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Universal extra dimensions and $Z \to b\bar{b}$

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Abstract

We study, at the one loop level, the dominant contributions from a single universal extra dimension to the process $Z \to b\bar{b}$. By resorting to the gaugeless limit of the theory we explain why the result is expected to display a strong dependence on the mass of the top-quark, not identified in the early literature. A detailed calculation corroborates this expectation, giving rise to a lower bound for the compactification scale which is comparable to that obtained from the ρ parameter. An estimate of the subleading corrections is furnished, together with a qualitative discussion on the difference between the present results and those derived previously for the non-universal case.

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I. INTRODUCTION

Models with large extra dimensions [1, 2, 3, 4] have been extensively studied in recent years, and have served as a major source of inspiration in the ongoing search of physics beyond the Standard Model (SM). The general idea behind these scenarios is that the ordinary four dimensional SM emerges as the low energy effective theory of more fundamental models living in five or more dimensions with the extra dimensions compactified. The effects of the extra dimensions are communicated to the four dimensional world through the presence of infinite towers of Kaluza-Klein (KK) modes, which modify qualitatively the behavior of the low-energy theory. In particular, the non-renormalizability of the theory is found when summing the infinite tower of KK states. The size of the extra dimensions can be surprisingly large without contradicting present experimental data (see for instance [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]). This offers the exciting possibility of testing these models in the near future, since the lowest KK states, if light enough, could be produced in the next generation of accelerators.

Extra dimensions may or may not be accessible to all known fields, depending on the specifics of the underlying, more fundamental theory. Scenarios where all SM fields live in higher dimensions have been the focal point of particular attention [12, 16]). This type of extra dimensions is refered to in the literature as "universal extra dimensions" (UED). From the phenomenological point of view, the most characteristic feature of such theories is the conservation of the KK number at each elementary interaction vertex [12, 16]. As a result, and contrary to what happens in the non-universal case, the coupling of any excited (massive) KK mode to two zero modes is prohibited. This fact alters profoundly their production mechanisms: using normal (zero-mode) particles as initial sates, such modes cannot be resonantly produced, nor can a single KK mode appear in the final states, but instead they must be pair-produced. In addition, the conservation of the KK number leads to the appearance of heavy stable (charged and neutral) particles, which seem to pose cosmological complications (e.g. nucleosynthesis) [16]; however, one-loop effects may overcome such problems [17]. Finally, this conservation yields the additional important feature that, the constraints on the size of the extra dimensions which are obtained from SM precision measurements are less stringent; this is so because the extra modes do not affect the tree-level predictions, and make their presence felt only through loop corrections. This last point merits particular attention, given its phenomenological importance, together with the fact that loop calculations in the context of such theories constitute a relatively unexplored territory.

In general the precision electroweak observables most sensitive to radiative corrections, whether from within the SM or from its extensions, are those enhanced by the large top-quark mass : R_b, or equivalently, the process $Z \to b\bar{b}$ [18, 19, 20, 21], the $B-\bar{B}$ -mixing [22], and the ρ parameter. These observables have already been considered in models with extra dimensions. Thus, R_b was considered, for instance, in [16, 23, 24], $B-\bar{B}$ was considered first in [23] and, recently, it has been studied in the context of UED in [25, 26], In the case of theories with UED the study of the corrections to the ρ parameter has yielded a lower bound on the size of the compactification scale, the inverse of the compactification radious, R, of about 300 GeV [16]. In this paper we will study in detail the bound obtained on the size of a single UED from the process $Z \to b\bar{b}$. Our experience with the radiative corrections induced by the SM particles suggests that the bounds obtained from this process could in principle be comparable to those extracted from the ρ parameter; the reason is that the aforementioned enhancement induced by the dependence on the mass of the top-quark takes place in both cases. A simple one-loop calculation, motivated by the behavior of the theory in its gaugeless limit and subsequently corroborated by a more detailed analysis, reveals that, contrary to what has been claimed in [16], the leading corrections to the left-handed $Zb\bar{b}$ coupling, g_L, due to the KK modes corresponding to a single UED, display a strong dependence on the mass of the top-quark (they are proportional to m_t^4). This fact makes the bounds obtained from R_b comparable to those obtained from the ρ parameter; in particular we find $R^{-1} > 300$ GeV at 95% CL.

The paper is organized as follows: In section II we start from the five-dimensional Lagrangian and after standard manipulations we derive the corresponding four-dimensional interactions and mass spectrum, paying particular attention to the interactions involving top and bottom quarks. In section III we first discuss the physical arguments which suggest a strong dependence of the result on the top-quark mass; then we present a more detailed one-loop calculation which demonstrates precisely the announced leading behavior. The subleading corrections, e.g. terms suppressed by an additional factor $\mathcal{O}(M_W^2/m_t^2)$ are also estimated. Finally, in section IV we present our conclusions.

II. THE LAGRANGIAN

We will concentrate on the electroweak part, $SU(2)_L \times U(1)_Y$, of the SM Lagrangian, written in five dimensions; we will denote by x the four normal coordinates, and by $y \equiv x^4$ the fifth one, which will undergo compactification.

The Lagrangian $\mathcal{L}$ assumes the form

$$\mathcal{L} = \int_0^{\pi R} dy (\mathcal{L}_A + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y) \,, \tag{2.1}$$

where

$$\begin{aligned} \mathcal{L}_A &= -\frac{1}{4} F^{MNa} F^a_{MN} - \frac{1}{4} F^{MN} F_{MN} \;, \\ \mathcal{L}_H &= (D_M \Phi)^\dagger D^M \Phi - V(\Phi) \;, \\ \mathcal{L}_F &= \overline{Q}(i\Gamma^M D_M) Q + \overline{U}(i\Gamma^M D_M) U + \overline{D}(i\Gamma^M D_M) D \;, \\ \mathcal{L}_Y &= -\overline{Q}\widetilde{Y}_u \Phi^c U - \overline{Q}\widetilde{Y}_d \Phi D + \text{h.c.} \end{aligned} \tag{2.2}$$

In the above formulas $M, N = 0, 1, 2, 3, 4$ are the five-dimensional Lorentz indices, $F^a_{MN} = \partial_M W^a_N - \partial_N W^a_M + g\epsilon^{abc} W^b_M W^c_N$ is the field strength associated with the $SU(2)_L$ gauge group, and $F_{MN} = \partial_M B_N - \partial_N B_M$ that of the $U(1)_Y$ group. The covariant derivative is defined as $D_M \equiv \partial_M - i\widetilde{g} W^a_M T^a - i\widetilde{g}' B_M Y$, where $\widetilde{g}$ and $\widetilde{g}'$ are the five-dimensional gauge coupling constants of $SU(2)_L$ and $U(1)_Y$, respectively, and T^a and Y are the corresponding generators. Γ_M denote the five dimensional gamma matrices, $\Gamma_\mu = \gamma_\mu$ and $\Gamma_4 = i\gamma_5$, and the metric convention is $g_{MN} = (+, -, -, -, -)$. The fermionic fields Q, D and U are four-component spinors and carry the same quantum numbers as the corresponding SM fields. $SU(2)$ and color indices have been suppressed. Finally, Φ and $\Phi^c = i\tau^2 \Phi^*$ denote the standard Higgs doublet and its charge conjugated field, and $\widetilde{Y}_u$ are the Yukawa matrices in the five dimensional theory; they mix different generations, whose indices are suppressed. We do not include lepton or gluon couplings because they are not relevant for our discussion.

Next, as usual, we assume that the fifth dimension is compactified on a circle of radius R in which the points y and $-y$ are identified (e.g. an orbifold $S^1/\mathbb{Z}_2$). Fields even under the $\mathbb{Z}_2$ symmetry will have zero modes and will be present in the low energy theory. Fields odd under $\mathbb{Z}_2$ will only have KK modes and will disappear from the low energy spectrum. One chooses the Higgs doublet to be even under the $\mathbb{Z}_2$ symmetry in order to have a standard zero mode Higgs field. Then we carry out the Fourier expansion of the fields,

$$\begin{aligned}
A_\mu(x,y) &= \frac{1}{\sqrt{\pi R}} A_\mu^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \cos\left(\frac{ny}{R}\right) \ , \\
A_5(x,y) &= \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} A_5^{(n)}(x) \sin\left(\frac{ny}{R}\right) \ , \\
Q(x,y) &= \frac{1}{\sqrt{\pi R}} Q_L^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[Q_L^{(n)}(x) \cos\left(\frac{ny}{R}\right) + Q_R^{(n)}(x) \sin\left(\frac{ny}{R}\right)\right] \ , \\
U(x,y) &= \frac{1}{\sqrt{\pi R}} U_R^{(0)}(x) + \frac{\sqrt{2}}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[U_R^{(n)}(x) \cos\left(\frac{ny}{R}\right) + U_L^{(n)}(x) \sin\left(\frac{ny}{R}\right)\right] \ , \qquad (2.3)
\end{aligned}$$

where the expansion for A_μ applies to any of the gauge fields and (after suppressing the Lorentz index μ) for the Higgs doublet, whereas that of A_5 applies to the fifth component of the gauge fields. Similarly, the expansion for U is valid also for D. The above expansions allow us to carry out the standard y integration in Eq. (2.1), and obtain the KK spectrum and the relevant interaction terms. We will mainly be interested in third generation quarks, thus, $Q_t^{(n)}$ and $Q_b^{(n)}$ will refer to the upper and lower parts of the doublet Q and the $U^{(n)}$ will be KK modes of right-handed top quarks. In particular, the relation between the mass- and gauge-eigenstates of the KK quarks can be expressed as

$$\begin{bmatrix} Q_t^{(n)} \\ U^{(n)} \end{bmatrix} = \begin{bmatrix} \gamma_5 \cos(\alpha_n^t) & \sin(\alpha_n^t) \\ -\gamma_5 \sin(\alpha_n^t) & \cos(\alpha_n^t) \end{bmatrix} \begin{bmatrix} Q_t'^{(n)} \\ U'^{(n)} \end{bmatrix} \ , \qquad (2.4)$$

and the mixing angle is given by $\tan(2\alpha_n^t) = m_t/m_n$, where $m_n \equiv n/R$. The case of $Q_b^{(n)}$ is similar but since we are neglecting all mass scales except m_t and m_n the mass eigenstate is simply $Q_b'^{(n)} = \gamma_5 Q_b^{(n)}$. The mass spectrum assumes the form (we remove the primes)

$$m_{Q_b^n} = m_n \,, \quad m_{Q_t^n} = m_{U^n} = \sqrt{m_t^2 + m_n^2}\,. \qquad (2.5)$$

The couplings between the quarks and the scalar modes are important to our purposes, because they are proportional to m_t. In contrast to what happens within the SM, they will be physical degrees of freedom, i.e. they cannot be gauged away by choosing, for example, a unitary-type of gauge. After dimensional reduction, the fifth components of the charged gauge fields, $W_5^{-(n)}$, mix with the KK modes of the charged component $\Phi^{-(n)}$ of the Higgs doublet. After diagonalization one obtains a physical boson, $\Phi_P^{-(n)}$, and a Goldstone boson

$\Phi_G^{-(n)}$ that will contribute to the mass of the KK gauge bosons. In particular,

$$\Phi_G^{-(n)} = \frac{m_n W_5^{-(n)} + i M_W \Phi^{-(n)}}{\sqrt{m_n^2 + M_W^2}} \overset{n\neq 0}{\underset{M_W \to 0}{\longrightarrow}} W_5^{-(n)} \;, \tag{2.6a}$$

$$\Phi_P^{-(n)} = \frac{i M_W W_5^{-(n)} + m_n \Phi^{-(n)}}{\sqrt{m_n^2 + M_W^2}} \overset{n\neq 0}{\underset{M_W \to 0}{\longrightarrow}} \Phi^{-(n)} \;. \tag{2.6b}$$

As seen from the expansion, the W_5 has no zero mode, there is no physical zero mode $\Phi_P^{-(0)}$ and the zero mode Goldstone boson comes entirely from the zero mode Higgs field. On the other hand, for $1/R \gg M_W$ the KK Goldstone bosons are mainly the $W_5^{-(n)}$, while the physical scalars are mainly the KK modes of the Higgs doublet $\Phi^{-(n)}$. Their couplings are exactly the same as those of the Goldstone bosons of the SM, e.g.

$$\mathcal{L}_Y = \frac{\sqrt{2}}{v} m_t V_{tj} \overline{U}_R^{(n)} Q_{j\,L}^{(0)} \Phi^{+(n)} + \text{h.c.} \;, \tag{2.7}$$

where we have written only the third quark generation and in the following we will neglect the mixings, $V_{tj} \approx \delta_{tj}$.

The electroweak symmetry breaking proceeds by minimizing a Higgs potential of the standard form, e.g. $V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \widetilde{\lambda}(\Phi^\dagger\Phi)^2$. The mass terms of the different KK scalar modes are given by $m_{\Phi^n}^2 = -\mu^2 + m_n^2$, in such a way that if $\mu < R^{-1}$ only the neutral component of the fundamental mode, $\Phi_0^{(0)}$, gets a vacuum expectation value (VEV), $\langle \Phi_0^{(0)} \rangle = v/\sqrt{2}$. At low energy, when no KK modes can be produced, and at tree-level this model coincides exactly with the SM. In particular, the VEV of the zero mode Higgs doublet induces mixing between $W_{\mu 3}^{(0)}$ and $B_\mu^{(0)}$ giving rise to a massless photon, $A_\mu^{(0)}$, and a massive Z boson, $Z_\mu^{(0)}$.

After a bit of algebra one arrives to the expression of the couplings of the Z boson with the KK modes of the rest of the fields, given by

$$\mathcal{L}_Z = \frac{g}{2c_w} Z_\mu^{(0)} [J^{\mu(0)} + J^{\mu(n)} + J_\Phi^{\mu(n)}] \;, \tag{2.8}$$

where the $J^{\mu(0)}$ is the usual SM neutral current, and

$$\begin{aligned} J^{\mu(n)} &= \left(1 - \frac{4}{3}s_w^2\right) \overline{Q}_t^{(n)} \gamma^\mu Q_t^{(n)} - \left(1 - \frac{2}{3}s_w^2\right) \overline{Q}_b^{(n)} \gamma^\mu Q_b^{(n)} - \left(\frac{4}{3}s_w^2\right) \overline{U}^{(n)} \gamma^\mu U^{(n)} + \dots \;, \\ J_\Phi^{\mu(n)} &= (-1 + 2s_w^2) \Phi^{+(n)} i\partial^\mu \Phi^{-(n)} + \text{h.c.} \;. \end{aligned} \tag{2.9}$$

Here $Q_t^{(n)}$, $Q_b^{(n)}$ and $U^{(n)}$ are Dirac spinors, and the ellipses denote the contribution of $D^{(n)}$ fields, which are not relevant for our calculation. Similarly, the interaction of the charged bosons with the quarks is given by

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}[\bar{b}_L \gamma^\mu W_\mu^{-(n)} Q_{tL}^{(n)} - i\bar{b}_L W_5^{-(n)} Q_{tR}^{(n)} + h.c.] \ . \tag{2.10}$$

The couplings of the photon may be derived similarly. From the above equations it is straightforward to extract the necessary Feynman rules for our calculations. The couplings of the photon may be derived similarly.

III. CALCULATING $Z \to b\bar{b}$

In this section we will compute the corrections to the effective $Zb\bar{b}$ coupling due to the presence of a single UED. Shifts in the $Zb\bar{b}$ coupling due to radiative corrections, either from within the SM or from new physics, affect observables such as the branching ratio $R_b = \Gamma_b/\Gamma_h$, where $\Gamma_b = \Gamma(Z \to b\bar{b})$ and $\Gamma_h = \Gamma(Z \to \text{hadrons})$, or the left right asymmetry A_b. These type of corrections can be treated uniformly by expressing them as a modification to the tree level couplings $g_{L(R)}$ defined as

$$\frac{g}{c_W}\bar{b}\gamma^\mu(g_L P_L + g_R P_R) b Z_\mu \ . \tag{3.1}$$

Z and b's are SM fields, $P_{L(R)}$ are the chirality projectors and

$$g_L = -\frac{1}{2} + \frac{1}{3}s_W^2 + \delta g_L^{\rm SM} + \delta g_L^{\rm NP} \ , \tag{3.2a}$$

$$g_R = \frac{1}{3}s_W^2 + \delta g_R^{\rm SM} + \delta g_R^{\rm NP} \ , \tag{3.2b}$$

where we have separated radiative corrections coming from SM contributions and from new physics, (NP). It turns out that, both within the SM as well as in most of its extensions, only g_L receives corrections proportional to m_t^2 at the one loop level, due to the difference in the couplings between the two chiralities. In particular, a shift δg_L^{NP} in the value of g_L due to new physics translates into a shift in R_b given by

$$\delta R_b = 2R_b(1 - R_b)\frac{g_L}{g_L^2 + g_R^2}\delta g_L^{\rm NP} \ , \tag{3.3}$$

and to a shift in the left-right asymmetry A_b given by

$$\delta A_b = \frac{4g_R^2 g_L}{(g_L^2 + g_R^2)^2}\delta g_L^{\rm NP} \ . \tag{3.4}$$

These equations, when compared with experimental data, will be used to set bounds on the compactification scale.

By far the easiest way to compute the leading top-quark-mass dependent one-loop corrections to δg_L from the SM itself, $\delta g_L^{\rm SM}$, is to resort to the gaugeless limit of the SM [27], e.g. the limit where the gauge couplings g and g', corresponding to the gauge groups $SU(2)_L$ and $U(1)_Y$ respectively, are switched off. In that limit the gauge bosons play the role of external sources and the only propagating fields are the quarks, the Higgs field, and the charged and neutral Goldstone bosons $G^\pm$ and G^0. As explained in [28, 29] one may relate the one-loop vertex $Zb\bar{b}$ to the corresponding $G^0 b\bar{b}$ vertex by means of a Ward identity; the latter is a direct consequence of current conservation, which holds for the neutral current before and after the Higgs doublet acquires a vacuum expectation value v.

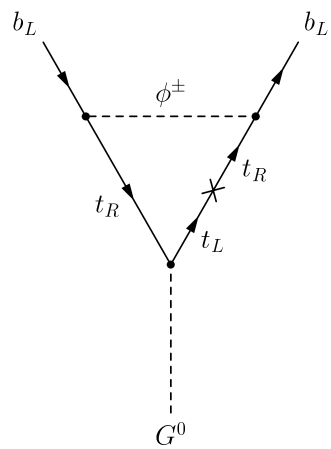

FIG. 1: The only diagram contributing to the SM $G^0 b\bar{b}$ vertex in the gaugeless limit for massless b-quarks.

In practice, carrying out the calculation in the aforementioned limit amounts to the elementary computation of the one-loop off-shell vertex $G^0 b\bar{b}$. In the gaugeless limit and for massless b-quarks the only contribution to this vertex is depicted in Fig. 1, where the cross in the top-quark line represents a top-quark mass insertion needed to flip chirality (an insertion in the other top-quark line is assumed). This diagram gives a derivative coupling of the Goldstone field to the b-quarks which can be gauged (or related to the Z vertex through the Ward identity) to recover the $Zb\bar{b}$ vertex. Then, one immediately finds

$$\delta g_L^{\rm SM} \approx \frac{\sqrt{2} G_F m_t^4}{(2\pi)^4} \int \frac{i d^4 k}{(k^2 - m_t^2)^2 k^2} = \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} \ , \tag{3.5}$$

where G_F is the Fermi constant, and the m_t^4 dependence coming from three Yukawa couplings and one mass insertion is partially compensated by the $1/m_t^2$ dependence coming from the loop integral.

In the case of a single UED this argument persists: one must simply consider the analog of diagram in Fig. 1, where now the particles inside the loop have been replaced by their KK modes, as shown in Fig. 2. If we denote by δg_L^{UED} the new physics contributions in the UED model (the SM contributions are not included) the result is

$$\begin{aligned}\delta g_L^{\rm UED} &\approx \frac{\sqrt{2}G_F m_t^4}{(2\pi)^4}\sum_{n=1}^{\infty}\int\frac{id^4k}{(k^2-m_{Q_t^n}^2)^2(k^2-m_n^2)} \\ &= \frac{\sqrt{2}G_F m_t^4}{(4\pi)^2}\sum_{n=1}^{\infty}\int_0^1\frac{dx\,x}{xm_t^2+m_n^2}\approx\frac{\sqrt{2}G_F m_t^4}{(4\pi)^2}\frac{\pi^2R^2}{12}\ ,\end{aligned}\tag{3.6}$$

and depends *quartically* on the mass of the top quark. Notice that there are several differences with respect to the SM: (i) The cross now represents the mixing mass term between $Q_t^{(n)}$ and $U^{(n)}$, which is proportional to m_t; (ii) The $\Phi^{\pm(n)}$, for $n\neq 0$, are essentially the physical KK modes of the charged Higgses as shown in Eq.(2.6b); (iii) From the virtual momentum integration one obtains now a factor $1/m_n^2$, instead of the factor $1/m_t^2$ of the SM case.

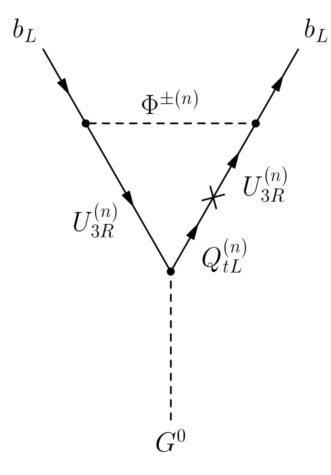

FIG. 2: The dominant diagram contributing to the UED $G^0b\bar{b}$ vertex in the gaugeless limit for massless b-quarks.

This simple calculation allows us to understand easily the leading corrections arising from extra dimensions.

A more standard calculation of the $Zb\bar{b}$ vertex in UED yields exactly the same result. In

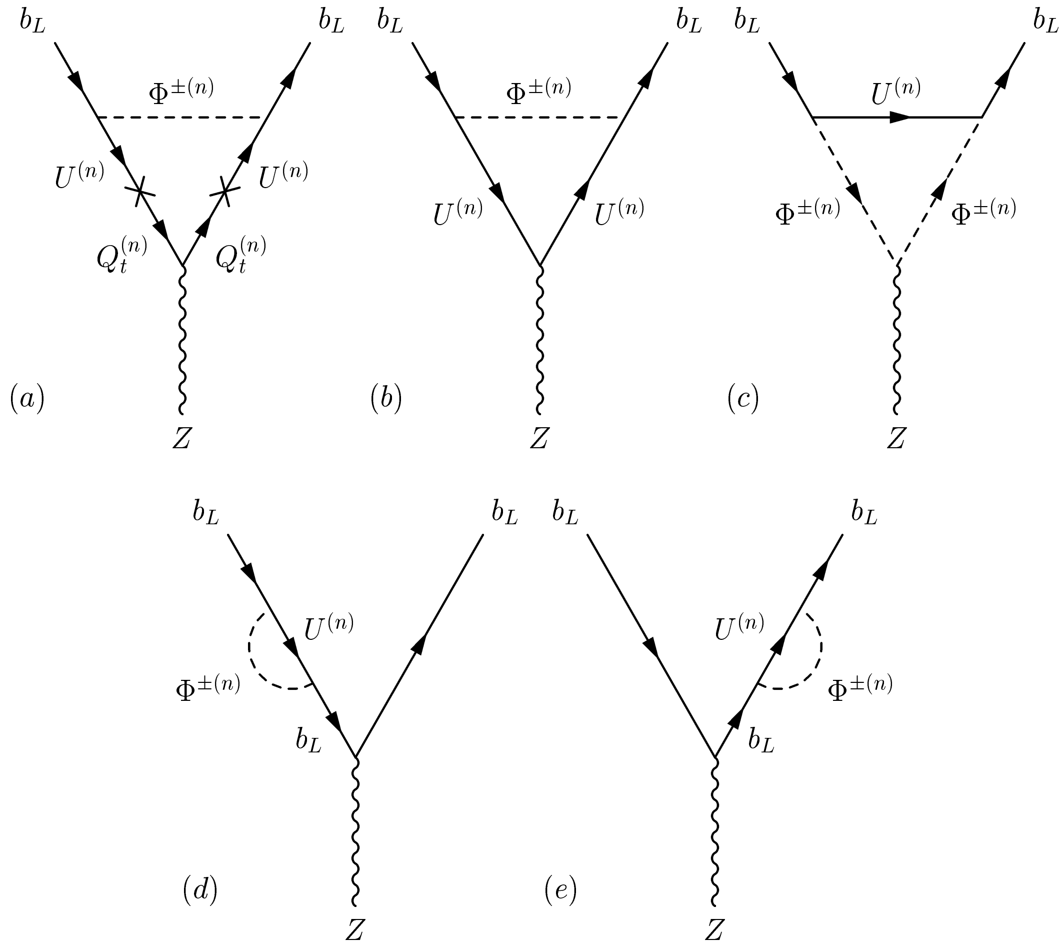

FIG. 3: Dominant UED contributions to the $Zb\bar{b}$ vertex.

this case the radiative corrections to the $Zb\bar{b}$ vertex stem from the diagrams of Fig. 3.

If we neglect the b-quark mass and take $M_Z \ll R^{-1}$, the result, for each mode, can be expressed in terms of a single function, $f(r_n)$, defined as

$$i\mathcal{M}^{(n)} = i\frac{g}{c_w}\frac{\sqrt{2}G_F m_t^2}{(4\pi)^2} f(r_n)\overline{u}'\gamma^\mu P_L u \epsilon_\mu \ , \tag{3.7}$$

where u and u' are the spinors of the b quarks and ϵ_μ stands for the polarization vector of the Z boson. The argument of the function f is $r_n = m_t^2/m_n^2$.

Although the complete result is finite, partial results are divergent and are regularized by using dimensional regularization. The contributions of the different diagrams in Fig. 3 are

$$
\begin{aligned}
f^{(a)}(r_n) &= \left(1-\frac{4}{3}s_w^2\right)\left[\frac{r_n-\log(1+r_n)}{r_n}\right] \,, \\
f^{(b)}(r_n) &= \left(-\frac{2}{3}s_w^2\right)\left[\delta_n - 1 + \frac{2r_n+r_n^2-2(1+r_n^2)\log(1+r_n)}{2r_n^2}\right] \,, \\
f^{(c)}(r_n) &= \left(-\frac{1}{2}+s_w^2\right)\left[\delta_n + \frac{2r_n+3r_n^2-2(1+r_n)^2\log(1+r_n)}{2r_n^2}\right] \,, \\
f^{(d)}(r_n)+f^{(e)}(r_n) &= \left(\frac{1}{2}-\frac{1}{3}s_w^2\right)\left[\delta_n + \frac{2r_n+3r_n^2-2(1+r_n)^2\log(1+r_n)}{2r_n^2}\right] \,, \qquad (3.8)
\end{aligned}
$$

where $\delta_n \equiv 2/\epsilon - \gamma + \log(4\pi) + \log(\mu^2/m_n^2)$, and μ is the 't Hooft mass scale. From Eq. (3.8) it is straightforward to verify that all terms proportional to δ_n cancel, and so do all terms proportional to s_w^2, as expected from the gaugeless limit result. Thus, finally, the only term which survives is the term in $f^{(a)}(r_n)$ not proportional to s_w^2, yielding the following (per mode) contribution to g_L:

$$
\delta g_L^{(n)} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\left[\frac{r_n-\log(1+r_n)}{r_n}\right] \,, \qquad (3.9)
$$

which is precisely the one obtained from the gaugeless limit calculation, e.g. Eq. (3.6) with the elementary integration over the Feynman parameter x already carried out. Notice also that the above result is consistent with the decoupling theorem since the contribution for each mode vanishes when its mass is taken to infinity, e.g. $r_n \to 0$.

In order to compute the effect of the entire KK tower, it is more convenient to first carry out the sum and then evaluate the Feynman parameter integral; this interchange is mathematically legitimate since the final answer is convergent. Thus,

$$
\delta g_L^{\mathrm{UED}} = \sum_{n=1}^{\infty}\delta g_L^{(n)} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}\int_0^1 dx \sum_{n=1}^{\infty}\frac{r_n x}{1+r_n x} = \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2}F_{\mathrm{UED}}(a) \,, \qquad (3.10)
$$

where $a = \pi R m_t$, and

$$
\begin{aligned}
F_{\mathrm{UED}}(a) &= -\frac{1}{2}+\frac{a}{2}\int_0^1 dx\, \sqrt{x}\coth(a\sqrt{x}) \\
&\approx \frac{a^2}{12}-\frac{a^4}{270}+\mathcal{O}(a^6) \,. \qquad (3.11)
\end{aligned}
$$

It is instructive to compare the above result with the one obtained in the context of models where the extra dimension is not universal. In particular, in the model considered in [23] the fermions live in four dimensions, and only the gauge bosons and the Higgs doublet live in five [6]. In this case there is no KK tower for the fermions, and therefore, in the loop-diagrams appear only the SM quarks interacting with the KK tower of the Higgs fields. The result displays a logarithmic dependence on the parameter a, which gives rise to a relatively tight lower bound on R^{-1}, of the order of 1 TeV. Specifically, the corresponding $F(a)$ is given by[1]

$$
\begin{aligned}
F(a) &= -1 + 2a \int_0^{\infty} dx \frac{x^2}{(1+x^2)^2} \coth(ax) \\
&\approx \left(\frac{2}{3} \log(\pi/a) - \frac{1}{3} - \frac{4}{\pi^2} \zeta'(2) \right) a^2 \ ,
\end{aligned} \tag{3.12}
$$

where the expansion on the second line holds for small values of a, and ζ' is the derivative of the Riemann Zeta function. The appearance of the $\log(a)$ in $F(a)$ and its absence from $F_{\rm UED}(a)$ may be easily understood from the effective theory point of view. Due to the KK-number conservation in UED models, the tree-level low energy effective Lagrangian when all KK modes are integrated out is exactly the Standard Model; there are no additional tree-level operators suppressed by the compactification scale. Since one-loop logarithmic contributions, $\log(a)$, can be obtained in the effective theory by computing the running of operators generated at tree level, it is clear that in the UED no $\log(a)$ can appear at one loop in low energy observables. The situation is completely different if higher dimension operators are already generated at tree level, as is the case of the model considered in ref. [23], where the leading logarithmic corrections can be computed by using the tree-level effective operators in loops.

We next turn to the bounds on R^{-1}. We will use the SM prediction $R_b^{\rm SM} = 0.21569 \pm 0.00016$ and the experimentally measured value $R_b^{\rm exp} = 0.21664 \pm 0.00068$. Combining Eq. (3.3) and Eq. (3.10), we obtain $F_{\rm UED}(a) = -0.24 \pm 0.31$, and making a weak signal treatment [30] we arrive at the 95% CL bound $F_{\rm UED}(a) < 0.39$. The results for a single UED can be easily derived from (3.11), yielding

$$
R^{-1} > 230 \text{ GeV} \ . \tag{3.13}
$$

The SM prediction for the left-right asymmetry $A_b^{SM} = 0.9347 \pm 0.0001$ and the measured value $A_b^{exp} = 0.921 \pm 0.020$ gives a looser bound.

[1] Note that, unlike in ref. [23], the $F(a)$ does not include the SM contribution.

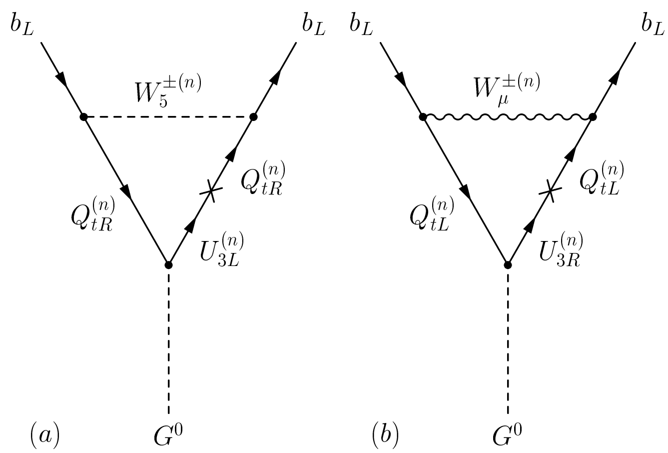

FIG. 4: Diagrams giving subleading contributions to the $G^0 b\bar{b}$ vertex.

Above we have computed only the leading contribution, which goes as $G_F m_t^4 R^2$. There are also formally subleading contributions, suppressed by (at least) an additional factor M_W^2/m_t^2; given that this factor is not so small such corrections could be numerically important, and should be estimated. The dominant contributions of this type come from diagrams with $W_\mu^{\pm(n)}$ and $W_5^{\pm(n)}$ running in the loops. Since these corrections are still proportional to m_t^2 they can be estimated using again the Ward identity that relates the G^0 couplings to the Z couplings. The relevant diagrams are shown in Fig. 4. Their contribution modify the value of $\delta g_L^{\rm UED}$ as follows:

$$\delta g_L^{\rm UED} = \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} F_{\rm UED}(a) \left(1 + 3 \frac{M_W^2}{m_t^2} \right) \ . \tag{3.14}$$

Taking these corrections into account leads to a slight modification of the bound on the compactification scale, $R^{-1} > 300$ GeV. Evidently, this bound is absolutely comparable to the one obtained from the ρ parameter.

IV. CONCLUSIONS

We have computed the leading contributions, for a large top-quark mass, to $Z \to b\bar{b}$ in a model with one universal extra dimension. These contributions depend *quartically* on the top-quark mass and can be evaluated easily in the gaugeless limit of the theory, where only one diagram contributes.

There are subleading corrections, formally suppressed by a factor M_W^2/m_t^2, which, in principle, can be important. We have estimated them by considering the diagrams with the KK modes of the $W^{-(n)}$ and the $W_5^{-(n)}$ running in the loop, and found that they contribute a +65% of the correction.

None of the contributions contains logarithmic ($\log(R)$) corrections. This can be understood from the KK-number conservation, which leads to the absence of tree-level low-energy operators (containing only SM fields). These results have been used to set a bound on the compactification scale $R^{-1} > 300$ GeV at 95% CL which is comparable to the bounds obtained from the contributions of KK modes to the ρ parameter [16] in this model, and which is much weaker than bounds obtained in models with no KK-number conservation [23].

What are the consequences of these results for further studies of UED in b-physics? In [31] it was shown that the vertex $Z \to b\bar{b}$ and $B - \bar{B}$ mixing are highly correlated, and that it is very difficult to obtain a relatively large contribution to $B - \bar{B}$ mixing evading the bounds coming from R_b. This was corroborated explicitly in [23] in a model with only scalars and gauge bosons in extra dimensions. Recently $B - \bar{B}$ mixing has also been considered in UED [25, 26]. Although the simple argument, developed in [31], was based on the logarithmic corrections and it is not applicable in the case of UED because of the absence of logarithmic corrections both in $Z \to b\bar{b}$ and in $B - \bar{B}$ mixing, we believe that some correlation still exists. This will be further explored in a future work.

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