

The Two-Loop Pinch Technique in the Electroweak Sector

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Abstract

The generalization of the two-loop Pinch Technique to the Electroweak Sector of the Standard Model is presented. We restrict ourselves to the case of conserved external currents, and provide a detailed analysis of both the charged and neutral sectors. The crucial ingredient for this construction is the identification of the parts discarded during the pinching procedure with well-defined contributions to the Slavnov-Taylor identity satisfied by the off-shell one-loop gauge-boson vertices; the latter are nested inside the conventional two-loop self-energies. It is shown by resorting to a set of powerful identities that the two-loop effective Pinch Technique self-energies coincide with the corresponding ones computed in the Background Feynman gauge. The aforementioned identities are derived in the context of the Batalin-Vilkovisky formalism, a fact which enables the individual treatment of the self-energies of the photon and the Z -boson. Some possible phenomenological applications are briefly discussed.

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I. INTRODUCTION

In strongly coupled theories such as QCD the need for addressing non-perturbative phenomena in the continuum through the study of Schwinger-Dyson equations has motivated the invention [1] and further development [2] of the diagrammatic method known as the Pinch Technique (PT), in an attempt to devise a self-consistent, physically meaningful truncation scheme. The fundamental underlying problem is that off-shell Green's function are in general unphysical, and reliable information may be extracted from them only when they are combined to form observables order-by-order in perturbation theory, which is certainly not the case when dealing with intrinsically non-perturbative equations. The PT reorganizes systematically a given physical amplitude into sub-amplitudes, which have the same kinematic properties as conventional n -point functions, (propagators, vertices, boxes) but, in addition, are endowed with desirable physical properties. Most importantly, at one-loop order they are independent of the gauge-fixing parameter and satisfy naive, (ghost-free) tree-level Ward Identities (WIs), instead of the usual Slavnov-Taylor Identities (STIs) [3]. These, and other important properties, are realized diagrammatically by exploiting the elementary Ward identities of the theory in order to enforce crucial cancellations. Given their special properties the PT n -point functions could thus serve, at least in principle, as the new building blocks of an improved set of manifestly gauge-invariant Schwinger-Dyson equations, a task which, however, still remains incomplete.

On the other hand, the generalization of the PT to the Electroweak sector of the Standard Model [4, 5, 6], has given rise to various applications. In particular, the physics of unstable particles, and the computation of resonant transition amplitudes has attracted significant attention in recent years, because it is both phenomenologically relevant and theoretically challenging [7]. At one-loop order, the resummation formalism based on the PT [8] has accomplished the simultaneous reconciliation of crucial physical requirements such as gauge-fixing parameter independence, gauge-invariance, renormalization-group invariance, and the optical and equivalence theorems [9]. Thus, the Breit-Wigner type of propagators so constructed give rise to Born-improved amplitudes free of any unphysical artifacts. Other applications include the correct definition of off-shell form-factors for extracting the anomalous AWW and ZWW couplings [10], the derivation of a gauge-invariant and process-independent neutrino charged radius [4, 11], the gauge-invariant formulation [12] of the STU

parameters [13], the manifestly gauge- and renormalization-group-invariant formulation of the precision electroweak corrections [14], the unambiguous definition of the universal part of the two-loop ρ parameter [15], the gauge-invariant formulation of resonant CP violation [16], and the resolution of issues related to gauge- and scheme-dependence of mixing matrix renormalization [17, 18].

The generalization of the PT beyond one-loop has been presented in [19] for the case of massless Yang-Mills. However, its extension to the Electroweak Sector has been pending, mainly due to the following two reasons: First, at the technical level, the direct application of the diagrammatic construction used in the QCD case to the Electroweak Sector, due to the proliferation of Feynman diagrams in the latter, would lead to a major book-keeping challenge. Second, at the conceptual level, the modification of the STIs used in intermediate steps, and in particular the non-transversality of the gauge-boson self-energies, complicates further the construction, and requires additional theoretical input, not needed in the QCD case.

Recently however significant progress has been made in formulating the PT non-diagrammatically, leading to an enormous simplification of the operational aspects of the PT construction, by allowing the collective treatment of entire sets of Feynman graphs, instead of the algebraic manipulation of individual graphs [20]. There are two basic facts which have enabled the aforementioned improvement: First, it has been realized that the PT constructions amounts to the judicious reallocation of well-defined contributions generated when the longitudinal momenta circulating inside the one- and two-loop graphs trigger the STI of the three-gluon vertex (tree-level and one-loop, respectively), which is *nested* in the aforementioned graphs. Thus, the parts of the one-loop and two-loop Feynman diagrams that are shuffled around during the pinching process are systematically identified in terms of well-defined field-theoretical objects, namely the ghost Green's functions which appear in the aforementioned STI. Second, the task of comparing the resulting PT effective Green's functions to those computed in the Feynman gauge of the Background Field Method (BFM) [21], in order to verify whether the known correspondence [19, 22] persists in the two-loop Electroweak case, is significantly facilitated by resorting to a set of non-trivial identities, the so-called Background-Quantum Identities (BQIs), which relate the BFM n -point functions to the corresponding conventional n -point functions computed in the covariant renormalizable gauges, to all orders in perturbation theory. These BQIs are derived in the context

of the Batalin-Vilkovisky (BV) formalism [23], and contain auxiliary Green’s functions of “anti-fields” and background sources.

In this paper we generalize the two-loop PT in the case of the Electroweak Sector by resorting to the aforementioned theoretical ingredients. In particular, we focus on the “intrinsic PT construction”, which represent a more economical alternative to the usual, more laborious, explicit “S-matrix” PT. We carry out the PT construction both for the charged and the neutral sector, thus defining the PT two-loop self-energies for the W - and Z -bosons respectively. To simplify the construction, without compromising the novel features we want to address, we restrict ourselves to the case where the external (charged and neutral) currents are conserved, *i.e.*, the external on-shell fermions are considered to be massless. One of the main ingredients of the two-loop construction are the STIs satisfied by the off-shell three-gauge-boson vertices appearing nested inside the WW , ZZ , AA , and AZ self-energies, *i.e.*, the vertices WWZ , and WWA . These STI are triggered by longitudinal momenta originating from other *elementary* three-gauge-boson vertices, appearing inside the same Feynman graphs. The STIs employed are directly derived in the framework of the BV formalism, which allows for an elegant unified treatment, and, in addition, facilitates the task of comparing the resulting PT gauge-boson self-energies to those of the BFM. Specifically, the BV formalism applied in the Electroweak Sector, and for the particular objectives we would like to achieve, proves more suitable than the Zinn-Justin approach [24], usually employed in the literature. The basic advantage of the BV approach in the present context is that, by treating on equal footing the photon and the Z , allows one to disentangle the BQI for the photon self-energy from the corresponding BQI for the Z -boson self-energy. Thus, one may compare the PT and BFM expressions for each of the two self-energies separately. Instead, the Zinn-Zustin approach yields a BQI involving both self-energies in a single expression, which, even though is sufficient for addressing issues of renormalization, is not particularly helpful to our purposes.

The paper is organized as follows: In Section II we present the BV formalism for the case of the Electroweak Sector of the Standard Model. In Section III we derive the basic ingredients, which will allow us both the definition of the two-loop intrinsic PT self-energies, and their easy comparison to the corresponding quantities defined in the Feynman gauge of the BFM. In particular, we derive the STI for the three-gauge-boson vertices, and the *disentangled* BQIs for the gauge-boson self-energies. In section IV we use the known one-loop

results derived in the context of the “ S -matrix” PT in order to familiarize ourselves with the correspondence between the two formalisms. In Section V we prepare the stage for the two-loop construction by studying the one-loop intrinsic PT, by explaining the role of the STI, which in this case coincides with the usual tree-level WI satisfied by the bare three-gauge-boson vertex. The next three sections are rather technical, and contain the main result of our paper, namely the Electroweak two-loop intrinsic PT construction. In particular, in Section VI present the general philosophy and methodology, which is subsequently applied in detail in the charged (Section VII) and neutral (Section VIII) sectors. In Section IX we present our conclusions, whereas Feynman rules for constructing perturbatively the auxiliary Green’s functions appearing in the BQIs are listed in the final Appendix.

II. THE BV FORMALISM IN THE ELECTROWEAK SECTOR

In this section we will briefly review the most salient features of the BV formalism [23] as it applies to the Electroweak Sector. In order to define the relevant quantities and set up the notation used throughout the paper, we begin by writing the classical (gauge invariant) Standard Model Lagrangian as

$$\mathcal{L}_{\text{SM}}^{\text{cl}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{H}}. \quad (2.1)$$

The gauge invariant $SU(2)_w \otimes U(1)_Y$ Yang-Mills part \mathcal{L}_{YM} consists of an isotriplet W_μ^a (with $a = 1, 2, 3$) associated with the weak isospin generators T_w^a , an isosinglet W_μ^4 with weak hypercharge Y_w associated to the group factor $U(1)_Y$; it reads

$$\begin{aligned} \mathcal{L}_{\text{YM}} &= -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \\ &= -\frac{1}{4} (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_w f^{abc} W_\mu^b W_\nu^c)^2 - \frac{1}{4} (\partial_\mu W_\nu^4 - \partial_\nu W_\mu^4)^2 + \mathcal{L}_\psi. \end{aligned} \quad (2.2)$$

The Higgs-boson part \mathcal{L}_{H} involves a complex $SU(2)_w$ scalar doublet field φ and its complex conjugate φ^\dagger given by

$$\varphi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(H + i\chi) \end{pmatrix}, \quad \varphi^\dagger = \begin{pmatrix} \frac{1}{\sqrt{2}}(H - i\chi) \\ -\phi^- \end{pmatrix}. \quad (2.3)$$

Here H denotes the physical Higgs field while ϕ^\pm and χ represents respectively charged and neutral unphysical degrees of freedom. Then \mathcal{L}_{H} takes the form

$$\mathcal{L}_{\text{H}} = (D_\mu \varphi)^\dagger (D^\mu \varphi) - V(\varphi) \quad (2.4)$$

with the covariant derivative D_μ defined as

$$D_\mu = \partial_\mu - ig_w T_W^a W_\mu^a + ig_1 \frac{Y_W}{2} W_\mu^4 \quad (2.5)$$

and the Higgs potential as

$$V(\varphi) = \frac{\lambda}{4} (\varphi^\dagger \varphi)^2 - \mu^2 (\varphi^\dagger \varphi). \quad (2.6)$$

The Higgs field H will give mass to all the Standard Model fields, by acquiring a vacuum expectation value v ; in particular the masses of the gauge fields are generated after absorbing the massless would-be Goldstone bosons ϕ^\pm and χ . The physical massive gauge-bosons W^\pm , Z and the photon A are then obtained by diagonalizing the mass matrix, and reads

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ W_\mu^4 \end{pmatrix}, \quad (2.7)$$

where

$$c_w = \cos \theta_w = \frac{g_w}{\sqrt{g_1^2 + g_w^2}} = \frac{M_W}{M_Z}, \quad s_w = \sin \theta_w = \sqrt{1 - c_w^2}, \quad (2.8)$$

and θ_w is the weak mixing angle. Finally, to the Lagrangian $\mathcal{L}_{\text{SM}}^{\text{cl}}$ must be added the matter Lagrangian \mathcal{L}_ψ ; its explicit form may be found in [25], but is not important for what follows.

For quantizing the theory, a gauge fixing term must be added to the classical Lagrangian $\mathcal{L}_{\text{SM}}^{\text{cl}}$. To avoid tree-level mixing between gauge and scalar fields, a renormalizable R_ξ gauge of the 't Hooft type is most commonly chosen; this is specified by one gauge parameter for each gauge-boson, and defined through the linear gauge fixing functions

$$\begin{aligned} F^\pm &= \partial^\mu W_\mu^\pm \mp i\xi_w M_W \phi^\pm, \\ F^Z &= \partial^\mu Z_\mu - i\xi_z M_Z \chi, \\ F^A &= \partial^\mu A_\mu, \end{aligned} \quad (2.9)$$

yielding to the R_ξ gauge fixing Lagrangian

$$\mathcal{L}_{\text{GF}} = (\xi_w B^+ B^- + B^+ F^- + B^- F^+) + \left[\frac{1}{2} \xi_z (B^Z)^2 + B^Z F^Z \right] + \left[\frac{1}{2} \xi_A (B^A)^2 + B^A F^A \right]. \quad (2.10)$$

The fields B^\pm , B^Z and B^A represent auxiliary, non propagating fields: they are the so called Nakanishy-Lautrup Lagrange multipliers for the gauge condition, and they can be eliminated through their equations of motion

$$B^\pm = -\frac{1}{\xi_w} F^\pm, \quad B^Z = -\frac{1}{\xi_z} F^Z, \quad B^A = -\frac{1}{\xi_A} F^A, \quad (2.11)$$

leading to the usual gauge fixing Lagrangian

$$\mathcal{L}_{\text{GF}} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi_A} (F^A)^2. \quad (2.12)$$

The corresponding Faddeev-Popov ghost sector reads then

$$\mathcal{L}_{\text{FPG}} = -\sum_{n,m} \bar{u}^n \frac{\delta F^n}{\delta \theta^m} u^m, \quad (2.13)$$

where $n, m = \pm, Z, A$, while $\delta F^n / \delta \theta^m$ denotes the variation of the gauge fixing functions under an infinitesimal gauge transformation (with gauge parameters θ^m).

The complete Standard Model Lagrangian in the R_ξ gauges reads then

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SM}}^{\text{cl}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FPG}}. \quad (2.14)$$

The full set of Feynman rules derived from this Lagrangian (together with the BFM gauge fixing procedure and the corresponding Feynman rules) can be found in [25], and will be used throughout the paper.

The starting point of the BV formalism is the introduction of an *external* field – called anti-field – $\Phi^{*,n}$ for *each* field Φ^n appearing in the Lagrangian, regardless of its transformation properties under the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [26]. This is to be contrasted with the approach proposed by Zinn-Justin [24], where one introduces anti-fields only for fields transforming non-linearly under the BRST transformation. This would mean that one would introduce the anti-field $W^{*,3}$ for the W^3 combination of the physical fields Z and A , but no anti-field $W^{*,4}$ for the W^4 combination, which transforms linearly. Moreover, no Weinberg rotation for the anti-fields should be introduced. While the Zinn-Justin approach encodes the necessary information for addressing issues of renormalization in the Standard Model, the STIs and BQIs of the neutral sector become entangled, and it is not evident how to extract the identities needed for constructing the *individual* PT two-loop self-energies (see Section VIII), and for subsequently comparing them with those of the BFM. The great advantage of the BV formalism in the present context is that it treats on the same footing all the gauge fields, avoiding the inconvenient entanglement of both the STIs as well as the BQIs.

The anti-fields $\Phi^{*,n}$ will carry the same Bose/Fermi statistic of the corresponding field Φ^n and a ghost number such that

$$gh \{ \Phi^{*,n} \} = -gh \{ \Phi^n \} - 1. \quad (2.15)$$

Thus, since the ghost number is equal to 1 for the ghost fields u^n , to -1 for the anti-ghost fields \bar{u}^n , and zero for the other fields, one has the assignment

$$gh \{V_\mu^{*,n}, u^{*,n}, \bar{u}^{*,n}, G^*, \psi^{*,I}, \bar{\psi}^{*,I}\} = \{-1, -2, 0, -1, -1, -1\}, \quad (2.16)$$

where we introduced the short-hand notation

$$\begin{aligned} V_\mu^n &= (W_\mu^+, W_\mu^-, Z_\mu, A_\mu), & V_\mu^{*,n} &= (W_\mu^{*,+}, W_\mu^{*, -}, Z_\mu^*, A_\mu^*), \\ u^n &= (u^+, u^-, u^Z, u^A), & u^{*,n} &= (u^{*,+}, u^{*, -}, u^{*,Z}, u^{*,A}), \\ G^n &= (\phi^+, \phi^-, \chi, H), & G^{*,n} &= (\phi^{*,+}, \phi^{*, -}, \chi^*, H^*). \end{aligned} \quad (2.17)$$

The original gauge invariant Lagrangian is then supplemented by a term coupling the fields Φ^n to the corresponding anti-fields $\Phi^{*,n}$, giving the modified Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{BV}} &= \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{H}} + \mathcal{L}_{\text{BRST}} \\ &= \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{H}} + \sum_n \Phi^{*,n} s\Phi^n, \end{aligned} \quad (2.18)$$

where s is the BRST operator. The BRST transformations of all the Standard Model fields can be found in [27].

The action $\Pi^{(0)}[\Phi, \Phi^*]$ which is built up from the new Lagrangian \mathcal{L}_{BV} will satisfy the *master equation*

$$\sum_n \int d^4x \left[\frac{\delta \Pi^{(0)}}{\delta \Phi^{*,n}} \frac{\delta \Pi^{(0)}}{\delta \Phi^n} \right] = 0, \quad (2.19)$$

which is just a consequence of the BRST invariance of the action and of the nilpotency of the BRST operator.

Since the anti-fields are external fields we must constrain them to suitable values before we can use the action $\Pi^{(0)}$ in the calculation of S -matrix elements. To this purpose one introduces an arbitrary fermionic functional $\Psi[\Phi]$ (with $gh \{\Psi[\Phi]\} = -1$) such that

$$\Phi^{*,n} = \frac{\delta \Psi[\Phi]}{\delta \Phi^n}. \quad (2.20)$$

Then the action becomes

$$\begin{aligned} \Pi^{(0)}[\Phi, \delta \Psi / \delta \Phi] &= \Pi^{(0)}[\Phi] + \sum_n (s\Phi^n) \frac{\delta \Psi[\Phi]}{\delta \Phi^n} \\ &= \Pi^{(0)}[\Phi] + s\Psi[\Phi], \end{aligned} \quad (2.21)$$

i.e., it is equivalent to the gauge fixed action of the Yang-Mills theory under scrutiny, since we can choose the fermionic functional Ψ to satisfy

$$s\Psi[\Phi] = \int d^4x (\mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FPG}}). \quad (2.22)$$

The fermionic functional Ψ is often referred to as the gauge fixing fermion.

Moreover, since the anti-ghost anti-fields $\bar{u}^{*,n}$ and the auxiliary fields B^n have linear BRST transformations, they form the so called *trivial pairs*: they enter, together with their anti-fields, bilinearly in the action

$$\mathbb{I}^{(0)}[\Phi, \Phi^*] = \mathbb{I}_{\text{min}}^{(0)}[V_\mu^n, u^n, G^n, V_\mu^{*,n}, u^{*,n}, G^{*,n}] - B^n \bar{u}^{*,n}. \quad (2.23)$$

The last term has no effect on the master equation, which will be in fact satisfied by the *minimal* action $\mathbb{I}_{\text{min}}^{(0)}$ alone. In what follows we will restrict our considerations to the minimal action (which depends on the *minimal variables* $V_\mu^n, u^n, G^n, V_\mu^{*,n}, u^{*,n}, G^{*,n}$), dropping the corresponding subscript.

It is well known that the BRST symmetry is crucial for providing the unitarity of the S -matrix and the gauge independence of physical observables; thus it must be implemented in the theory at all orders, not only at the classical level. This is provided by establishing the quantum corrected version of Eq.(2.19), in the form of the STI functional

$$\begin{aligned} \mathcal{S}(\mathbb{I})[\Phi, \Phi^*] &= \sum_n \int d^4x \left[\frac{\delta \mathbb{I}}{\delta \Phi^{*,n}} \frac{\delta \mathbb{I}}{\delta \Phi^n} \right] \\ &= \sum_n \int d^4x \left[\frac{\delta \mathbb{I}}{\delta V_\mu^{*,n}} \frac{\delta \mathbb{I}}{\delta V_\mu^n} + \frac{\delta \mathbb{I}}{\delta u^{*,n}} \frac{\delta \mathbb{I}}{\delta u^n} + \frac{\delta \mathbb{I}}{\delta G^{*,n}} \frac{\delta \mathbb{I}}{\delta G^n} \right. \\ &\quad \left. + \sum_I \left(\frac{\delta \mathbb{I}}{\delta \psi^{*,I}} \frac{\delta \mathbb{I}}{\delta \bar{\psi}^I} + \frac{\delta \mathbb{I}}{\delta \psi^I} \frac{\delta \mathbb{I}}{\delta \bar{\psi}^{*,I}} \right) \right] \\ &= \int d^4x \left[\frac{\delta \mathbb{I}}{\delta W_\mu^{*,+}} \frac{\delta \mathbb{I}}{\delta W_\mu^-} + \frac{\delta \mathbb{I}}{\delta W_\mu^{*, -}} \frac{\delta \mathbb{I}}{\delta W_\mu^+} + \frac{\delta \mathbb{I}}{\delta u^{*,+}} \frac{\delta \mathbb{I}}{\delta u^-} + \frac{\delta \mathbb{I}}{\delta u^{*, -}} \frac{\delta \mathbb{I}}{\delta u^+} \right. \\ &\quad + \frac{\delta \mathbb{I}}{\delta Z_\mu^*} \frac{\delta \mathbb{I}}{\delta Z_\mu} + \frac{\delta \mathbb{I}}{\delta u^{*,Z}} \frac{\delta \mathbb{I}}{\delta u^Z} + \frac{\delta \mathbb{I}}{\delta A_\mu^*} \frac{\delta \mathbb{I}}{\delta A_\mu} + \frac{\delta \mathbb{I}}{\delta u^{*,A}} \frac{\delta \mathbb{I}}{\delta u^A} \\ &\quad + \frac{\delta \mathbb{I}}{\delta \phi^{*,+}} \frac{\delta \mathbb{I}}{\delta \phi^-} + \frac{\delta \mathbb{I}}{\delta \phi^{*, -}} \frac{\delta \mathbb{I}}{\delta \phi^+} + \frac{\delta \mathbb{I}}{\delta \chi^*} \frac{\delta \mathbb{I}}{\delta \chi} + \frac{\delta \mathbb{I}}{\delta H^*} \frac{\delta \mathbb{I}}{\delta H} \\ &\quad \left. + \sum_I \left(\frac{\delta \mathbb{I}}{\delta \psi^{*,I}} \frac{\delta \mathbb{I}}{\delta \bar{\psi}^I} + \frac{\delta \mathbb{I}}{\delta \psi^I} \frac{\delta \mathbb{I}}{\delta \bar{\psi}^{*,I}} \right) \right] \\ &= 0 \end{aligned} \quad (2.24)$$

where $\mathbb{I}[\Phi, \Phi^*]$ is now the effective action, and the sum is extended over all the Standard Model fermions. Eq.(2.24) gives rise to the complete set of non linear STIs at all orders in

the perturbative theory, via the repeated application of functional differentiation. Notice that $gh\{\mathcal{S}(\mathbb{I})\} = +1$, and that Green functions with non-zero ghost charge vanish, since it is a conserved quantity. This implies that for obtaining non-trivial identities it is necessary to differentiate the expression (2.24) with respect to one ghost field (ghost charge +1), or with respect to two ghost fields and one anti-field (ghost charge +2 - 1 = +1 again). For example, for deriving the STI satisfied by the three-gauge-boson vertex, one has to differentiate Eq.(2.24) with respect to two gauge-boson fields and one ghost field (see Section III A below).

A technical remark is in order here. Recall that we have chosen to work with the minimal generating functional \mathbb{I} , from which the trivial pairs $(B^n, \bar{u}^{*,n})$ has been removed [28]. In the case of a linear gauge fixing, such as the one at hand, this is equivalent to working with the “reduced” functional \mathbb{I} , defined by subtracting from the complete generating functional \mathbb{I}^C the local term $\int d^4x \mathcal{L}_{GF}$ corresponding to the gauge fixing part of the Lagrangian. One should then keep in mind that the Green’s functions generated by the minimal effective action \mathbb{I} , or the complete one \mathbb{I}^C , are *not* equal [29]. At tree-level, one has for example that

$$\begin{aligned}\mathbb{I}_{W_\mu^\pm W_\nu^\mp}^{(0)}(q) &= \mathbb{I}_{W_\mu^\pm W_\nu^\mp}^C{}^{(0)}(q) + \frac{1}{\xi_w} q_\mu q_\nu \\ &= -i \left[(q^2 - M_w^2) g_{\mu\nu} - q_\mu q_\nu \right], \\ \mathbb{I}_{\phi^\pm \phi^\mp}^{(0)}(q) &= \mathbb{I}_{\phi^\pm \phi^\mp}^C{}^{(0)} + \xi_w M_w^2.\end{aligned}\tag{2.25}$$

At higher orders the difference depends only on the renormalization of the W field and of the gauge parameter (and, as such, is immaterial for our purposes). It should be noticed that, since we have eliminated the classical gauge-fixing fermion from the generating functional \mathbb{I} , we allow for tree-level mixing between the scalar and the gauge-boson sector: this means that $\mathbb{I}_{\phi^\pm W_\mu^\mp}$ and $\mathbb{I}_{\chi Z}$ do not vanish at tree-level. However the aforementioned mixing is not present at the propagator level, *i.e.*, when these particles circulate in loops [28, 29, 30]; therefore loops must be computed using the usual Feynman rules of the R_ξ gauges [31, 32].

Another important ingredient of the construction we carry out in what follows is to write down the STI functional in the BFM. For doing this we introduce the set of classical vector and scalar fields Ω_μ^n and Ω^{G^n}

$$\Omega_\mu^n = (\Omega_\mu^+, \Omega_\mu^-, \Omega_\mu^Z, \Omega_\mu^A), \quad \Omega^{G^n} = (\Omega^+, \Omega^-, \Omega^\chi, \Omega^H),\tag{2.26}$$

which carry the same quantum numbers of the corresponding vector and scalar fields V_μ^n

and G^n respectively, but ghost charge +1. Next, we implement the equations of motion of the background fields \widehat{V}_μ^n and \widehat{G}^n at the quantum level by extending the BRST symmetry to them through the equations

$$\begin{aligned} s\widehat{V}_\mu^n &= \Omega_\mu^n & s\Omega_\mu^n &= 0, \\ s\widehat{G}^n &= \Omega^{G^n} & s\Omega^{G^n} &= 0. \end{aligned} \quad (2.27)$$

Finally, in order to control the dependence of the Green's functions on the background fields we modify the STI functional of Eq.(2.24) as [30]

$$\begin{aligned} \mathcal{S}'(\mathbb{I}')[\Phi, \Phi^*] &= \mathcal{S}(\mathbb{I}')[\Phi, \Phi^*] + \sum_n \left[\Omega_\mu^n \left(\frac{\delta \mathbb{I}'}{\delta \widehat{V}_\mu^n} - \frac{\delta \mathbb{I}'}{\delta V_\mu^n} \right) + \Omega^{G^n} \left(\frac{\delta \mathbb{I}'}{\delta \widehat{G}^n} - \frac{\delta \mathbb{I}'}{\delta G^n} \right) \right] \\ &= \mathcal{S}(\mathbb{I}')[\Phi, \Phi^*] + \Omega_\mu^+ \left(\frac{\delta \mathbb{I}'}{\delta \widehat{W}_\mu^+} - \frac{\delta \mathbb{I}'}{\delta W_\mu^+} \right) + \Omega_\mu^- \left(\frac{\delta \mathbb{I}'}{\delta \widehat{W}_\mu^-} - \frac{\delta \mathbb{I}'}{\delta W_\mu^-} \right) \\ &\quad + \Omega_\mu^Z \left(\frac{\delta \mathbb{I}'}{\delta \widehat{Z}_\mu} - \frac{\delta \mathbb{I}'}{\delta Z_\mu} \right) + \Omega_\mu^A \left(\frac{\delta \mathbb{I}'}{\delta \widehat{A}_\mu} - \frac{\delta \mathbb{I}'}{\delta A_\mu} \right) + \Omega^+ \left(\frac{\delta \mathbb{I}'}{\delta \widehat{\phi}^+} - \frac{\delta \mathbb{I}'}{\delta \phi^+} \right) \\ &\quad + \Omega^- \left(\frac{\delta \mathbb{I}'}{\delta \widehat{\phi}^-} - \frac{\delta \mathbb{I}'}{\delta \phi^-} \right) + \Omega^\chi \left(\frac{\delta \mathbb{I}'}{\delta \widehat{\chi}} - \frac{\delta \mathbb{I}'}{\delta \chi} \right) + \Omega^H \left(\frac{\delta \mathbb{I}'}{\delta \widehat{H}} - \frac{\delta \mathbb{I}'}{\delta H} \right), \end{aligned} \quad (2.28)$$

where \mathbb{I}' denotes the effective action that depends on the background sources Ω_μ^n , and $\mathcal{S}(\mathbb{I}')[\Phi, \Phi^*]$ is the STI functional of Eq.(2.24). Differentiation of the STI functional Eq.(2.28) with respect to the background source and background or quantum fields, will then relate 1PI functions involving background fields with the ones involving quantum fields (see Section III B below).

The final ingredient we need to know for the actual computation of STIs are the coupling of the anti-fields and background sources to the other fields of the theory. The former are controlled by the Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{BRST}} &= W_\mu^{*,\pm} \left\{ \partial_\mu u^\mp \pm ig_w \left(W_\mu^\mp + \widehat{W}_\mu^\mp \right) \left(s_w u^A - c_w u^Z \right) \right. \\ &\quad \mp ig_w u^\mp \left[s_w \left(A_\mu + \widehat{A}_\mu \right) - c_w \left(Z_\mu + \widehat{Z}_\mu \right) \right] \left. \right\} + u^{*,\pm} \left[\pm \frac{ig_w}{2} u^\mp \left(s_w u^A - c_w u^Z \right) \right] \\ &\quad + Z_\mu^* \left\{ \partial_\mu u^Z - ig_w c_w \left[\left(W_\mu^+ + \widehat{W}_\mu^+ \right) u^- - \left(W_\mu^- + \widehat{W}_\mu^- \right) u^+ \right] \right\} - u^{*,Z} \left(ig_w c_w u^- u^+ \right) \\ &\quad + A_\mu^* \left\{ \partial_\mu u^A + ig_w s_w \left[\left(W_\mu^+ + \widehat{W}_\mu^+ \right) u^- - \left(W_\mu^- + \widehat{W}_\mu^- \right) u^+ \right] \right\} + u^{*,A} \left(ig_w s_w u^- u^+ \right) \\ &\quad + \phi^{*,\pm} \left\{ \mp \frac{ig_w}{2} \left[\left(H + \widehat{H} \right) + v \mp i \left(\chi + \widehat{\chi} \right) \right] u^\mp \right. \\ &\quad \left. \pm ig_w \left(\phi^\mp + \widehat{\phi}^\mp \right) \left(s_w u^A - \frac{c_w^2 - s_w^2}{2c_w} u^Z \right) \right\} \end{aligned}$$

$$\begin{aligned}
& + \chi^* \left\{ \frac{g_W}{2} \left[(\phi^+ + \widehat{\phi}^+) u^- + (\phi^- + \widehat{\phi}^-) u^+ \right] - \frac{g_W}{2c_W} \left[(H + \widehat{H}) + v \right] u^Z \right\} \\
& + H^* \left\{ \frac{ig_W}{2} \left[(\phi^+ + \widehat{\phi}^+) u^- - (\phi^- + \widehat{\phi}^-) u^+ \right] + \frac{g_W}{2c_W} (\chi + \widehat{\chi}) u^Z \right\} \\
& + \mathcal{L}_{\text{BRST}}^\psi,
\end{aligned} \tag{2.29}$$

where $\mathcal{L}_{\text{BRST}}^\psi$ stands for the term involving fermions and can be found in [30]. The Lagrangian \mathcal{L}_Ω coupling the background sources with the Standard Model fields is identical to the above one upon the replacement of the anti-fields for background sources and ghost fields for anti-ghost fields. Notice that all necessary Feynman rules coming from $\mathcal{L}_{\text{BRST}}$ and \mathcal{L}_Ω are listed in Appendix A.

III. THE BASIC TOOLS: STIs AND BQIs

After having reviewed the BV formalism as it applies to the electroweak sector of the Standard Model, we next proceed to derive the basic ingredients needed for the PT construction. In particular we will focus on two aspects: *(i)* the derivation of the STIs for the off-shell propagators and three-gauge-boson vertices; as we will see these STIs are of central importance for the intrinsic PT method, to be presented in Sections V and VI. *(ii)* the derivation of the BQIs relating the background and quantum two- and three-point functions. These identities facilitate significantly the eventual comparison between the effective PT Green's functions and the BFM Green's functions, computed at $\xi_Q = 1$. The crucial point is that the conventional Green's functions are related to the BFM ones by means of the same type of building blocks as those that appear in the STIs of the three-gauge-boson vertex, derived in *(i)*, namely auxiliary, unphysical Green's functions.

A. STIs

In the BV formalism, all the STIs satisfied by the 1PI n -point functions can be derived by appropriate functional differentiation of the STI functional of Eq.(2.24).

1. Gauge-boson two-point functions

The STI satisfied by the gauge-bosons two-point functions $\mathbb{I}\Gamma_{V_\mu^n V_\nu^m}$ can be obtained by considering the following functional differentiation

$$\left. \frac{\delta^2 \mathcal{S}(\mathbb{I}\Gamma)}{\delta u^i(p_1) \delta V_\beta^j(q)} \right|_{\Phi=0} = 0 \quad q + p_1 = 0, \quad (3.1)$$

which will provide us with the STI

$$\sum_n \left[\mathbb{I}\Gamma_{u^i V_\rho^{*,n}}(q) \mathbb{I}\Gamma_{V^{n,\rho} V_\beta^j}(q) + \mathbb{I}\Gamma_{u^i G^{*,n}}(q) \mathbb{I}\Gamma_{G^n V_\beta^j}(q) \right] = 0, \quad (3.2)$$

where (as always from now on) the sum over n is constrained by charge conservation. Different values of the indices i and j will determine which of the various possible STIs implicit in Eq.(3.2) we are considering. For example, the STI satisfied by the W s two-point function is obtained by choosing $i = \pm$, $j = \mp$, and reads

$$\mathbb{I}\Gamma_{u^\pm W_\rho^{*,\mp}}(q) \mathbb{I}\Gamma_{W^{\pm,\rho} W_\beta^\mp}(q) + \mathbb{I}\Gamma_{u^\pm \phi^{*,\mp}}(q) \mathbb{I}\Gamma_{\phi^\pm W_\beta^\mp}(q) = 0. \quad (3.3)$$

Choosing instead $i = Z, A$, and letting $V_\beta^j \equiv \mathcal{V}_\beta^j$ with $j = Z, A$, so that $\mathcal{V}_\beta^j = (Z_\beta, A_\beta)$ with $M_{\mathcal{V}^j} = (M_Z, 0)$, we obtain the STI satisfied by the neutral gauge-bosons two-point functions, *i.e.*,

$$\sum_n \mathbb{I}\Gamma_{u^i \mathcal{V}_\rho^{*,n}}(q) \mathbb{I}\Gamma_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}(q) + \mathbb{I}\Gamma_{u^i H^*}(q) \mathbb{I}\Gamma_{H \mathcal{V}_\beta^j}(q) + \mathbb{I}\Gamma_{u^i \chi^*}(q) \mathbb{I}\Gamma_{\chi \mathcal{V}_\beta^j}(q) = 0. \quad (3.4)$$

2. Gauge-boson three-point functions

The STI satisfied by the three-gauge-boson vertex, can be derived by considering the following functional differentiation:

$$\left. \frac{\delta^3 \mathcal{S}(\mathbb{I}\Gamma)}{\delta u^k(q_1) \delta V_\alpha^i(q_2) \delta V_\beta^j(q_3)} \right|_{\Phi=0} = 0 \quad q_1 + q_2 + q_3 = 0, \quad (3.5)$$

which in turn will give us the STI

$$\begin{aligned} & \sum_n \left[\mathbb{I}\Gamma_{u^k V_\rho^{*,n}}(-q_1) \mathbb{I}\Gamma_{V^{n,\rho} V_\alpha^i V_\beta^j}(q_2, q_3) + \mathbb{I}\Gamma_{u^k V_\rho^{*,n} V_\beta^j}(q_2, q_3) \mathbb{I}\Gamma_{V^{n,\rho} V_\alpha^i}(q_2) \right. \\ & + \mathbb{I}\Gamma_{u^k V_\rho^{*,n} V_\alpha^i}(q_3, q_2) \mathbb{I}\Gamma_{V^{n,\rho} V_\beta^j}(q_3) + \mathbb{I}\Gamma_{u^k G^{*,n}}(-q_1) \mathbb{I}\Gamma_{G^n V_\alpha^i V_\beta^j}(q_2, q_3) \\ & \left. + \mathbb{I}\Gamma_{u^k G^{*,n} V_\beta^j}(q_2, q_3) \mathbb{I}\Gamma_{G^n V_\alpha^i}(q_2) + \mathbb{I}\Gamma_{u^k G^{*,n} V_\alpha^i}(q_3, q_2) \mathbb{I}\Gamma_{G^n V_\beta^j}(q_3) \right] = 0, \quad (3.6) \end{aligned}$$

Different values of the indices i , j and k will determine on which leg (and with which four-momentum) we are contracting the three-gauge-boson vertex. For example, if we choose $k = \pm$, $i = \mp$ and let $V_\beta^j \equiv \mathcal{V}_\beta^j$, Eq.(3.6) gives

$$\begin{aligned}
& \mathbb{I}_{u^\pm W_\rho^{*,\mp}}(-q_1) \mathbb{I}_{W^{\pm,\rho} W_\alpha^\mp \mathcal{V}_\beta^j}(q_2, q_3) + \mathbb{I}_{u^\pm W_\rho^{*,\mp}}(q_2, q_3) \mathbb{I}_{W^{\pm,\rho} W_\alpha^\mp}(q_2) \\
& + \sum_n \mathbb{I}_{u^\pm \mathcal{V}_\rho^{*,n} W_\alpha^\mp}(q_3, q_2) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}(q_3) + \mathbb{I}_{u^\pm \phi^{*,\mp}}(-q_1) \mathbb{I}_{\phi^\pm W_\alpha^\mp \mathcal{V}_\beta^j}(q_2, q_3) \\
& + \mathbb{I}_{u^\pm \phi^{*,\mp} \mathcal{V}_\beta^j}(q_2, q_3) \mathbb{I}_{\phi^\pm W_\alpha^\mp}(q_2) + \mathbb{I}_{u^\pm H^* W_\alpha^\mp}(q_3, q_2) \mathbb{I}_{H \mathcal{V}_\beta^j}(q_3) \\
& + \mathbb{I}_{u^\pm \chi^* W_\alpha^\mp}(q_3, q_2) \mathbb{I}_{\chi \mathcal{V}_\beta^j}(q_3) = 0, \tag{3.7}
\end{aligned}$$

i.e., we get the STI satisfied by the three-gauge-boson vertex when contracting from the charged W^\pm legs.

The remaining two STIs, are obtained by choosing $k = Z, A$, $i = \pm$ and $j = \mp$, and read

$$\begin{aligned}
& \sum_n \mathbb{I}_{u^k \mathcal{V}_\rho^{*,n}}(-q_1) \mathbb{I}_{\mathcal{V}^{n,\rho} W_\alpha^\pm W_\beta^\mp}(q_2, q_3) + \mathbb{I}_{u^k W_\rho^{*,\pm} W_\beta^\mp}(q_2, q_3) \mathbb{I}_{W^{\mp,\rho} W_\alpha^\pm}(q_2) \\
& + \mathbb{I}_{u^k W_\rho^{*,\mp} W_\alpha^\pm}(q_3, q_2) \mathbb{I}_{W^{\pm,\rho} W_\beta^\mp}(q_3) + \mathbb{I}_{u^k H^*}(-q_1) \mathbb{I}_{H W_\alpha^\pm W_\beta^\mp}(q_2, q_3) \\
& + \mathbb{I}_{u^k \chi^*}(-q_1) \mathbb{I}_{\chi W_\alpha^\pm W_\beta^\mp}(q_2, q_3) + \mathbb{I}_{u^k \phi^{*,\pm} W_\beta^\mp}(q_2, q_3) \mathbb{I}_{\phi^\mp W_\alpha^\pm}(q_2) \\
& + \mathbb{I}_{u^k \phi^{*,\mp} W_\alpha^\pm}(q_3, q_2) \mathbb{I}_{\phi^\pm W_\beta^\mp}(q_3) = 0, \tag{3.8}
\end{aligned}$$

which correspond to the STI satisfied by the three-gauge-boson vertex when contracting on the neutral \mathcal{V}^i legs.

B. BQIs

Standard Model BQIs were first presented in [29, 30], in the context of the Zinn-Justin formalism. Here we present instead the *disentangled* BQIs, derived in the BV formalism; they may be derived by appropriate functional differentiation of the BFM STI functional of Eq.(2.28).

1. Gauge-boson two-point functions

The BQIs for the two-point functions involving the gauge-bosons can be obtained by considering the functional differentiations

$$\left. \frac{\delta^2 \mathcal{S}(\mathbb{I})}{\delta \Omega_\alpha^i(p_1) \delta \widehat{V}_\beta^j(q)} \right|_{\Phi=0} = 0 \quad q + p_1 = 0, \tag{3.9}$$

$$\left. \frac{\delta^2 \mathcal{S}(\mathbb{I})}{\delta \Omega_\alpha^i(p_1) \delta V_\beta^j(q)} \right|_{\Phi=0} = 0 \quad q + p_1 = 0, \quad (3.10)$$

$$\left. \frac{\delta^2 \mathcal{S}(\mathbb{I})}{\delta \Omega_\alpha^i(p_1) \delta G^j(q)} \right|_{\Phi=0} = 0 \quad q + p_1 = 0, \quad (3.11)$$

which will provide us the BQIs

$$\begin{aligned} \mathbb{I}_{\widehat{V}_\alpha^i \widehat{V}_\beta^j}(q) &= \sum_n \left\{ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i V_\rho^{*,n}}(q) \right] \mathbb{I}_{V^{n,\rho} \widehat{V}_\beta^j}(q) + \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(q) \mathbb{I}_{G^n \widehat{V}_\beta^j}(q) \right\}, \\ \mathbb{I}_{\widehat{V}_\alpha^i V_\beta^j}(q) &= \sum_n \left\{ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i V_\rho^{*,n}}(q) \right] \mathbb{I}_{V^{n,\rho} V_\beta^j}(q) + \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(q) \mathbb{I}_{G^n V_\beta^j}(q) \right\}, \\ \mathbb{I}_{\widehat{V}_\alpha^i G^j}(q) &= \sum_n \left\{ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i V_\rho^{*,n}}(q) \right] \mathbb{I}_{V^{n,\rho} G^j}(q) + \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(q) \mathbb{I}_{G^n G^j}(q) \right\}. \end{aligned} \quad (3.12)$$

We can now combine these three equations in such a way that all the two-point functions mixing background and quantum fields drop out, therefore obtaining the BQI

$$\begin{aligned} \mathbb{I}_{\widehat{V}_\alpha^i \widehat{V}_\beta^j}(q) &= \sum_{m,n} \left\{ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i V_\rho^{*,n}}(q) \right] \left[g_{\beta\sigma} \delta^{jm} + \mathbb{I}_{\Omega_\beta^j V_\sigma^{*,m}}(q) \right] \mathbb{I}_{V^{m,\sigma} V^{n,\rho}}(q) \right. \\ &+ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i V_\rho^{*,n}}(q) \right] \mathbb{I}_{\Omega_\beta^j G^{*,m}}(q) \mathbb{I}_{G^m V^{n,\rho}}(q) \\ &+ \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(q) \left[g_{\beta\sigma} \delta^{jm} + \mathbb{I}_{\Omega_\beta^j V_\sigma^{*,m}}(q) \right] \mathbb{I}_{V^{m,\rho} G^n}(q) \\ &\left. + \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(q) \mathbb{I}_{\Omega_\beta^j G^{*,m}}(q) \mathbb{I}_{G^m G^n}(q) \right\}. \end{aligned} \quad (3.13)$$

In what follows we will only consider the case of conserved massless currents. Then only the first line of the above equation will contribute, since all the other terms will be proportional to q_α or q_β , so that they will vanish when contracted with the corresponding external current.

The BQIs for different self-energies are obtained by different choices of the indices i and j appearing in Eq.(3.13). For example, choosing $i = \pm$ and $j = \mp$, we get the BQI for the W propagator

$$\begin{aligned} \mathbb{I}_{\widehat{W}_\alpha^\pm \widehat{W}_\beta^\mp}(q) &= \left[g_{\alpha\rho} + \mathbb{I}_{\Omega_\alpha^\pm W_\rho^{*,\mp}}(q) \right] \left[g_{\beta\sigma} + \mathbb{I}_{\Omega_\beta^\mp W_\sigma^{*,\pm}}(q) \right] \mathbb{I}_{W^{\pm,\rho} W^{\mp,\sigma}}(q) \\ &= \mathbb{I}_{W_\alpha^\pm W_\beta^\mp}(q) + \mathbb{I}_{\Omega_\alpha^\pm W_\rho^{*,\mp}}(q) \mathbb{I}_{W^{\pm,\rho} W_\beta^\mp}(q) + \mathbb{I}_{\Omega_\beta^\mp W_\sigma^{*,\pm}}(q) \mathbb{I}_{W_\alpha^\pm W^{\mp,\sigma}}(q) \\ &+ \mathbb{I}_{\Omega_\alpha^\pm W_\rho^{*,\mp}}(q) \mathbb{I}_{W^{\pm,\rho} W^{\mp,\sigma}}(q) \mathbb{I}_{\Omega_\beta^\mp W_\sigma^{*,\pm}}(q), \end{aligned} \quad (3.14)$$

while the BQIs involving the neutral gauge-bosons propagators are obtained by letting $V_\alpha^i \equiv \mathcal{V}_\alpha^i$ and $V_\beta^j \equiv \mathcal{V}_\beta^j$, and reads

$$\mathbb{I}_{\widehat{\mathcal{V}}_\alpha^i \widehat{\mathcal{V}}_\beta^j}(q) = \sum_{m,n} \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}(q) \right] \left[g_{\beta\sigma} \delta^{jm} + \mathbb{I}_{\Omega_\beta^j \mathcal{V}_\sigma^{*,m}}(q) \right] \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}^{m,\sigma}}(q)$$

$$\begin{aligned}
&= \mathbb{I}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}(q) + \sum_n \left[\mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}(q) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}(q) + \mathbb{I}_{\Omega_\beta^j \mathcal{V}_\sigma^{*,m}}(q) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}_\alpha^i}(q) \right] \\
&+ \sum_{m,n} \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}(q) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}^{m,\sigma}}(q) \mathbb{I}_{\Omega_\beta^j \mathcal{V}_\sigma^{*,m}}(q). \tag{3.15}
\end{aligned}$$

2. Gauge-boson-fermion-anti-fermion three-point functions

For the annihilation channel (one can study equally well the elastic channel) we consider the following functional differentiation

$$\left. \frac{\delta^3 \mathcal{S}(\mathbb{I})}{\delta \Omega_\alpha^i(q) \delta \bar{\psi}(Q') \delta \psi(Q)} \right|_{\Phi=0} = 0 \quad Q' + Q + q = 0, \tag{3.16}$$

which will furnish the BQI

$$\begin{aligned}
\mathbb{I}_{\widehat{V}_\alpha^i \bar{\psi} \psi}(Q', Q) &= \sum_n \left\{ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}(-q) \right] \mathbb{I}_{\mathcal{V}^{n,\rho} \bar{\psi} \psi}(Q', Q) + \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(-q) \mathbb{I}_{G^n \bar{\psi} \psi}(Q', Q) \right\} \\
&+ \mathbb{I}_{\bar{\psi} \psi}(-Q') \mathbb{I}_{\Omega_\alpha^i \bar{\psi}^* \psi}(Q', Q) - \mathbb{I}_{\Omega_\alpha^i \psi^* \bar{\psi}}(Q, Q') \mathbb{I}_{\bar{\psi} \psi}(Q). \tag{3.17}
\end{aligned}$$

We then sandwich the above equation between on-shell spinors, and make use of the Dirac equation of motion to eliminate the last two terms; thus we arrive at the on-shell BQIs

$$\mathbb{I}_{\widehat{V}_\alpha^i \bar{\psi} \psi}(Q', Q) = \sum_n \left\{ \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}(q) \right] \mathbb{I}_{\mathcal{V}^{n,\rho} \bar{\psi} \psi}(Q', Q) + \mathbb{I}_{\Omega_\alpha^i G^{*,n}}(q) \mathbb{I}_{G^n \bar{\psi} \psi}(Q', Q) \right\}. \tag{3.18}$$

The last term appearing in Eq. (3.18) will be absent when considering the case of massless conserved currents; moreover the BQIs involving charged and neutral gauge-bosons background fields are obtained, as usual, by choosing different values of the index i . Thus, for $i = \pm$ we obtain the BQI involving the background and quantum W s

$$\mathbb{I}_{\widehat{W}_\alpha^\pm \bar{\psi} \psi}(Q', Q) = \left[g_{\alpha\rho} + \mathbb{I}_{\Omega_\alpha^\pm W_\rho^{*,\mp}}(q) \right] \mathbb{I}_{W^{\pm,\rho} \bar{\psi} \psi}(Q', Q), \tag{3.19}$$

while letting $V_\alpha^i \equiv \mathcal{V}_\alpha^i$ we get the BQIs involving background and quantum neutral gauge-bosons, which reads

$$\mathbb{I}_{\widehat{V}_\alpha^i \bar{\psi} \psi}(Q', Q) = \sum_n \left[g_{\alpha\rho} \delta^{in} + \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}(q) \right] \mathbb{I}_{\mathcal{V}^{n,\rho} \bar{\psi} \psi}(Q', Q). \tag{3.20}$$

IV. THE ONE-LOOP S -MATRIX PT REVISITED.

In this section we will briefly review the one-loop S -matrix construction of the Electroweak sector in order to establish the correspondence between results already existing in the literature and the newly introduced BV language. In particular we will re-express the one-loop S -matrix PT results in terms of the BV building blocks, and will familiarize ourselves with the use of the BQI. Notice however that the two-loop construction, which is the main result of this paper will be carried out in the context of the intrinsic PT, whose one-loop preliminaries will be presented in the next section.

We will consider for concreteness the S -matrix element for a 2 fermion elastic scattering process $\psi(P)\psi(P') \rightarrow \psi(Q)\psi(Q')$ in the Electroweak sector of the Standard Model; we set $q = P' - P = Q' - Q$, and $s = q^2$ is the square of the momentum transfer. One could equally well study the annihilation channel of the process $\psi(P)\bar{\psi}(P') \rightarrow \psi(Q)\bar{\psi}(Q')$, in which case s would be the center-of-mass energy. We assume that the theory has been quantized using the renormalizable (R_ξ) gauges [31, 32], and, without loss of generality, we choose the Feynman gauge. Then, the only pinching contributions originate from the elementary three-gauge-boson vertices appearing inside vertex graphs. The bare tree-level three-gauge-boson vertex is given by the following expression (all momenta are incoming, *i.e.*, $q + p_1 + p_2 = 0$)

$$\begin{array}{c}
 V_\alpha \\
 \text{wavy} \\
 q
 \end{array}
 \begin{array}{c}
 V_\mu^1 \\
 \text{wavy} \\
 p_1
 \end{array}
 \begin{array}{c}
 V_\nu^2 \\
 \text{wavy} \\
 p_2
 \end{array}
 = \mathbb{I}_{V_\alpha V_\mu^1 V_\nu^2}^{(0)}(q, p_1, p_2) = ig_W C \Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2),$$

where

$$C_{V^i W^+ W^-} = \begin{cases} s_W, & \text{if } i = A \\ -c_W, & \text{if } i = Z \end{cases} \quad (4.1)$$

and, finally,

$$\Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2) = (q - p_1)_\nu g_{\alpha\mu} + (p_1 - p_2)_\alpha g_{\mu\nu} + (p_2 - q)_\mu g_{\alpha\nu}. \quad (4.2)$$

The Lorentz structure $\Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2)$ may be split into two parts [2, 33]

$$\Gamma_{\alpha\mu\nu}^{(0)}(q, p_1, p_2) = \Gamma_{\alpha\mu\nu}^F(q, p_1, p_2) + \Gamma_{\alpha\mu\nu}^P(q, p_1, p_2), \quad (4.3)$$

with

$$\begin{aligned}
 \Gamma_{\alpha\mu\nu}^F(q, p_1, p_2) &= (p_1 - p_2)_\alpha g_{\mu\nu} + 2q_\nu g_{\alpha\mu} - 2q_\mu g_{\alpha\nu}, \\
 \Gamma_{\alpha\mu\nu}^P(q, p_1, p_2) &= p_{2\nu} g_{\alpha\mu} - p_{1\mu} g_{\alpha\nu}.
 \end{aligned} \quad (4.4)$$

The vertex $\Gamma_{\alpha\mu\nu}^F(q, p_1, p_2)$ coincides with the Feynman gauge BFM bare vertex involving one background gauge-boson (carrying four-momentum q) and two quantum ones (carrying four-momenta p_1 and p_2). The above decomposition allows $\Gamma_{\alpha\mu\nu}^F$ to satisfy the WI

$$q^\alpha \Gamma_{\alpha\mu\nu}^F(q, p_1, p_2) = [(p_2^2 - M_{V_2}^2) - (p_1^2 - M_{V_1}^2) + (M_{V_1}^2 - M_{V_2}^2)]g_{\mu\nu}, \quad (4.5)$$

The first two terms on the right-hand side (RHS) are the difference of the two-inverse propagators appearing inside the one-loop vertex graphs (in the renormalizable Feynman gauge); the last term accounts for the difference in their masses, and is associated to the coupling of the corresponding would-be Goldstone bosons. The term $\Gamma_{\alpha\mu\nu}^P$ contains the pinching momenta; inside Feynman diagrams such as those of Fig.1 they trigger elementary WIs, which will eliminate the internal fermion propagator, resulting in an effectively propagator-like contribution. The propagator-like terms thusly generated are to be allotted to the conventional self-energy graphs, and will form part of the effective one-loop PT gauge-boson self-energy. On the other hand the remaining purely vertex-like parts define the effective PT gauge-boson-fermion-fermion three-point function $\widehat{\Pi}_{V_\alpha^n \bar{\psi}\psi}(Q', Q)$.

In the next two subsections we will carry out in detail the one-loop PT construction for both the charged as well as the neutral gauge-boson sector of the Standard Model.

A. The charged sector

In this case we will concentrate on the S -matrix element for the electron-neutrino elastic scattering process $e(P)\nu_e(P') \rightarrow e(Q)\nu_e(Q')$. Both the electron and its neutrino will be considered as strictly mass-less; (so that we can neglect longitudinal pieces); moreover, for definiteness, we will concentrate on the three-point function $\Pi_{W_\alpha^+ \bar{\psi}\psi}(Q', Q)$ (exactly the same results hold for the three-point function involving the W^- gauge-boson).

We then start by implementing (see Fig.1a and d) the vertex decomposition of Eq.(4.3), with $p_{1\mu} = -k_\mu$, $p_{2\nu} = (k - q)_\nu$, inside the $\Gamma_{W_\alpha^+ \bar{\psi}\psi}^{V^2(1)}(Q', Q)$ part of the full one-loop three-point function $\Pi_{W_\alpha^+ \bar{\psi}\psi}^{(1)}(Q', Q)$. The $\Gamma_{\alpha\mu\nu}^P(q, p_1, p_2)$ term triggers then the elementary WIs

$$\begin{aligned} \not{k} &= (\not{k} + \not{Q}) - \not{Q}, \\ \not{k} - \not{q} &= (\not{k} + \not{Q}) - \not{Q}', \end{aligned} \quad (4.6)$$

If the external fermions have non-vanishing masses the above WI is slightly modified. As has been explained in detail [4, 6] the resulting modification are compensated precisely by the

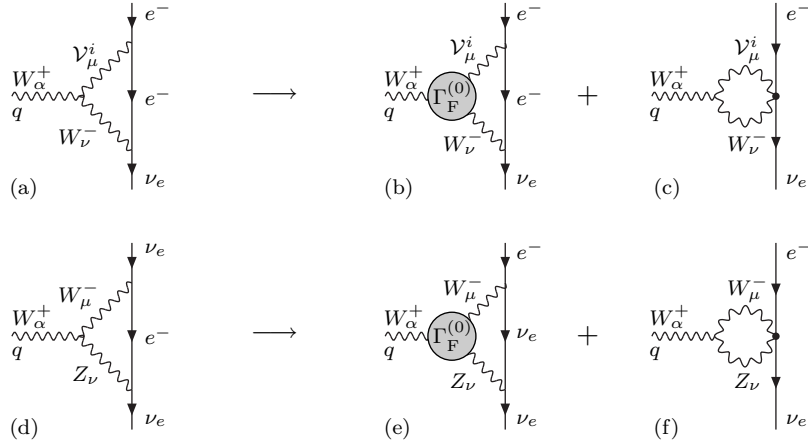


FIG. 1: Carrying out the fundamental vertex decomposition inside the three-point function $\Gamma_{W_\alpha^+ \psi \bar{\psi}}^{V^2(1)}$ (a), (d) contributing to $\Pi_{W_\alpha^+ \psi \bar{\psi}}^{(1)}$, gives rise to the genuine vertex $\widehat{\Gamma}_{W_\alpha^+ \psi \bar{\psi}}^{V^2(1)}$ (b), (e) and a self-energy-like contribution $V_{\alpha\rho}^{P(1)} \frac{\gamma^\rho P_L}{\sqrt{2}}$ (c), (f).

contributions of the would-be Goldstone bosons, which in the case of massive fermions must also be considered, allowing the generalization of the method to the case of non-conserved currents. The first terms on the RHS of the two WI identities listed in Eq.(4.6) generate two self-energy like pieces (Fig.1c and f), which are to be allotted to the conventional self-energy. In particular,

$$\begin{aligned} \Gamma_{W_\alpha^+ \psi \bar{\psi}}^{V^2(1)}(Q', Q) &= \widehat{\Gamma}_{W_\alpha^+ \psi \bar{\psi}}^{V^2(1)}(Q', Q) + V_{\alpha\rho}^{P(1)}(q) \frac{\gamma^\rho P_L}{\sqrt{2}} - X_{1\alpha}^{(1)}(Q', Q) \Sigma^{(0)}(Q') \\ &\quad - \Sigma^{(0)}(Q) X_{2\alpha}^{(1)}(Q', Q), \end{aligned} \quad (4.7)$$

where

$$\begin{aligned} \widehat{\Gamma}_{W_\alpha^+ \psi \bar{\psi}}^{V^2(1)}(Q', Q) &= ig_W^2 \int_{L_1} \Gamma_{\alpha\mu\nu}^F(q, -k, k-q) \frac{\gamma^\mu P_L}{\sqrt{2}} S^{(0)}(k+Q) \left[s_W^2 J_A(q, k) \gamma^\nu \right. \\ &\quad \left. + J_Z(q, k) \gamma^\nu \left(\frac{P_L}{2} - s_W^2 \right) + J'_Z(q, k) \gamma^\nu \frac{P_L}{2} \right], \\ V_{\alpha\rho}^{P(1)}(q) &= 2g_W^2 g_{\alpha\rho} \sum_i C_i^2 \int_{L_1} J_i(q, k) \\ &= 2g_W^2 g_{\alpha\rho} \int_{L_1} [s_W^2 J_A(q, k) + c_W^2 J_Z(q, k)], \end{aligned} \quad (4.8)$$

$C_i \equiv C_{\nu^i W^+ W^-}$ is defined in Eq.(4.1), and, finally,

$$\int_{L_1} \equiv \mu^{2\varepsilon} \int \frac{d^d k}{(2\pi)^d},$$

$$\begin{aligned}
J_i(q, k) &= d_W(k)d_{\nu^i}(k - q), \\
J'_i(q, k) &= d_W(k - q)d_{\nu^i}(k),
\end{aligned}
\tag{4.9}$$

with

$$\begin{aligned}
d_W(q) &= \frac{1}{q^2 - M_W^2}, \\
d_{\nu^i}(q) &= \frac{1}{q^2 - M_{\nu^i}^2}.
\end{aligned}
\tag{4.10}$$

Notice that the last two terms appearing in the RHS of Eq.(4.7) vanish for on-shell external fermions, and will be discarded in the analysis that follows.

The (dimension-less) self-energy-like contribution $V_{\alpha\rho}^{\text{P}(1)}(q)$, together with an equal contribution coming from the mirror vertex (not shown), after trivial manipulations gives rise to the dimensionful quantity

$$\Pi_{\alpha\beta}^{\text{P}(1)}(q) = 2d_W^{-1}(q)V_{\alpha\beta}^{\text{P}(1)}(q),
\tag{4.11}$$

which will be added to the conventional one-loop two-point function $\Pi_{W_\alpha^+ W_\beta^-}^{(1)}(q)$, to give rise to the PT two-point function $\widehat{\Pi}_{W_\alpha^+ W_\beta^-}^{(1)}(q)$:

$$\widehat{\Pi}_{W_\alpha^+ W_\beta^-}^{(1)}(q) = \Pi_{W_\alpha^+ W_\beta^-}^{(1)}(q) + \Pi_{\alpha\beta}^{\text{P}(1)}(q).
\tag{4.12}$$

Correspondingly, the PT one-loop three-point function $\widehat{\Pi}_{W_\alpha^+ \bar{\psi}\psi}^{(1)}(Q', Q)$ will be defined as

$$\begin{aligned}
\widehat{\Pi}_{W_\alpha^+ \bar{\psi}\psi}^{(1)}(Q', Q) &= \widehat{\Gamma}_{W_\alpha^+ \bar{\psi}\psi}^{V^2(1)}(Q', Q) + \Gamma_{W_\alpha^+ \bar{\psi}\psi}^{\bar{\psi}\psi(1)}(Q', Q) \\
&= \Pi_{W_\alpha^+ \bar{\psi}\psi}^{(1)}(Q', Q) - V_{\alpha\rho}^{\text{P}(1)}(q) \frac{\gamma^\rho P_L}{\sqrt{2}}.
\end{aligned}
\tag{4.13}$$

We can now compare these results with the ones that we get from the BQIs of Eqs.(3.14) and (3.19) found in the previous sections. At one-loop these BQIs read

$$\begin{aligned}
\Pi_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(1)}(q) &= \Pi_{W_\alpha^+ W_\beta^-}^{(1)}(q) + 2\Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}(q)\Pi_{W^{+,\rho} W_\beta^-}^{(0)}(q), \\
\Pi_{\widehat{W}_\alpha^+ \bar{\psi}\psi}^{(1)}(Q', Q) &= \Pi_{W_\alpha^+ \bar{\psi}\psi}^{(1)}(Q', Q) + \Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}(q)\Pi_{W^{+,\rho} \bar{\psi}\psi}^{(0)}(Q', Q).
\end{aligned}
\tag{4.14}$$

Moreover perturbatively one has

$$\Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}(q) = \text{Diagram 1} + \text{Diagram 2}$$

The diagrams are:

Diagram 1: A loop with a wavy line labeled W_μ at the top and a dashed line labeled u^i at the bottom. The left external line is labeled Ω_α^+ and the right external line is labeled W_ρ^{*-} .

Diagram 2: A loop with a wavy line labeled ν_μ^i at the top and a dashed line labeled u^+ at the bottom. The left external line is labeled Ω_α^+ and the right external line is labeled W_ρ^{*-} .

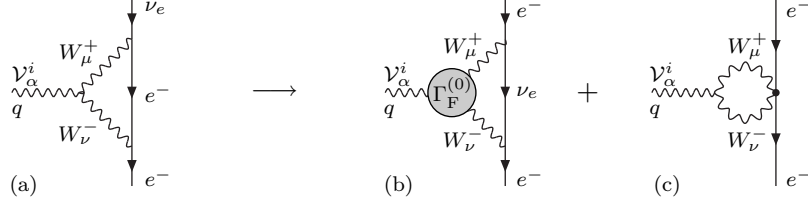


FIG. 2: Carrying out the fundamental vertex decomposition inside the three-point function $\Gamma_{\mathcal{V}_\alpha^i \psi \bar{\psi}}^{V^2(1)}$ (a), contributing to $\Pi_{\mathcal{V}_\alpha^i \psi \bar{\psi}}^{(1)}$, gives rise to the genuine vertex $\widehat{\Gamma}_{\mathcal{V}_\alpha^i \bar{\psi} \psi}^{V^2(1)}$ (b), and a self-energy-like contribution $V_{\alpha\rho}^{i,P(1)} \frac{\gamma^\rho P_L}{\sqrt{2}}$ (c).

Therefore, using the Feynman rules of Appendix A, and observing that $\Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)} = i\Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}$, we find

$$\begin{aligned} \Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}(q) &= 2ig_W^2 g_{\alpha\rho} \sum_i C_i^2 \int_{L_1} J_i(q, k) \\ &= iV_{\alpha\rho}^{P(1)}(q). \end{aligned} \quad (4.15)$$

Thus, after simple algebra, we find the results

$$\begin{aligned} 2\Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}(q) \Pi_{W^{+, \rho} W_\beta^-}^{(0)}(q) &= \Pi_{\alpha\beta}^{P(1)}(q), \\ \Pi_{\Omega_\alpha^+ W_\rho^{*-}}^{(1)}(q) \Pi_{W^{+, \rho} \bar{\psi} \psi}^{(0)}(Q', Q) &= -V_{\alpha\rho}^{P(1)}(q) \frac{\gamma^\rho P_L}{\sqrt{2}}, \end{aligned} \quad (4.16)$$

which will in turn automatically enforce the identifications

$$\begin{aligned} \widehat{\Pi}_{W_\alpha^+ W_\beta^-}^{(1)}(q) &\equiv \Pi_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(1)}(q), \\ \widehat{\Pi}_{W_\alpha^+ \bar{\psi} \psi}^{(1)}(Q', Q) &\equiv \Pi_{\widehat{W}_\alpha^+ \bar{\psi} \psi}^{(1)}(Q', Q). \end{aligned} \quad (4.17)$$

B. The neutral sector

In this case we will concentrate on the S -matrix element for the electron-electron elastic scattering process $e(P)e(P') \rightarrow e(Q)e(Q')$, where again the electrons will be treated as mass-less.

As in the charged sector case, we start by implementing (see Fig.2a) the vertex decomposition of Eq.(4.3), with $p_{1\mu} = -k_\mu$, $p_{2\nu} = (k - q)_\nu$, inside the $\Gamma_{\mathcal{V}_\alpha^i \bar{\psi} \psi}^{V^2(1)}(Q', Q)$ part of the full one-loop three-point functions $\Pi_{\mathcal{V}_\alpha^i \bar{\psi} \psi}^{(1)}(Q', Q)$. The $\Gamma_{\alpha\mu\nu}^P(q, p_1, p_2)$ term triggers then the elementary WIs of Eq.(4.6), so that two self-energy like pieces are generated (Figs.2c). In

particular,

$$\begin{aligned}\Gamma_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{V^2(1)}(Q', Q) &= \widehat{\Gamma}_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{V^2(1)}(Q', Q) + V_{\alpha\rho}^{i,P(1)}(q) \frac{\gamma^\rho P_L}{2} - X'_{1\alpha}(Q', Q) \Sigma^{(0)}(Q') \\ &\quad - \Sigma^{(0)}(Q) X'_{2\alpha}(Q', Q),\end{aligned}\tag{4.18}$$

where

$$\begin{aligned}\widehat{\Gamma}_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{V^2(1)}(Q', Q) &= ig_W^2 C_i \int_{L_1} \Gamma_{\alpha\mu\nu}^F(q, -k, k-q) \gamma^\mu P_L S^{(0)}(k+Q') \gamma^\nu \frac{1}{2} P_L J_W(q, k), \\ V_{\alpha\rho}^{i,P(1)}(q) &= 2g_W^2 C_i g_{\alpha\rho} \int_{L_1} J_W(q, k),\end{aligned}\tag{4.19}$$

and, finally,

$$J_W(q, k) = d_W(k) d_W(k-q).\tag{4.20}$$

Again the last two terms appearing in Eq.(4.18) will be discarded, since they vanish for on-shell fermions. The (dimension-less) self-energy-like piece $V_{\alpha\rho}^{i,P(1)}(q)$ must be allotted to the conventional one-loop ZZ , AZ , ZA and AA . To accomplish that we notice that the effective vertex $(\gamma^\mu P_L/2)$ in Eq.(4.18) may be written as a linear combination of the two Standard Model tree-level vertices (factoring out the electroweak coupling g_W)

$$\begin{aligned}\mathbb{I}_{A^\mu \bar{\psi}\psi}^{(0)} &= -is_W Q_\psi \gamma^\mu, \\ \mathbb{I}_{Z^\mu \bar{\psi}\psi}^{(0)} &= -i \frac{1}{c_W} \gamma^\mu [(s_W^2 Q_\psi - T_z^\psi) P_L + s_W^2 Q_\psi P_R],\end{aligned}\tag{4.21}$$

as follows

$$\frac{i}{2} \gamma^\mu P_L = - \left(\frac{s_W}{2T_z^\psi} \right) \mathbb{I}_{A^\mu \bar{\psi}\psi}^{(0)} + \left(\frac{c_W}{2T_z^\psi} \right) \mathbb{I}_{Z^\mu \bar{\psi}\psi}^{(0)}.\tag{4.22}$$

When the fermion ψ is an electron as in our case, $T_z^\psi = -1/2$, and we find

$$\begin{aligned}\frac{1}{2} \gamma^\mu P_L &= -is_W \mathbb{I}_{A^\mu \bar{\psi}\psi}^{(0)} + c_W \mathbb{I}_{Z^\mu \bar{\psi}\psi}^{(0)} \\ &= s_W (s_W \gamma^\mu) - c_W \left[-\frac{1}{c_W} \gamma^\mu \left(\frac{1}{2} P_L - s_W^2 \right) \right],\end{aligned}\tag{4.23}$$

so that Eq.(4.23) together Eq.(4.19) will fix the PT self-energy like contributions to be

$$V_{ij,\alpha\rho}^{P(1)}(q) = 2g_W^2 C_i C_j g_{\alpha\rho} \int_{L_1} J_W(q, k).\tag{4.24}$$

Adding an equal contribution coming from the mirror vertex (not shown) we find the dimensionful quantity

$$\Pi_{ij,\alpha\beta}^{P(1)}(q) = 2 [d_{\nu^i}^{-1}(q) + d_{\nu^j}^{-1}(q)] V_{ij,\alpha\beta}^{P(1)}(q),\tag{4.25}$$

which will be added to the conventional one-loop two-point function $\Pi_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(1)}(q)$, to give rise to the PT two-point function $\widehat{\Pi}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(1)}(q)$:

$$\widehat{\Pi}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(1)}(q) = \Pi_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(1)}(q) + \Pi_{ij, \alpha\beta}^{\text{P}(1)}(q). \quad (4.26)$$

Correspondingly, the PT one-loop three-point function $\widehat{\Pi}_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q)$ will be defined as

$$\begin{aligned} \widehat{\Pi}_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q) &= \widehat{\Gamma}_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{V^2(1)}(Q', Q) + \Gamma_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{\bar{\psi}\psi(1)}(Q', Q) \\ &= \Pi_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q) - V_{\alpha\rho}^{i, \text{P}(1)}(q) \frac{\gamma^\rho P_L}{2}. \end{aligned} \quad (4.27)$$

We can now compare these results with those obtained from the BQIs of Eqs.(3.15) and (3.20), reported in the previous sections. Expanding at the one-loop level these BQIs, we find

$$\begin{aligned} \Pi_{\widehat{\mathcal{V}}_\alpha^i \widehat{\mathcal{V}}_\beta^j}^{(1)}(q) &= \Pi_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(1)}(q) + \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,j}}^{(1)}(q) \Pi_{\mathcal{V}^{j,\rho} \mathcal{V}_\beta^j}^{(0)}(q) + \Pi_{\Omega_\beta^j \mathcal{V}_\rho^{*,i}}^{(1)}(q) \Pi_{\mathcal{V}^{i,\rho} \mathcal{V}_\alpha^i}^{(0)}(q), \\ \Pi_{\widehat{\mathcal{V}}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q) &= \Pi_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q) + \sum_n \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \bar{\psi}\psi}^{(0)}(Q', Q). \end{aligned} \quad (4.28)$$

In addition, perturbatively one has (with $a=i, j$, $b=j, i$ and $\lambda = \alpha, \beta$)

$$\Pi_{\Omega_\lambda^a \mathcal{V}_\rho^{*,b}}^{(1)}(q) = \frac{\Omega_\lambda^a}{\equiv} \text{[Diagram 1]} + \frac{\Omega_\lambda^a}{\equiv} \text{[Diagram 2]}$$

Therefore, using the Feynman rules of Appendix A, and observing that $\Pi_{\Omega_\lambda^a \mathcal{V}_\rho^{*,b}}^{(1)} = i\Pi_{\Omega_\lambda^a \mathcal{V}_\rho^b}^{(1)}$, we find

$$\begin{aligned} \Pi_{\Omega_\lambda^a \mathcal{V}_\rho^{*,b}}^{(1)}(q) &= 2ig_W^2 C_a C_b g_{\alpha\rho} \int_{L_1} J_W(q, k) \\ &= iV_{ab, \alpha\rho}^{\text{P}(1)}(q). \end{aligned} \quad (4.29)$$

Thus, after simple manipulations, we arrive at the results

$$\begin{aligned} \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,j}}^{(1)}(q) \Pi_{\mathcal{V}^{j,\rho} \mathcal{V}_\beta^j}^{(0)}(q) + \Pi_{\Omega_\beta^j \mathcal{V}_\rho^{*,i}}^{(1)}(q) \Pi_{\mathcal{V}^{i,\rho} \mathcal{V}_\alpha^i}^{(0)}(q) &= \Pi_{ij, \alpha\beta}^{\text{P}(1)}(q), \\ \sum_n \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \bar{\psi}\psi}^{(0)}(Q', Q) &= -V_{\alpha\rho}^{i, \text{P}(1)}(q) \frac{\gamma^\rho P_L}{2}, \end{aligned} \quad (4.30)$$

which automatically enforce the identifications

$$\begin{aligned} \widehat{\Pi}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(1)}(q) &\equiv \Pi_{\widehat{\mathcal{V}}_\alpha^i \widehat{\mathcal{V}}_\beta^j}^{(1)}(q), \\ \widehat{\Pi}_{\mathcal{V}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q) &\equiv \Pi_{\widehat{\mathcal{V}}_\alpha^i \bar{\psi}\psi}^{(1)}(Q', Q). \end{aligned} \quad (4.31)$$

V. ELECTROWEAK INTRINSIC PT AT ONE-LOOP

In the intrinsic PT construction one avoids the embedding of the PT objects into S -matrix elements; of course, all results of the intrinsic PT are identical to those obtained in the S -matrix PT context. The basic idea, is that the pinch graphs, which are essential in canceling the gauge dependences of ordinary diagrams, are always missing one or more propagators corresponding to the external legs of the improper Green's function in question. It then follows that the gauge-dependent parts of such ordinary diagrams must also be missing one or more external propagators. Thus the intrinsic PT construction goal is to isolate systematically the parts of 1PI diagrams that are proportional to the inverse propagators of the external legs and simply discard them. The important point is that these inverse propagators arise from the STIs satisfied by (i) the three-gauge-boson vertex and –a characteristic that distinguish the electroweak sector of the Standard Model case from the QCD case– (ii) the gauge-boson propagators appearing inside appropriate sets of diagrams, when they will be contracted by longitudinal momenta. The STIs triggered are nothing but the one appearing in Eqs.(3.2) and (3.6). Of course the momenta appearing in these STIs will now be related to virtual integration momenta appearing in the quantum loop.

In the context of QCD, this construction has been carried out at one-loop in [2] and recently generalized at the two-loop level in [20]. Here we present for the first time the two-loop generalization of this construction in the electroweak sector of the Standard Model, employing the one-loop versions of Eqs.(3.2) and (3.6).

The essential feature of the intrinsic PT construction is to arrive at the desired object, for example the effective gauge-boson self-energy, by discarding in a systematic way well-defined pieces from the conventional self-energy. The terms discarded originate from Eqs.(3.2) and (3.6), and they are all precisely known in terms of physical and unphysical Green's functions, appearing in the theory. Then, one can directly compare the result obtained by the intrinsic PT procedure to the corresponding BFM quantity (at $\xi_Q = 1$), employing the BQIs of Eqs.(3.14) and (3.20).

We start by reviewing the one-loop intrinsic PT construction, beginning again, without loss of generality, in the renormalizable Feynman gauge.

A. Charged sector

In the charged sector case, the quantity we want to construct is the one-loop W two-point function. In the absence of longitudinal momenta coming from internal gauge boson propagators (since we work in the Feynman gauge), the only diagram contributing to the W self-energy that can trigger an STI, is the one containing two three-gauge-boson vertices:

$$= \sum_i \int_{L_1} J'(q, k) \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{(0)}(q, -k, k - q) \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{(0)}(q, -k, k - q). \quad (5.1)$$

We next carry out the PT decomposition of Eq.(4.3) on *both* the three-gauge-boson vertices appearing in Eq.(5.1), *i.e.*, we write

$$\begin{aligned} \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{(0)} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{(0)} &= \left[\Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{\text{F}} + \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{\text{P}} \right] \left[\Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{\text{F}} + \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{\text{P}} \right] \\ &= \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{\text{F}} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{\text{F}} + \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{\text{P}} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{(0)} \\ &\quad + \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{(0)} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{\text{P}} - \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{\text{P}} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{\text{P}}, \end{aligned} \quad (5.2)$$

where, since it is important to know from which leg of the vertex we will finally trigger the STI, we have defined

$$\begin{aligned} \Pi_{W_\alpha^\pm W_\sigma^\mp \nu_\rho^i}^{\text{F}} &= \pm (ig_{\text{W}} C_i) \Gamma_{\alpha\sigma\rho}^{\text{F}}, \\ \Pi_{W_\alpha^\pm W_\sigma^\mp \nu_\rho^i}^{\text{P}} &= \pm (ig_{\text{W}} C_i) \Gamma_{\alpha\sigma\rho}^{\text{P}}, \end{aligned} \quad (5.3)$$

Of the four terms appearing in Eq.(5.2), the first and the last are left untouched, while for the second and the third we can write

$$\begin{aligned} &\Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{\text{P}} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{(0)} + \Pi_{W_\alpha^+ W_\sigma^- \nu_\rho^i}^{(0)} \Pi_{W_\beta^- W^+, \sigma \nu^{i, \rho}}^{\text{P}} \\ &= - (ig_{\text{W}} C_i) [k^\sigma g_\alpha^\rho + (k - q)^\rho g_\alpha^\sigma] \Pi_{W_\sigma^+ W_\beta^- \nu_\rho^i}^{(0)}(k, -q, -k + q) \\ &\quad + (ig_{\text{W}} C_i) [k^\sigma g_\beta^\rho + (k - q)^\rho g_\beta^\sigma] \Pi_{W_\sigma^- W_\alpha^+ \nu_\rho^i}^{(0)}(k, -q, -k + q). \end{aligned} \quad (5.4)$$

The longitudinal momenta k and $(k - q)$ appearing in the expression above will then trigger the tree-level version of the STIs of Eqs.(3.7) and (3.8) respectively, which read

$$\Pi_{u^\pm W_\sigma^*, \mp}^{(0)}(-k) \Pi_{W^\pm, \sigma W_\lambda^*, \mp \nu_\rho^i}^{(0)}(-q, -k + q) = -\Pi_{u^\pm W_\sigma^*, \mp \nu_\rho^i}^{(0)}(-q, -k + q) \Pi_{W^\pm, \sigma W_\lambda^*, \mp}^{(0)}(-q)$$

$$\begin{aligned}
& - \sum_n \mathbb{I}_{u^\pm \mathcal{V}_\sigma^{*,n} W_\lambda^\mp}^{(0)}(-k+q, -q) \mathbb{I}_{\mathcal{V}_\sigma^{n,\rho} \mathcal{V}_\alpha^i}^{(0)}(-k+q) \\
& - \mathbb{I}_{u^\pm \phi^{*,\mp}}^{(0)}(-k) \mathbb{I}_{\phi^\pm W_\lambda^\mp \mathcal{V}_\rho^i}^{(0)}(-q, -k+q), \\
\sum_n \mathbb{I}_{u^i \mathcal{V}_\rho^{*,n}}^{(0)}(k-q) \mathbb{I}_{\mathcal{V}_\sigma^{n,\rho} W_\sigma^\pm W_\lambda^\mp}^{(0)}(k, -q) & = -\mathbb{I}_{u^i W_\rho^{*,\pm} W_\lambda^\mp}^{(0)}(k, -q) \mathbb{I}_{W^\mp, \rho W_\sigma^\pm}^{(0)}(k) \\
& - \mathbb{I}_{u^i W_\rho^{*,\mp} W_\sigma^\pm}^{(0)}(-q, k) \mathbb{I}_{W^\pm, \rho W_\lambda^\mp}^{(0)}(-q), \tag{5.5}
\end{aligned}$$

where λ can be β or α depending on which term of Eq.(5.4) we are considering. Now, from the BRST Lagrangian of Eq.(2.29) we have the relations

$$\begin{aligned}
\mathbb{I}_{u^\pm W_\sigma^{*,\mp}}^{(0)}(-k) & = -ik_\sigma, \\
\mathbb{I}_{u^i \mathcal{V}_\rho^{*,n}}^{(0)}(k-q) & = i(k-q)_\rho \delta^{in}, \\
\mathbb{I}_{u^\pm \phi^{*,\mp}}^{(0)}(-k) & = \pm iM_W, \tag{5.6}
\end{aligned}$$

which, together with the Feynman rules of Appendix A, will in turn give us the tree-level STIs in their final form, *i.e.*,

$$\begin{aligned}
k^\sigma \mathbb{I}_{W_\sigma^\pm W_\lambda^\mp \mathcal{V}_\rho^i}^{(0)}(-q, -k+q) & = \pm (g_W C_i) \mathbb{I}_{W_\rho^\pm W_\lambda^\mp}^{(0)}(-q) \mp (g_W C_i) \mathbb{I}_{\mathcal{V}_\lambda^i \mathcal{V}_\rho^i}^{(0)}(-k+q) \\
& \pm M_W \mathbb{I}_{\phi^\pm W_\lambda^\mp \mathcal{V}_\rho^i}^{(0)}(-q, -k+q), \tag{5.7}
\end{aligned}$$

$$(k-q)^\rho \mathbb{I}_{\mathcal{V}_\rho^i W_\sigma^\pm W_\lambda^\mp}^{(0)}(k, -q) = \pm (g_W C_i) \mathbb{I}_{W_\sigma^\pm W_\lambda^\mp}^{(0)}(-q) \mp (g_W C_i) \mathbb{I}_{W_\lambda^\mp W_\sigma^\pm}^{(0)}(k). \tag{5.8}$$

The first term on the RHS of these two STIs is to be discarded from the W self-energy. Thus, the 1PI one-loop intrinsic PT self-energy, to be denoted as before by $\widehat{\mathbb{I}}_{W_\alpha^+ W_\beta^-}^{(1)}(q)$, is defined as

$$\widehat{\mathbb{I}}_{W_\alpha^+ W_\beta^-}^{(1)}(q) = \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(1)}(q) - \Pi_{\alpha\beta}^{\text{IP}(1)}(q), \tag{5.9}$$

where the superscript ‘‘IP’’ stands for ‘‘intrinsic pinch’’, and $\Pi_{\alpha\beta}^{\text{IP}(1)}$ is obtained by plugging the discarded term back into Eqs.(5.4) and (5.1), and has precisely the form

$$\begin{aligned}
\Pi_{\alpha\beta}^{\text{IP}(1)}(q) & = -4ig_W^2 \sum_i C_i^2 \int_{L_1} J'_i(q, k) \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(0)}(q) \\
& = -\Pi_{\alpha\beta}^{\text{P}(1)}(q). \tag{5.10}
\end{aligned}$$

At this point, following the original IP procedure [2], one should combine the first and last term on the RHS of Eq.(5.2) with the terms of the STIs of Eqs.(5.7) and (5.8) which have not been discarded, and add the remaining diagrams contributing to the W self-energy, in order to check that effectively $\widehat{\mathbb{I}}_{W_\alpha^+ W_\beta^-}^{(1)}(q)$ coincides with $\mathbb{I}_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(1)}(q)$. However in light of

the BQI of Eq.(3.14), this last identification is more immediate, in the sense that no further manipulation of the answer is needed: the difference between $\widehat{\Pi}_{W_\alpha^+ W_\beta^-}^{(1)}(q)$ and $\Pi_{W_\alpha^+ W_\beta^-}^{(1)}(q)$ is the same as the difference between $\Pi_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(1)}(q)$ and $\Pi_{W_\alpha^+ W_\beta^-}^{(1)}(q)$, as given by the BQI. Already at the one-loop level we can see that the use of the BQIs constitutes a definite technical advantage.

B. Neutral sector

In the neutral sector case, our aim is to construct the one-loop \mathcal{V}^i two-point functions. As before, in the absence of longitudinal momenta coming from the internal gauge-boson propagators, and recalling that we are working in the mass-less conserved current case, the only diagram contributing to the \mathcal{V}^i self-energy that can trigger an STI, is the one containing two three-gauge-boson vertices, *i.e.*,

$$= \int_{L_1} J_W(q, k) \Pi_{\mathcal{V}_\alpha^i W_\sigma^- W_\rho^+}^{(0)}(q, -k, k - q) \Pi_{\mathcal{V}_\beta^j W^{+, \sigma} W^{-, \rho}}^{(0)}(q, -k, k - q). \quad (5.11)$$

As in the charged sector case, we next carry out the PT decomposition of Eq.(4.3) on both the three-gauge-boson vertices appearing in Eq.(5.1), concentrating only on the term

$$\begin{aligned} & \Pi_{\mathcal{V}_\alpha^i W_\sigma^- W_\rho^+}^{\text{P}} \Pi_{\mathcal{V}_\beta^j W^{+, \sigma} W^{-, \rho}}^{(0)} + \Pi_{\mathcal{V}_\alpha^i W_\sigma^- W_\rho^+}^{(0)} \Pi_{\mathcal{V}_\beta^j W^{+, \sigma} W^{-, \rho}}^{\text{P}} \\ &= (ig_{\text{W}} C_i) [k^\sigma g_\alpha^\rho + (k - q)^\rho g_\alpha^\sigma] \Pi_{W_\sigma^+ W_\rho^- \mathcal{V}_\beta^j}^{(0)}(k, -k + q, -q) \\ &- (ig_{\text{W}} C_j) [k^\sigma g_\beta^\rho + (k - q)^\rho g_\beta^\sigma] \Pi_{W_\sigma^- W_\rho^+ \mathcal{V}_\alpha^i}^{(0)}(k, -k + q, -q). \end{aligned} \quad (5.12)$$

The longitudinal momenta k and $(k - q)$ appearing in the above equation will trigger in this case the following tree-level STIs

$$\begin{aligned} k^\sigma \Pi_{W_\sigma^\pm W_\rho^\mp \mathcal{V}_\alpha^a}^{(0)}(-k + q, -q) &= \mp (g_{\text{W}} C_a) \Pi_{\mathcal{V}_\beta^a \mathcal{V}_\lambda^a}^{(0)}(-q) \pm (g_{\text{W}} C_a) \Pi_{W_x^\pm W_\rho^\mp}^{(0)}(-k + q) \\ &\pm M_W \Pi_{\phi^\pm W_\mp \mathcal{V}_\lambda^a}^{(0)}(-k + q, -q), \\ (k - q)^\rho \Pi_{W_\rho^\mp W_\sigma^\pm \mathcal{V}_\alpha^a}^{(0)}(k, -q) &= \mp (g_{\text{W}} C_a) \Pi_{\mathcal{V}_\beta^a \mathcal{V}_\lambda^a}^{(0)}(-q) \pm (g_{\text{W}} C_a) \Pi_{W_x^\mp W_\sigma^\pm}^{(0)}(k) \\ &\pm M_W \Pi_{\phi^\mp W_\pm \mathcal{V}_\lambda^a}^{(0)}(k, -q), \end{aligned} \quad (5.13)$$

where $a = j, i$ and $\lambda = \beta, \alpha$ depending on which term of Eq.(5.11) we are considering.

As before, the first terms appearing in the RHS of these STIs will be discarded from the self-energy. Thus the 1PI one-loop intrinsic PT self-energy, to be denoted as before by $\widehat{\Pi}_{\nu_\alpha^i \nu_\beta^j}^{(1)}(q)$, is defined as

$$\widehat{\Pi}_{\nu_\alpha^i \nu_\beta^j}^{(1)}(q) = \Pi_{\nu_\alpha^i \nu_\beta^j}^{(1)}(q) - \Pi_{ij, \alpha\beta}^{\text{IP}(1)}(q), \quad (5.14)$$

where the quantity $\Pi_{ij, \alpha\beta}^{\text{IP}(1)}(q)$ is obtained by plugging the discarded terms back into Eq.(5.11), and has precisely the form

$$\Pi_{ij, \alpha\beta}^{\text{IP}(1)}(q) = -2ig_w^2 C_i C_j \int_{L_1} J_W(q, k) \left[\Pi_{\nu_\alpha^i \nu_\beta^i}^{(0)}(q) + \Pi_{\nu_\alpha^j \nu_\beta^j}^{(0)}(q) \right], \quad (5.15)$$

i.e., after trivial algebra, we find the identity

$$\Pi_{ij, \alpha\beta}^{\text{IP}(1)}(q) = -\Pi_{ij, \alpha\beta}^{\text{P}(1)}(q). \quad (5.16)$$

Again, since the difference between $\widehat{\Pi}_{\nu_\alpha^i \nu_\beta^j}^{(1)}(q)$ and $\Pi_{\nu_\alpha^i \nu_\beta^j}^{(1)}(q)$ is the same as the difference between $\Pi_{\widehat{\nu}_\alpha^i \widehat{\nu}_\beta^j}^{(1)}(q)$ and $\Pi_{\nu_\alpha^i \nu_\beta^j}^{(1)}(q)$ as given by the BQI of Eq.(3.15), we conclude that the PT result coincides with the BFM one.

VI. ELECTROWEAK INTRINSIC PT AT TWO-LOOPS

In the next three sections we will generalize the intrinsic PT construction presented above to two loops. The results presented are completely new, since the PT (intrinsic or S -matrix) has never been applied to the purely bosonic part of the Electroweak sector beyond one loop. The only two-loop results existing in the literature involve the special subset of contributions containing a fermion loop [34]. In this section we will outline the general formalism necessary for carrying out the two-loop construction in a concise and expeditious way, and in the next two sections we will present detailed constructions for the cases of the charged and neutral sectors.

The 1PI Feynman diagram contributing to the conventional two-loop gauge-boson self-energy both in the charged and neutral sector can be separated in three distinct sets [20]: (*i*) the set of diagrams that contains two external (tree-level) three-gauge-boson vertices, and thus can be written schematically (suppressing Lorentz indices) as $\Pi_{VVV}^{(0)}[\mathcal{K}_2] \Pi_{VVV}^{(0)}$, with \mathcal{K}_2 a kernel associated to these diagrams; (*ii*) the set of diagrams which has only one external (tree-level) three-gauge-boson vertex, and that therefore can be written as $\Pi_{VVV}^{(0)}[\mathcal{K}_1]$ or $[\mathcal{K}_1] \Pi_{VVV}^{(0)}$;

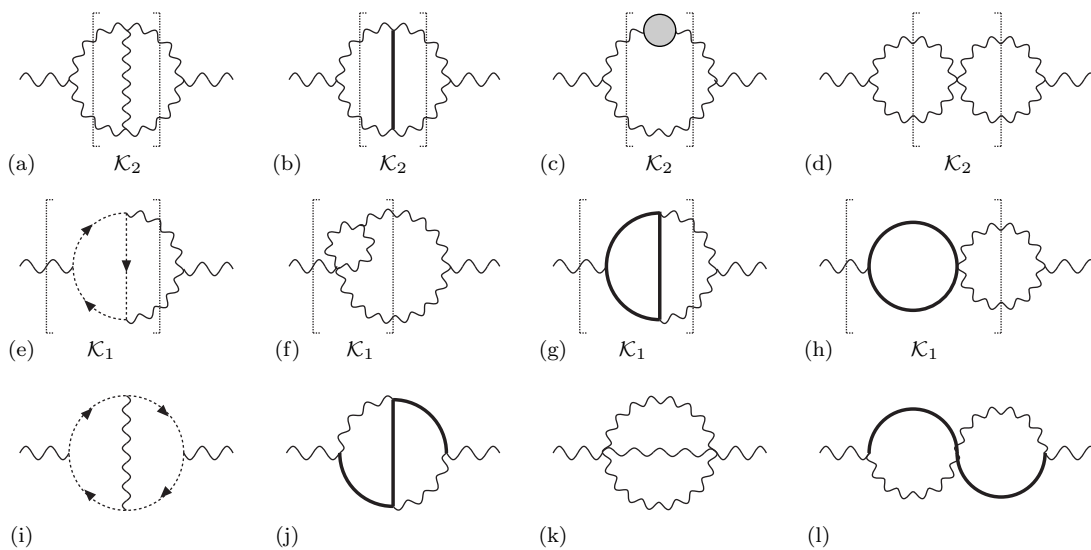
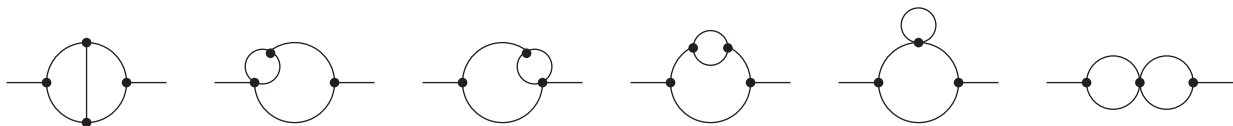


FIG. 3: Some of the Feynman diagrams contributing to the conventional two-loop gauge-boson self-energies $\Pi_{V^n V^m}^{(2)}$ in the R_ξ gauges. In the square brackets we indicate the associated kernel as explained in the text: thus diagrams (a)–(d) belongs to set (i), diagrams (e)–(h) to set (ii), and diagrams (i)–(l) to set (iii). The blob in diagram (c) denotes the one-loop correction to the gauge-boson propagator.

(iii) all the remaining diagrams, containing no external three-gauge-boson vertices. Notice that the diagrams belonging to this last set are simple “spectators” as far as this construction is concerned, since, due to the fact that they do not contain any longitudinal momenta, they cannot trigger any STI: therefore they will be left untouched. Out of all the 20 possible 1PI topologies (including seagull and tadpoles diagrams) that contribute to the two-loop gauge-boson self-energies [35], the only ones that furnish diagrams belonging to the sets (i) and (ii) are the following



Some of the diagrams arising from the above topologies, together with their associated kernels, are shown in Fig.3.

At this point we make the following observation [20]: if one were to carry out the decomposition of Eq.(4.3) to the pair of external vertices appearing in the diagrams of set (i) and to the external vertex appearing in the diagrams of set (ii), the longitudinal momenta

stemming from the pinching part of the vertices Γ^P would be triggering the one-loop version of the STIs of Eqs.(3.7) or (3.8), just as in the one-loop case one has been triggering the tree-level version of these STIs. The only exception are those diagrams of the set (i) which contain one-loop corrections to the internal propagators, such as the diagram of Fig.3c: the final vertex STIs triggered in this case are still the tree-level version of Eqs.(3.7) or (3.8). There is however an important difference between the QCD and the Electroweak case regarding the role of such graphs. In the case of QCD, out of the two possible longitudinal momenta originating from (either) Γ^P only one will reach the other side of the diagram, thus triggering the corresponding (tree-level) STI, whereas the other one will vanish when contracted with the transverse (one-loop) gluon self-energy. In the Electroweak case however, the corresponding (one-loop) gauge-boson self-energies are not transverse; thus the second longitudinal momentum will also reach the other side, after first triggering the one-loop version of the corresponding STIs, Eqs.(3.3) and (3.4). Thus, additional pinching contributions will be generated, which must be carefully determined.

The first step in the construction is to carry out the usual PT decomposition of the (tree-level) three-gauge-boson vertex of Eq.(4.3) to the diagrams of sets (i) and (ii). For the diagrams belonging to the set (i) we will generically write

$$\begin{aligned} \mathbb{I}_{VVV}^{(0)}[\mathcal{K}_2] \mathbb{I}_{VVV}^{(0)} &= \mathbb{I}_{VVV}^F[\mathcal{K}_2] \mathbb{I}_{VVV}^F + \mathbb{I}_{VVV}^P[\mathcal{K}_2] \mathbb{I}_{VVV}^{(0)} + \mathbb{I}_{VVV}^{(0)}[\mathcal{K}_2] \mathbb{I}_{VVV}^P \\ &- \mathbb{I}_{VVV}^P[\mathcal{K}_2] \mathbb{I}_{VVV}^P, \end{aligned} \quad (6.1)$$

and, as in the one-loop case, of the four terms appearing in the above equation, the first and the last are left untouched, and constitute part of the PT answer. Instead, to the second and third term of Eq.(6.1) corresponding to all the kernels \mathcal{K}_2 that *do not* contain one-loop corrections to the internal (gauge-boson) propagators (such as the ones associated to the diagrams of Fig.3a, b and d), we add the pinching part of all the diagrams belonging to set (ii), for which we generically write

$$\begin{aligned} \mathbb{I}_{VVV}^{(0)}[\mathcal{K}_1] &= \mathbb{I}_{VVV}^F[\mathcal{K}_1] + \mathbb{I}_{VVV}^P[\mathcal{K}_1], \\ [\mathcal{K}_1] \mathbb{I}_{VVV}^{(0)} &= [\mathcal{K}_1] \mathbb{I}_{VVV}^F + [\mathcal{K}_1] \mathbb{I}_{VVV}^P. \end{aligned} \quad (6.2)$$

Then, one arrives at the equation

$$\left\{ \mathbb{I}_{VVV}^{(0)} [\mathcal{K}_2] \mathbb{I}_{VVV}^{(0)} + \mathbb{I}_{VVV}^{(0)} [\mathcal{K}_1] + [\mathcal{K}_1] \mathbb{I}_{VVV}^{(0)} \right\}^P = \text{Diagram 1} + \text{Diagram 2} \equiv \mathcal{C}_{2L}, \quad (6.3)$$

with the blobs representing the full one-loop three-gauge-boson vertex.

For the second and third terms of Eq.(6.1), corresponding to the remaining kernels \mathcal{K}_2 that *do* contain one-loop corrections to the internal gauge-boson propagators (such as the one associated to the diagram of Fig.3c), we instead directly get

$$\left\{ \mathbb{I}_{VVV}^{(0)} [\mathcal{K}_2] \mathbb{I}_{VVV}^{(0)} \right\}^P = \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} \equiv \mathcal{C}_{1L}, \quad (6.4)$$

where now the blobs denote one-loop corrections to the gauge-boson propagator.

To carry out the generalization of the intrinsic PT to two-loops we must next isolate from Eqs.(6.3) and (6.4) those terms stemming from the triggering of the STIs of Eqs.(3.7), (3.8), (3.3) and (3.4) which are proportional to $\left[\Delta_\alpha^{(-1)\rho}(q) \right]^{(n)}$, with $n = 0, 1$; we will denote such contributions generically by $\Pi_{\alpha\beta}^{\text{IP}(2)}(q)$ in the charged sector case and $\Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q)$ in the neutral sector case. Thus the 1PI diagrams contributing to the two-loop gauge-boson self-energies, can be cast respectively in the form

$$\mathbb{I}_{W_\alpha^+ W_\beta^-}^{(2)}(q) = G_{W_\alpha^+ W_\beta^-}^{(2)}(q) + \Pi_{\alpha\beta}^{\text{IP}(2)}(q), \quad (6.5)$$

$$\mathbb{I}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) = G_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) + \Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q). \quad (6.6)$$

Notice however that the 1PR set of one-loop self-energy diagrams, must also be rearranged following the intrinsic PT procedure, and be converted into the equivalent string involving PT one-loop self-energies (which are known objects from the one-loop results). This treatment of the 1PR string will give rise, in addition to the PT strings, to (i) a set of contributions which are proportional to the inverse propagator of the external legs, and (ii) a set of contributions which is *effectively* 1PI, and therefore also belongs to the definition of the 1PI two-loop PT gauge-boson self-energies; we will denote these two sets of contributions collectively by $S_{\alpha\beta}^{\text{IP}(2)}(q)$ (charged sector) and $S_{ij,\alpha\beta}^{\text{IP}(2)}(q)$ (neutral sector).

Thus, the sum of the 1PI and 1PR contributions to the conventional two-loop gauge-boson

self-energies can be cast in the form

$$\begin{aligned} \mathbb{\Pi}_{W_\alpha^+ W_\beta^-}^{(2)}(q) - i\mathbb{\Pi}_{W_\alpha^+ W_\rho^-}^{(1)}(q)d_W(q)\mathbb{\Pi}_{W^{+\rho}W_\beta^-}^{(1)}(q) &= G_{W_\alpha^+ W_\beta^-}^{(2)}(q) - i\widehat{\mathbb{\Pi}}_{W_\alpha^+ W_\rho^-}^{(1)}(q)d_W(q)\widehat{\mathbb{\Pi}}_{W^{+\rho}W_\beta^-}^{(1)}(q) \\ &+ \Pi_{\alpha\beta}^{\text{IP}(2)}(q) + S_{\alpha\beta}^{\text{IP}(2)}(q), \end{aligned} \quad (6.7)$$

$$\begin{aligned} \mathbb{\Pi}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) - i\sum_n \mathbb{\Pi}_{\mathcal{V}_\alpha^i \mathcal{V}_\rho^n}^{(1)}(q)d_{\mathcal{V}^n}(q)\mathbb{\Pi}_{\mathcal{V}^{n\rho}\mathcal{V}_\beta^j}^{(1)}(q) &= G_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) - i\sum_n \widehat{\mathbb{\Pi}}_{\mathcal{V}_\alpha^i \mathcal{V}_\rho^n}^{(1)}(q)d_{\mathcal{V}^n}(q)\widehat{\mathbb{\Pi}}_{\mathcal{V}^{n\rho}\mathcal{V}_\beta^j}^{(1)}(q) \\ &+ \Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q) + S_{ij,\alpha\beta}^{\text{IP}(2)}(q). \end{aligned} \quad (6.8)$$

By definition of the intrinsic PT procedure, we will now discard from the above expressions all the terms which are proportional to the inverse propagator of the external legs (*i.e.*, $d_W^{-1}(q)$ or $d_{\mathcal{V}^i}^{-1}(q)$ and $d_{\mathcal{V}^j}^{-1}(q)$ in the charged, respectively neutral, sector), thus defining the quantities

$$R_{\alpha\beta}^{\text{IP}(2)}(q) = \Pi_{\alpha\beta}^{\text{IP}(2)}(q) + S_{\alpha\beta}^{\text{IP}(2)}(q), \quad (6.9)$$

$$R_{ij,\alpha\beta}^{\text{IP}(2)}(q) = \Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q) + S_{ij,\alpha\beta}^{\text{IP}(2)}(q), \quad (6.10)$$

where the primed functions are obtained from the unprimed ones appearing in Eqs.(6.7) and (6.8) by discarding the aforementioned terms.

Thus, making use of Eqs.(6.5)–(6.10), the 1PI two-loop intrinsic PT gauge-boson self-energies, to be denoted as before by $\widehat{\mathbb{\Pi}}_{W_\alpha^+ W_\beta^-}^{(2)}(q)$ and $\widehat{\mathbb{\Pi}}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q)$, will be defined as

$$\begin{aligned} \widehat{\mathbb{\Pi}}_{W_\alpha^+ W_\beta^-}^{(2)}(q) &= G_{W_\alpha^+ W_\beta^-}^{(2)}(q) + R_{\alpha\beta}^{\text{IP}(2)}(q) \\ &= \mathbb{\Pi}_{W_\alpha^+ W_\beta^-}^{(2)}(q) - \Pi_{\alpha\beta}^{\text{IP}(2)}(q) + R_{\alpha\beta}^{\text{IP}(2)}(q), \end{aligned} \quad (6.11)$$

$$\begin{aligned} \widehat{\mathbb{\Pi}}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) &= G_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) + R_{ij,\alpha\beta}^{\text{IP}(2)}(q) \\ &= \mathbb{\Pi}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) - \Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q) + R_{ij,\alpha\beta}^{\text{IP}(2)}(q). \end{aligned} \quad (6.12)$$

We next proceed to give the details of the two-loop construction in both the charged as well as the neutral gauge-boson sector.

VII. TWO-LOOP CHARGED SECTOR

As explained in the previous section, the starting point for the IP construction are Eqs.(6.3) and (6.4): from them, by using the two- and three-point functions STIs, we will isolate the 1PI parts that are proportional to the inverse propagator of the external legs,

and simply discard them. In the charged gauge-boson sector case these equations read

$$\begin{aligned} \mathcal{C}_{2L} &= -ig_w \sum_i C_i \int_{L_1} J'_i(q, k) \left\{ [k^\sigma g_\alpha^\rho + (k-q)^\rho g_\alpha^\sigma] \mathbb{\Pi}_{W_\sigma^+ W_\beta^- \nu_\rho^i}^{(1)}(k, -q, -k+q) \right. \\ &\quad \left. - [k^\sigma g_\beta^\rho + (k-q)^\rho g_\beta^\sigma] \mathbb{\Pi}_{W_\sigma^- W_\alpha^+ \nu_\rho^i}^{(1)}(k, -q, -k+q) \right\}, \end{aligned} \quad (7.1)$$

$$\begin{aligned} \mathcal{C}_{1L}^c &= -g_w \sum_i C_i \int_{L_1} J'_i(q, k) d_W(k) \times \\ &\quad \times \left\{ [k^\sigma g_\alpha^\rho + (k-q)^\rho g_\alpha^\sigma] \mathbb{\Pi}_{W_\sigma^+ W_\mu^-}^{(1)}(k) \mathbb{\Pi}_{W^{+, \mu} W_\beta^- \nu_\rho^i}^{(0)}(k, -q, -k+q) \right. \\ &\quad \left. - [k^\sigma g_\beta^\rho + (k-q)^\rho g_\beta^\sigma] \mathbb{\Pi}_{W_\sigma^- W_\mu^+}^{(1)}(k) \mathbb{\Pi}_{W^{-, \mu} W_\alpha^+ \nu_\rho^i}^{(0)}(k, -q, -k+q) \right\}, \end{aligned} \quad (7.2)$$

$$\begin{aligned} \mathcal{C}_{1L}^n &= g_w \sum_{i,j} \int_{L_1} J_i(q, k) d_{\nu^j}(k) \left\{ C_i [k^\mu g_\alpha^\sigma + (k-q)^\sigma g_\alpha^\mu] \mathbb{\Pi}_{\nu_\mu^i \nu_\rho^j}^{(1)}(k) \mathbb{\Pi}_{W_\sigma^+ W_\beta^- \nu^{j, \rho}}^{(0)}(-k+q, -q, k) \right. \\ &\quad \left. - C_j [k^\mu g_\beta^\sigma + (k-q)^\sigma g_\beta^\mu] \mathbb{\Pi}_{\nu_\mu^j \nu_\rho^i}^{(1)}(k) \mathbb{\Pi}_{W_\sigma^- W_\alpha^+ \nu^{i, \rho}}^{(0)}(-k+q, -q, k) \right\}, \end{aligned} \quad (7.3)$$

where $\mathcal{C}_{1L} = \mathcal{C}_{1L}^c + \mathcal{C}_{1L}^n$ and the superscript ‘‘c’’ and ‘‘n’’ stands for ‘‘charged’’ and ‘‘neutral’’, depending on which one-loop propagator the longitudinal momentum is hitting.

Let us start from the analysis of the \mathcal{C}_{2L} contribution, Eq.(7.1). For the two terms proportional to the longitudinal momentum k , the STI triggered will be the one-loop version of the STI of Eq.(3.7); writing only the terms that we are going to discard (as always from now on), this STI reads

$$\begin{aligned} k^\sigma \mathbb{\Pi}_{W_\sigma^\pm W_\lambda^\mp \nu_\rho^i}^{(1)}(-q, -k+q) &= -i \mathbb{\Pi}_{u^\pm W_\sigma^*, \mp}^{(1)}(-k) \mathbb{\Pi}_{W^{\pm, \sigma} W_\lambda^\mp \nu_\rho^i}^{(0)}(-q, -k+q) \\ &\quad - i \mathbb{\Pi}_{u^\pm W_\sigma^*, \mp \nu_\rho^i}^{(1)}(-q, -k+q) \mathbb{\Pi}_{W^{\pm, \sigma} W_\lambda^\mp}^{(0)}(-q) \\ &\quad - i \mathbb{\Pi}_{u^\pm W_\sigma^*, \mp \nu_\rho^i}^{(0)}(-q, -k+q) \mathbb{\Pi}_{W^{\pm, \sigma} W_\lambda^\mp}^{(1)}(-q), \end{aligned} \quad (7.4)$$

where λ can be β or α depending on which of the two terms we are considering.

Next, making use of the following (mutually inverse) relations

$$\begin{aligned} \mathbb{\Pi}_{u^\pm W_\sigma^*, \mp}(k) &= ik_\sigma \mathbb{\Pi}_{u^\pm W^*, \mp}(k), \\ \mathbb{\Pi}_{u^\pm W^*, \mp}(k) &= -i \frac{k^\sigma}{k^2} \mathbb{\Pi}_{u^\pm W_\sigma^*, \mp}(k), \end{aligned} \quad (7.5)$$

and observing that

$$\mathbb{\Pi}_{u^\pm W^*, \mp}^{(1)}(k) = -\frac{i}{k^2} L_V^{u(1)}(k), \quad (7.6)$$

where $L_V^{u(1)}(k)$ denotes the part of the one-loop $u^\pm \bar{u}^\pm$ ghost self-energy which involves an internal gauge-boson propagator, we find

$$-i\mathbb{\Gamma}_{u^\pm W_\sigma^{*\mp}}^{(1)}(-k)\mathbb{\Gamma}_{W_\sigma^\pm W_\lambda^\mp \nu_\rho^i}^{(0)}(-q, -k+q) = \pm (ig_w C_i) \frac{1}{k^2} L_V^{u(1)}(k) \mathbb{\Gamma}_{W_\rho^\pm W_\lambda^\mp}^{(0)}(-q). \quad (7.7)$$

This result, together with the Feynman rules listed in Appendix A, will finally give us the one-loop STI of Eq.(7.4) in its final form, *i.e.*,

$$\begin{aligned} k^\sigma \mathbb{\Gamma}_{W_\sigma^\pm W_\lambda^\mp \nu_\rho^i}^{(1)}(-q, -k+q) &= \pm (ig_w C_i) \frac{1}{k^2} L_V^{u(1)}(k) \mathbb{\Gamma}_{W_\rho^\pm W_\lambda^\mp}^{(0)}(q) \\ &\quad - i\mathbb{\Gamma}_{u^\pm W_\sigma^{*\mp} \nu_\rho^i}^{(1)}(-q, -k+q) \mathbb{\Gamma}_{W^{\pm, \sigma} W_\lambda^\mp}^{(0)}(q) \\ &\quad \pm (g_w C_i) \mathbb{\Gamma}_{W_\rho^\pm W_\lambda^\mp}^{(1)}(q). \end{aligned} \quad (7.8)$$

For the remaining two \mathcal{C}_{2L} terms, proportional to the longitudinal momentum $(k-q)$, the STI triggered will instead be the one-loop version of the STI of Eq.(3.8), *i.e.*,

$$\begin{aligned} (k-q)^\rho \mathbb{\Gamma}_{\nu_\rho^i W_\sigma^\pm W_\beta^\mp}^{(1)}(k, -q) &= i \sum_n \mathbb{\Gamma}_{u^i \nu_\rho^{*,n}}^{(1)}(k-q) \mathbb{\Gamma}_{\nu^{n, \rho} W_\sigma^\pm W_\lambda^\mp}^{(0)}(k, -q) \\ &\quad + i\mathbb{\Gamma}_{u^i W_\rho^{*\mp} W_\sigma^\pm}^{(1)}(-q, k) \mathbb{\Gamma}_{W^{\pm, \rho} W_\lambda^\mp}^{(0)}(-q) \\ &\quad + i\mathbb{\Gamma}_{u^i W_\rho^{*\mp} W_\sigma^\pm}^{(0)}(-q, k) \mathbb{\Gamma}_{W^{\pm, \rho} W_\lambda^\mp}^{(1)}(-q). \end{aligned} \quad (7.9)$$

We next make use of the following relations

$$\begin{aligned} \mathbb{\Gamma}_{u^i \nu_\rho^{*,n}}(k-q) &= i(k-q)_\rho \mathbb{\Gamma}_{u^i \nu^{*,n}}(k-q), \\ \mathbb{\Gamma}_{u^i \nu^{*,n}}(k-q) &= -i \frac{(k-q)^\rho}{(k-q)^2} \mathbb{\Gamma}_{u^i \nu_\rho^{*,n}}(k-q), \end{aligned} \quad (7.10)$$

and observe that

$$\mathbb{\Gamma}_{u^i \nu^{*,n}}^{(1)}(k-q) = -\frac{i}{(k-q)^2} L_V^{in(1)}(k-q), \quad (7.11)$$

where $L_V^{in(1)}(k-q)$ represents the part of the one-loop $u^i \bar{u}^n$ ghost self-energy which involves an internal gauge-boson propagator, to find

$$i \sum_n \mathbb{\Gamma}_{u^i \nu_\rho^{*,n}}^{(1)}(k-q) \mathbb{\Gamma}_{\nu^{n, \rho} W_\sigma^\pm W_\lambda^\mp}^{(0)}(k, -q) = \pm (ig_w C_n) \frac{1}{(k-q)^2} L_V^{in(1)}(k-q) \mathbb{\Gamma}_{W_\sigma^\pm W_\lambda^\mp}^{(0)}(q). \quad (7.12)$$

This result, together with the Feynman rules listed in Appendix A, will finally give us the STI of Eq.(7.9) in its final form, *i.e.*,

$$\begin{aligned} (k-q)^\rho \mathbb{\Gamma}_{\nu_\rho^i W_\sigma^\pm W_\beta^\mp}^{(1)}(k, -q) &= \pm ig_w \sum_n C_n \frac{1}{(k-q)^2} L_V^{in(1)}(k-q) \mathbb{\Gamma}_{W_\sigma^\pm W_\lambda^\mp}^{(0)}(q) \\ &\quad - i\mathbb{\Gamma}_{u^i W_\rho^{*\mp} W_\sigma^\pm}^{(1)}(-q, k) \mathbb{\Gamma}_{W^{\pm, \rho} W_\lambda^\mp}^{(0)}(q) \\ &\quad \pm (g_w C_i) \mathbb{\Gamma}_{W^{\pm, \rho} W_\lambda^\mp}^{(1)}(q). \end{aligned} \quad (7.13)$$

Using then the following properties of the BRST vertices

$$\begin{aligned}
\mathbb{\Gamma}_{u^\pm W_\sigma^* \mp \mathcal{V}_\rho^i}^{(1)}(-q, -k+q) &= \mathbb{\Gamma}_{u^\pm W^* \mp \mathcal{V}^i}^{(1)}(-q, -k+q) g_{\sigma\rho}, \\
\mathbb{\Gamma}_{u^i W_\rho^* \mp W_\sigma^\pm}^{(1)}(-q, k) &= \mathbb{\Gamma}_{u^i W^* \mp W^\pm}^{(1)}(-q, k) g_{\rho\sigma}, \\
\mathbb{\Gamma}_{u^+ W^* \cdot - \mathcal{V}^i}^{(1)}(-q, -k+q) &= -\mathbb{\Gamma}_{u^- W^* \cdot + \mathcal{V}^i}^{(1)}(-q, -k+q), \\
\mathbb{\Gamma}_{u^i W^* \cdot - W^+}^{(1)}(-q, k) &= -\mathbb{\Gamma}_{u^i W^* \cdot + W^-}^{(1)}(-q, k),
\end{aligned} \tag{7.14}$$

we can write the discarded terms coming from the \mathcal{C}_{2L} contribution as

$$\begin{aligned}
\mathcal{C}_{2L} &= 2g_w^2 \sum_i C_i \int_{L_1} J'_i(q, k) \left[C_i \frac{1}{k^2} L_V^u(k) + \sum_j C_j \frac{1}{(k-q)^2} L_V^{ij}(k-q) \right] \mathbb{\Gamma}_{W_\alpha^+ W_\beta^-}^{(0)}(q) \\
&- 2g_w \sum_i C_i \int_{L_1} J'_i(q, k) \left[\mathbb{\Gamma}_{u^+ W_\sigma^* \cdot - \mathcal{V}_\alpha^i}^{(1)}(-q, -k+q) + \mathbb{\Gamma}_{u^i W_\sigma^* \cdot - W_\alpha^+}^{(1)}(-q, k) \right] \mathbb{\Gamma}_{W^+ \cdot \sigma W_\beta^-}^{(0)}(q) \\
&- 4ig_w^2 \sum_i C_i^2 \int_{L_1} J'_i(q, k) \mathbb{\Gamma}_{W_\alpha^+ W_\beta^-}^{(1)}(q).
\end{aligned} \tag{7.15}$$

We next consider the lower order corrections, *i.e.*, the contributions coming from the \mathcal{C}_{1L} terms, beginning from the charged contribution \mathcal{C}_{1L}^c of Eq.(7.2).

For the two terms proportional to the longitudinal momentum k , the STI triggered will be the one-loop version of Eq.(3.3), which reads

$$\begin{aligned}
k^\sigma \mathbb{\Gamma}_{W_\sigma^\pm W_\mu^\mp}^{(1)}(k) &= i\mathbb{\Gamma}_{u^\pm W_\sigma^* \mp}^{(1)}(k) \mathbb{\Gamma}_{W^\pm \cdot \sigma W_\mu^\mp}^{(0)}(k) + i\mathbb{\Gamma}_{u^\pm \phi^* \mp}^{(1)}(k) \mathbb{\Gamma}_{\phi^\pm W_\mu^\mp}^{(0)}(k) \\
&+ i\mathbb{\Gamma}_{u^\pm \phi^* \mp}^{(0)}(k) \mathbb{\Gamma}_{\phi^\pm W_\mu^\mp}^{(1)}(k).
\end{aligned} \tag{7.16}$$

Using then the tree-level version of the STIs of Eqs.(3.3) and (3.7), we find

$$i\mathbb{\Gamma}_{u^\pm W_\sigma^* \mp}^{(1)}(k) \mathbb{\Gamma}_{W^\pm \cdot \sigma W_\mu^\mp}^{(0)}(k) \mathbb{\Gamma}_{W^\pm \cdot \mu W_\lambda^\mp \mathcal{V}_\rho^i}^{(0)}(k, -q, -k+q) = \mp (g_w C_i) M_W^2 \frac{1}{k^2} L_V^u(k) \mathbb{\Gamma}_{W_\rho^\pm W_\lambda^\mp}^{(0)}(q). \tag{7.17}$$

Moreover observing that

$$M_W \mathbb{\Gamma}_{u^\pm \phi^* \mp}^{(1)} = \mp L_G^u(k), \tag{7.18}$$

where $L_G^u(k)$ is the part of the one-loop $u^\pm \bar{u}^\pm$ ghost self-energy involving an internal Goldstone boson propagator, we find

$$i\mathbb{\Gamma}_{u^\pm \phi^* \mp}^{(1)}(k) \mathbb{\Gamma}_{\phi^\pm W_\mu^\mp}^{(0)}(k) \mathbb{\Gamma}_{W^\pm \cdot \mu W_\lambda^\mp \mathcal{V}_\rho^i}^{(0)}(k, -q, -k+q) = \mp (g_w C_i) L_G^u(k) \mathbb{\Gamma}_{W_\rho^\pm W_\lambda^\mp}^{(0)}(q). \tag{7.19}$$

Finally, the last term appearing in Eq. (7.16) is part of the PT answer, and it is precisely the term responsible for converting the conventional $W^\pm \phi^\mp \mathcal{V}^i$ vertex into the corresponding BFM vertex $\widehat{W}^\pm \phi^\mp \mathcal{V}^i$.

For the remaining two \mathcal{C}_{1L}^c terms proportional to the longitudinal momentum $(k - q)$, the STI triggered will simply be the one of Eq. (5.8); thus, the discarded term stemming from \mathcal{C}_{1L}^c , reads

$$\begin{aligned} \mathcal{C}_{1L}^c &= 2g_w^2 \sum_i C_i^2 \int_{L_1} J'_i(q, k) d_w(k) \left[\frac{M_W^2}{k^2} L_V^{u(1)}(k) + L_G^{u(1)}(k) \right] \mathbb{\Pi}_{W_\alpha^+ W_\beta^-}^{(0)}(q) \\ &\quad - 2g_w^2 \sum_i C_i^2 \int_{L_1} J'_i(q, k) d_w(k) \mathbb{\Pi}_{W_\alpha^+ W_\mu^-}^{(1)}(k) \mathbb{\Pi}_{W^{+\mu} W_\beta^-}^{(0)}(q). \end{aligned} \quad (7.20)$$

In the case of the neutral contributions \mathcal{C}_{1L}^n , the STI triggered by the two terms proportional to the longitudinal momentum k , will be the one-loop version of Eq.(3.4). Taking into account that the Green's functions $\mathbb{\Pi}_{\mathcal{V}_\rho^H}$ vanish at tree-, one- and two-loop level as they violate CP invariance, we arrive at the one-loop STI

$$\begin{aligned} k^\mu \mathbb{\Pi}_{\mathcal{V}_\mu^a \mathcal{V}_\rho^b}^{(1)}(k) &= i \sum_n \mathbb{\Pi}_{u^a \mathcal{V}_\mu^{*,n}}^{(1)}(k) \mathbb{\Pi}_{\mathcal{V}^{n,\mu} \mathcal{V}_\rho^b}^{(0)}(k) + i \mathbb{\Pi}_{u^a \chi^*}^{(1)}(k) \mathbb{\Pi}_{\chi \mathcal{V}_\rho^b}^{(0)}(k) \\ &\quad + i \mathbb{\Pi}_{u^a \chi^*}^{(0)}(k) \mathbb{\Pi}_{\chi \mathcal{V}_\rho^b}^{(1)}(k), \end{aligned} \quad (7.21)$$

where $a = i, j$, $b = j, i$ and depending on which of the two terms we are considering.

Using then the tree-level version of the STIs of Eqs.(3.4) and (3.8), we find

$$i \sum_n \mathbb{\Pi}_{u^a \mathcal{V}_\mu^{*,n}}^{(1)}(k) \mathbb{\Pi}_{\mathcal{V}^{n,\mu} \mathcal{V}_\rho^b}^{(0)}(k) \mathbb{\Pi}_{W_\sigma^\pm W_\lambda^\mp \mathcal{V}_\rho^b}^{(0)}(-k + q, -q, k) = \pm (g_w C_b) M_{\mathcal{V}^b}^2 \frac{1}{k^2} \mathbb{\Pi}_{W_\sigma^\pm W_\lambda^\mp}^{(0)}(q). \quad (7.22)$$

Moreover, observing that

$$i M_{\mathcal{V}^b} \mathbb{\Pi}_{u^a \chi^*}^{(1)}(k) = L_G^{ab(1)}(k), \quad (7.23)$$

being $L_G^{ab(1)}(k)$ the part of the one-loop $u^a \bar{u}^b$ ghost self-energy involving an internal Goldstone boson propagator, we find

$$i \mathbb{\Pi}_{u^a \chi^*}^{(1)}(k) \mathbb{\Pi}_{\chi \mathcal{V}_\rho^b}^{(0)}(k) \mathbb{\Pi}_{W_\sigma^\pm W_\lambda^\mp \mathcal{V}_\rho^b}^{(0)}(-k + q, -q, k) = \pm (g_w C_b) L_G^{ab(1)}(k) \mathbb{\Pi}_{W_\sigma^\pm W_\lambda^\mp}^{(0)}(q). \quad (7.24)$$

Finally, the last term appearing in Eq. (7.21) is part of the PT answer, and it is precisely the term responsible for generating the characteristic BFM vertex $\chi \widehat{W}^\pm W^\mp$ (recall that the tree-level coupling $\chi W^\pm W^\mp$ does not exist in the conventional R_ξ gauges).

For the remaining two \mathcal{C}_{1L}^n terms proportional to the longitudinal momentum $(k - q)$, the STI triggered will simply be

$$(k - q)^\sigma \mathbb{\Pi}_{W_\sigma^\pm W_\lambda^\mp \mathcal{V}_\rho^a}^{(0)}(-q, k) = \mp (g_w C_a) \mathbb{\Pi}_{W_\rho^\pm W_\lambda^\mp}^{(0)}(q). \quad (7.25)$$

Thus, the discarded term originating from \mathcal{C}_{1L}^n , reads

$$\begin{aligned} \mathcal{C}_{1L}^n &= 2g_w^2 \sum_{i,j} C_i C_j \int_{L_1} J_i(q, k) d_{\nu^j}(k) \left[\frac{M_V^2}{k^2} L_V^{ij(1)}(k) + L_G^{ij(1)}(k) \right] \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(0)}(q) \\ &\quad - 2g_w^2 \sum_{i,j} C_i C_j \int_{L_1} J_i(q, k) d_{\nu^j}(k) \mathbb{I}_{\nu_\alpha^i \nu_\beta^j}^{(1)}(k) \mathbb{I}_{W^+, \rho W_\beta^-}^{(0)}(q). \end{aligned} \quad (7.26)$$

Having analyzed in detail all the contributions, we can now add them up to construct the quantity $\Pi_{\alpha\beta}^{\text{IP}(2)}(q)$; after some algebra, and using the relations

$$\begin{aligned} \frac{M_V^2}{k^2(k^2 - M_V^2)} + \frac{1}{k^2} &= \frac{1}{k^2 - M_V^2}, \\ L_V^u(1)(k) + L_G^u(1)(k) &= L^u(1)(k), \\ L_V^{ij(1)}(k) + L_G^{ij(1)}(k) &= L^{ij(1)}(k), \end{aligned} \quad (7.27)$$

the discarded terms furnished by the intrinsic PT algorithm will give rise to the quantity

$$\begin{aligned} \Pi_{\alpha\beta}^{\text{IP}(2)}(q) &= 2g_w^2 \sum_i C_i \int_{L_1} \left[C_i J'_i(q, k) d_W(k) L^u(1)(k) + \sum_j C_j J_i(q, k) d_{\nu^j}(k) L^{ij(1)}(k) \right] \times \\ &\quad \times \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(0)}(q) \\ &\quad - 2g_w^2 \sum_i C_i \int_{L_1} \left[C_i J'_i(q, k) d_W(k) \mathbb{I}_{W_\alpha^+ W_\mu^-}^{(1)}(k) + \sum_j C_j J_i(q, k) d_{\nu^j}(k) \mathbb{I}_{\nu_\alpha^i \nu_\mu^j}^{(1)}(k) \right] \times \\ &\quad \times \mathbb{I}_{W^+, \mu W_\beta^-}^{(0)}(q) \\ &\quad - 2g_w \sum_i C_i \int_{L_1} J'_i(q, k) \left[\mathbb{I}_{u^+ W_\sigma^*, -\nu_\alpha^i}^{(1)}(-q, -k + q) + \mathbb{I}_{u^i W_\sigma^*, -W_\alpha^+}^{(1)}(-q, k) \right] \times \\ &\quad \times \mathbb{I}_{W^+, \sigma W_\beta^-}^{(0)}(q) \\ &\quad - 4ig_w^2 \sum_i C_i^2 \int_{L_1} J'_i(q, k) \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(1)}(q). \end{aligned} \quad (7.28)$$

The last step consists in comparing this result to the one coming from the BQI of Eq.(3.14). At two loops this BQI reads

$$\begin{aligned} \mathbb{I}_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(2)}(q) &= \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(2)}(q) + 2\mathbb{I}_{\Omega_\alpha^+ W_\rho^*, -}^{(2)}(q) \mathbb{I}_{W^+, \rho W_\beta^-}^{(0)}(q) + 2\mathbb{I}_{\Omega_\alpha^+ W_\rho^*, -}^{(1)}(q) \mathbb{I}_{W^+, \rho W_\beta^-}^{(1)}(q) \\ &\quad + \mathbb{I}_{\Omega_\alpha^+ W_\rho^*, -}^{(1)}(q) \mathbb{I}_{W^+, \rho W^-, \sigma}^{(0)}(q) \mathbb{I}_{\Omega_\beta^- W_\sigma^*, +}^{(1)}(q), \end{aligned} \quad (7.29)$$

and the diagrams contributing to the two-loop two-point function $\mathbb{I}_{\Omega_\alpha^+ W_\rho^*, -}^{(2)}$ are shown in Fig.4. Using the one-loop result of Eq.(4.15), it is then easy to prove that

$$2\mathbb{I}_{\Omega_\alpha^+ W_\rho^*, -}^{(1)}(q) \mathbb{I}_{W^+, \rho W_\beta^-}^{(1)}(q) \equiv 4ig_w^2 \sum_i C_i^2 \int_{L_1} J'_i(q, k) \mathbb{I}_{W_\alpha^+ W_\beta^-}^{(1)}(q), \quad (7.30)$$

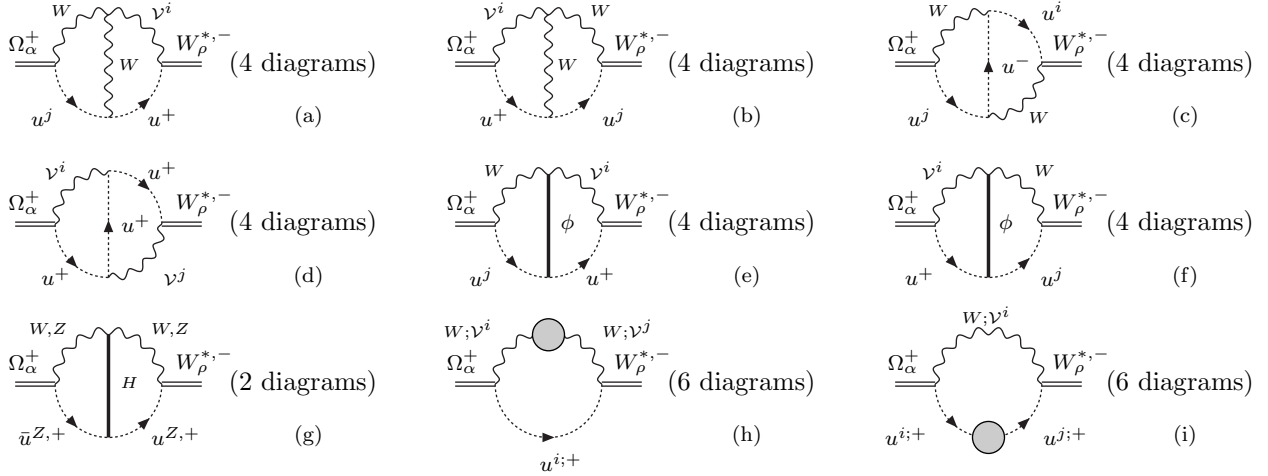


FIG. 4: Two loop contributions to the two-point function $\Pi_{\Omega_{\alpha}^{+}W_{\rho}^{*-,-}}^{(2)}(q)$. The blobs represent one-loop corrections to the corresponding propagator.

while it is a long but straightforward exercise to demonstrate that

$$\begin{aligned}
& 2\Pi_{\Omega_{\alpha}^{+}W_{\rho}^{*-,-}}^{(2)}(q)\Pi_{W^{+,\rho}W_{\beta}^{-}}^{(0)}(q) \\
& \equiv -2g_{\text{W}}^2 \sum_i C_i \int_{L_1} \left[C_i J'_i(q, k) d_W(k) L^{u(1)}(k) + \sum_j C_j J_i(q, k) d_{\nu^j}(k) L^{ij(1)}(k) \right] \times \\
& \times \Pi_{W_{\alpha}^{+}W_{\beta}^{-}}^{(0)}(q) \\
& + 2g_{\text{W}}^2 \sum_i C_i \int_{L_1} \left[C_i J'_i(q, k) d_W(k) \Pi_{W_{\alpha}^{+}W_{\mu}^{-}}^{(1)}(k) + \sum_j C_j J_i(q, k) d_{\nu^j}(k) \Pi_{\nu_{\alpha}^i \nu_{\mu}^j}^{(1)}(k) \right] \times \\
& \times \Pi_{W^{+,\mu}W_{\beta}^{-}}^{(0)}(q) \\
& + 2g_{\text{W}} \sum_i C_i \int_{L_1} J'_i(q, k) \left[\Pi_{u^{+}W_{\sigma}^{*-,-}\nu_{\alpha}^i}^{(1)}(-q, -k + q) + \Pi_{u^i W_{\sigma}^{*-,-}W_{\alpha}^{+}}^{(1)}(-q, k) \right] \Pi_{W^{+,\sigma}W_{\beta}^{-}}^{(0)}(q).
\end{aligned} \tag{7.31}$$

In particular the first term on the RHS of the above equation gives the diagrams of Fig.4i, the second term the ones of Fig.4h, and the last term all the remaining diagrams of Fig.4 (a–g).

The last term appearing in Eq.(7.29) will be finally generated in the conversion of the 1PR strings into the 1PR PT strings, as follows. After treating the conventional 1PR diagrams involving charged gauge-boson self-energies along the same lines explained in [20] for the QCD case, one arrives, after discarding the terms proportional to the inverse propagator of

the external legs, to the result

$$S_{\alpha\beta}^{\text{IP}(2)}(q) = 2iV_{\alpha\rho}^{\text{P}(1)}(q)\mathbb{\Pi}_{W^+,\rho W_\beta^-}^{(1)}(q) + \mathbb{\Pi}_{\Omega_\alpha^+ W_\rho^{*,-}}^{(1)}(q)\mathbb{\Pi}_{W^+,\rho W^-, \sigma}^{(0)}(q)\mathbb{\Pi}_{\Omega_\beta^- W_\sigma^{*,+}}^{(1)}(q). \quad (7.32)$$

On the other hand, from Eq.(7.28) we have

$$\mathbb{\Pi}_{\alpha\beta}^{\text{IP}(2)}(q) = -2iV_{\alpha\rho}^{\text{P}(1)}(q)\mathbb{\Pi}_{W^+,\rho W_\beta^-}^{(1)}(q), \quad (7.33)$$

so that adding by parts these two equations we obtain

$$R_{\alpha\beta}^{\text{IP}(2)}(q) = \mathbb{\Pi}_{\Omega_\alpha^+ W_\rho^{*,-}}^{(1)}(q)\mathbb{\Pi}_{W^+,\rho W^-, \sigma}^{(0)}(q)\mathbb{\Pi}_{\Omega_\beta^- W_\sigma^{*,+}}^{(1)}(q). \quad (7.34)$$

Thus, finally, the quantity $-\mathbb{\Pi}_{\alpha\beta}^{\text{IP}(2)}(q) + R_{\alpha\beta}^{\text{IP}(2)}(q)$ will provide precisely all the terms appearing in the two-loop version of the relevant BQI [Eq.(8.9)], *i.e.*, one has

$$\begin{aligned} -\mathbb{\Pi}_{\alpha\beta}^{\text{IP}(2)}(q) + R_{\alpha\beta}^{\text{IP}(2)}(q) &= 2\mathbb{\Pi}_{\Omega_\alpha^+ W_\rho^{*,-}}^{(2)}(q)\mathbb{\Pi}_{W^+,\rho W_\beta^-}^{(0)}(q) + 2\mathbb{\Pi}_{\Omega_\alpha^+ W_\rho^{*,-}}^{(1)}(q)\mathbb{\Pi}_{W^+,\rho W_\beta^-}^{(1)}(q) \\ &+ \mathbb{\Pi}_{\Omega_\alpha^+ W_\rho^{*,-}}^{(1)}(q)\mathbb{\Pi}_{W^+,\rho W^-, \sigma}^{(0)}(q)\mathbb{\Pi}_{\Omega_\beta^- W_\sigma^{*,+}}^{(1)}(q). \end{aligned} \quad (7.35)$$

Then, the difference between $\widehat{\mathbb{\Pi}}_{W_\alpha^+ W_\beta^-}^{(2)}(q)$ and $\mathbb{\Pi}_{W_\alpha^+ W_\beta^-}^{(2)}(q)$ given by the intrinsic PT definition of Eq.(6.11), is the same as the difference between $\mathbb{\Pi}_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(2)}(q)$ and $\mathbb{\Pi}_{W_\alpha^+ W_\beta^-}^{(2)}(q)$ given by the BQI, thus proving that

$$\widehat{\mathbb{\Pi}}_{W_\alpha^+ W_\beta^-}^{(2)}(q) \equiv \mathbb{\Pi}_{\widehat{W}_\alpha^+ \widehat{W}_\beta^-}^{(2)}(q). \quad (7.36)$$

VIII. TWO-LOOP NEUTRAL SECTOR

In the neutral gauge-boson sector the starting point will be the following expressions

$$\begin{aligned} \mathcal{C}_{2L}^{ij} &= ig_w \int_{L_1} J_W(q, k) \left\{ C_i [k^\sigma g_\alpha^\rho + (k-q)^\rho g_\alpha^\sigma] \mathbb{\Pi}_{W_\sigma^+ W_\rho^- \nu_\beta^j}(k, -k+q, -q) \right. \\ &\quad \left. - C_j [k^\sigma g_\beta^\rho + (k-q)^\rho g_\beta^\sigma] \mathbb{\Pi}_{W_\sigma^- W_\rho^+ \nu_\alpha^i}(k, -k+q, -q) \right\}, \end{aligned} \quad (8.1)$$

$$\begin{aligned} \mathcal{C}_{1L}^{ij} &= 2g_w \int_{L_1} J_W(q, k) d_W(q) \times \\ &\quad \times \left\{ C_i [k^\sigma g_\alpha^\rho + (k-q)^\rho g_\alpha^\sigma] \mathbb{\Pi}_{W_\sigma^+ W_\mu^-}(k) \mathbb{\Pi}_{W^+,\mu W_\rho^- \nu_\beta^j}(k, -k+q, -q) \right. \\ &\quad \left. - C_j [k^\sigma g_\beta^\rho + (k-q)^\rho g_\beta^\sigma] \mathbb{\Pi}_{W_\sigma^- W_\mu^+}(k) \mathbb{\Pi}_{W^-, \mu W_\rho^+ \nu_\alpha^i}(k, -k+q, -q) \right\}. \end{aligned} \quad (8.2)$$

We then start the analysis from the \mathcal{C}_{2L}^{ij} contributions, Eq.(8.1). For the two terms proportional to the longitudinal momentum k , the STI triggered will be the one-loop version

of the STI of Eq.(3.7), which, making use of Eqs.(7.5) and (7.6) and the Feynman rules of Appendix A, in this case reads

$$\begin{aligned}
k^\sigma \mathbb{I}_{W^\pm, \sigma W_\rho^\mp \nu_\lambda^a}^{(1)}(-k+q, -q) &= \mp (ig_w C_a) \frac{1}{k^2} L_V^{u(1)}(k) \mathbb{I}_{\nu_\rho^a \nu_\lambda^a}^{(0)}(q) \\
&\quad - i \mathbb{I}_{u^\pm \nu_\sigma^{*,a} W_\rho^\mp}^{(1)}(-q, -k+q) \mathbb{I}_{\nu_\rho^a, \sigma \nu_\lambda^a}^{(0)}(q) \\
&\quad \mp \sum_n (g_w C_n) \mathbb{I}_{\nu_\rho^n \nu_\lambda^a}^{(1)}(q). \tag{8.3}
\end{aligned}$$

For the two remaining \mathcal{C}_{2L}^{ij} terms, proportional to the longitudinal momentum $(k-q)$ the STI triggered will read instead

$$\begin{aligned}
(k-q)^\rho \mathbb{I}_{W^\mp, \rho W_\sigma^\pm \nu_\lambda^a}^{(1)}(k, -q) &= \mp (ig_w C_a) \frac{1}{(k-q)^2} L_V^{u(1)}(k-q) \mathbb{I}_{\nu_\sigma^a \nu_\lambda^a}^{(0)}(q) \\
&\quad + i \mathbb{I}_{u^\mp \nu_\rho^{*,a} W_\sigma^\pm}^{(1)}(-q, k) \mathbb{I}_{\nu_\sigma^a, \rho \nu_\lambda^a}^{(0)}(q) \\
&\quad \mp \sum_n (g_w C_n) \mathbb{I}_{\nu_\sigma^n \nu_\lambda^a}^{(1)}(q). \tag{8.4}
\end{aligned}$$

Thus, the discarded terms stemming from the \mathcal{C}_{2L}^{ij} contributions, can be written as

$$\begin{aligned}
\mathcal{C}_{2L}^{ij} &= 2g_w^2 C_i C_j \int_{L_1} J_W(q, k) \frac{1}{k^2} L_V^{u(1)}(k) \left[\mathbb{I}_{\nu_\alpha^i \nu_\beta^j}^{(0)}(q) + \mathbb{I}_{\nu_\alpha^j \nu_\beta^i}^{(0)}(q) \right] \\
&\quad + 2g_w \int_{L_1} J_W(q, k) \left[C_i \mathbb{I}_{u^+ \nu_\sigma^{*,j} W_\alpha^-}^{(1)}(-q, -k+q) \mathbb{I}_{\nu_j, \sigma \nu_\beta^j}^{(0)}(q) \right. \\
&\quad \left. - C_j \mathbb{I}_{u^- \nu_\sigma^{*,i} W_\beta^+}^{(1)}(-q, -k+q) \mathbb{I}_{\nu_i, \alpha \nu_i, \sigma}^{(0)}(q) \right] \\
&\quad - 2ig_w^2 \int_{L_1} J_W(q, k) \sum_n C_n \left[C_i \mathbb{I}_{\nu_\alpha^n \nu_\beta^j}^{(1)}(q) + C_j \mathbb{I}_{\nu_\alpha^i \nu_\beta^n}^{(1)}(q) \right]. \tag{8.5}
\end{aligned}$$

We next consider the lower order corrections coming from the \mathcal{C}_{1L}^{ij} contributions, Eq.(8.2). For the two terms proportional to the longitudinal momentum k , the STI triggered will be precisely the one appearing in Eq.(7.16). However, for the first two terms of this STI we now find the following results

$$\begin{aligned}
i \mathbb{I}_{u^\pm W_\sigma^{*,\mp}}^{(1)}(k) \mathbb{I}_{W^\pm, \sigma W_\mu^\mp}^{(0)}(k) \mathbb{I}_{W^\pm, \mu W_\rho^\mp \nu_\lambda^a}^{(0)}(k, -k+q, -q) &= \pm (g_w C_a) M_W^2 \frac{1}{k^2} L_V^{u(1)}(k) \mathbb{I}_{\nu_\rho^a \nu_\lambda^a}^{(0)}(q), \\
i \mathbb{I}_{u^\pm \phi^{*,\mp}}^{(1)}(k) \mathbb{I}_{\phi^\pm W_\mu^\mp}^{(0)}(k) \mathbb{I}_{W^\pm, \mu W_\rho^\mp \nu_\lambda^a}^{(0)}(k, -k+q, -q) &= \pm (g_w C_a) L_G^{u(1)}(k) \mathbb{I}_{\nu_\rho^a \nu_\lambda^a}^{(0)}(q), \tag{8.6}
\end{aligned}$$

while, as before, the last term of Eq.(7.16) will part of the PT answer.

Finally, for the remaining two \mathcal{C}_{1L}^{ij} terms proportional to the longitudinal momentum $(k-q)$, the STI triggered will simply be the one of Eq.(5.13); thus, the discarded terms

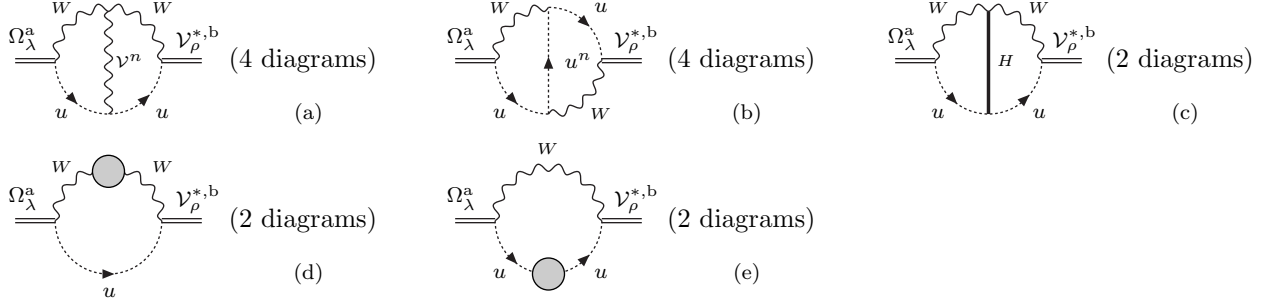


FIG. 5: Two loop contributions to the two-point function $\Pi_{\Omega_\lambda^a \nu_\rho^{*,b}}^{(2)}(q)$. The blobs represent one-loop corrections to the corresponding propagator.

stemming from the \mathcal{C}_{1L}^{ij} contributions, will read

$$\begin{aligned} \mathcal{C}_{1L}^{ij} &= 2g_W^2 C_i C_j \int_{L_1} J_W(q, k) d_W(k) \left[\frac{M_W^2}{k^2} L_W^{u(1)}(k) + L_G^{u(1)}(k) \right] \left[\Pi_{\nu_\alpha^i \nu_\beta^j}^{(0)}(q) + \Pi_{\nu_\alpha^j \nu_\beta^i}^{(0)}(q) \right] \\ &- 2g_W^2 C_i C_j \int_{L_1} J_W(q, k) d_W(k) \Pi_{W_\alpha^+ W_\mu^-}^{(1)}(k) \left[\Pi_{\nu_\alpha^i \nu_\beta^j}^{(0)}(q) + \Pi_{\nu_\alpha^j \nu_\beta^i}^{(0)}(q) \right]. \end{aligned} \quad (8.7)$$

The two contributions analyzed will then add up to give the quantity $\Pi_{ij, \alpha\beta}^{\text{IP}(2)}(q)$ which reads, after trivial manipulations,

$$\begin{aligned} \Pi_{ij, \alpha\beta}^{\text{IP}(2)}(q) &= 2g_W^2 C_i C_j \int_{L_1} J_W(q, k) d_W(k) L^{u(1)}(k) \left[\Pi_{\nu_\alpha^i \nu_\beta^j}^{(0)}(q) + \Pi_{\nu_\alpha^j \nu_\beta^i}^{(0)}(q) \right] \\ &- 2g_W^2 C_i C_j \int_{L_1} J_W(q, k) d_W(k) \Pi_{W_\alpha^+ W_\mu^-}^{(1)}(k) \left[\Pi_{\nu_\alpha^i \nu_\beta^j}^{(0)}(q) + \Pi_{\nu_\alpha^j \nu_\beta^i}^{(0)}(q) \right] \\ &+ 2g_W \int_{L_1} J_W(q, k) \left[C_i \Pi_{u^+ \nu_\sigma^{*,j} W_\alpha^-}^{(1)}(-q, -k + q) \Pi_{\nu_\beta^j \nu_\sigma^j}^{(0)}(q) \right. \\ &- \left. C_j \Pi_{u^- \nu_\sigma^{*,i} W_\beta^+}^{(1)}(-q, -k + q) \Pi_{\nu_\alpha^i \nu_\sigma^i}^{(0)}(q) \right] \\ &- 2ig_W^2 \int_{L_1} J_W(q, k) \sum_n C_n \left[C_i \Pi_{\nu_\alpha^n \nu_\beta^j}^{(1)}(q) + C_j \Pi_{\nu_\alpha^i \nu_\beta^n}^{(1)}(q) \right]. \end{aligned} \quad (8.8)$$

The last step is then to compare this result to the one coming from the BQI of Eq.(3.15). At two loops this BQI reads

$$\begin{aligned} \Pi_{\hat{\nu}_\alpha^i \hat{\nu}_\beta^j}^{(2)}(q) &= \Pi_{\nu_\alpha^i \nu_\beta^j}^{(2)}(q) + \Pi_{\Omega_\alpha^i \nu_\rho^{*,j}}^{(2)}(q) \Pi_{\nu_\beta^j \nu_\rho^j}^{(0)}(q) + \Pi_{\Omega_\beta^j \nu_\rho^{*,i}}^{(2)}(q) \Pi_{\nu_\alpha^i \nu_\rho^i}^{(0)}(q) \\ &+ \sum_n \left[\Pi_{\Omega_\alpha^i \nu_\rho^{*,n}}^{(1)}(q) \Pi_{\nu_\beta^j \nu_\rho^j}^{(1)}(q) + \Pi_{\Omega_\beta^j \nu_\rho^{*,n}}^{(1)}(q) \Pi_{\nu_\alpha^i \nu_\rho^i}^{(1)}(q) \right] \\ &+ \sum_n \Pi_{\Omega_\alpha^i \nu_\rho^{*,n}}^{(1)}(q) \Pi_{\nu_\beta^j \nu_\rho^j \nu_\sigma^i}^{(0)}(q) \Pi_{\Omega_\beta^j \nu_\sigma^i}^{(1)}(q), \end{aligned} \quad (8.9)$$

and the diagrams contributing to the two-loop two-point function $\Pi_{\Omega_\lambda^a \mathcal{V}_\rho^b}^{(2)}(q)$ are shown in Fig.5. Using the one-loop result of Eq.(4.29), it is then easy to show that

$$\begin{aligned} & \sum_n \left[\Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}^{(1)}(q) + \Pi_{\Omega_\beta^j \mathcal{V}_\rho^{*,n}}^{(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\alpha^i}^{(1)}(q) \right] \\ & \equiv 2ig_W^2 \int_{L_1} J_W(q, k) \sum_n C_n \left[C_i \Pi_{\mathcal{V}_\alpha^n \mathcal{V}_\beta^j}^{(1)}(q) + C_j \Pi_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^n}^{(1)}(q) \right], \end{aligned} \quad (8.10)$$

while it is a long but straightforward exercise to check that

$$\begin{aligned} & \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,j}}^{(2)}(q) \Pi_{\mathcal{V}^{j,\rho} \mathcal{V}_\beta^j}^{(0)}(q) + \Pi_{\Omega_\beta^j \mathcal{V}_\rho^{*,i}}^{(2)}(q) \Pi_{\mathcal{V}^{i,\rho} \mathcal{V}_\alpha^i}^{(0)}(q) \\ & \equiv -2g_W^2 C_i C_j \int_{L_1} J_W(q, k) d_W(k) L^u(1)(k) \left[\Pi_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(0)}(q) + \Pi_{\mathcal{V}_\alpha^j \mathcal{V}_\beta^i}^{(0)}(q) \right] \\ & + 2g_W^2 C_i C_j \int_{L_1} J_W(q, k) d_W(k) \Pi_{W_\alpha^+ W_\mu^-}^{(1)}(k) \left[\Pi_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(0)}(q) + \Pi_{\mathcal{V}_\alpha^j \mathcal{V}_\beta^i}^{(0)}(q) \right] \\ & - 2g_W \int_{L_1} J_W(q, k) \left[C_i \Pi_{u^+ \mathcal{V}_\sigma^{*,j} W_\alpha^-}^{(1)}(-q, -k + q) \Pi_{\mathcal{V}^{j,\sigma} \mathcal{V}_\beta^j}^{(0)}(q) \right. \\ & \left. - C_j \Pi_{u^- \mathcal{V}_\sigma^{*,i} W_\beta^+}^{(1)}(-q, -k + q) \Pi_{\mathcal{V}^{i,\alpha} \mathcal{V}_\sigma}^{(0)}(q) \right]. \end{aligned} \quad (8.11)$$

In particular the first term on the RHS of the above equation gives the diagrams of Fig.5e, the second term the ones of Fig.5d, and the last term all the remaining diagrams of Fig.5 (a-c).

The last term appearing in Eq.(8.9) will be finally generated in the conversion of the 1PR strings into the 1PR PT strings, as follows. After treating the conventional 1PR diagrams involving neutral gauge-bosons self-energies along the same lines explained in [20] for the QCD case, one arrives, after discarding the terms proportional to the inverse propagator of the external legs, to the result

$$\begin{aligned} S_{ij,\alpha\beta}^{\text{IP}(2)}(q) & = i \sum_n \left[V_{in,\alpha\rho}^{\text{P}(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}^{(1)}(q) + V_{jn,\beta\rho}^{\text{P}(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\alpha^i}^{(1)}(q) \right] \\ & + \sum_n \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\sigma}^{(0)}(q) \Pi_{\Omega_\beta^j \mathcal{V}_\sigma^{*,n}}^{(1)}(q). \end{aligned} \quad (8.12)$$

On the other hand from Eq.(8.8) we have

$$\Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q) = -i \sum_n \left[V_{in,\alpha\rho}^{\text{P}(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}^{(1)}(q) + V_{jn,\beta\rho}^{\text{P}(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\alpha^i}^{(1)}(q) \right] \quad (8.13)$$

so that adding by parts these two equations we obtain

$$R_{ij,\alpha\beta}^{\text{IP}(2)}(q) = \sum_n \Pi_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \Pi_{\mathcal{V}^{n,\rho} \mathcal{V}_\sigma}^{(0)}(q) \Pi_{\Omega_\beta^j \mathcal{V}_\sigma^{*,n}}^{(1)}(q). \quad (8.14)$$

Thus, finally, the quantity $-\Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q) + R_{ij,\alpha\beta}^{\text{IP}(2)}(q)$ will provide precisely all the terms appearing in the two-loop version of the relevant BQI [Eq.(7.29)], *i.e.*, one has

$$\begin{aligned}
-\Pi_{ij,\alpha\beta}^{\text{IP}(2)}(q) + R_{ij,\alpha\beta}^{\text{IP}(2)}(q) &= \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,j}}^{(2)}(q) \mathbb{I}_{\mathcal{V}^{j,\rho} \mathcal{V}_\beta^j}^{(0)}(q) + \mathbb{I}_{\Omega_\beta^j \mathcal{V}_\rho^{*,i}}^{(2)}(q) \mathbb{I}_{\mathcal{V}^{i,\rho} \mathcal{V}_\alpha^i}^{(0)}(q) \\
&+ \sum_n \left[\mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}_\beta^j}^{(1)}(q) + \mathbb{I}_{\Omega_\beta^j \mathcal{V}_\rho^{*,n}}^{(1)}(q) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}_\alpha^i}^{(1)}(q) \right] \\
&+ \sum_n \mathbb{I}_{\Omega_\alpha^i \mathcal{V}_\rho^{*,n}}^{(1)}(q) \mathbb{I}_{\mathcal{V}^{n,\rho} \mathcal{V}^{n,\sigma}}^{(0)}(q) \mathbb{I}_{\Omega_\beta^j \mathcal{V}_\sigma^{*,n}}^{(1)}(q). \tag{8.15}
\end{aligned}$$

Then, the difference between $\widehat{\mathbb{I}}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q)$ and $\mathbb{I}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q)$ as given by the intrinsic PT definition of Eq.(6.12), is the same as the difference between $\mathbb{I}_{\widehat{\mathcal{V}}_\alpha^i \widehat{\mathcal{V}}_\beta^j}^{(2)}(q)$ and $\mathbb{I}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q)$ as given by the BQI, thus proving that

$$\widehat{\mathbb{I}}_{\mathcal{V}_\alpha^i \mathcal{V}_\beta^j}^{(2)}(q) \equiv \mathbb{I}_{\widehat{\mathcal{V}}_\alpha^i \widehat{\mathcal{V}}_\beta^j}^{(2)}(q). \tag{8.16}$$

IX. DISCUSSION AND CONCLUSIONS

In this paper we have extended the two-loop PT construction to the Electroweak sector of the Standard Model. This generalization has been a pending problem, mainly due to the proliferation of Feynman diagrams as compared to the QCD two-loop case, as well as due to the conceptual complication arising from the non-transversality of the massive gauge-boson one-loop self-energies appearing inside the two-loop diagrams. The aforementioned logistical problems have been solved by resorting to the recently introduced intrinsic PT construction by means of the STI satisfied by the one-loop three-gauge boson vertices; the latter are nested inside the two-loop Feynman graphs determining the gauge-boson self-energies at the same order. Thus, instead of manipulating algebraically individual Feynman diagrams, entire classes of diagrams may be simultaneously addressed. In the construction we have restricted ourselves to the operationally simpler case of massless-external fermions; thus, only the parts of the self-energies proportional to $g_{\mu\nu}$ have been considered. For the same reason longitudinal contributions to the STIs employed have been consistently discarded throughout. The final outcome of this construction are gauge-independent two-loop self-energies for both the charged (W) and neutral gauge-bosons (A, Z).

The comparison of the resulting PT expressions with those of the BFM in the Feynman gauge, constitutes an almost obligatory exercise, given the well-known correspondence established in the literature. The task of carrying out this comparison is greatly simplified

by employing the BQIs; the latter relate the BFM n -point functions with the conventional (quantum) ones, by means of a set of well-defined auxiliary Green's functions, definable in the BV framework. The non-trivial step in this exercise is to establish that the pieces removed from the conventional self-energy, following the strict rules of PT procedure, are precisely those accounting for the difference between the BFM and conventional two-point functions as captured in the corresponding BQI. Thus, the correspondence between the PT and the BFM results in the Feynman gauge persists unaltered in the case of the two-loop Electroweak construction.

The generalization to the case of massive external fermions of the construction presented here, *i.e.*, the case of non-conserved external currents, is technically more involved for the following reasons [4, 6]: First of all, in the case of the S -matrix construction, the tree-level WIs listed in Eq.(4.6) are modified by the presence of masses, giving rise to additional terms. These terms will combine non-trivially with the additional graphs containing the would-be Goldstone bosons, in order to give rise to the necessary cancellations. In addition, the propagator-like corrections which will be obtained from the vertices must be judiciously allotted to not only the corresponding gauge-boson self-energies, but also to the self-energies describing the various higher order mixings, such as $\mathbb{I}\Gamma_{\phi^\pm W_\beta^\mp}(q)$, $\mathbb{I}\Gamma_{\phi^\pm \phi^\mp}(q)$, $\mathbb{I}\Gamma_{\chi \nu_\beta^j}(q)$, $\mathbb{I}\Gamma_{H \nu_\beta^j}(q)$, $\mathbb{I}\Gamma_{\chi\chi}(q)$, and $\mathbb{I}\Gamma_{H\chi}(q)$, whose PT counterparts to the given order must also be constructed [6]. In the case of the intrinsic PT, where the vertex diagrams are essentially inert, the complications from the fact that the currents are not-conserved are mainly due to the appearance of the aforementioned mixing self-energies in the corresponding three-gauge-boson STIs of Eq.(3.6). Moreover, in both the S -matrix PT and the intrinsic PT the presence of the mixing self-energies in the relevant BQIs [*viz.* Eq.(3.13)] further complicates the final comparison of the results between the PT and the BFM. The point we would like to emphasize however, is that, despite all these technical issues discussed above, no additional conceptual difficulties are expected in the non-conserved current case.

The distinction between the S -matrix PT and intrinsic PT warrants some further comments. In this paper we have focused on the intrinsic PT construction, because of the realization that the parts discarded correspond to very precise terms in the STI satisfied by the one-loop three-gauge-boson vertex; thus the algorithm presented here constitutes the natural generalization to two-loops of the one-loop intrinsic PT construction presented in Section VI. On the other hand we have refrained from carrying out the two-loop S -matrix

PT construction *explicitly, i.e.*, by directly rearranging the two-loop vertex graphs, as has been done in [19, 20]; at present this would constitute an arduous diagrammatic task, since the equivalent of the three-gauge-boson vertex STI, whose use has been crucial in obtaining compact results in the intrinsic PT case, still eludes us. This issue is currently under investigation, and we hope to report further progress in the near future.

At the phenomenological level, and especially in the field of precision Electroweak physics, the two-loop construction presented here may serve as a starting point for complementing existing two-loop calculations [36]. In particular, the current accuracy in the measurement of M_W is $M_W = 80.451 \pm 0.033$ GeV [37], and it is supposed to be further improved in the final LEP II analysis and the Tevatron Run II, each giving an error of $\delta M_W \approx 30$ MeV. Furthermore, at the LHC the error is expected to be as low as $\delta M_W \approx 15$ MeV [38]. Even more impressively, high-luminosity linear colliders operating at the W^+W^- threshold could reduce the error to $\delta M_W \approx 6$ MeV [39, 40]. In order to match the expected experimental precision, quantities such as Δr or the two-loop ρ parameter must be determined with high theoretical accuracy [36]; in particular, purely bosonic two-loop corrections may have to be calculated eventually. The theoretical framework put forward in the present paper sets up the stage for carrying out such a task in a systematic way [41]. Aside of these possibilities however, the two-loop construction presented here renders the various one-loop results of the past (listed in the Introduction) conceptually far more robust, demonstrating that the special field-theoretic properties achieved by means of the PT method are not a fortuitous one-loop accident.

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a. Gauge-boson sector

$\Omega_\alpha^\pm \rightarrow \bar{u}^A$ via W_β^\mp : $= \pm i g_W s_W g_{\alpha\beta}$
 $\Omega_\alpha^\pm \rightarrow \bar{u}^Z$ via W_β^\mp : $= \mp i g_W c_W g_{\alpha\beta}$
 $\Omega_\alpha^\pm \rightarrow \bar{u}^\pm$ via A_β : $= \mp i g_W s_W g_{\alpha\beta}$
 $\Omega_\alpha^\pm \rightarrow \bar{u}^\pm$ via Z_β : $= \pm i g_W c_W g_{\alpha\beta}$
 $\Omega_\alpha^Z \rightarrow \bar{u}^\pm$ via W_β^\pm : $= \mp i g_W c_W g_{\alpha\beta}$
 $\Omega_\alpha^A \rightarrow \bar{u}^\pm$ via W_β^\pm : $= \pm i g_W s_W g_{\alpha\beta}$

b. Scalar sector

$\Omega^\pm \rightarrow \bar{u}^A$ via ϕ^\mp : $= \pm i g_W s_W$
 $\Omega^\pm \rightarrow \bar{u}^Z$ via ϕ^\mp : $= \mp i g_W \frac{c_W^2 - s_W^2}{2c_W}$
 $\Omega^\pm \rightarrow \bar{u}^\pm$ via χ : $= -\frac{g_W}{2}$
 $\Omega^\pm \rightarrow \bar{u}^\pm$ via H : $= \mp i \frac{g_W}{2}$
 $\Omega^X \rightarrow \bar{u}^\pm$ via ϕ^\pm : $= \frac{g_W}{2}$
 $\Omega^X \rightarrow \bar{u}^Z$ via H : $= -\frac{g_W}{2c_W}$
 $\Omega^H \rightarrow \bar{u}^\pm$ via ϕ^\pm : $= \pm i \frac{g_W}{2}$
 $\Omega^H \rightarrow \bar{u}^Z$ via χ : $= \frac{g_W}{2c_W}$

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