Bounds on models with one latticized extra dimension

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Abstract

We study an extension of the standard model with one latticized extra dimension accessible to all fields. The model is characterized by the size of the extra dimension and the number of sites, and contains a tower of massive particles. At energies lower than the mass of the new particles there are no tree-level effects. Therefore, bounds on the scale of new physics can only be set from one-loop processes. We calculate several observables sensitive to loop-effects, such as the ρ parameter, $b \to s\gamma$, $Z \to b\bar{b}$, and the $B^0 \rightleftharpoons \overline{B}^0$ mixing, and use them to set limits on the lightest new particles for different number of sites. It turns out that the continuous result is rapidly reached when the extra dimension is discretized in about 10 to 20 sites only. For small number of sites the bounds placed on the usual continuous scenario can be reduced by roughly a factor of 10%–25%, which means that the new particles can be as light as 320 GeV. Finally, we briefly discuss an alternative model in which fermions do not have additional modes.

I. INTRODUCTION

The interest on the possible existence of additional spatial dimensions [1, 2, 3] has been renewed in the last years, when it was realized that many long standing problems in particle physics and cosmology could be addressed from an entirely different perspective. The mass spectrum of the fermions [4, 5, 6, 7, 8, 9], novel neutrino oscillation scenarios [10, 11, 12, 13, 14], possible grand unification at low scales [15, 16, 17], as well as new ways to understand the family puzzle [18], are just some examples of the fields were extra dimensions could be relevant.

One common prediction of extra-dimensional scenarios is the existence of a tower of Kaluza-Klein (KK) modes on top of each degree of freedom propagating in the bulk. The towers modify the low energy predictions, a fact that has been exploited to set bounds on the size of the extra dimensions. Many precision observables sensitive to these modifications have been used in the literature for the case of continuous extra dimensions [19, 20, 21, 22, 23, 24, 25, 26, 27]. In fact, the lowest KK states, if sufficiently light, could be produced in the next generation of accelerators. As far as this last possibility is concerned, the models with universal extra dimensions (UED) [20], i.e. with all the Standard Model (SM) fields propagating in the bulk, seem to be particularly promising. In these models, due to the conservation of the KK-number (momentum-conservation in the extra dimension) the only effects at low energy (i.e. below the threshold of production of new particles) arise at one loop. This allows for substantially lower bounds, compared to other models: the masses of the new particles can be as low as 400 GeV without contradicting present experimental data.

Gauge theories in extra dimensions are not renormalizable and suffer from the standard ambiguities when one tries to use them beyond their range of applicability [17]. This has motivated the search of theories that are better behaved at high energies, and reduce to extra-dimensional models at low energy (or simulate sufficiently their spectrum and interactions). Among the possible candidates we will concentrate on the so-called "deconstructed extra dimensions"[28] and "latticized extra dimensions"[29, 30]. The former constitute UV completions of higher dimensional field theories, with gravity decoupled: at very high energies one starts with particularly constructed four-dimensional theories, which are renormalizable, and in most cases even asymptotically free. Then an extra (latticized) dimension is gener-

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ated dynamically at low energies, through the condensation of fermions, which transform appropriately under the various gauge groups [31]. In the context of these theories a new way for solving the hierarchy problem has been pointed out [31, 32]; the Higgs is understood as a pseudo-Goldstone boson associated to a symmetry that has an extra-dimensional analogue. The most economical form of these models, known under the name of "little Higgs" models, are currently the object of an intense study, mainly in order to determine to which extent the cancellation of quadratic divergences can be achieved without fine-tunning [33]. On the other hand, the latticized theories [29, 30] focus on a manifestly gauge-invariant effective Lagrangian description of the KK modes in 3+1 dimensions. Such a description is particularly useful when dealing with non-abelian gauge theories, because it evades complications with gauge-invariance, arising when hard momentum cutoffs are used in loop expansions, which is essentially what the usual truncation of the KK-tower amounts to. The common feature of both types of theories, at least for our purposes, is that they mimic to some extent an extra-dimensional behavior and share an interpretation in terms of a discretized extra dimension.

An important phenomenological difference between continuous and latticized scenarios is the structure of the KK tower. Specifically, whereas in the continuous scenarios the KK towers are infinite, in the latticized versions they are finite, due to the presence of a minimum physical distance, namely the distance between sites in the extra dimensions[1]. As commented above, in the case of universal extra dimensions the bounds on the masses of the lowest KK-particles is rather low, a fact which offers the challenging possibility of (pair)-producing them in upcoming experiments. Therefore, it is important to study how this picture changes when the extra dimensions are latticized. In this paper we perform an analysis, similar to that of the continuous cases, to determine the modifications to the bounds on the masses of new particles when one universal extra dimension is latticized.

As the continuum theory, the latticized version of the universal extra dimensional model has no new tree-level effects at low energy, and bounds can only be set from loop processes. We will try to place limits on the new physics scale by studying several well measured quantities that in the standard model depend strongly on the top-quark mass[2]: the ρ parameter,

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[1] In latticized scenarios the replicated degrees of freedom are not usually called KK modes, nevertheless we will call them simply *"modes"* since this stresses the similarity between latticized and continuous cases.

[2] It is natural to look for dependences on the top-quark mass because those are less suppressed. In fact, if

$b \to s\gamma$, $Z \to b\bar{b}$ and the $B^0 \rightleftharpoons \overline{B}^0$ mixing. We focus on the dominant pieces of the radiative corrections for a large top-quark mass.

For comparison we discuss briefly some results for an alternative scenario in which fermions do not propagate in the latticized extra dimension. In this model the effects of new particles appear already at tree level. In particular, they modify the Fermi constant which allow for much stronger limits on the masses of new particles. Some one-loop processes also provide interesting bounds but they are not competitive with the tree-level bounds.

The paper is organized as follows: in Sec. II we describe a full latticized version of the standard model focusing on the relevant pieces for our calculations, the electroweak sector in general and top-quark couplings in particular. In Sec. III the contributions of the new physics are computed for the different observables and the bounds one can set on the mass of new physics particles are discussed. In Sec. IV we summarize the main results of our study. Finally, in Appendix A we collect some details of the derivation of the spectrum of the model.

II. THE MODEL

In this section we will specify the field content of the model and its Lagrangian, and extract the mass spectrum and couplings necessary for computing the relevant observables.

Following [29, 30], the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_H + \mathcal{L}_Y \,, \tag{2.1}$$

where the various pieces are defined as follows. The gauge piece, $\mathcal{L}_G$, associated to the gauge group[3] $G = \Pi_{i=0}^{N-1} SU(2)_i \times U(1)_{iY}$, reads

$$
\begin{aligned}
\mathcal{L}_G &= \sum_{i=0}^{N-1} -\frac{1}{4} F_{i\mu\nu}^a F^{i\mu\nu a} - \frac{1}{4} F_{i\mu\nu} F^{i\mu\nu} \\
&+ \sum_{i=1}^{N-1} \mathrm{Tr}\{(D_\mu \Phi_i)^\dagger (D^\mu \Phi_i)\} + (D_\mu \phi_i)^\dagger (D^\mu \phi_i) - V(\Phi, \phi) \,,
\end{aligned} \tag{2.2}
$$

---

the new physics decouple, as it is the case, the scale of new physics should appear in the denominator of physical observables. This scale must be compensated by another mass and the largest mass available in the SM is the top-quark mass

[3] We will not consider the contributions associated with strong interactions.

where $F_{i\mu\nu}^a$ is the strength tensor associated with the gauge field of the i-th $SU(2)_i$ and $F_{i\mu\nu}$ is the one for $U(1)_{iY}$. Φ_i and ϕ_i are elementary scalars that will acquire a VEV (common to every i), due to the potential term $V(\Phi, \phi)$. Each of them become effectively nonlinear σ model fields that can be parametrized as usual in terms of the scalar fields π_i and π_i^a

$$\phi_i = \frac{v_1}{\sqrt{2}}e^{i\pi_i/v_1} \qquad \Phi_i = v_2 e^{i\pi_i^a \tau^a/2v_2}, \qquad (2.3)$$

where v_1 and v_2 are the VEVs of ϕ_i and Φ_i respectively and τ^a are the usual Pauli matrices. In this paper we will concentrate on the so called "*aliphatic model*" [29], in which the Φ_i fields transform as $(2_i, \bar{2}_{i-1})$ under the groups $SU(2)_i$ and $SU(2)_{i-1}$, as singlets for the rest, and carry no $U_{iY}(1)$ charge, while the ϕ_i fields are singlets under all the $SU(2)$ groups and they are charged only under $U(1)_{iY}$ and $U(1)_{(i-1)Y}$ (all hypercharges Y_i will be eventually set to $Y_i = 1/3$ [30]). Thus, the covariant derivative assumes the form

$$D_\mu \Phi_i = \partial_\mu \Phi_i - i\mathcal{W}_{\mu,i}\Phi_i + i\Phi_i \mathcal{W}_{\mu,i-1}, \qquad (2.4)$$

where $\mathcal{W}_{\mu,i} = \tilde{g}W_{\mu\,i}^a T_i^a$, T_i^a are the generators of the $SU_i(2)$ and $\tilde{g}$ is the dimensionless gauge coupling constant that is assumed to be the same for all the $SU(2)$ groups. The $U(1)$ covariant derivative for the ϕ_i can be constructed similarly.

The fermionic piece, $\mathcal{L}_F$, contains the following fields (generational indices are suppressed)

$$Q_i = \begin{bmatrix} Q_{ui} \\ Q_{di} \end{bmatrix}, \qquad U_i, \qquad D_i, \qquad i = 0, \dots, N-1 \qquad (2.5)$$

Q_i transforms as a doublet under $SU(2)_i$ and as a singlet for the rest of $SU(2)$ groups, and among the $U(1)$ fields it is only charged under $U(1)_i$, with hypercharge $Y_Q = 1/3$. U_i and D_i are only charged under $U(1)_i$, with hypercharges $Y_U = 4/3$ and $Y_D = -2/3$. They are all vector-like fields with right- and left-handed chiral components except for $i = 0$. In this case Q is left-handed and U and D are right-handed, which is equivalent to imposing $Q_{0R} = 0$, $U_{0L} = 0$, and $D_{0L} = 0$. Then one sets $\mathcal{L}_F = \mathcal{L}_Q + \mathcal{L}_U + \mathcal{L}_D$, where

$$\mathcal{L}_Q = \sum_{i=0}^{N-1} \overline{Q}_{iL} i \slashed{D} Q_{iL} + \overline{Q}_{iR} i \slashed{D} Q_{iR} - M_f \overline{Q}_{iL}\left(\frac{\sqrt{2}\Phi_{i+1}^\dagger \phi_{i+1}^\dagger}{v_1 v_2}Q_{i+1R} - Q_{iR}\right) + \text{h.c.},$$

$$\mathcal{L}_U = \sum_{i=0}^{N-1} \overline{U}_{iR} i \slashed{D} U_{iR} + \overline{U}_{iL} i \slashed{D} U_{iL} + M_f \overline{U}_{iR}\left(\frac{\phi_{i+1}^{4\dagger}}{(v_1/\sqrt{2})^4}U_{i+1L} - U_{iL}\right) + \text{h.c.}, \qquad (2.6)$$

and $\mathcal{L}_D$ can be extracted from $\mathcal{L}_U$ making the next substitutions, $U \to D$, $\phi_i \to \phi_i^\dagger$ and the exponent should be replaced $4 \to 2$. $\not{D}$ is the usual covariant derivative associated with the gauge group G, and M_f is a generic mass that in principle could depend on i but usually it is set equal for all of them.

The next piece in the Lagrangian, $\mathcal{L}_H$, is the one associated with the Higgs doublet [30]

$$\mathcal{L}_H = \sum_{i=0}^{N-1} (D_\mu H_i)^\dagger (D^\mu H_i) - M_0^2 \left| H_{i+1} - \left(\frac{\Phi_{i+1}\phi_{i+1}^3}{(v_1/\sqrt{2})^3 v_2} \right) H_i \right|^2 - V(H_i) , \qquad (2.7)$$

where H_i is a doublet under $SU(2)_i$ and singlet for $SU(2)_{j\neq i}$ with hypercharges $Y_i = 1$ and $Y_{j\neq i} = 0$. Following [25] we parametrize its components as

$$H_i = \begin{bmatrix} i\chi_i^+ \\ \frac{1}{\sqrt{2}}(\psi_i - i\chi_i^3) \end{bmatrix} \qquad i = 0, 1, \ldots, N-1 , \qquad (2.8)$$

and the form of the potential it is chosen $V(H_i) = -m^2 H_i^\dagger H_i + \frac{\tilde{\lambda}}{2}(H_i^\dagger H_i)^2$.

The Yukawa sector, $\mathcal{L}_Y$, will be taken with the Yukawa matrices independent of i

$$\mathcal{L}_Y = \sum_{i=0}^{N-1} \overline{Q_i} \widetilde{Y}_u H_i^c U_i + \sum_{i=0}^{N-1} \overline{Q_i} \widetilde{Y}_d H_i D_i + \text{h.c.} , \qquad (2.9)$$

where $H_i^c \equiv i\tau^2 H_i^*$ is the usual Higgs doublet conjugate.

Finally, we choose to work in an arbitrary R_ξ-covariant gauge, in the spirit of [34]; this means that the π fields will be maintained explicitly in our spectrum.[4]

Extracting the bilinear terms is straightforward but tedious. The mass-eigenstate fields, before spontaneous symmetry breaking, will be denoted by a "tilde" and are related to the gauge-eigenstate ones by

$$Q_{iL} = a_{ij}\widetilde{Q}_{jL}, \quad U_{iR} = a_{ij}\widetilde{U}_{jR}, \quad Q_{iR} = b_{ij}\widetilde{Q}_{jR}, \quad U_{iL} = b_{ij}\widetilde{U}_{jL},$$

$$W_{\mu i}^a = a_{ij}\widetilde{W}_{\mu j}^a, \quad \pi_i^a = b_{ij}\widetilde{W}_{5j}^a, \quad H_i = a_{ij}\widetilde{H}_j,$$

where a_{ij} and b_{ij} are $N \times N$ and $(N-1) \times (N-1)$ matrices, respectively, given by

$$b_{ij} = \sqrt{\frac{2}{N}} \sin\left(ij\frac{\pi}{N}\right), \qquad a_{ij} = \begin{cases} j = 0 & \sqrt{1/N} \\ j \neq 0 & \sqrt{2/N}\cos\left(\frac{2i+1}{2}\frac{j\pi}{N}\right) \end{cases} \qquad (2.10)$$

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[4] Alternatively, one may follow the approach of [29, 30], and remove the π fields from the spectrum by resorting to a unitary-gauge type of gauge-fixing.

and the vector-like fields are defined as $\widetilde{Q}_i = \widehat{Q}_{iR} + \widehat{Q}_{iL}$, and similarly for $\widetilde{U}_i$. The masses of the gauge bosons, to be denoted by M_i, are given by

$$M_i = 2\tilde{g}v_2 \sin\left(\frac{i\pi}{2N}\right). \tag{2.11}$$

In the limit $N \to \infty$ one recovers the spectrum of the KK-modes of a continuous extra dimension, i.e. $M_i = \tilde{g}v_2 i\pi/N$, provided that one identifies the length of the extra dimension as $\pi R \approx N/\tilde{g}v_2$. For a finite N one can define $d \equiv 1/(\tilde{g}v_2)$ and $\pi R \equiv (N-1)d$, where πR can be identified with the length of a discretized extra dimension with N sites and d as the lattice spacing. For $N = 1$ we recover the SM. Note that the massless vector bosons $\widetilde{W}^a_{\mu 0}$ and $\widetilde{B}_{\mu 0}$ are associated to the SM model gauge bosons, i.e. they are the gauge bosons of the unbroken diagonal group, which is identified with the SM gauge group. The continuum limit of the fermion masses is obtained by setting $M_f = d^{-1}$ and their values coincide with those of the gauge bosons. In addition, the Higgs masses squared are given by $M^2(\widetilde{H}_i) = M_i^2 - m^2$. Eventually $\widetilde{H}_0$ will break the gauge symmetry spontaneously; in fact it will be identified with the SM Higgs doublet, therefore $\langle 0|\widetilde{H}_0|0\rangle = v/\sqrt{2}$ with $v = 246 \; GeV$.

After symmetry breaking further diagonalizations are required in order to determine the final spectrum and mass-eigenstates of the theory. One can verify that, at least for the degrees of freedom we will be interested in, the spectrum of this model coincides with that of a continuous universal extra dimension with $m_n = n/R$ replaced by M_n. In particular, the mass $M(\widetilde{W}^\pm_{\mu i})$ of the charged eigen-states $\widetilde{W}^\pm_{\mu i}$ is given by $M(\widetilde{W}^\pm_{\mu i}) = \sqrt{M_W^2 + M_i^2}$. The corresponding would-be Goldstone bosons $G_i^\pm$ and the physical scalar $a_i^\pm$ are given by

$$G_i^\pm = \frac{M_i \widetilde{W}^\pm_{5i} + M_W \widetilde{\chi}^\pm_i}{\sqrt{M_i^2 + M_W^2}} \xrightarrow{g\to 0} \widetilde{W}^\pm_{5i}, \qquad a_i^\pm = \frac{-M_W \widetilde{W}^\pm_{5i} + M_i \widetilde{\chi}^\pm_i}{\sqrt{M_i^2 + M_W^2}} \xrightarrow{g\to 0} \widetilde{\chi}^\pm_i \tag{2.12}$$

In the gaugeless limit, the Goldstone bosons and the physical scalars can be directly identified with $\widetilde{W}^\pm_{5i}$ and $\chi_i^\pm$, respectively. Notice also that tree-level mixings between gauge-bosons and would-be Goldstone bosons are removed through the choice of the R_ξ gauge [19, 35].

The determination of the final spectrum and the corresponding mass-eigenstates in the fermionic sector is slightly more involved; the details are presented in the Appendix A. Note however that for the rest of this paper we will only use the "tilded" fermion fields $\widetilde{Q}_i$ and $\widetilde{U}_i$ (i.e. the fields right before breaking the last $SU(2) \times U(1)_Y$). The fact that the $\widetilde{Q}_i$ and $\widetilde{U}_i$ are not eigenstates of the Lagrangian after the final symmetry breaking reflects itself in

the form of the tree-level propagators:

$$
\begin{bmatrix}
\overrightarrow{\quad\widetilde{Q}_{ti}\quad} & \overrightarrow{\quad\widetilde{Q}_{ti}\;\times\;\widetilde{U}_i\;} \\
\overrightarrow{\;\widetilde{U}_i\;\times\;\widetilde{Q}_{ti}\;} & \overrightarrow{\quad\widetilde{U}_i\quad}
\end{bmatrix}
=
\begin{bmatrix}
i\frac{\slashed{p}+M_i}{p^2-m_Q^2} & i\frac{m_t}{p^2-m_Q^2} \\
i\frac{m_t}{p^2-m_Q^2} & i\frac{\slashed{p}-M_i}{p^2-m_Q^2}
\end{bmatrix}
\tag{2.13}
$$

where $m_Q^2 = M_i^2 + m_f^2$, and the "cross" denotes tree-level mixings, proportional to m_f/M_i (see Appendix A).

We end this section presenting the interaction terms relevant for our calculations; note in particular that we work in the limit of large top-quark mass.

From the Yukawa interaction, the couplings proportional to m_t are

$$
\mathcal{L}_Y = \sum_{i=1}^{N-1} i m_t \frac{\sqrt{2}}{v} V_{tb} \overline{\widetilde{U}}_{tiR} \chi_i^+ b_L + \text{h.c.} \,.
\tag{2.14}
$$

The couplings with the Z-boson can be obtained from $\mathcal{L}_Z = (g/2c_w)[J_{SM}^\mu + J_F^\mu + J_\chi^\mu]Z_\mu^{(0)}$, where J_{SM}^μ is the usual SM current, and

$$
\begin{aligned}
J_F^\mu &= \sum_{i=1}^{N-1}\left(1-\frac{4}{3}s_w^2\right)\overline{\widetilde{Q}}_{ti}\gamma^\mu\widetilde{Q}_{ti} - \frac{4}{3}s_w^2\overline{\widetilde{U}}_{it}\gamma^\mu\widetilde{U}_{it}\,, \\
J_\chi^\mu &= \sum_{i=1}^{N-1}(-1+2s_w^2)\widetilde{\chi}_i^+ i\partial^\mu\widetilde{\chi}_i^- + \text{h.c.}
\end{aligned}
\tag{2.15}
$$

For the couplings with the photon, it easy to check that the electromagnetic current can be written as $j_{em}^\mu = j_{SM}^\mu + j_F^\mu + j_\chi^\mu$, where the new currents are

$$
\begin{aligned}
j_F^\mu &= \sum_{i=1}^{N-1}\frac{2}{3}\overline{\widetilde{Q}}_{ti}\gamma^\mu\widetilde{Q}_{ti} - \frac{1}{3}\overline{\widetilde{U}}_{it}\gamma^\mu\widetilde{U}_{it}\,, \\
j_\chi^\mu &= \sum_{i=1}^{N-1}-\widetilde{\chi}_i^+ i\partial^\mu\widetilde{\chi}_i^- + \text{h.c.}
\end{aligned}
\tag{2.16}
$$

III.   BOUNDS ON THE NEW PHYSICS

In this section we will set lower bounds on the mass of the lightest new particles of the model, M_1, as defined in Eq. (2.11), which for simplicity is going to be denoted M in what follows. As commented in the introduction in models with universal dimensions, continuous or latticized, due to the KK-number conservation the only effects of new particles at low energies appear at the one-loop level. Since the new physics decouples when $M \to \infty$, the

new contributions must scale as some inverse power of the new physics scale, which must be compensated by another scale from within the SM. The largest available scale in the SM is the top-quark mass. Thus, we expect large contributions in all observables that involve the top quark in loops. In particular, the new contributions to such observables will be suppressed only by factors m_t^2/M^2 with respect to the SM contributions.

In the SM there are several well-measured observables which are very sensitive to the top-quark mass: the decay rates $b \to s \gamma$ and $Z \to b\bar{b}$, the ρ parameter, and the rates of $B^0 \rightleftharpoons \overline{B^0}$. These observables will next be computed within the model we consider, with the expectation that they will turn out to be rather sensitive to the top-quark mass.

A.   Radiative corrections to $b \to s\gamma$

The experimental observable is the semi-inclusive decay $B(B \to X_s\gamma)$. Using the heavy quark expansion it is found that, up to small bound state corrections, this decay agrees with the parton model rates for the underlying decays of the b quark [36, 37], $b \to s\gamma$. This flavor violating transition is a very good place to look for new physics, because in the SM it is forbidden at tree level due to gauge symmetry, thus it can only proceed through radiative corrections. The transition can be parametrized by the following effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i \mathcal{O}_i \,, \tag{3.1}$$

where $\mathcal{O}_7$ is the operator that drives the transition $b \to s\gamma$,

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2} m_b \bar{s} \sigma^{\mu\nu} P_R b F_{\mu\nu} \,, \tag{3.2}$$

and C_7 is a coefficient to be computed in the specific model. In the SM, and at the scale of the W-boson mass, $C_7^{SM}(M_W) = -1/2 \, A(m_t^2/M_W^2)$, where $x_t = m_t^2/M_W^2$, and

$$A(x) = x \left[\frac{\frac{2}{3}x^2 + \frac{5}{12}x - \frac{7}{12}}{(x-1)^3} - \frac{\left(\frac{3}{2}x^2 - x\right) \ln x}{(x-1)^4} \right] \,. \tag{3.3}$$

The other operators included in (3.1) do not contribute directly to $b \to s\gamma$; however, QCD radiative corrections mix all the operators, and $\mathcal{O}_7$ end up receiving contributions from the other operators as well. These contributions can be conveniently calculated, resummed by using the renormalization group, and encapsulated in the evolution of the Wilson coefficients
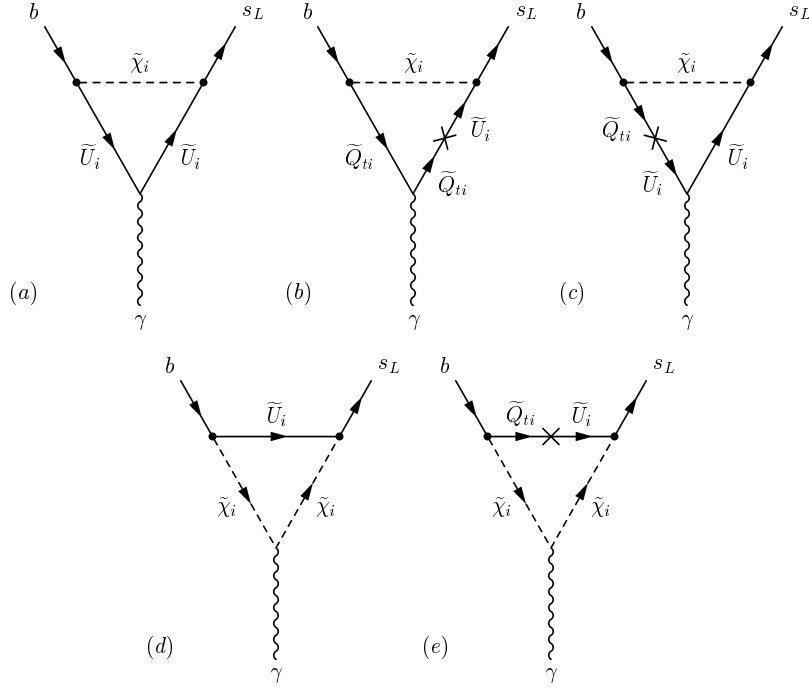
FIG. 1: Diagrams that contribute to $\mathcal{O}_7$.

$C_i(\mu)$ from M_W to m_b. It turns out that the corrections are numerically important [24, 38]:

$$C_7(m_b) \approx 0.698\, C_7(M_W) - 0.156\, C_2(M_W) + 0.086\, C_8(M_W)\,, \tag{3.4}$$

where C_2 and C_8 are the coefficients of the operators

$$\mathcal{O}_2 = [\bar{c}_{L\alpha}\gamma_\mu b_{L\alpha}][\bar{s}_{L\beta}\gamma^\mu c_{L\beta}]\,, \tag{3.5a}$$

$$\mathcal{O}_8 = \frac{g_s}{(4\pi)^2}m_b\bar{s}_{L\alpha}\sigma^{\mu\nu}T^a_{\alpha\beta}b_{R\beta}G^a_{\mu\nu}\,, \tag{3.5b}$$

and α, β are color indices. In the case of the SM, the contribution of $\mathcal{O}_8$ is negligible because it is generated at one loop, $C_8^{SM}(M_W) = -0.097$ [38], but that of $\mathcal{O}_2$, $C_2^{SM}(M_W) = 1$, is important because it appears at tree level. Equation (3.4) is valid for any theory, as long as one assumes that there is no new physics below M_W. Then, the running from M_W to m_b is the standard QCD running, and all the new physics is included in the boundary conditions for the Wilson coefficients at the scale M_W.

In the model we consider, the transition proceeds through the same effective Hamiltonian, but the coefficient C_7 is modified by the diagrams of Fig. 1. There are also diagrams in which the $\tilde{\chi}_i$ are replaced by $\widetilde{W}_{\mu i}$ and by the non physical degrees of freedom $\widetilde{W}_{5i}$ but since the

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couplings of these are reduced by a factor $(M_W/m_t)^2 \approx 0.22$ we will ignore them and work at this level of precision.

The contribution of the i-th mode to the C_7 coefficient can be written in the form [24]

$$C_{7\,i}(M_W) = \frac{r_i}{1+r_i}\left[B\left(1+r_i\right) - \frac{1}{6}A\left(1+r_i\right)\right]\,,\qquad (3.6)$$

where $r_i \equiv m_t^2/M_i^2$, and $B(x)$ is given by

$$B(x) = \frac{x}{2}\left[\frac{\frac{5}{6}x - \frac{1}{2}}{(x-1)^2} - \frac{\left(x - \frac{2}{3}\right)\log x}{(x-1)^3}\right]\,.\qquad (3.7)$$

An expansion of $C_{7\,i}$ reveals that it does not contain logarithms of the two different mass scales M_i and m_t,

$$C_{7\,i}(M_W) = \frac{23}{144}r_i - \frac{13}{120}r_i^2 + \mathcal{O}(r_i^3)\,,\qquad (3.8)$$

a feature that can be understood from an effective field theory point of view. Specifically, when the heavy degrees of freedom are integrated out, the tree-level effective Lagrangian is exactly the SM Lagrangian; there are no additional tree-level operators suppressed by powers of M_i^{-1}. It is well known that the dominant logarithms of the two different scales can be recovered from the running of the operators in the low energy effective Lagrangian induced by the presence of the additional operators. Thus, since in our case there are no additional tree-level operators, no logarithms can appear in Eq. (3.8). As a matter of fact this is also an inherited property from UED, where no such logarithms appear either, for the same reason. Finally, all contributions must be put together,

$$C_7(M_W) = C_7^{SM}(M_W) + \sum_{i=1}^{N-1} C_{7\,i}(M_W)\,,\qquad (3.9)$$

where we have neglected the running between m_t and M_W, i.e. $C_{7\,i}(m_t) \approx C_{7\,i}(M_W)$.

Since the model we consider does not generate additional contributions at tree level, we basically obtain the SM result for C_2. On the other hand, the C_8 coefficient can get corrections, which will be comparable to those of the SM; however, since the latter are negligible, so are the former.

For comparing the predictions for this process with the experimental result, it is convenient to use the ratio $\widetilde{\Gamma} = \Gamma(b \to s\gamma)/\Gamma(b \to cl\nu)$, which depends much less on m_b, and, therefore, presents a smaller uncertainty [39]: 10% for the theoretical value in the SM, while

the uncertainty in the the experimental determination is about 15% (both at 1σ), and central values agree quite well with the SM calculations. In fact, current determinations only allow for new physics contributions which are about 36% of the SM value (at 95% CL) [24], i.e. $|\widetilde{\Gamma}^{total}/\widetilde{\Gamma}^{SM} - 1| \leq 0.36$. Since the process $b \to cl\nu$ is only modified at one loop by the new physics the previous equation can be translated into the more useful one

$$\left| \frac{|C_7^{total}(m_b)|^2}{|C_7^{SM}(m_b)|^2} - 1 \right| < 0.36 \qquad 95\% \text{ CL}. \tag{3.10}$$

The C_7 coefficients at the scale m_b are obtained from Eq. (3.4). The final bounds that one can set from this process are shown in Fig. 4.

B.   The $Z \to b\bar{b}$ process.

Radiative corrections coming from new physics affect the branching ratio $R_b = \Gamma_b/\Gamma_h$, where $\Gamma_b = \Gamma(Z \to b\bar{b})$ and $\Gamma_h = \Gamma(Z \to \text{hadrons})$ and also the left-right asymmetry A_b. Both can be treated uniformly by expressing them as a modification to the tree-level couplings $g_{L(R)}$ (it is understood that we refer only to the couplings of the b-quark) defined as

$$\frac{g}{c_W} \bar{b}\gamma^\mu (g_L P_L + g_R P_R) b Z_\mu, \tag{3.11}$$

Z and b's are SM fields, $P_{L(R)}$ are the chirality projectors, and

$$g_L = -\frac{1}{2} + \frac{1}{3}s_W^2 + \delta g_L^{SM} + \delta g_L^{NP}, \tag{3.12a}$$

$$g_R = \frac{1}{3}s_W^2 + \delta g_R^{SM} + \delta g_R^{NP}, \tag{3.12b}$$

where we have separated radiative corrections coming from SM contributions and from new physics (NP). It turns out that, both within the SM as well as in most of its extensions, only g_L receives corrections proportional to m_t^2 at the one-loop level, due to the difference in the couplings between the two chiralities. In particular, a shift δg_L^{NP} in the value of g_L due to new physics translates into a shift in R_b given by

$$\delta R_b = 2R_b(1 - R_b)\frac{g_L}{g_L^2 + g_R^2}\delta g_L^{NP}, \tag{3.13}$$

and to a shift in the left-right asymmetry A_b given by

$$\delta A_b = \frac{4g_R^2 g_L}{(g_L^2 + g_R^2)^2}\delta g_L^{NP}. \tag{3.14}$$

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These equations, when compared with experimental data, will provide bounds on the new physics. A possible way of parametrize δg_L^{NP} is defining the function $F(a)$ through the relation

$$\delta g_L^{NP} = \delta g_L^{SM} F(a) = \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} F(a) , \tag{3.15}$$

where $a = \pi R m_t$ and G_F is the Fermi constant.

In the model we consider, the new contributions stem from the set of diagrams displayed in Fig. 2. Following Ref. [26] we parametrize the different contributions as
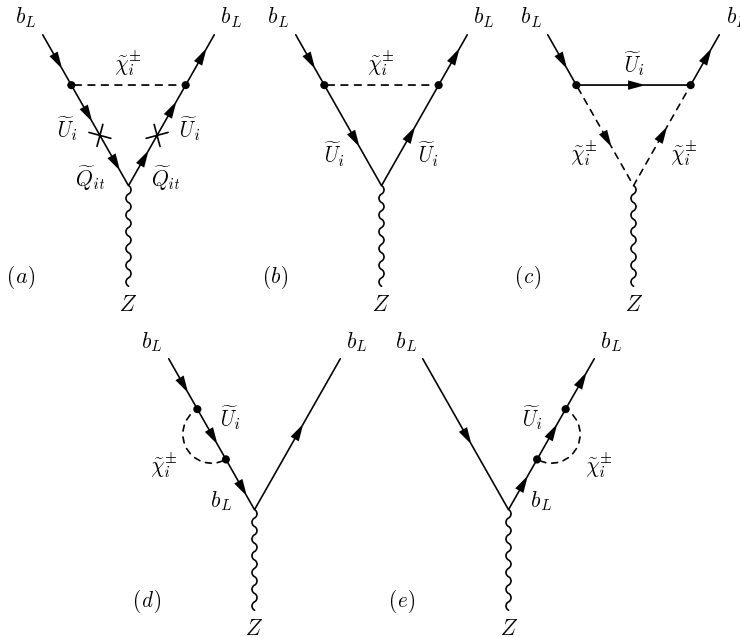

FIG. 2: Diagrams contributing to $Z \to b\bar{b}$.

$$i\mathcal{M}_i = i \frac{g}{c_w} \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} f(r_i) \overline{u}' \gamma^\mu P_L u \epsilon_\mu , \tag{3.16}$$

where u and u' are the spinors of the b quarks and ϵ_μ stands for the polarization vector of the Z boson. Although each of the contributions is divergent, the sum is finite; the divergences cancel, and so do all terms proportional to s_w^2. Thus, finally, the only term which survives is the term from diagram Fig. 2a not proportional to s_w^2, yielding the following contribution

$$\delta g_{Li} = \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} \left[\frac{r_i - \log(1 + r_i)}{r_i} \right] . \tag{3.17}$$

A way to understand this result is by resorting the so called gauge-less limit [40, 41], as was done in Ref. [26] in the context of the UED model. Notice also here the absence of logarithms in Eq. (3.17) when $r_i \to 0$.

The full contribution $\delta g_L^{NP} = \sum_{i=1}^{N-1} \delta g_{Li}$ expressed in terms of $F(a)$ can be written in the form

$$F(a) = \int_0^1 dx \sum_{i=1}^{N-1} \frac{a^2 x}{4(N-1)^2 \sin^2(i\pi/2N) + a^2 x} . \tag{3.18}$$

This function captures the correction proportional to m_t^2, the full one-loop result could be adapted from [25] by replacing $m_n \to M_i$ as explained above.

Now we will extract the experimental bound on $F(a)$ and translate it into bounds on M. The maximum experimentally allowed value of $F(a)$ could be extracted from the modifications induced to R_b and A_b via δg_L^{NP}, see Eqs. (3.13–3.14). The SM prediction for the left-right asymmetry $A_b^{SM} = 0.9347 \pm 0.0001$ and the measured value $A_b^{exp} = 0.921 \pm 0.020$ give a looser bound than the one coming from R_b, $R_b^{SM} = 0.21569 \pm 0.00016$ and $R_b^{exp} = 0.21664 \pm 0.00068$ which turns out to be $F(a) - 1 = -0.24 \pm 0.31$. Making a weak signal treatment [42] is easy to obtain the 95% CL bound $F(a) - 1 < 0.39$, from which the results displayed in Fig. 4 follow.

C. ρ parameter

The ρ parameter can be defined as the ratio of the relative strength of neutral to charged current interactions at low momentum transfer. In the SM, and at tree level ,it is predicted to be unity as a consequence of the custodial symmetry of the Higgs potential:

$$\rho \equiv \frac{G_{NC}(0)}{G_{CC}(0)} \approx \frac{M_W^2}{c_W^2 M_Z^2} = 1 . \tag{3.19}$$

However, since the SM contains couplings that violate this symmetry, the Yukawa couplings and the U(1) coupling g', radiative corrections modify ρ. At one loop the ρ defined above receives corrections from vertex, box and gauge-boson self-energy diagrams; however the dominant contributions, proportional to m_t^2, come from the top-quark loops inside the gauge boson self-energies. Keeping only these contributions, one has

$$\rho = 1 + \frac{\Sigma_W(0)}{M_W^2} - \frac{\Sigma_Z(0)}{M_Z^2} \approx 1 + \frac{1}{M_W^2}\left(\Sigma_1(0) - \Sigma_3(0)\right) \approx 1 + N_c \frac{\sqrt{2}G_F m_t^2}{(4\pi)^2} . \tag{3.20}$$

$\Sigma_W(0)$ and $\Sigma_Z(0)$ are co-factors of the $g^{\mu\nu}$ in the one-loop self-energies of the W and Z bosons, evaluated at $q^2 = 0$, and $\Sigma_1(0)$ and $\Sigma_3(0)$ are the equivalent functions for the W_1 and W_3 components of the SU(2) gauge bosons. In arriving at the above formula one uses
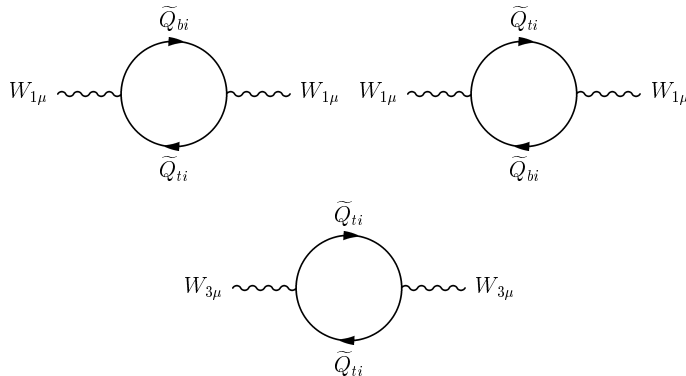
FIG. 3: Diagrams modifying the ρ parameter.

the fact that the photon-Z self-energy $\Sigma_{AZ}^{\mu\nu}$ is transverse, i.e. $\Sigma_{AZ}(0) = 0$; this last property holds *only* for the subset of graphs containing fermion-loops, but is no longer true when gauge-bosons are considered inside the loops of Σ_{AZ} [43, 44]. Finally, N_c is the number of colors.

Let us compute now the leading (m_t^2) corrections to ρ in the model we consider. To that end we need the couplings of W_1 and W_3 components (or alternatively the contributions of the new modes to the J_1 and J_3 currents),

$$\mathcal{L}_\rho = \frac{g}{2} \sum_{i=1}^{N-1} W_\mu^1 \left[\overline{\widetilde{Q}}_{it} \gamma^\mu \widetilde{Q}_{ib} + \overline{\widetilde{Q}}_{ib} \gamma^\mu \widetilde{Q}_{it} \right] + W_\mu^3 \left[\overline{\widetilde{Q}}_{it} \gamma^\mu \widetilde{Q}_{it} \right] , \qquad (3.21)$$

where we have already used the relation $g = \widetilde{g}/\sqrt{N}$ [30]. Thus, the couplings in this basis are the same as in the SM, but the propagators of the fields are modified as described in the previous section. The relevant diagrams are shown in Fig. 3. Since only the $\widetilde{Q}$ fields couple to W gauge bosons, we use the component Q-Q in the propagator of Eq. (2.13).

The contribution to the ρ parameter of each mode is finite and given by

$$\begin{aligned} \Delta\rho_i &= \frac{4}{g^2 v^2} \left[\Sigma_{1\,i}(0) - \Sigma_{3\,i}(0) \right] \qquad\qquad (3.22) \\ &= 2N_c \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} \left[1 - \frac{2}{r_i} + \frac{2}{r_i^2} \log(1 + r_i) \right] . \end{aligned}$$

The final result is found by summing the contributions of all the modes, $\Delta\rho = \Delta\rho^{SM} + \sum_{i=1}^{N-1} \Delta\rho_i$. In the limit $N \to \infty$ we recover the UED result[45].

In order to discriminate between the corrections coming from the SM and the ones coming exclusively from new physics we use the T parameter as defined in PDG [46]. This parameter
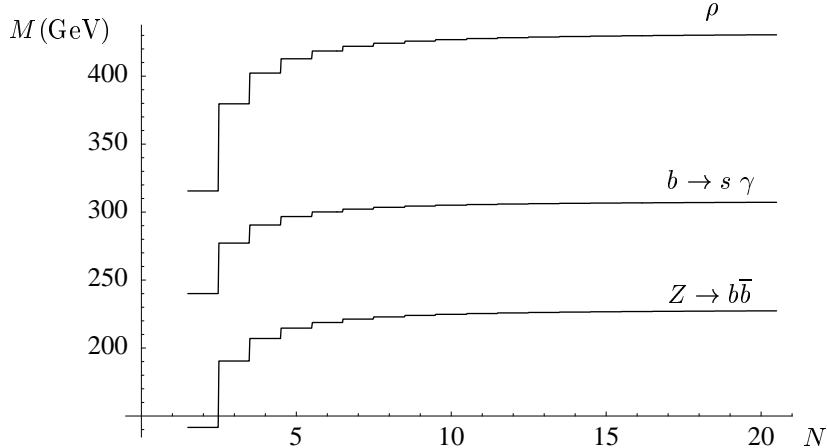
FIG. 4: Bounds on the mass scale of the new physics coming from different precision observables as a function of the number of sites N.

contains only the corrections to ρ coming from new physics, $\alpha(M_Z)T \equiv \Delta\rho^{NP}$. T is bounded to be $T < 0.4$ at 95 % CL. As in the UED case [20], this is the most restrictive observable. In Fig. 4 the resulting (95% CL) bounds from the ρ parameter are displayed for different number of sites, N. In the limit of relatively large N we find a limit of 430 GeV for the mass of the lightest new mode. This is in agreement with the latest results obtained in the UED model for a light Higgs boson. [5] If the number of sites is small, the bound can be reduced by a factor of about 10%–25%, thus allowing new modes ranging between 320–380 GeV.

D. $B^0 \rightleftharpoons \overline{B}^0$ mixing

The model we consider has the same flavor structure as the SM, i.e. the flavor violation is entirely governed by the CKM matrix. In models of this type, both $B^0 \rightleftharpoons \overline{B}^0$ mixings and the CP-violating parameter ε_K can be parametrized by a single function $S(x_t)$ [47], with $x_t = m_t^2/M_W^2$. $S(x_t)$ is defined through

$$\mathcal{H}_{eff}^{\Delta B=2} = \frac{M_W^2 G_F^2 (V_{tb}V_{td}^*)^2}{(4\pi)^2} S(x_t) [\overline{d}\gamma^\mu(1-\gamma_5)b][\overline{d}\gamma_\mu(1-\gamma_5)b]. \qquad (3.23)$$

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[5] In Fig. 3 of [45] a limit of 500 GeV at %90 CL for $m_H = 114$ GeV is quoted; this amounts to a limit of about 400 GeV at %95 CL.
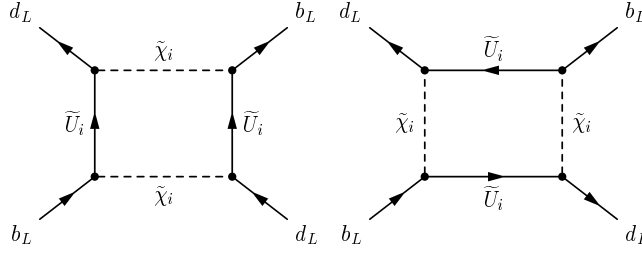
FIG. 5: Diagrams for the dominant corrections to $S(x_t)$

In the SM $S_{SM}(x_t)$ is dominated by the box diagrams with longitudinal W exchanges and the top-quark running inside the loop

$$S_{SM}(x_t) = \frac{x_t}{4}\left[1 + \frac{9}{1 - x_t} - \frac{6}{(1 - x_t)^2} - \frac{6x_t^2 \log(x_t)}{(1 - x_t)^3}\right].\qquad(3.24)$$

Using the running top-quark mass $m_t(m_t) = 167 \pm 5$ GeV, one obtains $S_{SM}(x_t) = 2.39 \pm 0.12$ [35]. If we split $S(x_t)$ in SM plus new physics contributions, $S(x_t) = S_{SM}(x_t) + S_{NP}(x_t)$, the latter can be encoded in a function defined as $G(a) = S_{NP}/S_{SM}$. For our estimates we will use the limit of large top-quark mass. Then, $S_{SM}(x_t) \approx x_t/4$, and

$$S_{NP}(x_t) = \frac{x_t}{4}(G(a) - 1).\qquad(3.25)$$

The dominant contributions to this function stem from the diagrams in Fig. 5. Notice that only the $\widetilde{U}_i$ fields are important because the couplings of $\widetilde{Q}_i$ fields with the $\widetilde{\chi}_i$ are proportional to the mass of the b quark, and therefore provide subdominant corrections. After the usual Fierz reordering and a bit of combinatorics we obtain the result Eq. (3.26). For comparison, we also present the results in the continuous scenarios [22, 25]:

$$\begin{aligned}
G(a) &= 1 + 2\int_0^1 \sum_{i=1}^{N-1} \frac{a^2 x(1 - x)\, dx}{4(N - 1)^2 \sin^2(i\pi/2N) + a^2 x},\\
G_{UED}(a) &= \int_0^1 dx\,(1 - x)\left[a\sqrt{x}\coth(a\sqrt{x}) + 1\right] = 1 + \frac{a^2}{18} - \frac{a^4}{540} + \mathcal{O}(a^6).\quad(3.26)
\end{aligned}$$

The expression for $G(a)$ is obtained following arguments very similar to the ones used in Ref. [26]. The full one-loop calculation has been previously carried out in Ref. [25, 48], and reduces to Eq. (3.26) when only the dominant corrections, for large m_t, are retained; the full one-loop result for $G(a)$ can be easily computed by adapting the results in Ref. [25]. Notice that the contributions in all cases are positive.

The last analyses of $S(x_t)$ [35] furnish a value which agrees well with the SM expectations

$$1.3 \leq S(x_t) \leq 3.8 \qquad 95\% \,; \tag{3.27}$$

the possible positive contributions have been lowered with respect to previous determinations [47], yielding better bounds.

Given the future improvements on the experimental determinations of $\sin 2\beta$ by BaBar and BELLE, and in particular of the mass splitting ΔM_s for the B sector in LHC and FNAL one may use this observable to predict possible deviations from the SM predictions. It turns out that, to an excellent accuracy [25], the aforementioned deviations in the case of ΔM_s are governed by $G(a)$

$$G_{NP}(a) = \frac{(\Delta M_s)_{NP}}{(\Delta M_s)_{SM}} > 1 \,. \tag{3.28}$$

The larger values of $G(a)$ occur for small N, but they are at most $G(a) \leq 1.14$, which represents too small a deviation to be discriminated experimentally. In fact, this observable is of the same order of magnitude as in the UED model [25].

The bounds one can set on the masses of the new modes are below the W mass, and are therefore irrelevant compared to previously discussed bounds. The virtue of this results is that the possible existence of extra latticized dimensions will not pollute the extraction of the CKM matrix parameters from the future improvements in the determination of the unitarity triangle.

E.   An alternative model with no new fermionic modes

A popular scenario involving extra dimensions [49, 50] is to assume that only gauge bosons and Higgs scalars propagate in the extra dimensions, while SM fermions are confined in the four dimensions. It is interesting, therefore, to study the latticized version of such models, which for brevity we will denote as LHG models. The main difference compared to the universal models is that they generate new low energy physics already at tree level. In particular, the exchange of extra W modes among SM fermions modifies the rate of muon decay, and, therefore, the definition of G_F, while leaving unmodified, at tree level, the well measured decay rates of the Z boson. This mismatch can be used to set stringent bounds
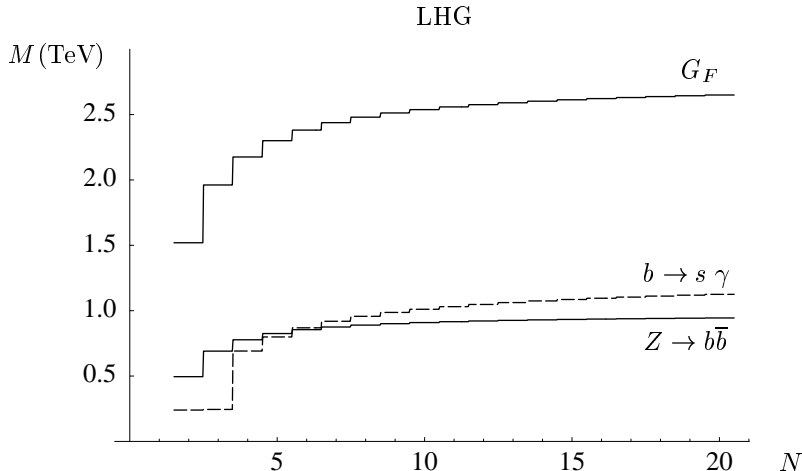
FIG. 6: Bounds on the mass scale of the new physics in the LHG models for different number of modes N.

on the masses of new particles in these scenarios [6].

The shift in G_F produced by the modes $\widetilde{W}_{\mu\,i}$ is

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \left[1 + \sum_{i=1}^{N-1} \frac{M_W^2}{M_i^2} 2\cos^2\left(\frac{i\pi}{2N}\right) \right] \; , \tag{3.29}$$

which reduces slightly the decay rate $\Gamma(Z \to \ell\bar{\ell})$

$$\frac{\Gamma^{LHG}}{\Gamma^{SM}} = 1 - \frac{(M_W\pi R)^2}{2(N-1)^2} \sum_{i=1}^{N-1} \cot^2\left(\frac{i\pi}{2N}\right) \; . \tag{3.30}$$

The maximum allowed deviation is very small [46], $|\Gamma^{NP}/\Gamma^{SM} - 1| < 0.0028$ at 95%, and it provides very stringent bounds, shown in Fig. 6, which are of the order of a few TeV. For comparison we also display the limits we obtained from $Z \to b\bar{b}$ and $b \to s\gamma$.

IV.   OUTLOOK AND CONCLUSIONS

We have studied a five-dimensional extension of the SM in which the extra spatial dimension is latticized, and all SM fields propagate in it. The model has the property that there are no tree-level effects below the threshold of production of new particles. Therefore, to set a lower bound on the scale of the new physics one should consider one-loop processes.

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[6] The argument is the same in the continuous scenarios[21, 51].

We considered a number of well-measured observables, and which depend strongly on the top-quark mass: the ρ parameter, $b \to s\gamma$, $Z \to b\bar{b}$, and the $B^0 \rightleftharpoons \overline{B}^0$ mixing. The dominant corrections, i.e. those proportional to the top-quark mass, have been computed, and compared with the ones obtained when only the SM is considered. It is found that the known bounds for the continuous version (UED) are rapidly reached when the extra dimension is latticized by only about 10 to 20 (four dimensional) sites. However, when a smaller number of sites is considered, the bounds on the scale of new physics is lowered by roughly a factor of 10%–25%, as can be seen in Fig. 4. Then, the limits on new particles are about 320–380 GeV. The bounds shown there correspond to the mass of the lightest modes, defined in Eq. (2.11).

We have also briefly discussed a latticized version of a model in which fermions are confined to the four-dimensional subspace. In these models, there are contributions that modify the muon decay rate already at tree level, allowing for much stronger bounds (of the order of 1 TeV).

To summarize, we found that models with one universal latticized extra dimension provide new interesting physics which could be well within the reach of the next generation of accelerators.

APPENDIX A: THE SPECTRUM OF THE FERMIONS

After the spontaneous breaking of the remaining $SU(2) \times U(1)$, i.e. the usual SM symmetry group, the fermionic mass-eigenstates must be re-defined. In particular, the fermion masses will receive additional contributions from the Yukawa piece $\mathcal{L}_Y$. The Yukawa piece can be written in terms of the tilde fields as

$$\mathcal{L}_Y = \sum_{i=0}^{N-1} \overline{\widetilde{Q}}_i \frac{\widetilde{Y}_u}{\sqrt{N}} \widetilde{H}_0^c \widetilde{U}_i + \overline{\widetilde{Q}}_i \frac{\widetilde{Y}_d}{\sqrt{N}} \widetilde{H}_0 \widetilde{D}_i + \text{h.c.} , \qquad (A1)$$

20

where we have concentrated on the terms containing the Higgs doublet $\widetilde{H}_0$. From the first term in the sum of Eq. (A1) is easy to convince oneself that $Y_u \equiv \widetilde{Y}_u/\sqrt{N}$ is the SM Yukawa matrix. When the Higgs doublet acquires a VEV one must diagonalize Y_u using the same field redefinitions as in SM, $\widetilde{Q}_{ui} \rightarrow U_u^\dagger \widetilde{Q}_{ui}$, $\widetilde{U}_i \rightarrow V_u^\dagger \widetilde{U}_i$. At the end the mass matrix for fermions will be[7]

$$
\left(\overline{\widetilde{U}}_{if} \; \overline{\widetilde{Q}}_{if} \right)
\begin{pmatrix} -M_i & m_f \\ m_f & M_i \end{pmatrix}
\begin{pmatrix} \widetilde{U}_{if} \\ \widetilde{Q}_{if} \end{pmatrix} ;
\tag{A2}
$$

f is a flavor index, $f = u, c, t$, and m_f are the masses of the SM up-type quarks. The above mass matrix coincides with the one obtained in the analogous model with one continuous extra dimension, after making the substitution $M_n \rightarrow m_n$ [20]; $m_n = n/R$ is the mass of the n-th KK mode of the field in the absence of Yukawa couplings. As a consequence, the mixing between the $\widetilde{Q}$ and $\widetilde{U}$ is the same as in the aforementioned continuous case. Denoting the new mass eigenstates by "primes" we have that

$$
\begin{pmatrix} \widetilde{U}_{if} \\ \widetilde{Q}_{if} \end{pmatrix} =
\begin{pmatrix} -\gamma^5 \cos\alpha_{if} & \sin\alpha_{if} \\ \gamma^5 \sin\alpha_{if} & \cos\alpha_{if} \end{pmatrix}
\begin{pmatrix} U'_{if} \\ Q'_{if} \end{pmatrix}
\tag{A3}
$$

where $\tan(2\alpha_{if}) = m_f/M_i$. The masses are given by $M(Q'_{if}) = \sqrt{M_i^2 + m_f^2} \equiv m_Q$. Notice that the zero-th modes have exactly the same masses as in SM, all of them coming purely from the Yukawa piece Eq. (A1) which, as said, for the zero-th modes coincides exactly with the SM Yukawa sector. In fact the same happens for the rest of the pieces in the Lagrangian and one can safely identify the zero-th tilded fields, $\widetilde{Q}_{0L}$, $\widetilde{U}_{0R}$ and $\widetilde{D}_{0R}$, with the SM ones.

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