

# B Physics and Extra Dimensions

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## Abstract

We compute the dominant new physics contributions to the processes  $Z \rightarrow b\bar{b}$  and  $B-\bar{B}$  in the context of two representative models with extra dimensions. The main thrust of the calculations focuses on how to control the effects of the infinite tower of Kaluza-Klein modes inside the relevant one-loop diagrams. By comparing the results with the existing experimental data, most importantly those for  $R_b$ , we show that one may derive interesting lower bounds on the size of the compactification scale  $M_c$ .

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# 1 Introduction

In the last years there has been a revival of interest in models where the ordinary four dimensional Standard Model (SM) arises as a low energy effective theory of models living in five or more dimensions, with the extra dimensions compactified. [1, 2, 3, 4]. Such models arise naturally in string scenarios, and merit a serious study, mainly because of the plethora of theoretical and phenomenological ideas associated with them, and the flexibility they offer for realizing new, previously impossible, field-theoretic constructions. Models with compact extra dimensions are in general not renormalizable, and one should regard them as low-energy manifestations of some more fundamental theory. The effects of the extra dimensions are communicated to the four dimensional world through the presence of infinite towers of KK modes, which modify qualitatively the behavior of the low-energy theory. In particular, the non-renormalizability of the theory is found when summing the infinite tower of KK states. The size of the extra dimensions can be surprisingly large without contradicting present experimental data (see for instance [5, 6, 7, 8, 9, 10, 11, 12, 13]). This opens the door to the possibility of testing these models in the near future. Most importantly, the lowest KK states, if light enough, could be produced in the next generation of accelerators.

$B$  phenomenology can provide important generic tests for new physics ([14, 15] and references therein). For example, the present experimental value of  $R_b$ ,  $R_b^{\text{exp}} = 0.2164 \pm 0.00073$  is perfectly compatible with the standard model [16], which predicts  $R_b^{\text{SM}} = 0.2157 \pm 0.0002$ . Deviation due to new physics must be accommodated in this small window, a fact which furnishes non-trivial constraints on the possible models.

In the SM the most important corrections are those enhanced by the large top-quark mass in  $Z \rightarrow b\bar{b}$  and  $B - \bar{B}$  mixing [17, 18, 19, 20, 21, 22], and  $\rho$  parameter. In this talk we will show that, due to this same enhancement, one may derive valuable information on the size of the extra dimension(s), through the study of the one-loop contributions of the KK modes to the processes  $Z \rightarrow b\bar{b}$  and  $B - \bar{B}$  mixing [23].

## 2 $Z \rightarrow b\bar{b}$ and New Physics

The effective  $Zb\bar{b}$  vertex is usually parametrized as

$$\frac{g}{c_W} \bar{b}\gamma^\mu (g_L P_L + g_R P_R) b Z_\mu \quad (1)$$

where  $P_L = (1 - \gamma_5)/2$  and  $P_R = (1 + \gamma_5)/2$  are, respectively, the left and right chirality projectors. At one loop, the dominant corrections amount to shifts in the coupling  $g_L$  and  $g_R$ . Thus, we will write  $g_L = -\frac{1}{2} + \frac{1}{3}s_W^2 + \delta g_L^{\text{SM}} + \delta g_L^{\text{NP}}$  which contains the SM tree level contribution, the SM one-loop contributions  $\delta g_L^{\text{SM}}$ , and the contributions coming from new physics  $\delta g_L^{\text{NP}}$ , and similarly for the right-handed couplings  $g_R = \frac{1}{3}s_W^2 + \delta g_R^{\text{SM}} + \delta g_R^{\text{NP}}$ . However, in general,  $g_R$  obtains sub-dominant corrections only (not proportional to the top quark mass), in both the SM and new physics. The dominant SM contribution comes

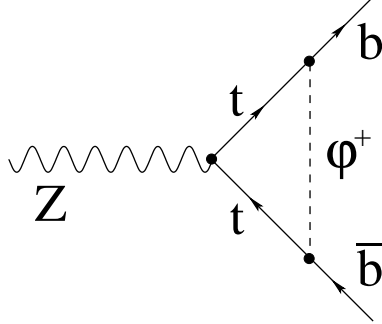


Figure 1: The leading contribution in the SM case

from the Goldstone boson diagrams running in the loop (see Fig. 1) and reads

$$\delta g_L^{\text{SM}} \approx \sqrt{2} G_F m_t^4 i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - m_t^2)^2 k^2} = \frac{\sqrt{2} G_F m_t^2}{(4\pi)^2} \quad (2)$$

A shift in the  $Zb\bar{b}$  couplings gives a shift in  $R_b = \Gamma_b/\Gamma_h$  (here  $\Gamma_b = \Gamma(Z \rightarrow b\bar{b})$  and  $\Gamma_h = \Gamma(Z \rightarrow \text{hadrons})$ ) given by

$$R_b = R_b^{\text{SM}} \frac{1 + \delta_{bV}^{\text{NP}}}{1 + R_b^{\text{SM}} \delta_{bV}^{\text{NP}}}, \quad (3)$$

where

$$\delta_{bV}^{\text{NP}} = \frac{\delta\Gamma_b}{\Gamma_b^{\text{SM}}} \approx 2 \frac{g_L}{(g_L)^2 + (g_R)^2} \delta g_L^{\text{NP}} \approx -4.6 \delta g_L^{\text{NP}}. \quad (4)$$

gives the shift in  $\Gamma_b$  due to vertex corrections coming from new physics,  $\Gamma_b = \Gamma_b^{\text{SM}} + \delta\Gamma_b$ . Here, quantities with the superscript ‘‘SM’’ represent standard model quantities including complete radiative corrections. Note that non-vertex corrections are universal for all quarks and cancel in the ratio  $R_b$ .

### 3 A model with fermions on the brane

Extra dimensions may or may not be accessible to all known fields, depending on the specifics of the underlying, more fundamental theory. Here we consider the simplest generalization of the SM, the so-called 5DSM, with fermions living in four dimensions, and gauge bosons and a single scalar doublet propagating in five dimensions [6]. The relevant pieces of the five dimensional Lagrangian are ( $\mu = 0, 1, 2, 3$  are four dimensional indices and  $M = 0, 1, 2, 3, 5$  are five dimensional ones)

$$L = \int d^5 x \left( \partial_M \varphi^\dagger \partial^M \varphi - (\bar{Q}_L Y_u u_R \varphi \delta(x^5) + \text{h.c.}) + \dots \right) \quad (5)$$

where  $\varphi(x^M)$  is the  $SU(2)$  Higgs doublet which lives in five dimensions.  $Q_L(x^\mu)$  and  $u_R(x^\mu)$  are the standard left-handed quark doublet and right-handed singlet, respectively, which live in four dimensions (brane); this is enforced by the presence of the  $\delta(x^5)$ . We assume that  $x^5$  is compactified on an orbifold  $S^1/Z_2$ . Fields even under the  $Z_2$  symmetry have zero modes and are present in the low energy theory, whereas fields that are odd have only KK modes and disappear from the low energy spectrum. One chooses the Higgs doublet to be even under the aforementioned symmetry in order to have the standard Higgs boson. Fourier-expanding the Higgs field as

$$\varphi(x^\mu, x^5) = \sum_{n=0}^{\infty} \varphi_n(x^\mu) \cos \frac{nx^5}{R}, \quad (6)$$

substituting in the fifth dimensional Lagrangian, and integrating on the fifth component, we obtain the following four dimensional Lagrangian for the KK modes  $\varphi_n(x)$ :

$$\begin{aligned} \mathcal{L} = & \partial_\mu \varphi_0^\dagger \partial^\mu \varphi_0 - \left( \bar{Q}_L Y_u u_R \varphi_0 + \text{h.c.} \right) \\ & + \sum_{n=1}^{\infty} \left( \partial_\mu \varphi_n^\dagger \partial^\mu \varphi_n - \frac{n^2}{R^2} \varphi_n^\dagger \varphi_n - \left( \bar{Q}_L Y_u u_R \sqrt{2} \varphi_n + \text{h.c.} \right) \right) \end{aligned} \quad (7)$$

The additional factor  $\sqrt{2}$  comes from the normalization of the zero mode in the Fourier series.

As in the SM case, the dominant contributions to  $Z \rightarrow b\bar{b}$  for large  $m_t$  come from the diagram Fig. 2 where charged KK modes are running in the loop. The zero mode provides just the standard model contribution due to the exchange of the Goldstone boson while the exchange of the KK modes of the charged components of the Higgs doublet will give an extra contribution. Since the coupling is universal summing all these contributions amounts to replacing the propagator of the Goldstone boson (in the Euclidean)

$$\frac{1}{k_E^2} \rightarrow \frac{1}{k_E^2} + 2 \sum_{n=1}^{\infty} \frac{1}{k_E^2 + n^2/R^2} = \sum_{n=-\infty}^{\infty} \frac{1}{k_E^2 + n^2/R^2} = \pi R \frac{\coth(k_E \pi R)}{k_E}. \quad (8)$$

This effective propagator reduces for small  $k_E$  to the standard Goldstone propagator. However, for large  $k_E$  it goes as  $1/k_E$ , which means that the ultraviolet behavior of this theory is worse than in the SM by one power of  $k_E$ . Even though this propagator provides a convergent result for this particular diagram, its soft high energy behaviour will trigger eventually the non-renormalizability of the theory.

Adding the KK modes we obtain

$$\delta g_L^{\text{NP}} \approx \delta g_L^{\text{SM}} (F(a) - 1), \quad (9)$$

where  $a = \pi R m_t$ , and

$$F(a) = 2a \int_0^\infty dx \frac{x^2}{(1+x^2)^2} \coth(ax) \quad (10)$$

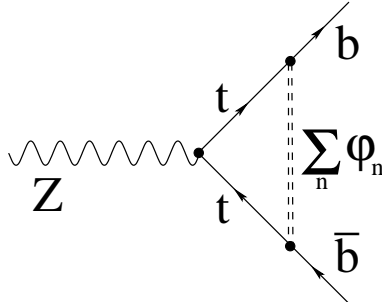


Figure 2: The KK tower corresponding to the Higgs doublet living in 5-D

is the ratio of the non-standard/standard integrals. Expanding for small  $a$  we obtain

$$F(a) \approx 1 + a^2 \left( -\frac{1}{3} - \frac{4}{\pi^2} \zeta'(2) - \frac{2}{3} \log(a/\pi) \right) \approx 1 + a^2 \left( 0.80979 - \frac{2}{3} \log(a) \right) \quad (11)$$

where  $\zeta'$  is just the derivative of the Riemann Zeta function. Contributions from the KK towers of gauge bosons running in the loop are not enhanced by the top quark mass and are suppressed by factors  $(m_W/m_t)^2$  with respect to the contribution considered. Although such contributions are important for obtaining the standard model value [18] at the required precision, they can be neglected when estimating bounds on new physics.

One finds  $F(a) - 1 = -0.24 \pm 0.31$ ; since  $F(a)$  is always larger than 1, translating this result into an upper bound on  $F(a) - 1$  is subtle. For this purpose we used the prescription of ref. [24] and found the 95% CL limit of  $F(a) - 1 < 0.39$ , which, after evaluation of the integral of Eq.(10), translates into an upper bound on  $a$ , yielding  $a < 0.56$ . This amounts to a lower bound  $M_c > 0.98$  TeV on the compactifications scale  $M_c = 1/R$ , which is comparable to those obtained from tree level processes [8, 9, 10, 11, 12, 13].

## 4 Models with universal extra dimensions

Scenarios where all SM fields live in higher dimensions have also been considered (see for example [12, 25]); we will refer to this type of extra dimension(s) as “universal”. From the phenomenological point of view, the most characteristic feature of such theories is the conservation of the KK number at each elementary interaction vertex [12, 25]. As a result, and contrary to what happens in the non-universal case, the coupling of any excited (massive) KK mode to two zero modes is prohibited. This fact alters profoundly their production possibilities: using normal (zero-mode) particles as initial states, such modes cannot be resonantly produced, nor can a single KK mode appear in the final states, but must be *pair-produced*. In addition, the conservation of the KK number leads to the appearance of heavy stable (charged and neutral) particles, which may pose cosmological complications (e.g. nucleosynthesis) [25]; however, one-loop effects may overcome such problems [26].

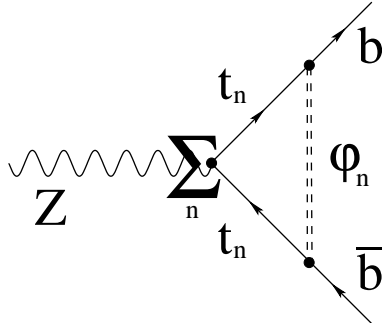


Figure 3: The sum goes over all particle in the loop which live in 5-D

It turns out that in the universal case the process  $Z \rightarrow b\bar{b}$  furnishes a less stringent bound compared to the non-universal one [25]. The main difference now is that in the relevant Feynman graph one should sum over *all* particles inside the loop, Fig. 3. The end result of the calculation is a lower bound for  $M_c$  of about 300 GeV.

## 5 $B$ - $\bar{B}$ mixing

In the SM, the mixing between the  $B^0$  meson and its anti-particle is also completely dominated by the top-quark contribution. The explicit  $m_t$  dependence of the box diagrams is given by the loop function [21]

$$S(x_t)_{\text{SM}} = \frac{x_t}{4} \left[ 1 + \frac{9}{1-x_t} - \frac{6}{(1-x_t)^2} - \frac{6x_t^2 \log(x_t)}{(1-x_t)^3} \right], \quad x_t \equiv \frac{m_t^2}{M_W^2}, \quad (12)$$

which contains the hard  $m_t^2$  term, i.e.  $x_t/4$ , induced by the longitudinal  $W$  exchanges. The same function controls the top-quark contribution to the  $K$ - $\bar{K}$  mixing parameter  $\varepsilon_K$ . The measured top-quark mass,  $m_t = 175$  GeV, implies  $S(x_t)_{\text{SM}} \sim 2.5$ .

Returning to the 5DSM model considered in section 3, the KK modes of the charged components of the doublet also contribute to the box diagram of Fig. 4. The total dominant contribution, SM plus KK modes, can be obtained by substituting the SM scalar propagator by the effective one of Eq. (8). However, since this modified propagator behaves as  $1/k_E$  for large  $k_E$ , and therefore, the insertion of two propagators of this type turns the box diagram into UV divergent. We write the correction to  $S(x_t)$  as

$$S(x_t) = S(x_t)_{\text{SM}} + \delta S(x_t), \quad \delta S(x_t) = \frac{x_t}{4} (G(a) - 1), \quad (13)$$

where  $G(a)$  is again the ratio of the non-standard to standard box integrals, i.e.

$$G(a) = 2a^2 \int_0^\infty dx \frac{x^3}{(1+x^2)^2} \coth^2(ax); \quad (14)$$

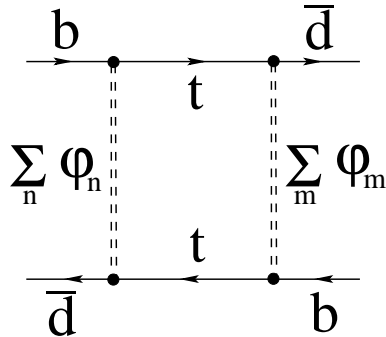


Figure 4: The box in the presence of KK corrections

it is clearly divergent for  $x \rightarrow \infty$ .

To evaluate  $G(a)$  we cut off the integral at  $x \approx n_s/a$ , where  $n_s$  is related to the scale at which new physics enters to regulate the five dimensional theory. In particular,  $M_s \sim n_s M_c$  and  $n_s \gg 1$ . We obtain

$$G(a) \approx 1 + a^2 \left( -1.14314 - \frac{4}{3} \log(a) + 2 \log(n_s) \right). \quad (15)$$

For moderate values of  $a \sim 0.2$  and  $n_s \sim 10$  the new physics correction is only about 0.2. For more extreme values (for instance  $a \sim 0.6$  and  $n_s \sim 100$ ), the corresponding contribution to  $G(a)$  is about 3. Notice also that the presence of diagrams with gauge boson KK modes could modify the bounds on  $M_c$  by a factor of about 20%. However, given the uncertainty in the calculation of the box diagrams due to the dependence on the scale  $M_s$ , estimating such effects seems superfluous. The important point, however, is that the contribution from extra dimensions to the function  $S(x_t)$  is always positive.

We can use the measured  $B_d^0 - \bar{B}_d^0$  mixing to infer the experimental value of  $S(x_t)$  and set a limit on  $\delta S(x_t)$ . The explicit dependence on the quark-mixing parameters can be resolved by combining the constraints from  $\Delta M_{B_d^0}$ ,  $\varepsilon_K$ , and  $\Gamma(b \rightarrow u)/\Gamma(b \rightarrow c)$ . In ref. [22] a complete analysis of the allowed values for  $S(x_t)$  was performed by varying all parameters in their allowed regions. The final outcome of such an analysis is that  $S(x_t)$  could take values within a rather large interval, namely  $1 < S(x_t) < 10$ . Since most of the errors come from uncertainties in theoretical calculations, it is rather difficult to assign confidence levels to the bounds quoted above. The lower limit is very stable under changes of parameters, while the upper limit could be modified by a factor of 2 by simply doubling some of the errors.

Given that the standard model value for  $S(x_t)$  is  $S(x_t)_{\text{SM}} = 2.5$ , *positive* contributions can be comfortably accommodated, whereas negative contributions are more constrained. As we have seen, extra dimensions result in *positive* contributions to  $S(x_t)$ ; in fact one can obtain values that could approach the upper limit of  $S(x_t)$  only for rather small values of the compactification scale  $M_c$  and large values of the scale of new physics,  $M_s$ . It seems



therefore that, at present, the above bounds do not provide good limits on  $M_c$ . On the other hand, if future experiments combined with theoretical improvements were to furnish a value for  $S(x_t)$  exceeding that of the SM, our analysis shows that such a discrepancy could easily be accommodated in models with large extra dimensions.

In conclusions, we have seen that the existing results from the processes  $Z \rightarrow b\bar{b}$  and  $B - \bar{B}$  mixing can furnish valuable lower bounds on the size of the possible extra dimensions. These bounds are model-dependent, varying between 0.3 GeV to 1 TeV, a fact which corroborates the expectation that the corresponding KK states should be well within the reach of the LHC.

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