CP violation through particle mixing and the H-A lineshape

J. Bernabéu[a], D. Binosi[b] and J. Papavassiliou[a]

[a]*ECT*, Villa Tambosi, Strada delle Tabarelle 286 I-38050 Villazzano (Trento), Italy*

[b]*Departamento de Fìsica Teòrica, and IFIC Centro Mixto,*

Universidad de Valencia-CSIC, E-46100, Burjassot, Valencia, Spain

Abstract

We consider the possibility of looking for CP-mixing effects in two-Higgs doublet models (and particularly in the MSSM) by studying the lineshape of the CP-even (H) and CP-odd (A) neutral scalars. In most cases H and A come quite degenerate in mass, and their s-channel production would lead to nearly overlapping resonances. CP-violating effects may connect these two Higgs bosons, giving origin to one-loop particle mixing, which, due to their mass proximity, can be resonantly enhanced. The corresponding transition amplitude contains then CP-even and CP-odd components; besides the signal of interereference between both amplitudes, leading to a CP-odd asymmetry, we propose to look for the mixing probability itself, a quantity which, although CP-even, can originate only from a CP-odd amplitude. We show that, in general, the effect of such a mixing probability cannot be mimicked by (or be re-absorbed into) a simple redefinition of the H and A masses in the context of a CP-conserving model. Specifically, the effects of the CP-mixing are such that, either the mass-splitting of the H and A bosons cannot be accounted for in the absence of CP-mixing, and/or the detailed energy dependence of the produced lineshape is clearly different from the one obtained by redefining the masses, but not allowing any mixing. This analysis suggests that the detailed study of the lineshape of this Higgs system may provide valuable information on the CP nature of the underlying theory.

PACS numbers: 11.30.Er,12.60.-i,14.80.Cp

I. INTRODUCTION

Despite the impressive success of the Standard Model (SM) in describing a plethora of high-precision experimental data [1], a number of important theoretical issues related to its structure remain largely unexplored. Perhaps the most prominent among them is the nature of the very mechanism which endows the elementary particles with their observed masses. The best established way for introducing masses at tree-level, without compromising renormalizability, is the Higgs mechanism [2, 3, 4], where the crucial ingredient is the coupling of all would-be massive particles to a complex scalar field with a non-vanishing vacuum expectation value. The most characteristic physical remnant of this procedure is a massive scalar particle in the spectrum of the theory, the Higgs boson. In addition to the minimal SM, the Higgs mechanism is employed in many popular new physics scenaria, most notably in the Minimal Supersymmetric Standard Model (MSSM) and its variants [5], leading to yet richer scalar sectors. However, to date, neither the Higgs boson of the minimal SM, nor the additional scalars predicted by its supersymmetric extensions have been observed, and their discovery and subsequent detailed study is considered as one of the main priorities for the upcoming collider experiments.

Several of the aforementioned scalars, and especially the "standard" Higgs boson, are expected to be discovered at the LHC [6]. In addition, in the past few years a significant amount of technical and theoretical research has been invested in the evaluation of the feasibility and physics potential of muon colliders [7, 8]. Such machines will have the particularly appealing feature of variable centre-of-mass energy, which will allow the resonant enhancement of s-channel interactions in order to produce Higgs bosons copiously. Thus, muon colliders are expected to operate as Higgs factories, in the energy range of up to .5 TeV. Given the additional features of small energy spread and precise energy determination, these machines offer a unique possibility for studying the line-shape of the SM Higgs boson, and determining its mass and width with an accuracy of about .1 MeV and .5 MeV, respectively. In addition, if supersymmetric particles will be discovered at the LHC, then the muon colliders would play the role of a precision machine for studying their main properties, such as masses, widths, and couplings, and delineating their line-shapes [9, 10, 11, 12, 13].

In the two-Higgs doublet models in general [14, 15, 16, 17, 18], and in most SUSY scenaria in particular [5], the extended scalar sector contains five physical fields: a couple

of charged Higgs bosons ($H^{\pm}$), a CP-odd scalar A, and two CP-even scalars h (the lightest, which is to be identified with the SM Higgs) and H (the heaviest). The detailed study of the H-A system is particularly interesting because in most *beyond the SM* scenarios (such as SUSY) the masses of these two particles are almost degenerate, and therefore their resonant production is expected to give rise to nearly overlapping resonances [9].

Specifically, the tree-level mass eigenvalues of the Higgs mass matrices are given by

$$
\begin{aligned}
m_{H^{\pm}}^2 &= m_A^2 + M_W^2, \\
m_{h,H}^2 &= \frac{1}{2}\left[M_Z^2 + m_A^2 \mp \sqrt{(M_Z^2 + m_A^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}\right],
\end{aligned}
\tag{1.1}
$$

and therefore one will have the tree-level bounds $M_Z \in (m_h, M_H)$ and $M_W < m_{H^{\pm}}$. Furthermore, in the decoupling limit $M_A \gg M_Z$, the neutral sector (tree-level) masses satisfy the relations [19]

$$
\begin{aligned}
m_h^2 &\approx M_Z^2 \cos^2 2\beta, \\
m_H^2 &\approx m_A^2 + M_Z^2 \sin^2 2\beta,
\end{aligned}
\tag{1.2}
$$

which, for $\tan\beta \geq 2$ (and thus $\cos^2 2\beta \approx 1$), imply the degeneracies $m_h \approx M_Z$ and $m_H \approx m_A$.

In general, the inclusion of radiative corrections is known to modify significantly the above tree-level relations, mainly due to the large Yukawa coupling of the top-quark. Such corrections are particularly important for the lower bound on the mass of the lightest Higgs boson h: for the largest bulk of the parameter space, h is in fact heavier than the Z boson, and can be as heavy as 130 GeV [20]. It is important to emphasize, however, that the inclusion of radiative corrections does *not* lift the mass degeneracy in the H-A system under consideration, especially in the parameter space region where $m_A > 2M_Z$ and $\tan\beta \geq 2$ [21]. In particular, as we will see in detail below, quantum effects do shift the values of m_H and m_A, but by amounts that are numerically comparable; as a result, the small mass splitting between H and A remains still valid (but is not described quantitatively by Eq. (1.2)).

In addition, as was shown by Pilaftsis [22], the near degeneracy of the H-A system may give rise to a resonant enhancement of CP violation due to particle mixing. Specifically, if the CP symmetry is exact, the H cannot mix with A, at any given order. However, in the presence of a CP-violating interactions [23, 24, 25, 26, 27, 28, 29, 30], in addition to a variety of interesting implications [31], the H can mix with the A already at one-loop level, giving rise to a non-vanishing mixing self-energy $\Pi_{HA}(s)$. Such mixing, in turn, can be measured
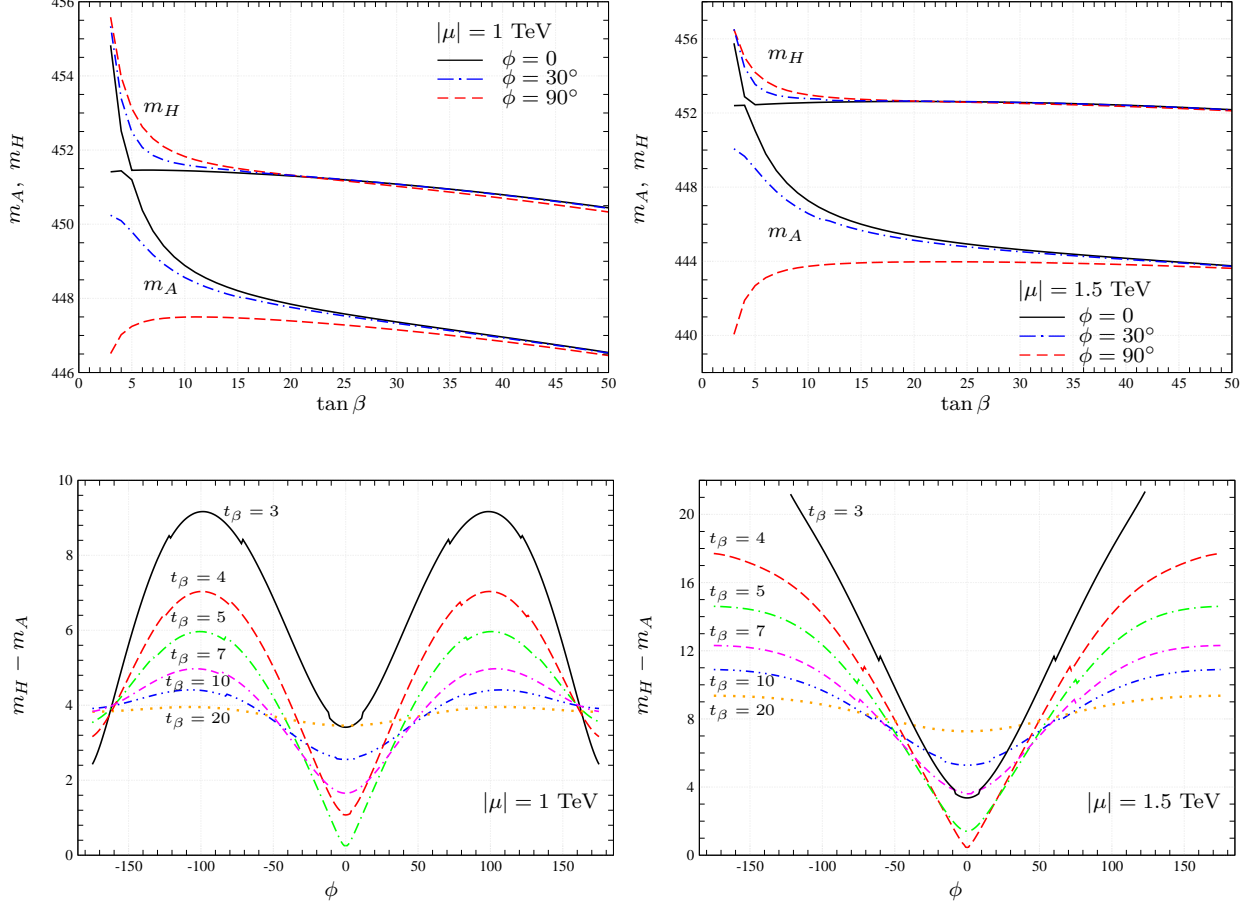
FIG. 1: *The mass spectrum of the $H - A$ system in the MSSM with and without CP-breaking effects. Upper panels: the variation of m_A and m_H as a function of $\tan\beta$, for different values of ϕ and $|\mu|$; Lower panels: the variation of $\delta m = m_H - m_A$ as a function of ϕ, for different values of $\tan\beta$ and $|\mu|$ (spikes appearing in the lower panels are code artifacts and should be ignored).*

through the study of appropriate CP-odd observables, such as left-right asymmetries. As has been explained in [22], in general the CP-violating amplitude is particularly enhanced near resonace, if the two mixing particles are nearly degenerate, a condition which is naturally fullfilled in the H-A system. Furthermore, the mixing between H and A has an additional profound effect for the two masses: the near degeneracy of the two particles is lifted, and the pole masses move further apart [32]; as a result, the originally overlapping resonances of the CP-invariant theory tend to be separated.

Using the code CPsuperH [33], one can study the $H - A$ system mass spectrum in the CP-invariant ($\phi = 0$) and CP-breaking ($\phi \neq 0$) limits of the MSSM. For our purposes we have assumed (motivated by SUGRA models) universal squarks soft masses M_0 (with

$M_0 = .5$ TeV), trilinear couplings A (with $|A| = 1$ TeV) and CP breaking phases $\phi_A = \phi$, with the MSSM μ parameter real and varying between 1.0 and 1.5 TeV. Finally we set $m_{H^\pm} = .4571$ TeV which sets the tree-level mass of the CP-odd Higgs at the value of .450 TeV. In the first two panels of Fig. 1, we plot the neutral CP-even and CP-odd scalar masses as a function of $\tan\beta$ for three values of the CP-breaking phase and two different values of μ ($|\mu| = 1$ TeV in the left panel, while $|\mu| = 1.5$ TeV in the right one). When the CP-breaking phases are absent (black continuous curves), the H-A resonance can be clearly seen at $\tan\beta = 5$ and $\tan\beta = 4$ respectively. The degeneracy is lifted when the phases are switched on (blue dashed-dotted and red dahsed curves, corresponding to $\phi = 30°$ and $\phi = 90°$ degrees, respectively), their effect becoming rapidly smaller for larger $\tan\beta$ values: from $\tan\beta \gtrsim 20$ they cannot be disentangled from radiative effects calculated in the CP-invariant limit of the theory. A more in depth view of how the CP-breaking phases affect the H-A mass spectrum can be achieved by studying the mass difference $\delta m = m_H - m_A$ versus ϕ for different values of $\tan\beta$, as it is shown in the two lower panels of Fig.1 (as before, $|\mu| = 1$ TeV in the left panel, while $|\mu| = 1.5$ TeV in the right one). In both cases we see that when $\tan\beta \gtrsim 20$ (orange dotted lines) the difference between the CP-invariant and CP-breaking case is barely visibile. There are some differences, however, depending on the value of the μ parameter. For $|\mu| = 1$ TeV, the mass difference shows a maximum around $\phi = 100°$; moreover in the CP-invariant limit the mass splitting is bigger for lower values of $\tan\beta$. When $|\mu| = 1.5$ TeV, instead, δm has no maximum and in the CP-invariant limit the splitting is bigger for larger $\tan\beta$ values.

The above analysis suggests that, when studying the lineshape of the H-A system, one may envisage two, physically very different, scenarios. In the first one, the CP symmetry is exact, with the position of resonances determined by Eq.(1.2) plus its radiative corrections; the relative position between the two resonance will then specify $\tan\beta$. In the second scenario, CP is violated, resulting in mixing at one-loop level between CP-even and CP-odd states wich translates into a non-vanishing off-shell transition amplitude $\Pi_{HA}(s)$. Then, for the same mass splitting δm as in the previous case, one may not reach the same conclusion for the value of $\tan\beta$, because one could have started out with the two masses almost degenerate, corresponding to a different value for $\tan\beta$ (lower or higher depending on the value of the μ parameter), and the observed separation between m_H and m_A may be due to the lifting of the degeneracy produced by the presence of the aforementioned $\Pi_{HA}(s)$.

To address these questions in detail, we then turn to the MSSM, in which the one-loop mixing between the CP-even and CP-odd Higgs will be due to third-family squarks circulating in the loop of Fig.2, and study the H-A lineshape, which will be derived explicitly at one-loop level for general values of $\Pi_{HH}(s)$, $\Pi_{AA}(s)$, and $\Pi_{HA}(s)$. Our main results may be then summarized as follows. In the cases for which the CP-breaking phases are sizeable (we will set $\phi = 90°$ for this case), one can always clearly distinguish between the CP-invariant and CP-beaking scenarios, either because the mass splitting in the latter case is just too big for being due to CP-invariant radiative corrections only, or because of the different energy dependence of the cross section. For smaller values of the CP-breaking phases (say $\phi = 30°$), one can still distinguish between the CP-invariant and CP-breaking case only when $\tan \beta$ is small; when $\tan \beta \gtrsim 10$ one cannot tell the two cases apart simply by studying the lineshape.

We therefore conclude that the experimental determination of the Higgs line-shape, in conjunction with CP-odd asymmetries and other suitable observables, may provide valuable information for settling the issue regarding CP-mixing effects in two-Higgs models.

II.   THE LINE-SHAPE IN THE PRESENCE OF H-A MIXING

In this section we will concentrate on the calculation of the line-shape for the resonant process $\mu^+ \mu^- \to A^*$, $H^* \to f \bar{f}$ in the presence of the CP-violating one-loop mixing diagrams of Fig.1 (a) between the CP-even (H) and the CP-odd (A) Higgses. The amplitude for such process can be written as

$$T = T^s + T^t = V_i^P \widehat{\Delta}_{ij} V_j^D + T^t, \tag{2.1}$$

where $i, j = A, H$, V_i^P (V_j^D) is the production (decay) amplitude of the process $\mu^+ \mu^- \to i$ ($j \to \bar{f} f$), $\widehat{\Delta}_{ij}$ is the propagator matrix in the presence of mixing, and T^t represents collectively all the t-channel and box diagrams.

The propagator matrix $\widehat{\Delta}_{ij}(s)$ appearing in the equation above is constructed from self-energies resummed in the pinch technique framework [34]; they are gauge-independent, display only physical thresholds, have the correct unitarity, analiticity, and renormalization group properties, and satisfy the equivalence theorem. In particular, one has

$$\widehat{\Delta}_{AA}(s) = [s - M_H^2 + \widehat{\Pi}_{HH}(s)]\widehat{\Delta}^{-1}(s),$$
$$\widehat{\Delta}_{HH}(s) = [s - M_A^2 + \widehat{\Pi}_{AA}(s)]\widehat{\Delta}^{-1}(s),$$

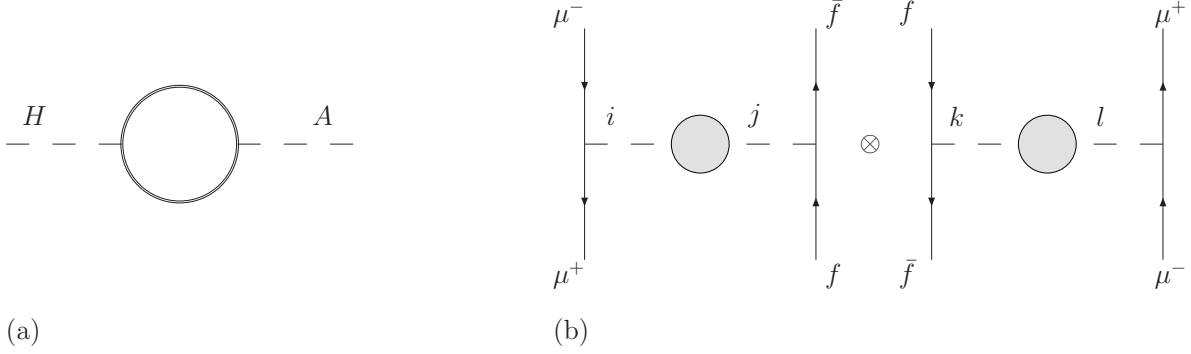(a)   (b)

FIG. 2: (a) *The one-loop AH mixing responsible for the CP-violating effects. Particles circulating in the loop may vary according to the model (in our case they are either squarks or heavy Majorana neutrinos). (b) The cross section for σ_{kl}^{ij} the resonant process $\mu^+\mu^- \to A^*$, $H^* \to f\bar{f}$. In the resonant region and at the one-loop level the only diagram contributing is the one shown. Notice that i, j, k, l may be either A or H (with A-H mixing due to the one-loop diagram (a)); however due to the chirality difference between A and H, the only non-zero combinations are obtained when $i = l$ and $j = k$.*

$$\widehat{\Delta}_{HA}(s) = \widehat{\Delta}_{AH}(s) = -\widehat{\Pi}_{AH}(s)\widehat{\Delta}^{-1}(s), \tag{2.2}$$

with $\widehat{\Delta}(s)$ the determinat of the propagator matrix, i.e.,

$$\widehat{\Delta}(s) = [s - M_H^2 + \widehat{\Pi}_{HH}(s)][s - M_A^2 + \widehat{\Pi}_{AA}(s)] - \widehat{\Pi}_{AH}(s)\widehat{\Pi}_{HA}(s). \tag{2.3}$$

In the resonant region on which we concentrate here, $M_A \approx M_H$, the t-channel and box diagrams are subdominant, with $T^t \ll T^s$, and can be safely ignored; moreover, at the one-loop level, the production and decay amplitudes V_i^P and V_j^D coincide with the corresponding tree-level (model dependent) vertices $\Gamma^{(0)}_{\mu^+\mu^-i} = \chi_{\mu i}$ and $\Gamma^{(0)}_{jf\bar{f}} = \chi_{jf}$. Finally, in the resonant region, the dispersive (real) part of the self-energies are very small, so that one can neglect them and concentrate only on their absorptive (imaginary) parts. The cross sections leading to the lineshape we look for, can then be calculated by just squaring the single diagram shown in Fig. 2 (b); helicity mismatches limit the possible combinations to the factors

$$
\begin{aligned}
D_{AA}^{AA}(s) &= \left[(s - M_H^2)^2 + (\mathrm{Im}\,\widehat{\Pi}_{HH}(s))^2\right] D^{-1}(s), \\
D_{HH}^{HH}(s) &= \left[(s - M_A^2)^2 + (\mathrm{Im}\,\widehat{\Pi}_{AA}(s))^2\right] D^{-1}(s), \\
D_{HA}^{AH}(s) &= D_{AH}^{HA} = \widehat{\Pi}_{AH}^2(s)D^{-1}(s),
\end{aligned}
\tag{2.4}
$$

with

$$D(s) = \left[(s - M_A^2)(s - M_H^2) - \mathrm{Im}\,\widehat{\Pi}_{AA}(s)\mathrm{Im}\,\widehat{\Pi}_{HH}(s) - \widehat{\Pi}_{AH}^2(s)\right]^2$$
$$+ \left[(s - M_A^2)\mathrm{Im}\,\widehat{\Pi}_{HH}(s) + (s - M_H^2)\mathrm{Im}\,\widehat{\Pi}_{AA}(s)\right]^2. \tag{2.5}$$

In the formulas above, the standard thresholds are given by

$$\mathrm{Im}\,\widehat{\Pi}_{HH}^{(f\bar{f})}(s) = \frac{\alpha_\mathrm{w} N_c^f}{8}\chi_{Hf}^2 \frac{m_f^2}{M_W^2} s \left(1 - \frac{4m_f^2}{s}\right)^{3/2} \theta(s - 4m_f^2),$$

$$\mathrm{Im}\,\widehat{\Pi}_{AA}^{(f\bar{f})}(s) = \frac{\alpha_\mathrm{w} N_c^f}{8}\chi_{Af}^2 \frac{m_f^2}{M_W^2} s \left(1 - \frac{4m_f^2}{s}\right)^{1/2} \theta(s - 4m_f^2),$$

$$\mathrm{Im}\,\widehat{\Pi}_{HH}^{(VV)}(s) = \frac{n_V \alpha_\mathrm{w}}{32}\chi_{HV}^2 \frac{M_H^4}{M_W^2} \left(1 - \frac{4M_V^2}{s}\right)^{1/2}$$

$$\times \left[1 + 4\frac{M_V^2}{M_H^2} - 4\frac{M_V^2}{M_H^4}(2s - 3M_V^2)\right]\theta(s - 4M_V^2), \tag{2.6}$$

where, $\alpha_\mathrm{w} = g^2/4\pi$, $n_V = 2$ (respectively 1) for $V \equiv W$ (respectively $V \equiv Z$), and $N_c^f = 1$ for leptons and 3 for quarks. Restoring the couplings, summing over final states and averaging over initial polarizations, one is then left with the cross-sections

$$\sigma(s) = \sum_{i,j} \sigma_{ji}^{ij}(s) = \frac{\pi \alpha_\mathrm{w}^2 m_\mu^2 m_b^2}{16 M_W^4} s \sum_{i,j} \chi_{i\mu}^2 \chi_{jb}^2 D_{ji}^{ij}(s), \tag{2.7}$$

where we have specialized to the case in which the final fermions are bottom quarks.

III. NUMERICAL RESULTS

In this section we present numerical results for the cross section of Eq.(2.7), in order to see how CP violating effects through the one-loop mixing of Fig.2 (a) affects the Higgs line-shape.

As anticipated in the introduction, the MSSM the Higgs sector is composed by two Higgs doublets (A_1, A_2) and (H_1, H_2). After performing an orthogonal rotation by the angles β (with $\tan\beta = v_2/v_1$ and v_i the VEVs of the Higgs doublets) and α (with $\tan\alpha \propto \tan\beta$), the doublets end up in the physical basis (G^0, A) and (h, H), with masses m_A, m_h and m_H (G_0 turns out to be the true would-be Goldstone boson, absorbed by the longitudinal part of the Z boson). Here we use the same conventions of [30], to which the reader is referred for details concerning the model [and the model dependent factor of, for example, Eq.(2.6)].

It turns out that the AH mixing is dominated by the large CP-violating Yukawa couplings to the top and bottom squarks, for which one has (see again [30], where all the definitions of the quantities appearing in the formula below can be found)

$$\Pi_{H_iA}(s) = \frac{1}{16\pi^2} \sum_{q=t,b} N_c^q \, \text{Im}(h_1^q) \left\{ \frac{r_i s_{2q}}{s_\beta v} \Delta M_{\tilde{q}}^2 \left[B_0(s, M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2) - B_0(0, M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2) \right] \right.$$
$$+ \, \text{Re}(h_i^q) \frac{s_{2q}^2}{s_\beta} \left[B_0(s, M_{\tilde{q}_1}^2, M_{\tilde{q}_1}^2) + B_0(s, M_{\tilde{q}_2}^2, M_{\tilde{q}_2}^2) - 2B_0(s, M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2) \right]$$
$$+ \, \frac{s_{2q}}{s_\beta} \left\{ (c_q^2 g_i^{L,q} + s_q^2 g_i^{R,q}) \left[B_0(s, M_{\tilde{q}_1}^2, M_{\tilde{q}_1}^2) - B_0(s, M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2) \right] \right.$$
$$+ \left. \left. (s_q^2 g_i^{L,q} + c_q^2 g_i^{R,q}) \left[B_0(s, M_{\tilde{q}_2}^2, M_{\tilde{q}_2}^2) - B_0(s, M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2) \right] \right\} \right\}, \tag{3.1}$$

with $s = q^2$, and $B_0(s, m_1, m_2)$ the standard Passarino-Veltman function defined as

$$B_0(s, m_1, m_2) = C_{\text{UV}} + 2 - \ln(m_1 m_2) + \frac{1}{s} \left[(m_2^2 - m_1^2) \ln\left(\frac{m_1}{m_2}\right) \right.$$
$$+ \left. \sqrt{\lambda(s, m_1^2, m_2^2)} \cosh^{-1}\left(\frac{m_1^2 + m_2^2 - s}{2m_1 m_2}\right) \right],$$
$$B_0(0, m_1, m_2) = C_{\text{UV}} + 1 - \ln(m_1 m_2) + \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \ln\left(\frac{m_2}{m_1}\right), \tag{3.2}$$

where $C_{\text{UV}} = 1/\epsilon - \gamma_E + \ln(4\pi\mu^2)$ is the UV cutoff of dimensional regularization, and $\lambda(x, y, z) = (x - y - z)^2 - 4yz$.

As emphasized in [30], the tadpole contribution $B_0(0, M_{\tilde{q}_1}^2, M_{\tilde{q}_2}^2)$ is crucial for accomplishing the UV-finiteness of the H_iA self-energies in question. Notice also that the $\Pi_{H_iA}(s)$ given by (3.1) are real, because, due to the heaviness of the squarks appearing in the loops, no particle thresholds can open. The CP violating self-energy transition HA is then obtained through an orthogonal rotation of the above result, giving

$$\Pi_{HA}(s) = -\sin\alpha \, \Pi_{H_1A}(s) + \cos\alpha \, \Pi_{H_2A}(s). \tag{3.3}$$

Notice that hA mixing is also possible, but $m_h \ll m_A$, m_H and therefore its contribution is negligible, since one is far from the resonant enhancement region.

The kinematic regime analized in this section is that where $m_A > 2M_Z$ and $\tan\beta \geq 2$; then since $\tan\beta \approx \tan\alpha$, Eq.(3.3) reduces to $\Pi_{HA}(s) \approx -\Pi_{H_1A}(s)$. Also, the rotation angle of the scalar top and bottom quarks are equal to $\pi/4$, i.e. $s_{2q} \approx 1$. As already mentioned before we assume universal squarks soft masses ($\tilde{M}_Q = \tilde{M}_t = \tilde{M}_b = M_0 = .5$ TeV) and trilinear couplings ($A_t = A_b = A$ with $|A| = 1$). Finally $m_{H^\pm}$ will be fixed at the value of .4571 TeV.
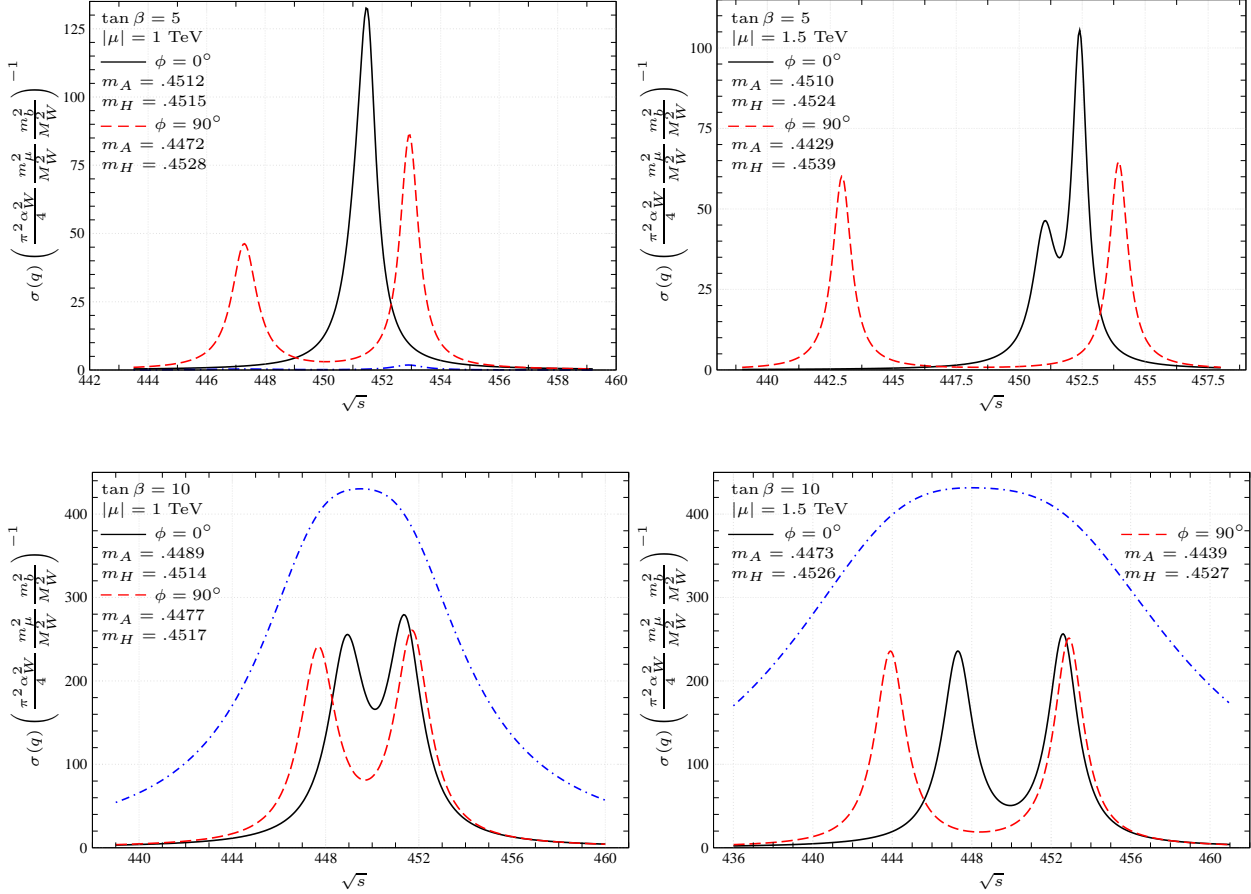
FIG. 3: *The MSSM Higgs lineshape for different values of $|\mu|$, $\tan\beta$ and ϕ. The black continuous curves correspond to the CP-invariant limit of the theory ($\phi = 0°$), while the red dashed one correspond to the case in which CP-breaking phases have been switched on (with $\phi = 90°$). Finally, the dashed-dotted blue curve, when present, correspond to the CP-invariant limit of the theory re-calculated for a different value of $\tan\beta$ to accommodate the mass-splitting observed in the CP-breaking case.*

For relatively big values of the CP-breaking phase (we set $\phi = 90°$) our results are shown in Fig.3.

As anticipated in the analysis carried out in Section I, we can see that as $\tan\beta$ decreases (upper panels, where $\tan\beta = 5$), the difference between the two line-shapes (the CP-invariant limit, black continuous curves, and the CP-breaking phase, red dashed curves) increases, the overlapping resonance get resolved and the two mass peaks are clearly separated. At this point one can determine directly from the lineshape the new pole mass of the Higgs bosons, which turn out to be always in good agreement with the masses ob-
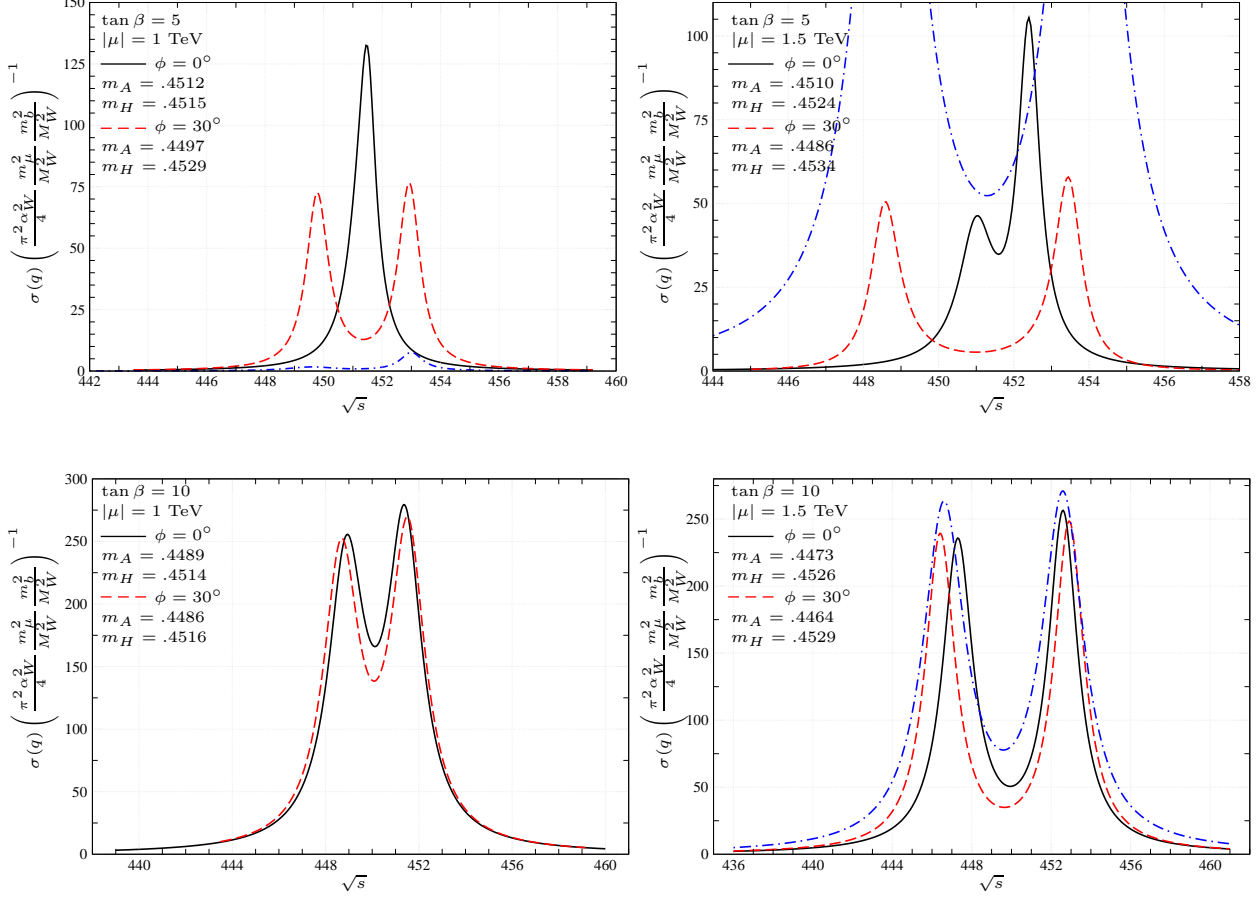
FIG. 4: *Same as before but for the CP-breaking phase which is now set to $\phi = 30°$. Notice that in the upper left panel, the same masses could have been also reached from a CP-invariant theory with $\tan\beta \sim 25$, which however would give a much higher cross section which is not shown.*

tained algebraically by re-diagonalizing the one-loop propagator matrix, *i.e.* by solving the characteristic equation $\Delta(s) = 0$, yielding

$$s_i = M_i^2 - \widehat{\Pi}_{ii}(s_i) + \frac{\widehat{\Pi}_{ij}(s_i)\widehat{\Pi}_{ji}(s_i)}{s_i - M_j^2 + \widehat{\Pi}_{jj}(s_i)}, \qquad (3.4)$$

where $i, j = A, H$ and $i \neq j$ [32].

In particular, notice that when $|\mu| = 1.5$ TeV(upper-right panel) the mass splitting $\delta m \sim 11$ GeV and cannot be explained in terms of CP-invariant radiative corrections only, no matter how big $\tan\beta$ is chosen. In the remaining panels, δm allows for smaller (upper-left panel, $\tan\beta \sim 2.5$) or larger (lower panels, left $\tan\beta \sim 20$ and right $\tan\beta \sim 30$) values of $\tan\beta$ which are consequently plotted in th blue dashed-dotted curves. Also in these cases one can distinguish the CP-invariant and CP-breaking phase of the theory due to the very

11

different energy behavior of the cross section; also notice that too lower values of $\tan\beta$ predicts, in the parameter range chosen here, a too low mass for the lightest Higgs, and therefore would be in any case excluded (*e.g.* when $\tan\beta \sim 2.5$, $m_h = 94.8$ GeV). For smaller values of the CP-breaking phase ϕ we get a somewhat different situation. In Fig.4, the same set of plots as before is shown for $\phi = 30°$: for small values of $\tan\beta$ (upper panels) it is still possible to discriminate the phase of the theory, while for larger values (lower panels) the effects of $A - H$ mixing vanishes much more rapidly than in the previous case.

IV. CONCLUSIONS

The CP-even and CP-odd neutral Higgs scalars, H and A, respectively, together with their charged companions and the "standard" Higgs boson, constitute the extended scalar sector of generic two-Higgs doublet models, and appear naturally in supersymmetric extensions of the SM. Therefore it is expected that their discovery and subsequenet study of their fundamental physical properties should be of central importance in the next decades. In addition to the LHC, where their primary discovery might take place, further detailed studies of their characteristics have been proposed in recent years, most notably in the context of muon-[7, 8] and photon colliders [35, 36].

Of particular interest is the possibility of encountering CP violation in the Higgs sector of two-Higgs doublet models, induced by the one-loop mixing of H and A. The latter is produced due to CP-violating (tree-level) interactions of each one of these two Higgs bosons with particles such as scalar quarks or Majorana neutrinos, to name a few, which will circulate in the (otherwise vanishing) H-A transition amplitude (Fig.1).

A most appealing feature of the H-A system is that, due to their relative small mass-difference, possible CP-mixing effects undergo resonant enhancement [22]. Such effect may be observed in appropriately constructed left-right asymmetries, as proposed in [22]. Since these observables are odd under CP, a non-vanishing experimental result would constitute an unequivocal signal of CP violation.

In this paper we have explored the possibility of detecting the presence of CP-mixing between H and A through the detailed study of the cross-section of the s-channel process $\mu^+\mu^- \to A^*, H^* \to f\bar{f}$ as function of the center-of-mass energy. Although the lineshape is a CP-even quantity, there are certain characteristics that signal the presence of a CP-violating

12

mixing. In the context of the photon colliders, Ref.[36] also discusses both CP-even and CP-odd observables in terms of photon polarizations.

As has been explained in a series of papers [9, 10, 11, 12, 13], the high-resolution scanning of the lineshape of this process is expected to reveal two relatively closely spaced, or even superimposed, resonances, corresponding to the nearly degenerate H and A. As was recognized in [32], the presence of CP-mixing between H and A modifies the position of the pole mass, and tends to push the two resonances further apart. Of course, this fact by itself could not serve as a signal for the presence of CP-mixing; one could simply envisage the situation where all fundamental interactions are CP-conserving and there is no CP-mixing, and the masses of H and A assume simply from the beginning their shifted values.

The result of our analysis is very positive and represents, in our opinion, a significant step in the search of the CP-properties of two scalar-doublet models: *either the mass-splitting of the H and A Higgs boson, in a CP-mixing scenario, cannot be accounted for in absence of CP-mixing, and/or the detailed energy dependence of the line-shape allows to discriminate between both scenarios.*

Even though in our analysis we have considered only the MSSM due to its phenomenological relevance, we however expect that our analysis will hold in general for two Higgs doublet models, independently from the specific type of mechanism which will give rise to the CP-breaking loops of Fig.2 (a). For example similar results are expected in the case of a two Higgs doublet model in which the particles circulating in the loop are three generations of heavy Majorana neutrinos N_i ($i = 1, 2, 3$) for which the mixing amplitude will be given by [22]

$$\Pi_{AH}(s) = -\frac{\alpha_w \, s}{4\pi} \chi_{Au} \chi_{Hu} \sum_{j>i}^{3} \mathrm{Im}(C_{ij}^2) \sqrt{\lambda_i \lambda_j} \left[B_0(s/M_W^2, \lambda_i, \lambda_j) + 2 B_1(s/M_W^2, \lambda_i, \lambda_j) \right],$$

$$B_1(s, m_i, m_j) = \frac{m_j^2 - m_i^2}{2s} B_0(s, m_i, m_j) - B_0(0, m_i, m_j) - \frac{1}{2} B_0(s, m_i, m_j), \qquad (4.1)$$

with $\lambda_i = m_i^2/M_W^2$, and $\mathrm{Im}(C_{ij}^2)$ playing the role of the MSSM CP-breaking phase ϕ.

Thus, the combined study of CP-odd observables, such as left-right asymmetries, together with the H-A lineshape, may furnish a powerful panoply for attacking the isssue of CP violation in two-Higss doublet models.

---

[1] See, for example, S. Eidelman *et al.* [Particle Data Group], Phys. Lett. B **592** (2004); G. Altarelli and M. W. Grunewald, Phys. Rept. **403-404**, 189 (2004); [arXiv:hep-ph/0404165]. A. Sirlin, J. Phys. G **29**, 213 (2003) [arXiv:hep-ph/0209079].

[2] P. W. Higgs, Phys. Lett. **12**, 132 (1964).

[3] F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964).

[4] G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964).

[5] For reviews, see, H. P. Nilles, Phys. Rept. **110** (1984) 1; H. E. Haber and G. L. Kane, Phys. Rept. **117** (1985) 75; A. B. Lahanas and D. V. Nanopoulos, Phys. Rept. **145**, 1 (1987); J. F. Gunion, H. E. Haber, G. L. Kane and S. Dawson, "The Higgs Hunter's Guide," (Addison-Wesley, Reading, MA, 1990).

[6] See, for example, J. Haller [ATLAS Collaboration], arXiv:hep-ex/0512042; J. Tanaka, Nucl. Phys. Proc. Suppl. **144** (2005) 341; R. Kinnunen, S. Lehti, A. Nikitenko and P. Salmi, J. Phys. G **31**, 71 (2005); M. Schumacher, arXiv:hep-ph/0410112; U. Ellwanger, J. F. Gunion, C. Hugonie and S. Moretti, arXiv:hep-ph/0401228.

[7] D. B. Cline, AIP Conf. Proc. **352**, 70 (1996).

[8] W. J. Marciano, AIP Conf. Proc. **441**, 347 (1998).

[9] V. D. Barger, M. S. Berger, J. F. Gunion and T. Han, Phys. Rept. **286**, 1 (1997) [arXiv:hep-ph/9602415].

[10] T. Binoth and A. Ghinculov, Phys. Rev. D **56**, 3147 (1997) [arXiv:hep-ph/9704299].

[11] M. Krawczyk, arXiv:hep-ph/9803484.

[12] B. Grzadkowski, arXiv:hep-ph/0005170.

[13] C. Blochinger *et al.*, arXiv:hep-ph/0202199.

[14] T. D. Lee, Phys. Rev. D **8**, 1226 (1973).

[15] G. C. Branco and M. N. Rebelo, Phys. Lett. B **160**, 117 (1985).

[16] J. Liu and L. Wolfenstein, Nucl. Phys. B **289**, 1 (1987).

[17] S. Weinberg, Phys. Rev. D **42**, 860 (1990).

[18] Y. L. Wu and L. Wolfenstein, Phys. Rev. Lett. **73**, 1762 (1994)

[19] See, for instance, the last item of [5], and references therein, and H. E. Haber, arXiv:hep-ph/9707213.

[20] S.P. Li and M. Sher, Phys. Lett. **B140**, 339 (1984); Y. Okada, M. Yamaguchi and T. Yanagida, Phys. Lett. **B262**, 54 (1991); H.E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991); J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. **B257**, 83 (1991); R. Barbieri, M. Frigeni and F. Caravaglios, Phys. Lett. **B258**, 167 (1991); D.M. Pierce, J.A. Bagger, K.T. Matchev and R.-J. Zhang, Nucl. Phys. **B491**, 3 (1997).

[21] See, *e.g.*, G.L. Kane, C. Kolda, L. Roszkowksi and J.D. Wells, Phys. Rev. **D49**, 6173 (1994).

[22] A. Pilaftsis, Nucl. Phys. B **504**, 61 (1997)

[23] J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez and J. W. F. Valle, Phys. Lett. B **187**, 303 (1987).

[24] A. Pilaftsis, Phys. Lett. B **285**, 68 (1992).

[25] J. G. Korner, A. Pilaftsis and K. Schilcher, Phys. Rev. D **47**, 1080 (1993); Phys. Lett. B **300**, 381 (1993).

[26] J. Bernabeu, J. G. Korner, A. Pilaftsis and K. Schilcher, Phys. Rev. Lett. **71**, 2695 (1993).

[27] A. Ilakovac, B. A. Kniehl and A. Pilaftsis, Phys. Lett. B **317**, 609 (1993) [Phys. Lett. B **320**, 329 (1994)]. [arXiv:hep-ph/9308318].

[28] J. Bernabeu and A. Pilaftsis, Phys. Lett. B **351**, 235 (1995).

[29] D. Tommasini, G. Barenboim, J. Bernabeu and C. Jarlskog, Nucl. Phys. B **444**, 451 (1995).

[30] A. Pilaftsis, Phys. Lett. B **435**, 88 (1998)

[31] A. Pilaftsis, Phys. Rev. D **58**, 096010 (1998); A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **553**, 3 (1999); D. A. Demir, Phys. Rev. D **60**, 055006 (1999); S. Y. Choi, M. Drees and J. S. Lee, Phys. Lett. B **481**, 57 (2000); M. Carena, J. R. Ellis, A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **586**, 92 (2000); G. L. Kane and L. T. Wang, Phys. Lett. B **488**, 383 (2000); S. Y. Choi, K. Hagiwara and J. S. Lee, Phys. Rev. D **64**, 032004 (2001); T. Ibrahim and P. Nath, Phys. Rev. D **63**, 035009 (2001).

[32] M. Carena, J. R. Ellis, A. Pilaftsis and C. E. Wagner, Nucl. Phys. B **625**, 345 (2002).

[33] J. S. Lee, A. Pilaftsis, M. Carena, S. Y. Choi, M. Drees, J. R. Ellis and C. E. M. Wagner, Comput. Phys. Commun. **156**, 283 (2004)

[34] J. Papavassiliou and A. Pilaftsis, Phys. Rev. Lett. **75**, 3060 (1995); Phys. Rev. D **53**, 2128 (1996); Phys. Rev. D **54**, 5315 (1996); D. Binosi and J. Papavassiliou, Phys. Rev. D **66**,

076010 (2002); D. Binosi, J. Phys. G **30**, 1021 (2004).

[35] I. F. Ginzburg, M. Krawczyk and P. Osland, arXiv:hep-ph/0101331; Nucl. Instrum. Meth. A **472**, 149 (2001); arXiv:hep-ph/0101208; W. Khater and P. Osland, Nucl. Phys. B **661**, 209 (2003); W. Bernreuther, A. Brandenburg and M. Flesch, arXiv:hep-ph/9812387; P. Niezurawski, A. F. Zarnecki and M. Krawczyk, JHEP **0502**, 041 (2005); M. Krawczyk, arXiv:hep-ph/0512371.

[36] S. Y. Choi, J. Kalinowski, Y. Liao and P. M. Zerwas, Eur. Phys. J. C **40**, 555 (2005).

[37] D. Binosi and L. Theussl, Comput. Phys. Commun. **161**, 76 (2004).