysical vertices. That this must be so follows from the same PT rules mentioned above: due to the absence of external three-gluon vertices the right-hand side cannot be pinched further, i.e. its separation into propagator- and vertex-like graphs is unambiguous, since there is no possibility (without violating the PT rules) to obtain further mixing. Thus, the right-hand side of Eq.(8) serves as a practical definition of the PT to all orders.

After these observations, we proceed to the PT construction to all orders. Once the effective Green's functions have been derived, they will be compared to the corresponding Green's functions obtained in the Feynman gauge of the background field method (BFG for

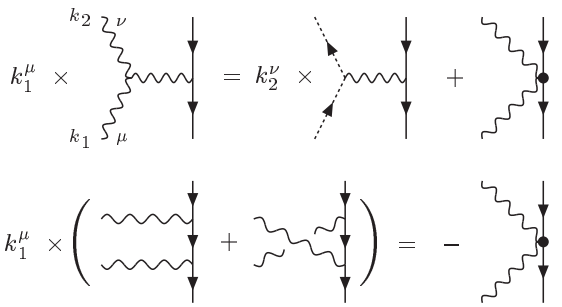

FIG. 3: Diagrammatic representation of the tree-level inequality of Eq.ÎNEQ.

short) in order to establish whether the known correspondence persists to all orders; as we will see, this is indeed the case (for an extended list of related references see [9]).

To begin with, it is immediate to recognize that in the RFG box diagrams of arbitrary order n, to be denoted by $B^{[n]}$, coincide with the PT boxes $\widehat{B}^{[n]}$, since all three-gluon vertices are "internal", *i.e.*, they do not provide longitudinal momenta. Thus, they coincide with the BFG boxes, $\tilde{B}^{[n]}$, *i.e.*, $\widehat{B}^{[n]} = B^{[n]} = \tilde{B}^{[n]}$ for every n.

We then continue with the construction of the 1PI PT gluon-quark–anti-quark vertex $\widehat{\Gamma}^e_\alpha$. We start from the corresponding vertex in the RFG, to be denoted by Γ^e_α, and focus only on the class of vertex diagrams containing an *external* bare three-gluon vertex; we will denote this subset by $\Gamma^e_{A^3,\alpha}$ [Fig.4(a)]. All other types of graphs contributing to Γ^e_α are inert as far as the PT procedure is concerned, because they do not furnish pinching momenta [7]. The next step is to carry out the vertex decomposition of Eq.(1) to the external three-gluon vertex $\Gamma^{eab,[0]}_{\alpha\mu\nu}$ appearing in $\Gamma^e_{A^3,\alpha}$. This will result in the obvious separation $\Gamma^e_{A^3,\alpha} = \Gamma^{\mathrm{F},e}_{A^3,\alpha} + \Gamma^{\mathrm{P},e}_{A^3,\alpha}$. The part $\Gamma^{\mathrm{F},e}_{A^3,\alpha}$ is also inert, and will be left untouched. Thus, the only quantity to be further manipulated is $\Gamma^{\mathrm{P},e}_{A^3,\alpha}$; it reads

$$\Gamma^{\mathrm{P},e}_{A^3,\alpha} = g f^{eba} \int [(k-q)^\mu g^\nu_\alpha + k^\nu g^\mu_\alpha] \left[\mathcal{T}^{ab}_{\mu\nu}\right]_{t,\mathrm{I}}, \tag{9}$$

where $\int \equiv \mu^{2\varepsilon} \int d^dk/(2\pi)^d$, $d = D - 2\varepsilon$, D is the space-time dimension, and μ is the 't Hooft mass. Following the discussion presented above, the pinching action amounts to the replacement $k^\nu[\mathcal{T}^{ab}_{\mu\nu}]_{t,\mathrm{I}} \to [k^\nu \mathcal{T}^{ab}_{\mu\nu}]_{t,\mathrm{I}} = \left[\mathcal{S}^{ab}_{2\mu}(-k+q,k)\right]_{t,\mathrm{I}}$ and similarly for the term coming from the momentum $(k-q)^\mu$, *i.e.*, $[(k-q)^\mu \mathcal{T}^{ab}_{\mu\nu}]_{t,\mathrm{I}} = -\left[\mathcal{S}^{ab}_{1\nu}(-k+q,k)\right]_{t,\mathrm{I}}$, or, equivalently,

$$\Gamma^{\mathrm{P},e}_{A^3,\alpha}(q) \to g f^{eba} \int \Big([\mathcal{S}^{ab}_{2\alpha}]_{t,\mathrm{I}} - [\mathcal{S}^{ab}_{1\alpha}]_{t,\mathrm{I}} \Big). \tag{10}$$

At this point the construction of the effective PT vertex $\widehat{\Gamma}^e_\alpha$ has been completed. The next

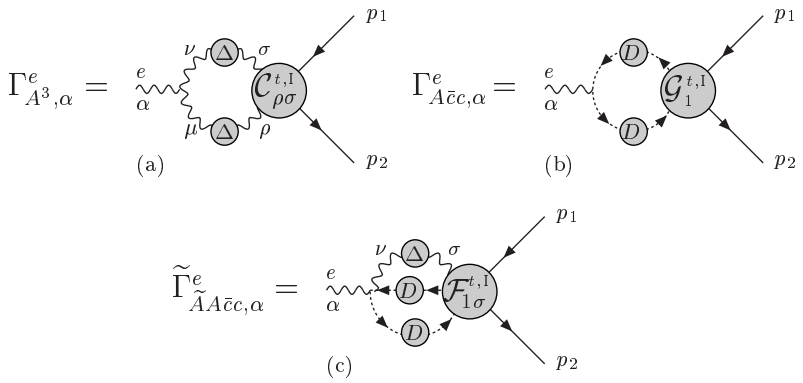

FIG. 4: The Green's functions identified in the construction of the all order PT vertex $\widehat{\Gamma}^e_\alpha$. The Green's functions (b) and (c) receive a contribution from similar terms with the ghost arrows reversed (not shown).

important point is to study the connection between $\widehat{\Gamma}^e_\alpha$ and the vertex $\widetilde{\Gamma}^e_\alpha$ in the BFG. To begin with, all "inert" terms contained in the original Γ^e_α carry over to the same sub-groups of graphs obtained in the BFG; most notably, the $\Gamma^{\mathrm{F},e}_{A^3,\alpha}$ is precisely the $\widetilde{\Gamma}^e_{\widetilde{A}A^2,\alpha}$ part of $\widetilde{\Gamma}^e_\alpha$, where $\widetilde{A}$ is the background gluon. The only exception are the ghost-diagrams contributing to Γ^e_α [Fig.4(b)]; the latter do *not* coincide with the corresponding ghost contributions in the BFG.

The important step is to recognize that the BFG ghost sector is provided precisely by combining the RFG ghosts with the right-hand side of Eq.(8). Specifically, one arrives at both the *symmetric* vertex $\widetilde{\Gamma}^e_{\widetilde{A}\bar{c}c}$, characteristic of the BFG, as well as at the four-particle ghost vertex $\widetilde{\Gamma}^e_{\widetilde{A}A\bar{c}c}$, which is totally absent in the conventional formalism [Fig.4(c)]. Indeed we find (omitting the spinors)

$$\begin{aligned}\int \left[\mathcal{S}^{ab}_{1\alpha}\right]_{t,\mathrm{I}} &= \int D(-k+q)\left\{-k_\alpha\left[\mathcal{G}^{ab}_1(-k+q,k)\right]_{t,\mathrm{I}} D(k)\right.\\ &+ \left. igf^{bcd}\left[\mathcal{Q}^{acd}_{1\alpha}(-k+q,k)\right]_{t,\mathrm{I}}\right\}. \end{aligned} \tag{11}$$

A similar equation, in which we have to trade the $\mathcal{G}^{ab}_1$ and $\mathcal{Q}^{acd}_{1\alpha}$ Green's functions for their Bose symmetric ones $\mathcal{G}^{ab}_2$ and $\mathcal{Q}^{acd}_{2\alpha}$, holds for the $\mathcal{S}_{2\alpha}$ term. It is then easy to show that

$$\begin{aligned}\widetilde{\Gamma}^e_{\widetilde{A}\bar{c}c,\alpha}(q) &\equiv \Gamma^e_{A\bar{c}c,\alpha}(q) + gf^{eba}\int\left\{k_\alpha\left[\mathcal{G}^{ab}_1(-k+q,k)\right]_{t,\mathrm{I}}\right.\\ &+ \left.(k-q)_\alpha\left[\mathcal{G}^{ab}_2(-k+q,k)\right]_{t,\mathrm{I}}\right\} D(-k+q)D(k),\\ \widetilde{\Gamma}^e_{\widetilde{A}A\bar{c}c,\alpha}(q) &\equiv ig^2 f^{eba}\int\left\{f^{acd}\left[\mathcal{Q}^{cdb}_{2\alpha}(-k+q,k)\right]_{t,\mathrm{I}} D(k)\right.\end{aligned}$$

$$- \; f^{bcd}\left[\mathcal{Q}^{acd}_{1\alpha}(-k+q,k)\right]_{t,\mathrm{I}} D(-k+q)\Big\}\,. \tag{12}$$

This concludes the proof that $\widehat{\Gamma}^{e}_{\alpha} \equiv \widetilde{\Gamma}^{e}_{\alpha}$. We emphasize that the sole ingredient in the above construction has been the STI of Eq.ônshSTI; in particular, at no point have we employed *a priori* the background formalism. Instead, its special ghost sector has arisen *dynamically*, once the PT rearrangement has taken place.

The final step is to construct the (all orders) PT gluon self-energy $\widehat{\Pi}^{ab}_{\mu\nu}$. Notice that at this point one would expect that it too coincides with the BFG gluon self-energy $\widetilde{\Pi}^{ab}_{\mu\nu}$, since both the boxes as well as the vertex do coincide with the corresponding quantities in BFG, and the S-matrix is unique (renormalization may be carried out order-by-order without any complications, see second paper in [7]). We will carry out a proof based on the strong induction principle, which states that a given predicate $P(n)$ on $\mathbb{N}$ is true $\forall\; n \in \mathbb{N}$, if $P(k)$ is true whenever $P(j)$ is true $\forall\; j \in \mathbb{N}$ with $j < k$. We will use a schematic notation, suppressing Lorentz, color, and momentum indices. At one- and two-loop, we know that the result is true [4, 7]. Assuming then that the PT construction has been successfully carried out up to the order $n-1$, we will show that the PT gluon self-energy is equal to the BFG gluon self-energy at order n, hence proving that this equality holds true at any given n. From the inductive hypothesis, we know that $\widehat{\Pi}^{[\ell]} \equiv \widetilde{\Pi}^{[\ell]}$, $\widehat{\Gamma}^{[\ell]} \equiv \widetilde{\Gamma}^{[\ell]}$, and $\widehat{B}^{[\ell]} \equiv \widetilde{B}^{[\ell]} \equiv B^{[\ell]}$, with $\ell = 1, \ldots, n-1$. Now, the S-matrix element of order n, to be denoted as $S^{[n]}$, assumes the form $S^{[n]} = \{\Gamma\Delta\Gamma\}^{[n]} + B^{[n]}$. Moreover, since it is unique, regardless if it is written in the RFG, in the BFG, as well as before and after the PT rearrangement, we have that $S^{[n]} \equiv \widehat{S}^{[n]} \equiv \widetilde{S}^{[n]}$. Using then the fact that $\widehat{B}^{[\ell]} \equiv \widetilde{B}^{[\ell]}$ holds true even when $\ell = n$, we find that $\{\Gamma\Delta\Gamma\}^{[n]} \equiv \{\widehat{\Gamma}\widehat{\Delta}\widehat{\Gamma}\}^{[n]} \equiv \{\widetilde{\Gamma}\widetilde{\Delta}\widetilde{\Gamma}\}^{[n]}$. These amplitudes can then be split into 1PR and 1PI parts; in particular, the 1PR part after the PT rearrangement coincides with the 1PR part written in the BFG, since $\{\Gamma\Delta\Gamma\}^{[n]}_{\mathrm{R}} = \Gamma^{[n_1]}\Delta^{[n_2]}\Gamma^{[n_3]}$ with $n_1, n_2, n_3 < n$, and $n_1 + n_2 + n_3 = n$. This implies in turn the equivalence of the 1PI parts, *i.e.*,

$$\left(\widehat{\Gamma}^{[n]} - \widetilde{\Gamma}^{[n]}\right)\Delta^{[0]}\Gamma^{[0]} + \Gamma^{[0]}\Delta^{[0]}\left(\widehat{\Gamma}^{[n]} - \widetilde{\Gamma}^{[n]}\right) + \Gamma^{[0]}\Delta^{[0]}\left(\widehat{\Pi}^{[n]} - \widetilde{\Pi}^{[n]}\right)\Delta^{[0]}\Gamma^{[0]} \equiv 0. \tag{13}$$

At this point, by means of the *explicit* construction presented for the vertex, we have that $\widehat{\Gamma}^{[n]} \equiv \widetilde{\Gamma}^{[n]}$, so that one immediately gets $\widehat{\Pi}^{[n]} \equiv \widetilde{\Pi}^{[n]}$. Hence, by strong induction, the above relation is true for any given perturbative order n, *i.e.*, we have $\widehat{\Pi}^{ab}_{\mu\nu} \equiv \widetilde{\Pi}^{ab}_{\mu\nu}$, *q.e.d.*

In conclusion, we have shown that the use of the STI ônshSTI satisfied by the special Green's function ŝgf, allows for the generalization of the PT procedure to all orders. It would

be interesting to further explore the physical meaning of the n-point functions obtained [10], and establish possible connections with related formalisms [11].

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