Gluon mass and freezing of the QCD coupling

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Abstract.

Infrared finite solutions for the gluon propagator of pure QCD are obtained from the gauge-invariant non-linear Schwinger-Dyson equation formulated in the Feynman gauge of the background field method. These solutions may be fitted using a massive propagator, with the special characteristic that the effective "mass" employed drops asymptotically as the inverse square of the momentum transfer, in agreement with general operator-product expansion arguments. Due to the presence of the dynamical gluon mass the strong effective charge extracted from these solutions freezes at a finite value, giving rise to an infrared fixed point for QCD.

1. Introduction

The systematic study of Schwinger-Dyson equations (SDE) in the framework of the pinch technique (PT) has led to the conclusion that the non-perturbative QCD dynamics generate an effective, mometum-dependent mass for the gluon, while preserving the local $SU(3)_c$ invariance of the theory [1, 2, 3]. This picture is further corroborated by lattice simulation and a variety of theoretical and phenomenological works [4]. One of the most important consequences of this picture is that this dynamical mass tames the Landau singularity associated with the perturbative β function, giving rise to a strong effective charge "freezing" at a finite value in the infrared. In this talk we report recent progress in the study of a non-linear SDE for the gluon propagator [3].

2. The non-linear SDE

The relevant SDE for $\Delta_{\mu\nu}(q)$ is shown in Fig.(1). Due to the special properties of the truncation scheme based on the PT [1, 5](and its connection with the Feynman gauge of the background field method (BFM) [6]), this equation is gauge-invariant despite the omission of ghost loops or higher order graphs [2]. Dropping for simplicity the longitudinal momenta, i.e. setting $\Delta_{\mu\nu}(q) = -ig_{\mu\nu}\Delta(q^2)$, one looks for solutions where $\Delta(q^2)$ reaches a finite (non-vanishing) value in the deep infrared; such solutions may be fitted by "massive" propagators of the form $\Delta^{-1}(q^2) = q^2 + m^2(q^2)$, where $m^2(q^2)$ is not "hard", but depends non-trivially on the momentum transfer q^2. The tree-level expressions for the three- and four-gluon vertices appearing in the two graphs of Fig.(1) are given in the first item of [6]. For the full three-gluon vertex, $\widetilde{\rm I\!\Gamma}$, denoted by the white blob in graph (a_1), we employ a gauge technique Ansatz, expressing it as a functional of Δ, in such a way as to satisfy (by construction) the all-order Ward identity

$$q^{\mu}\widetilde{\rm I\!\Gamma}_{\mu\alpha\beta}(q,p_1,p_2) = i[\Delta^{-1}_{\alpha\beta}(p_1) - \Delta^{-1}_{\alpha\beta}(p_2)]\,, \tag{1}$$

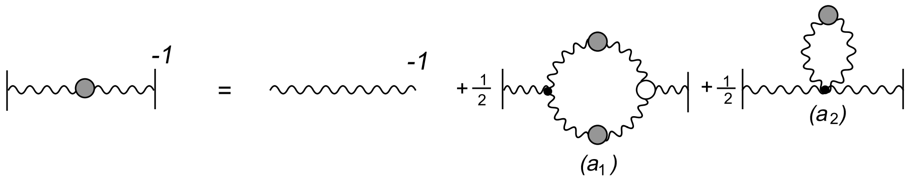

Figure 1. The gluonic "one-loop dressed" contributions to the SDE.

characteristic of the PT-BFM. Specifically, we use the following closed form for the vertex [3]:

$$
\begin{aligned}
\widetilde{\mathbb{I}\Gamma}^{\mu\alpha\beta} &= \widetilde{\Gamma}^{\mu\alpha\beta} + ig^{\alpha\beta}\frac{q^\mu}{q^2}\left[\Pi(p_2)-\Pi(p_1)\right] - i\frac{c_1}{q^2}\left(q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta}\right)\left[\Pi(p_1)+\Pi(p_2)\right] \\
&\quad - ic_2\left(q^\beta g^{\mu\alpha} - q^\alpha g^{\mu\beta}\right)\left[\frac{\Pi(p_1)}{p_1^2}+\frac{\Pi(p_2)}{p_2^2}\right]. \qquad (2)
\end{aligned}
$$

with $\widetilde{\Gamma}_{\mu\alpha\beta}(q,p_1,p_2) = (p_1-p_2)_\mu g_{\alpha\beta} + 2\,q_\beta g_{\mu\alpha} - 2\,q_\alpha g_{\mu\beta}$, and $i\Pi(q^2) = \Delta^{-1}(q^2) - q^2$.

Defining the renormalization-group invariant quantity [5] $d(q^2) = g^2\Delta(q^2)$, we arrive at

$$
d^{-1}(x) = K'x + \tilde{b}\sum_{i=1}^{8}\widehat{A}_i(x) + d^{-1}(0)\,, \qquad (3)
$$

with

$$
\begin{aligned}
\widehat{A}_1(x) &= -\left(1+\frac{6c_2}{5}\right)x\int_x^\infty dy\, y\,\mathcal{L}^2(y)d^2(y)\,, \\
\widehat{A}_2(x) &= \frac{6c_2}{5}x\int_x^\infty dy\,\mathcal{L}(y)d(y)\,, \\
\widehat{A}_3(x) &= -\left(1+\frac{6c_2}{5}\right)x\,\mathcal{L}(x)d(x)\int_0^x dy\, y\,\mathcal{L}(y)d(y)\,, \\
\widehat{A}_4(x) &= \left(-\frac{1}{10}-\frac{3c_2}{5}+\frac{3c_1}{5}\right)\int_0^x dy\, y^2\,\mathcal{L}^2(y)d^2(y)\,, \\
\widehat{A}_5(x) &= -\frac{6}{5}\Big(1+c_1\Big)\mathcal{L}(x)d(x)\int_0^x dy\, y^2\,\mathcal{L}(y)d(y)\,, \\
\widehat{A}_6(x) &= \frac{6c_2}{5}\int_0^x dy\, y\,\mathcal{L}(y)d(y)\,, \\
\widehat{A}_7(x) &= \frac{2}{5}\mathcal{L}(x)\frac{d(x)}{x}\int_0^x dy\, y^3\,\mathcal{L}(y)d(y)\,, \\
\widehat{A}_8(x) &= \frac{1}{5x}\int_0^x dy\, y^3\,\mathcal{L}^2(y)d^2(y)\,, \qquad (4)
\end{aligned}
$$

where $x=q^2$. The renormalization constant K' is fixed by the condition $d^{-1}(\mu^2)=\mu^2/g^2$, (with $\mu^2 \gg \Lambda^2$), and $\mathcal{L}(q^2) \equiv \tilde{b}\ln\left(q^2/\Lambda^2\right)$, where Λ is QCD mass scale. Due to the poles contained in the Ansatz for $\widetilde{\mathbb{I}\Gamma}^{\mu\alpha\beta}$, $d^{-1}(0)$ does not vanish, and is given by the (divergent) expression

$$
d^{-1}(0) = \frac{3\tilde{b}}{5\pi^2}\left[2(1+c_1)\int d^4k\,\mathcal{L}(k^2)\,d(k^2) - (1+2c_1)\int d^4k\,k^2\,\mathcal{L}^2(k^2)\,d^2(k^2)\right], \qquad (5)
$$

which can be made finite using dimensional regularization, and assuming that $m^2(q^2)$ drops sufficiently fast in the UV [2].

3. Results

The way to extract from $d(q^2)$ the corresponding $m^2(q^2)$ and $g^2(q^2)$ is by casting the numerical solutions into the form [1]

$$d(q^2) = \frac{g^2(q^2)}{q^2 + m^2(q^2)}\,, \quad g^2(q^2) = \left[\tilde{b}\ln\left(\frac{q^2 + f(q^2, m^2(q^2))}{\Lambda^2}\right)\right]^{-1}. \tag{6}$$

with

$$f(q^2, m^2(q^2)) = \rho_1 m^2(q^2) + \rho_2 \frac{m^4(q^2)}{q^2 + m^2(q^2)} + \rho_3 \frac{m^6(q^2)}{[q^2 + m^2(q^2)]^2}\,, \tag{7}$$

The functional form used for the running mass is

$$m^2(q^2) = \frac{m_0^4}{q^2 + m_0^2}\left[\ln\left(\frac{q^2 + \rho\, m_0^2}{\Lambda^2}\right) \Big/ \ln\left(\frac{\rho\, m_0^2}{\Lambda^2}\right)\right]^{\gamma_2 - 1}\,, \tag{8}$$

where $\gamma_2 = \frac{4}{5} + \frac{6c_1}{5}$; ρ, ρ_1, ρ_2, and ρ_3 are adjustable constants. Evidently, $m^2(q^2)$ is dropping in the deep ultraviolet as an *inverse power* of the momentum, as expected from general operator-product expansion calculations [7]. Note that $f(q^2, m^2(q^2))$ is such that $f(0, m^2(0)) > 0$; as a result, $g^2(q^2)$ reaches a finite positive value at $q^2 = 0$, leading to an *infrared fixed point* [1, 8, 9].

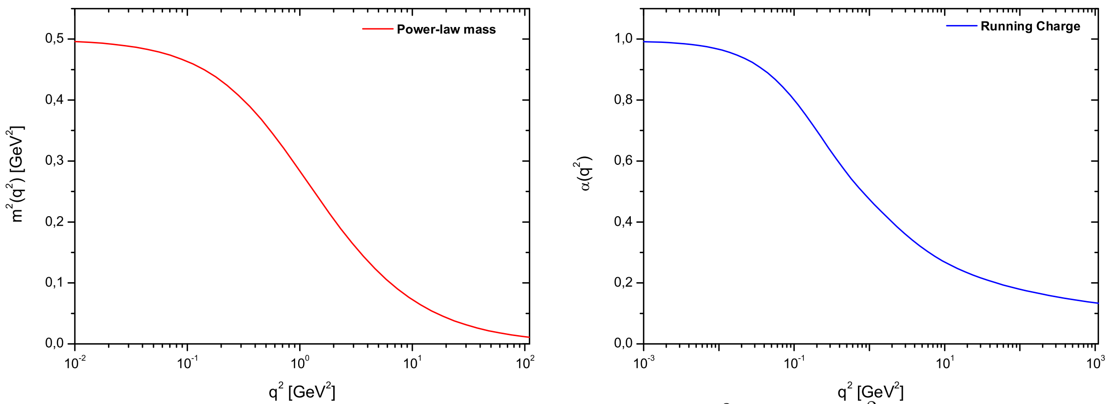

Figure 2. Left: dynamical mass with power-law running, for $m_0^2 = 0.5$ GeV ^{2} and $\rho = 1.046$ in Eq.(8). Right: the running charge, $\alpha(q^2) = g^2(q^2)/4\pi$.

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References

[1] J. M. Cornwall, Phys. Rev. D **26**, 1453 (1982); J. M. Cornwall and W. S. Hou, Phys. Rev. D **34**, 585 (1986).
[2] A. C. Aguilar and J. Papavassiliou, JHEP **0612**, 012 (2006).
[3] A. C. Aguilar and J. Papavassiliou, arXiv:0708.4320 [hep-ph].
[4] For an extensive list of citations, see [2].
[5] J. M. Cornwall and J. Papavassiliou, Phys. Rev. D **40** (1989) 3474; D. Binosi and J. Papavassiliou, Phys. Rev. D **66**, 111901 (2002); J. Phys. G **30**, 203 (2004).
[6] L. F. Abbott, Nucl. Phys. B **185**, 189 (1981); R. B. Sohn, Nucl. Phys. B **273**, 468 (1986); A. Hadicke, JENA-N-88-19.
[7] M. Lavelle, Phys. Rev. D **44**, 26 (1991).
[8] A. C. Aguilar, A. A. Natale and P. S. Rodrigues da Silva, Phys. Rev. Lett. **90**, 152001 (2003); A. C. Aguilar, A. Mihara and A. A. Natale, Phys. Rev. D **65**, 054011 (2002); Int. J. Mod. Phys. A **19** (2004) 249.
[9] S. J. Brodsky, arXiv:hep-ph/0703109.

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