Breit-Wigner formalism
for non-Abelian theories

arXiv:hep-ph/0102149v1 13 Feb 2001

Joannis Papavassiliou

Departamento de Física Teórica and IFIC, Universidad de Valencia
E-46100 Burjassot (Valencia), Spain

Abstract

The consistent description of resonant transition amplitudes within the framework of perturbative field theories necessitates the definition and resummation of off-shell Green's functions, which must respect several crucial physical requirements. In particular, the generalization of the usual Breit-Wigner formalism in a non-Abelian context constitutes a highly non-trivial problem, related to the fact that the conventionally defined Green's functions are unphysical. We briefly review the main field-theoretical difficulties arising when attempting to use such Green's functions outside the confines of a fixed order perturbative calculation, and explain how this task has been successfully accomplished in the framework of the pinch technique.

1

1 The Breit-Wigner resummation

The physics of unstable particles and the computation of resonant transition amplitudes has attracted significant attention in recent years, because it is both phenomenologically relevant and theoretically challenging. Throughout the nineties A.Sirlin and collaborators [1] have addressed various important issues related to the proper definition of masses and widths of unstable particles. During the same period A.Pilaftsis and I have developed the formalism which allows for the proper generalization of the Breit-Wigner resummation in a non-Abelian context [2, 3]. In what follows I will outline the conceptual and practical difficulties appearing when dealing with non-Abelian resonances, and will briefly explain how the field-theoretical method known as the pinch technique (PT) [4, 5] allows for a consistent description of resonant transition amplitudes.

The mathematical expressions for computing transition amplitudes are ill-defined in the vicinity of resonances, because the tree-level propagator of the particle mediating the interaction, i.e. $\Delta = (s - M^2)^{-1}$, becomes singular as the center-of-mass energy $\sqrt{s} \sim M$. The standard way for regulating this physical kinematic singularity is to use a Breit-Wigner type of propagator, which essentially amounts to the replacement $(s - M^2)^{-1} \rightarrow (s - M^2 + iM\Gamma)^{-1}$, where Γ is the width of the unstable (resonating) particle. The field-theoretic mechanism which enables this replacement is the Dyson resummation of the (one-loop) self-energy $\Pi(s)$ of the unstable particle, which leads to the substitution $(s - M^2)^{-1} \rightarrow [s - M^2 + \Pi(s)]^{-1}$; the running width of the particle is then defined as $M\Gamma(s) = \Im m \Pi(s)$, whereas the usual (on-shell) width is simply its value at $s = M^2$.

It is well-known that, to any *finite order*, the conventional perturbative expansion gives rise to expressions for physical amplitudes which are endowed with crucial properties. For example, the amplitudes are independent of the gauge-fixing parameter (GFP) chosen to quantize the theory, they are gauge-invariant (in the sense of current conservation), they are unitary (in the sense of probability conservation), and well behaved at high energies. The above properties are however not always encoded into the individual Green's functions which are the building blocks of the aforementioned perturbative expansion; indeed, the simple fact that Green's functions depend in general explicitly on the GFP, indicates that they are void of any physical meaning. Evidently, when going from unphysical Green's functions to physical amplitudes subtle field-theoretical mechanisms are at work, which implement highly non-trivial cancellations among the various Green's functions appearing at a given order.

The happy state of affairs described above is guaranteed within the frame-

work of the conventional perturbative expansion, provided that one works at a given fixed order. It is relatively easy to realize however that the Breit-Wigner procedure is in fact equivalent to a reorganization of the perturbative series; indeed, resumming the self-energy Π amounts to removing a particular piece from each order of the perturbative expansion, since from all the Feynman graphs contributing to a given order n we only pick the part that contains n self-energy bubbles Π, and then take $n \to \infty$. However, given that a non-trivial cancellation involving the unphysical Green's function is generally taking place at any given order of the conventional perturbative expansion, the act of removing one of them from each order may or may not distort those cancellations. To put it differently, if Π contains unphysical contributions (which would eventually cancel against other pieces within a given order) resumming it naively would mean that these unphysical contributions have also undergone infinite summation (they now appear in the denominator of the propagator Δ). In order to remove them one has to add the remaining perturbative pieces to an infinite order, clearly an impossible task, since the latter (boxes and vertices) do not constitute a resumable set. Thus, if the resumed Π happened to contain such unphysical terms, one would finally arrive at predictions for the physical amplitude close to the resonance which would be plagued with unphysical artifacts. It turns out that, while in scalar field theories and Abelian gauge theories Π does not contain such unphysical contributions, this seizes to be true in the case of non-Abelian gauge theories.

The most obvious signal revealing that the conventionally defined non-Abelian self-energies are not good candidates for resummation comes from the simple calculational fact that the bosonic radiative corrections to the self-energies of vector (γ, W, Z) or scalar (Higgs) bosons induce a non-trivial dependence on the GFP used to define the tree-level bosonic propagators appearing in the quantum loops. This is to be contrasted to the radiative corrections due to fermion loops, which, even in the context of non-Abelian gauge theories behave as in quantum electrodynamics (QED), *i.e.*, they are GFP-independent. In addition, formal field-theoretic considerations as well as direct calculations show that, contrary to the QED case, the non-Abelian Green's functions do not satisfy their naive, tree-level Ward identities , after bosonic one-loop corrections are included. A careful analysis shows that this fundamental difference between Abelian and non-Abelian theories has far-reaching consequences; the naive generalization of the Breit-Wigner method to the latter case gives rise to Born-improved amplitudes, which do not faithfully capture the underlying dynamics. Most notably, due to violation of the optical theorem, unphysical thresholds and artificial resonances appear, which distort the line-shapes of the resonating particles. In addition, the

high energy properties of such amplitudes are altered, and are in direct contradiction to the equivalence theorem (ET) [6].

In order to address these issues, a new approach to resonant transition amplitudes has been developed over the past few years [2, 3], which is based on the the pinch technique (PT) [4, 5]; the latter is a diagrammatic method whose main thrust is to exploit the symmetries built into physical amplitudes in order to construct off-shell sub-amplitudes which are kinematically akin to conventional Green's functions, but, unlike the latter, are also endowed with several crucial properties:

(a) They are independent of the GFP, within any gauge-fixing scheme chosen to quantize the theory.

(b) They satisfy naive (ghost-free) tree-level Ward identities instead of the usual Slavnov-Taylor identities.

(c) They display physical thresholds only [2].

(d) They satisfy individually the optical and equivalence theorems [2, 3].

(e) The effective two-point functions constructed are universal (process-independent) , Dyson-resumable [2], and do not shift the position of the gauge-independent complex pole [2]. In addition, one may use them to construct "effective charges", i.e process-independent and renormalization-group-invariant objects [3].

(f) The PT effective Green's functions *coincide* with the conventional Green's functions defined in the framework of the background field method [7], when the latter are computed in the Feynman gauge [8].

The crucial novelty introduced by the PT is that the resummation of graphs must take place only *after* the amplitude of interest has been cast via the PT algorithm into manifestly physical sub-amplitudes, with distinct kinematic properties, order by order in perturbation theory. Put in the language employed earlier, the PT ensures that all unphysical contributions contained inside Π have been identified and properly discarded, before Π undergoes resummation. It is important to emphasize that the only ingredient needed for constructing the PT effective Green's functions is the full exploitation of elementary Ward-identities , which are a direct consequence of the BRS [9] symmetry of the theory, and the proper use of the unitarity and analyticity of the S-matrix. In what follows I will describe some of the salient features of this method.

4

2 The Pinch Technique.

Within the PT framework, the transition amplitude $T(s, t, m_i)$ of a $2 \to 2$ process, can be decomposed as

$$T(s, t, m_i) = \widehat{T}_1(s) + \widehat{T}_2(s, m_i) + \widehat{T}_3(s, t, m_i), \qquad (2.1)$$

in terms of three individually gauge-invariant quantities: a propagator-like part $(\widehat{T}_1)$, a vertex-like piece $(\widehat{T}_2)$, and a part containing box graphs $(\widehat{T}_3)$. The important observation is that vertex and box graphs contain in general pieces, which are kinematically akin to self-energy graphs of the transition amplitude (Fig.1) The PT is a systematic way of extracting such pieces and appending them to the conventional self-energy graphs. In the same way, effective gauge invariant vertices may be constructed, if after subtracting from the conventional vertices the propagator-like pinch parts we add the vertex-like pieces coming from boxes. The remaining purely box-like contributions are then also gauge invariant. The way to identify the pieces which are to be reassigned, all one has to do is to resort to the fundamental PT cancellation, which is in turn a direct consequence of the elementary Ward identities of the theory.

There are two basic ingredients in the PT construction. The first is the identification of all longitudinal momenta involved, i.e. the momenta which can trigger the elementary Ward identities. There are two sources of such momenta: The tree-level expressions for the gauge boson propagators appearing inside Feynman diagrams and the tri-linear gauge boson vertices. For example, in the case of QCD, the tree-level gluon propagator reads

$$i\Delta_{\mu\nu}(q) = \frac{-i}{q^2}\left(g_{\mu\nu} - (1-\xi)\frac{q_\mu q_\nu}{q^2}\right), \qquad (2.2)$$

and the longitudinal momenta are simply those multiplying $(1-\xi)$. The identification of the longitudinal momenta stemming from the three-gluon vertex is slightly more subtle. To see how they emerge, one must split the Bose-symmetric three-gluon vertex in the following Bose-asymmetric way:

$$
\begin{aligned}
\Gamma_{\lambda\mu\nu}(q, -k_1, -k_2) &= (k_1 - k_2)_\lambda g_{\mu\nu} + (q + k_2)_\mu g_{\lambda\nu} - (q + k_1)_\nu g_{\lambda\nu} \\
&= [(k_1 - k_2)_\lambda g_{\mu\nu} + 2q_\mu g_{\lambda\nu} - 2q_\nu g_{\lambda\mu}] + (k_{2\nu} g_{\lambda\mu} - k_{1\mu} g_{\lambda\nu}) \\
&= \Gamma^F_{\lambda\mu\nu}(q, -k_1, -k_2) + \Gamma^P_{\lambda\mu\nu}(q, -k_1, -k_2). \qquad (2.3)
\end{aligned}
$$

The $\Gamma^P_{\lambda\mu\nu}(q, -k_1, -k_2)$ contains the aforementioned longitudinal momenta. The vertex $\Gamma^F_{\lambda\mu\nu}(q, -k_1, -k_2)$ is Bose-symmetric only with respect to the μ and ν legs. The first term in $\Gamma^F_{\lambda\mu\nu}(q, -k_1, -k_2)$ is a convective vertex

describing the coupling of a gluon to a scalar field, whereas the other two terms originate from gluon spin or magnetic moment. Evidently the above decomposition assigns a special rôle to the q-leg, and allows $\Gamma^F_{\lambda\mu\nu}(q, -k_1, -k_2)$ to satisfy the elementary Ward identity

$$q^\mu \Gamma^F_{\mu\alpha\beta}(q, -k_1, -k_2) = (k_1^2 - k_2^2)g_{\alpha\beta}. \tag{2.4}$$

The second PT ingredient is the following: One has to use all longitudinal momenta identified above in order to trigger a fundamental, BRS-driven cancellation involving graphs of different kinematic dependence. In particular, let us consider the amplitude $q\bar{q} \to gg$, to be denoted by $\mathcal{T} = \langle q\bar{q}|T|gg\rangle$. Diagrammatically, the amplitude $\mathcal{T}$ consists of two distinct parts: t and u-channel graphs that contain an internal quark propagator, $\mathcal{T}^{ab}_{t\mu\nu}$, as shown in Figs. 4(d), and an s-channel amplitude, $\mathcal{T}^{ab}_{s\mu\nu}$, which is given in Fig. 4(a). The subscript "s" and "t" refers to the corresponding Mandelstam variables, i.e. $s = q^2 = (p_1 + p_2)^2 = (k_1 + k_2)^2$, and $t = (p_1 - k_1)^2 = (p_2 - k_2)^2$. Specifically,

$$\mathcal{T}^{ab}_{\mu\nu} = \mathcal{T}^{ab}_{s\mu\nu} + \mathcal{T}^{ab}_{t\mu\nu}, \tag{2.5}$$

with

$$\mathcal{T}^{ab}_{s\mu\nu} = -g^2 \bar{v}(p_2) \frac{\lambda^c}{2} \gamma_\rho u(p_1) f^{abc} \left(\frac{1}{q^2}\right) \Gamma_{\rho\mu\nu}(q, -k_1, -k_2),$$

$$\mathcal{T}^{ab}_{t\mu\nu} = -g^2 \bar{v}(p_2)\left(\frac{\lambda^b}{2}\gamma^\nu S(\not{p}_1 - \not{k}_1)\frac{\lambda^a}{2}\gamma^\mu + \frac{\lambda^a}{2}\gamma^\mu S(\not{p}_1 - \not{k}_2)\gamma^\nu \frac{\lambda^b}{2}\right)u(p_1) \tag{2.6}$$

where

$$S(p) = \frac{i}{\not{p} - m} \tag{2.7}$$

is the quark propagator, and λ are the Gell-Mann matrices. It is elementary to verify that the action of the longitudinal momenta k_1^μ or k_2^ν leads to a non-trivial cancellation between the $\mathcal{T}_s$ and $\mathcal{T}_t$ amplitudes, as shown in Fig.4, which is a direct consequence of the BRS symmetry. Its one-loop implementation necessitates only the use of the following basic Ward identities

$$k_1^\mu \Gamma_{\rho\mu\nu}(q, -k_1, -k_2) = (q^2 g_{\rho\nu} - q_\rho q_\nu) - (k_2^2 g_{\rho\nu} - k_{2\rho}k_{2\nu}),$$

$$k_1^\mu \gamma_\mu = -i\left(S^{-1}(k_1 + p) - S^{-1}(p)\right), \tag{2.8}$$

triggered by the action of k_1^μ (or k_2^ν) on $\mathcal{T}^{ab}_{s\mu\nu}$ and $\mathcal{T}^{ab}_{t\mu\nu}$, respectively.

After carrying out the above s-cannel – t-channel cancellation, one is left with a set of "pure" propagator-like contributions which define the effective PT vacuum polarization of the gluon, denoted by $\widehat{\Pi}_{\mu\nu}(q)$, given by [4] (Fig.2)

$$\widehat{\Pi}_{\mu\nu}(q) = \frac{g^2}{16\pi^2} \frac{11c_A}{3} (q^2 g_{\mu\nu} q^2 - q_\mu q_\nu) \left[\ln\left(-\frac{q^2}{\mu^2}\right) + C_{UV} \right]. \qquad (2.9)$$

Here, $C_{UV} = 1/\epsilon - \gamma_E + \ln 4\pi + C$, where C is a GFP-independent constant and μ is the subtraction point. Notice that, as happens in QED, $\widehat{\Pi}_{\mu\nu}(q)$ captures the one-loop leading logarithmic corrections, i.e. the coefficient $\frac{11c_A}{3}$ multiplying the logarithm coincides with the coefficient of the one-loop β function of quark-less QCD.

Similarly one may define the GFP-independent one-loop quark-gluon vertex $\widehat{\Gamma}_\alpha^{(1)}(Q,Q')$ (Fig.3). In addition to being GFP-independent, by virtue of Eq. (2.4) $\widehat{\Gamma}_\alpha^{(1)}(Q,Q')$ satisfies the following QED-like Ward-identity

$$q^\alpha \widehat{\Gamma}_\alpha^{(1)}(Q,Q') = \widehat{\Sigma}^{(1)}(Q) - \widehat{\Sigma}^{(1)}(Q'), \qquad (2.10)$$

where $\widehat{\Sigma}^{(1)}$ is the PT one-loop quark self-energy, which coincides with the conventional one computed in the Feynman gauge.

The construction presented above goes through without major conceptual modifications (but with minor operational adjustments) in the context of non-Abelian gauge theories, such as the electroweak part of the Standard Model, where the gauge fields have been endowed with masses through the usual Higgs mechanism.

3   Conclusions

We have seen that the Breit-Wigner resummation formalism can be self-consistently extended to the case of non-Abelian gauge theories, provided that one resorts to the pinch technique rearrangement of the physical amplitude. To accomplish this one needs invoke only the full exploitation of the elementary Ward-identities of the theory, in conjunction with unitarity, analyticity, and renormalization group invariance.

From the phenomenological point of view the above framework enables the construction of Born-improved amplitudes in which all relevant physical information has been correctly encoded. This in turn will be useful for the detailed study of the physical properties of particles, most importantly the correct extraction of their masses, widths, and line shapes.

The formalism described in this paper has been recently extended at the two-loop level [10], leading to the exact replication of all the desirable properties listed above (items [(a)]–[(f)]).

Acknowledgements

FIGURE CAPTIONS

Fig.1: The basic pinch technique rearrangement of the one-loop vertex (a) into a purely vertex-like piece (b) and a propagator-like piece (c).

Fig.2: The effective pinch technique gluon self-energy.

Fig.3: The effective pinch technique one-loop gluon-quark vertex.

Fig.4: The fundamental BRS-driven s-channel – t-channel cancellation.

References

[1] A. Sirlin, Phys. Rev. Lett. **67**, 2127 (1991); Phys. Lett. **B267**, 240 (1991). K. Philippides and A. Sirlin; Phys. Lett. B367, 377 (1996); K. Philippides and A. Sirlin; Nucl. Phys. B477 59, (1996); M. Passera and A. Sirlin, Phys. Rev. **D58** (1998) 113010; B. A. Kniehl and A. Sirlin Phys. Rev. Lett. **81**, 1373 (1998).

[2] J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **75**, 3060 (1995); Phys. Rev. **D53**, 2128 (1996); Phys. Rev. **D54**, 5315 (1996);

[3] J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **80**, 2785 (1998); Phys. Rev. **58**, (1998) 53002 .

[4] J.M. Cornwall, Phys. Rev. **D26**, 1453 (1982).

[5] J.M. Cornwall and J. Papavassiliou, Phys. Rev. **D40**, 3474 (1989); J. Papavassiliou, Phys. Rev. **D 41**, 3179 (1990); G. Degrassi and A. Sirlin, Phys. Rev. **D 46**, 3104 (1992).

[6] J.M. Cornwall, D.N. Levin, and G. Tiktopoulos, Phys. Rev. **D10**, 1145 (1974); **E11**, 972 (1975); C.E. Vayonakis, Lett. Nuovo Cim. **17**, 383 (1976); M.S. Chanowitz and M.K. Gaillard, Nucl. Phys. **B261**, 379 (1985).

[7] See for example L.F. Abbott, Nucl. Phys. **B185**, 189 (1981), and references therein.

[8] A. Denner, S. Dittmaier, and G. Weiglein, Phys. Lett. **B333**, 420 (1994); S. Hashimoto, J. Kodaira, Y. Yasui, and K. Sasaki, Phys. Rev. **D50**, 7066 (1994);

[9] C. Becchi, A. Rouet, and R. Stora, Ann. Phys. (NY) **98**, 287 (1976).

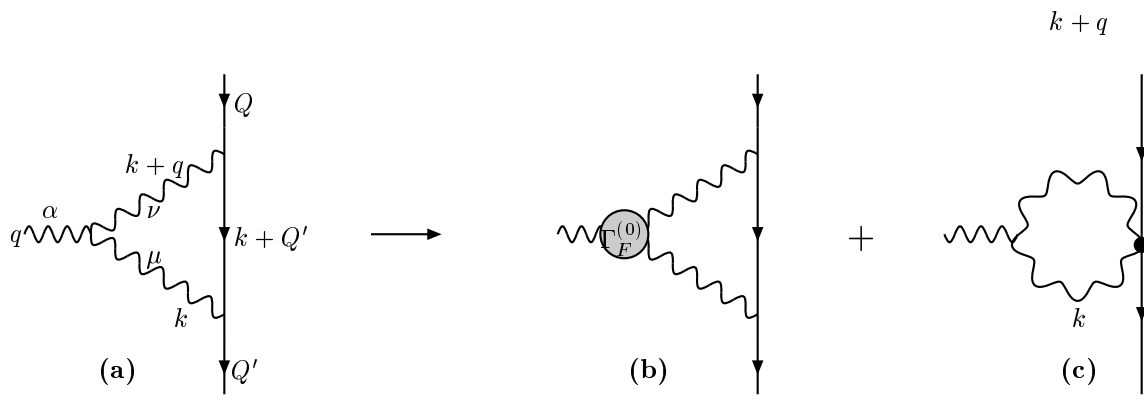[10] J. Papavassiliou, Phys. Rev. Lett. **84**, 2782 (2000); Phys. Rev. **D62**:045006,2000.
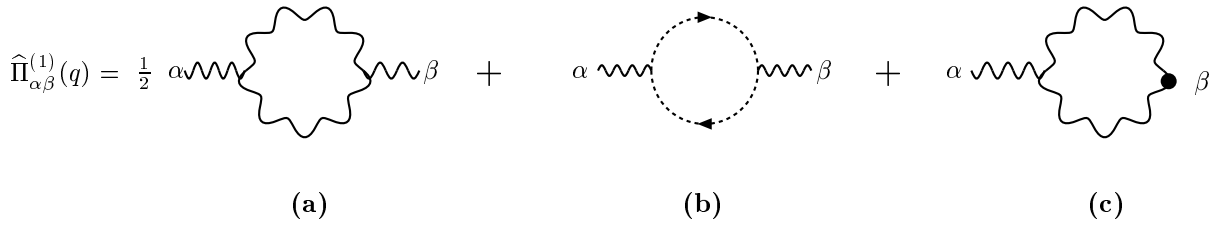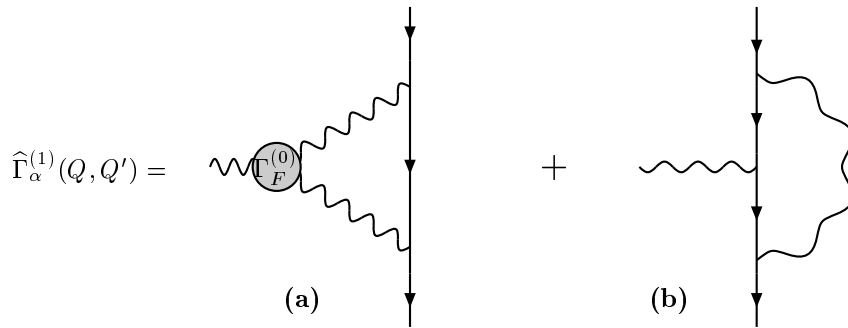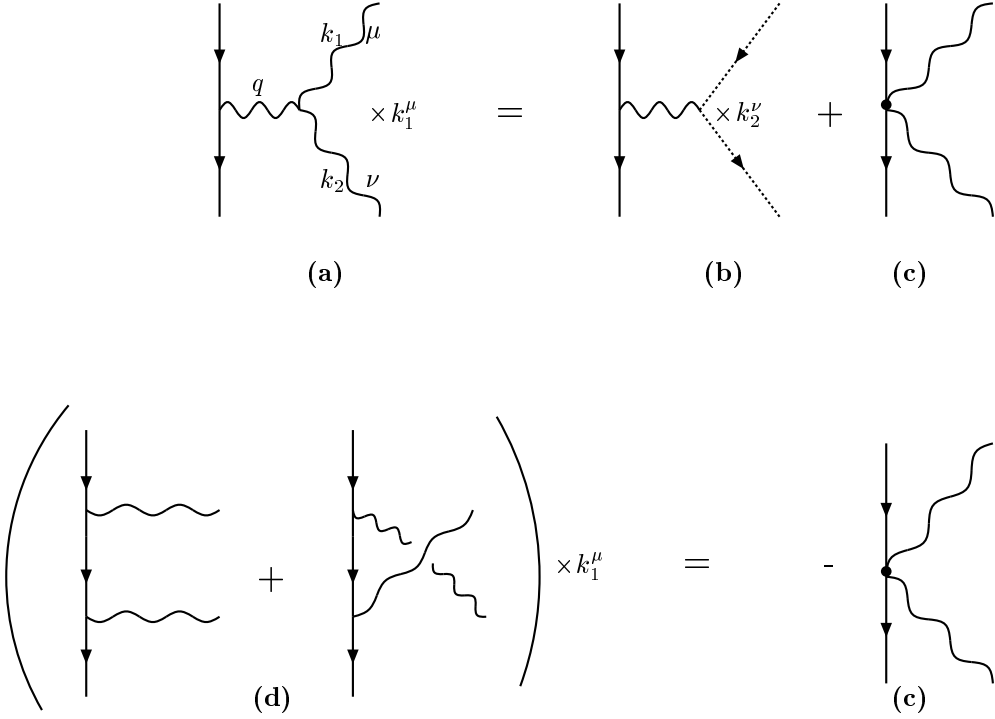
Fig. 1

Fig. 2

Fig. 3

$$\times k_1^\mu \qquad = \qquad \times k_2^\nu \qquad +$$

(a)   (b)   (c)

$$\left(\qquad + \qquad \right) \times k_1^\mu \qquad = \qquad -$$

(d)   (c)

Fig. 4