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The Self-Couplings of Vector Bosons: does LEP-1 obviate LEP-2?

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ABSTRACT

Theories beyond the standard model ("meta-theories") are severely constrained by the current body of data and must necessarily respect, we insist, the standard gauge symmetry. We analyze the constraints on two generic types of meta-theory, in which fundamental scalars do or do not exist. The novel low energy effects may be comprehensively described by grafting onto the standard Lagrangian new operators that –in the sense of a Taylor expansion– ought to form a complete set. Completeness calls for consideration of previously discarded operators, and for a thorough exploitation of the equations of motion. We illustrate the current strictures by focusing on the allowed range of departures from the most crucial, untested, precise standard prediction: the size and structure of the triple gauge-boson vertices. We conclude that their direct measurement at LEP-2 is, alas, most unlikely to provide new information.

DEDICATED TO MARTINUS (TINI) VELTMAN ON HIS $60^{th}\frac{1}{2}$ BIRTHDAY.

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1. Introduction

A precise measurement of the electron or muon magnetic moment that happens to agree with the QED prediction has two redeeming values. First, and to the incomprehensible dismay of the experimentalist, it tests the theory. Second, and not to the delight of the model-builder, it excludes large domains of "new physics" possibilities, provided their potential contributions to these particular observables do not accidentally cancel. As a consequence of the success of the theory, many conceivable higher-energy experiments are rendered unlikely to provide surprises. An important question is the degree to which one can exclude these novel effects in a sufficiently systematic and rigorous fashion. In this respect, a crucial role is played by certain extremely well-motivated theoretical assumptions. An example is electromagnetic gauge invariance, without which not only QED would self-destroy as a predictive theory, but the study of possible "new physics" would be wildly different. Recall how significantly the analysis of putative QED modifications (characterized by new dimensional constants in vertices and propagators) changed with the realization [1] that the original ansatze were not gauge invariant or current-conserving. The gauge symmetry is a very strong yoke, it not only makes the theory tenable, but it has the well-known consequence of taming the conceivable deviations from its predictions.

The present status of the standard model begins to resemble suspiciously that of QED in yestern years. To a large extent this paper is a paraphrase of the preceding paragraph, with QED substituted for the standard lore, with the magnetic moments and other low-energy QED observables traded for our current body of standard tests, and with the proper extension and discussion of the local gauge invariance. Particular effort is devoted, admittedly along well-known avenues, to characterizing in comprehensive generality the "meta-theories" that may supersede the standard model, rather than to studying in detail some particularly attractive examples. A complete analysis along these lines of all possible deviations in all planned experiments would be a Cyclopean task. To see the trees in the forest, we focus on a very important issue: the LEP-2 potential to investigate the couplings of gauge bosons to themselves. Even this focused effort turns out to be elaborate, and it embodies many of the practical lessons to be learned in a more general realm.

The standard electroweak theory fixes the couplings between its four vector bosons ($W^{\pm}$, Z^0 and γ). The trilinear gauge vertices $Z^0W^+W^-$ and γW^+W^- (to which we often refer by their initials: TGV) can be directly measured at LEP-2, and perhaps even at the Tevatron collider. Consensus has it [2] that the LEP-2 measurements will constitute a sensitive test of conceivable deviations from the standard lore, and of new physics beyond the weak scale. In this paper we challenge this consensus.

Our thesis is based on two facts. First, the standard predictions are favourably tested at energies up to $\sim M_Z$, at a level that includes one-loop radiative effects. Second, the whole edifice of the standard theory would crumble, should its SU(2)⊗U(1) gauge invariance not be an exact symmetry above the weak symmetry-breaking scale. Sensible ados to the standard dynamics, likewise, must respect this local symmetry. The subtle cancellations in quantum corrections that are necessary to render the theory renormalizable (i.e. consistent and predictive) require [3] that the algebraic relations between the various couplings imposed by the theory's gauge invariance be exactly satisfied. Deviations from a rigorous compliance to this algebra are deadly, their degree of lethality increasing with the dimensionality of the culprit non-gauge-invariant coupling(s). It is well known that even the softest possible gauge-breaking addition (a non-spontaneously induced vector-boson mass term) offsets the predictive power of the theory.

We take good care to modify the standard dynamics in a manner that, above its symmetry-breaking scale, fully respects the gauge symmetry. The gauge-invariant pedigree of these modifications implies relations between the various non-standard deviations, both above *and below* the symmetry-breaking scale. This memory of an inescapable ancestral gauge invariance has rather often been overlooked in the literature, leading to suspect and overly optimistic expectations concerning the "new physics" sensitivity of future machines. It is easier to discuss how inconceivable a breakdown of the standard gauge symmetries is after the completion of a full gauge-respecting analysis; we treat the issue of non-gauge invariance in a later chapter on Quantum Suicide.

We shall see that the imposition of gauge invariance on putative deviations from the standard model, and the constraints implied by their obeyance of the existing low-energy tests, suffice to conclude that the expected statistical precision of LEP-2, but for particularly contrived circumstances, will be insufficient to fulfil one of the main perceived duties of the machine: to add significant information on the nature of the electroweak gauge theory via a direct measurement of the triple-gauge couplings. We either know too much, or we have understood next to nothing.

Let our definition of the Standard Model (SM) be sharpened to describe its minimal linear version. "Minimal" refers to the assumed non-gauge particle content: three complete fermion families and a single scalar doublet Φ. "Linear" refers to the adopted realization of the gauge-symmetry breakdown and Higgs mechanism: the mass term and quartic self-coupling in the scalar potential are sufficiently small for the complete theory to be perturbatively treatable. The surviving physical scalar is a relatively narrow "elementary" object not much heavier than the weak symmetry breaking scale $v = <\Phi_0> = 2\,M_W\,\sin\theta/e \sim 240$ GeV, all "as in the books".

Our limits on "new physics" are sensitive to the unknown standard parameters: m_t and M_H. We find it useful to devote a chapter to a discussion of the relevant observables and their status in the standard picture, including one-loop radiative corrections. This is not much more than yet another rendering of the classic work of Veltman and others [4] (or the more recent one of Kennedy and Lynn [5]) currently oft-exploited, though less often properly referenced. Our conclusions on the (non)observability of novel effects at LEP-2 are much more dependent on the planned luminosity of the machine than on the very precise shape of the various standard cross sections. In reaching our conclusions one may therefore finesse the drudgery of considering the radiative corrections to the standard predictions for $e^+e^- \to W^+W^-$ production.

We know of three main avenues for possible non-minimal or non-standard effects:

♣ The addition of extra repetitive families and/or extra scalars in triplet or higher-dimensional representations. We consider this possibility "trivial" and disregard it, except for the assumption that its effects do not unnaturally cancel against the less trivial ones that we discuss.

♠ The existence of truly "new physics" (extra particles, supersymmetry, larger gauge symmetries, compositeness,...) characterized by a mass scale Λ comparable to, or larger than the physically distinct scale v. The unspecified heavy objects of this type of meta-theory are assumed not to acquire their masses from the standard machinery of symmetry breaking [6], though they may well be involved in the mechanisms that trigger it [7].

♢ The possibility that the scalar sector of the SM may be strongly interacting and the (currently) consequent necessity to treat the vector-boson longitudinal degrees of freedom in a manner [8, 9] reminiscent of chiral perturbation theory [10]. In this case the "new physics", to which we shall often refer as "chiral", is that underlying the mechanism of gauge-symmetry breakdown⋆ .

In short, our procedure and conclusions are the following. We describe the low energy effects of the "meta-theory" that may lie beyond the standard model in terms of towers of effective interactions constrained only by the standard symmetries [3]. Existing data are then used to limit the coefficients of the relevant terms in these

⋆ For transparent and practically up-to-date discussions, see [11, 12].

towers[†] . These constraints, barring a pandemic spread of unnatural cancellations between independent terms in a handful of relevant observables, suffice to limit below observable levels the potential deviations of the triple vector boson vertices from their standard form. The exceptions to this general conclusion are few and marginal; they are discussed in due detail. The limits that we systematically derive are also useful in constraining specific theories past the standard model, a point we do not elaborate in detail in this paper.

The low energy effects of a "meta-theory" can be expressed in terms of effective Lagrangians that are linear combinations of operators constructed with the fields of the Standard Model. A graft is "a shoot or scion inserted in a slit of another stock, so as to allow the sap of the latter to circulate through the former". The addition of new effective operators to the standard Lagrangian acts in every respect as a graft: not only do these operators foretell a new growth of novel dynamics, but their effects often permeate into the pre-existing standard masses and couplings. Thus, we shall often refer to the effective Lagrangian additions as "GRAFTS"[*] . A graft is also "a practice intended to secure the means of making illicit profit"; perhaps this particular meaning has not always been irrelevant.

The grafts, in the case that the SM is linearly realized (and contains elementary fields describing narrow and relatively light scalars), are to be organized in a "Taylor" expansion in their "naive-" or field-dimensionality, d. Their effects on current observables are expressible as a power series in v^2/Λ^2. The dominant terms have $d = 6$ and their number is finite [13, 14], but runs into the dozens. As we narrow our scope to possible deviations from the standard predictions for TGVs, we find that only six $d = 6$ operators are relevant (four of these operators have been previously discussed in somewhat different contexts [15, 16], the rest involve fermion fields and must be taken into account in a comprehensive treatment). The possible complication of the scales v and Λ being of comparable magnitude is discussed in a section on "Form Factors".

In the linear realization of the SM, our six operators constitute a "complete basis" for the analysis of non-standard TGVs, in the sense that all other relevant $d = 6$ grafts can be written, via a permissible and advisable use of the equations of motion, as linear combinations of the elements of the basis. There is an embarrassment of choices in the ensemble of operators that may be used as a basis; it is wise to select the ones best

† For our purposes it is more illuminating to look straight at the face of these operators, rather than to take a more oblique look at their possible collective effects on objects such as vector-boson two-point functions.

* Our GRAFTS could also be pedantically interpreted as "Gauge-Restricted Anomalous Foretelling Terms".

constrained by current data. Some grafts are "tree-grafts", they affect well-measured observables at tree level and are the obvious candidates for a most useful basis. Some of the operators not belonging to our chosen basis are "blind directions": linear combinations of the basic grafts with coefficients such that all tree-level effects on current observables exactly cancel. There is no known symmetry or dynamical reason why a meta-theory would be so contrived as to generate a low-energy effective Lagrangian pointing exclusively in blind directions. Thus, one is not particularly motivated to find bounds on the coefficients of the corresponding operators. Yet, the existing tests of the standard model are so accurate that it is possible to derive significant limits on operators pointing in blind directions (quantum grafts?) from their one-loop effects on current observables. This we do for a particularly interesting blind direction, just as an indicative example.

If the mechanism of spontaneous symmetry breakdown is non-linearly realized, the scale v and the "new physics" scale Λ are not independent. The low-energy non-standard effects are in this case describable in terms of the SU(2)$\otimes$U(1) gauge invariant effective operators occurring in a Taylor expansion in "chiral" dimensions d_χ, or equivalently in powers of k/Λ, with k the momenta of the particles involved. We find that the relevant $d_\chi \leq 4$ chiral-effective interactions fall into pairs of three distinct types. Two chiral operators are, in our framework, equivalent to two elements of our linear $d = 6$ basis, and their coefficients are severely constrained. Two others are equivalent to blind directions of naive dimensionality $d = 6$. The last two are equivalent to operators of naive dimensionality $d = 8$; one of them describes a blind direction, the other is a tree graft that we study in detail. Since there is every reason in this case to expect the coefficients of all six relevant chiral operators to be of comparable magnitude, we have not fully analyzed the chiral blind-direction operators.

This paper is divided into a multitude of Chapters and Sections, whose titles should suffice to convey an idea of its organization.

2. THE MINIMAL STANDARD MODEL IN THE Z-SCHEME

In discussions of the use of the standard model, the jejune question of what is the best renormalization and regularization scheme, has received implausible attention. To be explicit, we specify what our chosen procedure is. Five parameters of the minimal model are relevant to our discussion: g, g', v, m_t and M_H, in an established notation. There is no question that the last two parameters must, for the time being, be left free. There is no doubt that the very well-determined values of α and the Fermi constant G_F ought to be used to constrain the first three parameters (the fine structure constant is known to 0.045 parts per million from the low-energy classical limit of Thompson scattering and the Fermi constant as extracted from the muon decay rate by the removal [17] [18] of the standard process-dependent box, bremstrahlung and vertex corrections is known to 17 ppm). The hottest scheme-controversies concerned the "best" definition of the weak mixing angle θ, extracted in various ways from the data and used as the remaining known parameter. Should θ run with the energy scale? Does it depend significantly on m_t? Do alternatives have smaller errors? With the advent of a very precise and still fast-improving measurement of M_Z (a current 230 ppm) it is becoming generally admitted [19] that one can answer all of the above questions with a blaring "no", by simply defining θ_Z so that:

$$M_Z \equiv \frac{\left(\pi\,\alpha/\sqrt{2}\,G_F\right)^{1/2}}{\sin\theta_Z\,\cos\theta_Z} = \frac{37.281\,\mathrm{GeV}}{\sin\theta_Z\,\cos\theta_Z}. \qquad (1)$$

Numerically,

$$\sin^2\theta_Z = 0.21223 \pm 0.00014. \qquad (2)$$

This parameter is totally equivalent to the input $M_Z = 91.175 \pm 0.021$ GeV [20], but is useful as an auxiliary quantity* in terms of which to express further results.

We call a "Z-scheme" a framework in which the values of α, G_F and M_Z are kept as input parameters, fixed at their measured values. We use such a scheme both in the standard model and beyond it.

In deriving standard predictions beyond tree level, a concrete scheme must be chosen as an intermediate-stage working tool. The conceptually simplest scheme is the "on-shell" one [21], in which θ_W is a fixed quantity defined by

$$\cos\theta_W \equiv \frac{M_W}{M_Z} \qquad (3)$$

and

$$M_Z = \frac{\left(\pi\,\alpha/\sqrt{2}\,G_F\right)^{1/2}}{\sin\theta_W\,\cos\theta_W\,(1-\Delta r/2)}. \qquad (4)$$

The quantity Δr is a known function of m_t, M_H and the relatively poorly-determined

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$\star$ θ_Z is much better determined than any traditional alternative weak angle. If any derived quantity deserves mention in the data tables, $\sin^2\theta_Z$ is the weak mixing parameter of choice.

value of α_s. Comparison of (1) and (4) to first-order in Δr results in:

$$\sin^2\theta_W \simeq \sin^2\theta_Z + \frac{\cos^2\theta\,\sin^2\theta}{1-2\,\sin^2\theta}\,\Delta r. \tag{5}$$

With the exception of large-m_t effects in boson propagators, that are iterated in the consuetudinary manner, we do not work beyond first order corrections, either radiative or non-standard. Thus, here and in what follows we drop second-order effects, such as a distinction between θ_W and θ_Z in the second term of the r.h.s. of the preceding expression. For fixed m_t and M_H, $\sin^2\theta_W$ is determined with a precision comparable† to that of $\sin^2\theta_Z$ in (2), allowing one to predict other observables with comparable accuracy. Of all the relevant predictions, only the Z-widths and forward-backward asymmetries, the W mass and the inclusive neutrino cross sections are measured at the level of precision of the standard one-loop predictions. Recent results, that we have not exploited, on parity violation in atomic physics [22] and on τ-polarization asymmetries in Z-decay are reaching a competitive level of accuracy.

In practice, the standard predictions are significantly dependent on the value of m_t in the range 80 to 300 GeV, and marginally dependent on M_H in the range 40 to 1000 GeV. We choose to keep the m_t-dependence of our results explicit and allow M_H to span the range from 40 GeV to 1 TeV, with this "theoretical uncertainty" (and that in α_s) linearly summed to the experimental ones. It has become usual to combine theoretical and experimental errors in quadrature. Since we are dealing with bounds on novel effects, which ought to be pessimized, we adopt the safer route of combining the two types of uncertainties linearly. As a working kit, we use the programs* of Bardin *et al.*, [23], that we have checked in an approximation wherein the radiative corrections are quadratic in m_t.

The observables we use to constrain deviations from the standard model are:

◇ The W-mass, determined most precisely in current practice from the ratio M_W/M_Z measured at $p\bar{p}$ colliders [24] and the LEP value for M_Z. The standard prediction [4] is simply:

$$M_W = M_Z\cos\theta_W, \tag{6}$$

with θ_W determined by eq.(5).

† This is true, it goes without saying, in any scheme, but it is impractical to emphasize an auxiliary quantity, e.g. the weak mixing angle, when it is defined via a relation such as (3), that does not correspond to the tightest experimental input.

* We are indebted to Misha Bilenki for use of his programs and results prior to publication.

♣ The ratio of inclusive neutral- to charged-current neutrino cross sections on approximately isoscalar targets:

$$R_\nu = \frac{\sigma(\nu N \to \nu + \cdots)}{\sigma(\nu N \to \mu + \cdots)} \tag{7}$$

at momentum transfers negligible relative to the intermediate vector boson masses. The discussion of R_ν is more elaborate than that of other observables, given the necessary attention to be paid to the problems of minimizing the corrections away from a naive parton model [25] and of dealing with charm-threshold effects [26]. Moreover the constraints on standard and novel effects imposed by this observable, that we have computed with the necessary toil, turn out to be weaker than those of other observables‡ . We therefore omit the detailed expressions [26, 25] concerning R_ν.

♡ The leptonic and hadronic widths of the Z, as extracted from a fit to the data of an $e^+e^- \to f\bar{f}$ total cross section which, prior to corrections for initial-state bremstrahlung, reads:

$$\sigma_f(q^2) \simeq \frac{4\,\pi\, q^2\, N_c}{(q^2 - M_Z^2)^2 + \frac{q^2\,\Gamma^2}{M_Z^2}} \left[\frac{3\,\Gamma_e\,\Gamma_f}{M_Z^2\, N_c} + \frac{I_f^{Z\gamma}\,\bar{\alpha}\,(q^2 - M_Z^2)}{M_Z\,\sqrt{N_c}\,q^2}\right] + \frac{4\,\pi Q_f^2\,\bar{\alpha}^2 N_c}{3\,q^2}\,, \tag{8}$$

with $N_c = 3$ and 1 for quarks and leptons, and $\bar{\alpha} = \alpha(M_Z^2)$. We shall see that there are cases in which the standard approximation of energy-independent Γ's is called into question♮ . The radiatively-corrected standard predictions on Γ_f can be cast in the form:

$$\Gamma_f \equiv \Gamma_f^R + \Gamma_f^L \simeq \frac{G_F\, M_Z^3}{3\,\pi\sqrt{2}}\, N_c\,(1 + \delta\gamma_f)\,\left(\left[c_f^R\right]^2 + \left[c_f^L\right]^2\right)\cdot \tag{9}$$

It is useful to introduce the redundant vector and axial couplings

$$\begin{aligned} \bar{g}_f^V &\simeq \left(1 + \frac{\delta\gamma_f}{2}\right)\left(c_f^L + c_f^R\right)\,, \\ \bar{g}_f^A &\simeq \left(1 + \frac{\delta\gamma_f}{2}\right)\left(c_f^L - c_f^R\right)\cdot \end{aligned} \tag{10}$$

‡ This is true for the $2\,\sigma$ level of statistical confidence we shall choose to work with, and less true at the $1\,\sigma$ level.

♮ The Breit-Wigner form of (8) is only a currently sufficient approximation. At a soon to be reached precision, extra care is needed in relating theory and experiment and it is convenient to insist in M_Z to denote the real part of the pole of the propagator . We are indebted to Bryan Lynn and Robin Stuart for discussions on this point.

The standard values of the tree-level quantities are:

$$\begin{aligned} c^R_{f0} &\equiv \frac{1}{2}\ \left[g^V_{f0} - g^A_{f0}\right] = -Q_f \sin^2\theta\ , \\ c^L_{f0} &\equiv \frac{1}{2}\ \left[g^V_{f0} + g^A_{f0}\right] = T^f_3 + c^R_{f0}\,. \end{aligned} \tag{11}$$

It is convenient° to define $\delta\gamma_f$ so that the coefficient of T^f_3 in the left-handed couplings is unchanged by radiative corrections, and

$$\begin{aligned} c^R_f &\equiv \frac{1}{2}\ \left[\bar{g}^V_f - \bar{g}^A_f\right]\ \left(1 - \frac{\delta\gamma_f}{2}\right) = -(1+\delta\kappa_f)\,Q_f \sin^2\theta_Z\ , \\ c^L_f &\equiv \frac{1}{2}\ \left[\bar{g}^V_f + \bar{g}^A_f\right]\ \left(1 - \frac{\delta\gamma_f}{2}\right) = T^f_3 + c^R_f\,. \end{aligned} \tag{12}$$

The standard one-loop corrections $\delta\gamma_f$, $\delta\kappa_f$ are known functions of m_t and M_H. Also useful to our inquiries are the axial Z-couplings to electrons and muons. These are extracted from Γ_l and the forward-backward asymmetries A_l. To the present high accuracy of the experiments, this procedure is elaborate; it is described in detail in [28].

The non-standard corrections to Z-widths and asymmetries can also be conveniently cast in precisely the same form as the standard corrections. That is, the novel effects on these observables are described by explicit values of $\delta\gamma_f$ and $\delta\kappa_f$. For given values of the latter, the effects on the $Z \to f\bar{f}$ width and axial coupling for a fermion f are:

$$\frac{\delta\Gamma_f}{\Gamma_f} \simeq \delta\gamma_f + 2\,\delta\kappa_f\,\frac{c^R_f(c^R_f + c^L_f)}{\left[c^R_f\right]^2 + \left[c^L_f\right]^2}\ , \tag{13}$$

$$\frac{\delta\bar{g}^A_f}{\bar{g}^A_f} = \frac{1}{2}\,\delta\gamma_f\,. \tag{14}$$

For the interesting cases $f = l^-$ (a given charged lepton) and $f = h$ (the sum over all quarks but t) Eq.(13) corresponds to the numerical results:

$$\frac{\delta\Gamma_l}{\Gamma_l} \simeq \delta\gamma_l - 0.250\ \delta\kappa_l\ , \tag{15}$$

$$\frac{\delta\Gamma_h}{\Gamma_h} \simeq \delta\gamma_q - 0.318\ \delta\kappa_q\ , \tag{16}$$

where in (16) we have assumed (as we will, in all cases but one, be led to) common

° Our $1+\delta\gamma_f$ is often called ρ or $\rho_0\rho^f$, as in [26]. The symbol ρ has been endowed in this tessitura with so many not quite equivalent meanings that we refrain from using it.

values of $\delta\gamma_q$ and $\delta\kappa_q$ for all flavours.

Let $Q_f^{L,R}$ denote the left- and right-handed couplings of fermions to photons. The interference term in (8) is

$$I_f^{Z\gamma} = \left(Q_e^R\sqrt{\Gamma_e^R} - Q_e^L\sqrt{\Gamma_e^L}\right)\left(Q_f^R\sqrt{\Gamma_f^R} - Q_f^L\sqrt{\Gamma_f^L}\right), \tag{17}$$

with $\Gamma_f^{R,L}$ as in (9), while in the purely photonic term

$$Q_f^2 = \frac{1}{2}\left[\left(Q_e^L Q_f^L\right)^2 + \left(Q_e^R Q_f^R\right)^2\right]. \tag{18}$$

The distinction between Q^L and Q^R need only be made for operators not contributing at tree level to current observables, as we discuss in Section 3.7.

The experimental central values and $1\,\sigma$ errors we use are:

$$\begin{aligned}
M_Z &= 91.175 \pm 0.021\text{GeV [input] [20]}\\
R_\nu &= 0.308 \pm 0.002 \quad [29]\\
M_W &= 80.13 \pm 0.31 \text{ GeV} \quad [24]\\
\Gamma_l &= 83.2 \pm 0.4 \text{ MeV} \quad [20]\\
\Gamma_e &= 83.0 \pm 0.5 \text{ MeV} \quad [20]\\
\Gamma_\mu &= 83.8 \pm 0.8 \text{ MeV} \quad [20]\\
\Gamma_h &= 1740 \pm 9 \text{ MeV} \quad [20]\\
(\bar{g}_l^A)^2 &= .2492 \pm 0.0012 \quad [20]\\
(\bar{g}_e^A)^2 &= .251 \pm 0.003 \quad [30]\\
(\bar{g}_\mu^A)^2 &= .244 {}^{+0.005}_{-0.004} \quad [30]
\end{aligned} \tag{19}$$

To the experimental error in R_ν we have added a theoretical error of 1% to reflect the theoretical uncertainty associated with the charm threshold in the charged currents, discussed and estimated in [26]. Once again, we adopt the safe recipe of adding linearly the theoretical uncertainties to the experimental errors. We shall choose to perform our analysis at the 2σ level, the inputs to our limits on novel effects are twice the quoted errors.

We are concerned with putative modifications of the $e^+e^- \to W^+W^-$ amplitudes, and of the ZWW and γWW vertices in particular. To recall what these vertices are, let W_μ and $\hat{W}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ be the potential and the Abelian field strength of the W field, and let V_μ and $V_{\mu\nu}$ with $V = \gamma, Z$ be the corresponding entities for the physical neutral fields. The tree-level trilinear boson couplings in the unmodified standard Lagrangian $\mathcal{L}_0$ are:

$$\mathcal{L}_0^{(3)}(V) = -i\, e\, g_V \left[\left(\hat{W}^\dagger_{\mu\nu} W^\mu - \hat{W}_{\mu\nu} W^{\dagger\mu} \right) V^\nu + \kappa_V\, W^\dagger_\mu W_\nu V^{\mu\nu} \right] \tag{20}$$

with $g_\gamma = 1$ as a consequence of electromagnetic $U_\gamma(1)$ gauge invariance, and

$$\begin{aligned} g_Z &= \cot\theta\,, \\ \kappa_\gamma &= \kappa_Z = 1\,. \end{aligned} \tag{21}$$

The gauge non-Abelian form of the interactions in (20) and the values of the couplings in (21) are the most profound of the very few precise predictions of the standard model not yet subject to a direct experimental test. For our purposes, we need not consider $O(\alpha)$ corrections to the SM cross sections for W-pair production as computed with use of (20) and (21), since the observability or non-observability of the novel effects we study turns out to be a clear-cut issue, but for a couple of marginal cases. For these, the marginality is much more affected by the assumed statistics than it is by the (small) radiative effects.

Not all of the new TGV interactions we shall encounter are describable as modifications δg_Z, $\delta\kappa_\gamma$ and/or $\delta\kappa_Z$ of the standard results (21). One must also consider an additional interaction containing a triple boson coupling of an entirely non-standard form, customarily parametrized as:

$$\mathcal{L}_W^{(3)}(V) = -i\, e\, g_V \frac{\lambda_V}{M_W^2} \left[V^{\mu\nu} \hat{W}^\dagger_{\nu\rho} \hat{W}^\rho_{\ \mu} \right], \tag{22}$$

accompanied by vertices with more than three bosons that the gauge-invariant nature of our non-standard effects will fully determine. The $SU(2) \otimes U(1)$ gauge symmetry also implies

$$\lambda_\gamma = \lambda_Z \equiv \lambda\,, \tag{23}$$

and we shall see that disaster strikes when this or other gauge relations are forsaken.

3. Physics beyond a linear standard model

3.1 The $d = 6$ operators and the equations of motion

Assume the standard model to be realized in its minimal linear form, with a perturbatively treatable scalar sector. Thus Φ will stand for the doublet scalar field responsible for the observed W and Z masses, and H_0 for the surviving physical scalar (generalizations to more than one doublet are rather straightforward and, in our framework, totally innocuous). Let $\mathcal{L}_0$ denote the complete renormalizable standard Lagrangian (that includes the undiscovered top quark and elementary scalar). Imagine that there is life beyond this model: awaiting discovery there exists some SU(2)⊗U(1) gauge invariant dynamics (extra particles, higher gauge symmetries, supersymmetry, compositeness,...) characterized by a mass scale Λ larger than the weak symmetry-breaking scale $v = <\Phi_0> \sim 240$ GeV.

At energies lower than Λ, it is permissible and advisable to integrate "in" the heavier degrees of freedom and to embody their effects in effective operators exclusively constructed with the fields of the standard model. At a scale above v the exact gauge relations between the operators already existing in the unbroken standard model remain unscathed. But new operators with dimension $d \geq 6$ appear, suppressed by coefficients of order Λ^{4-d}.

At all scales of $O(\Lambda)$ or smaller, the most relevant novel operators have $d=6$ and the effective action* is constructed by grafting additive pieces to $\mathcal{L}_0$:

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \sum_j \mathcal{L}_j \tag{24}$$

with

$$\mathcal{L}_j = \frac{\alpha_j}{\Lambda^2} \mathcal{O}_j + O(1/\Lambda^4). \tag{25}$$

Complete lists of the scores of independent $d=6$ objects, $\mathcal{O}_j$, compatible with the SM symmetries and constructed out of its fields have been reported in [13, 14]. Of these grafts, the CP-odd ones are particularly intriguing; they have been previously discussed [31, 32] and will not be dealt with here.

⋆ For our purposes we need not dwell in the fine distinctions [6] between effective action and effective Lagrangian.

Not all of the possible $d = 6$ operators appear in a complete basis, for one is allowed to trade terms that add up to a total derivative, and to eliminate other terms by use of the equations of motion that $\mathcal{L}_0$ implies, as discussed in [14] for the case at hand, or in [33, 34] for the case of the chiral description of a strongly interacting longitudinal gauge sector.

We shall argue that one can exploit to one's advantage the freedom to choose a basis of grafts describing departures from the SM, given that the empirical information is obviously not of comparable quality for all measured processes. In analyzing the present strictures on linear meta-theories and the potentiality of future experiments, it is more than advisable to choose a basis all of whose elements are currently well constrained.

We are concerned here with those of the CP-even $d=6$ operators in (24) that are relevant to the triple gauge vertices accessible at LEP-2 and at hadron colliders. To introduce them, let $\vec{W}_{\mu\nu}$ be the (complete, non-Abelian) SU(2) gauge-field strength:

$$W^a_{\mu\nu} \equiv \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\,\epsilon_{abc} W^b_\mu W^c_\nu \tag{26}$$

and $B_{\mu\nu}$ its U(1) counterpart (with g and g' the respective coupling strengths) and let D_ρ be the SU(2)⊗U(1) covariant derivative

$$D_\rho \equiv \partial_\rho + i\,g\,\frac{\sigma_a}{2}\,W^a_\rho + i\,g'\,Y\,B_\rho\,. \tag{27}$$

It proves useful to introduce the weak isospin and hypercharge currents, $\vec{J}$ and J, of the scalar and matter fields:

$$\begin{aligned} \vec{J}_\rho(\Phi) &\equiv \Phi^\dagger\,\frac{i\vec{\sigma}}{2} D_\rho\Phi - (D_\rho\Phi)^\dagger\,\frac{i\vec{\sigma}}{2}\Phi\,, \\ \vec{J}_\rho(L_f) &\equiv \bar{L}_f\,\gamma_\rho \frac{\vec{\sigma}}{2}\,L_f\,, \\ J_\rho(\Phi) &\equiv \frac{i}{2}\,\Phi^\dagger D_\rho\Phi - \frac{i}{2}\,(D_\rho\Phi)^\dagger\,\Phi\,, \\ J_\rho(f) &\equiv y_f\,\bar{f}\gamma_\rho f\,, \end{aligned} \tag{28}$$

with L_f the left-handed fermion isodoublets and y_f the fermion hypercharges.

There are various kinds of grafts whose different roles, which we now sketch, should become increasingly clear as we discuss them in turn and in detail. Not all of the operators to be presently discussed survive as elements of our complete basis or are blind directions currently unconstrained at tree-level. For ease of reference, we gather in a table those operators that, one way or the other, will survive our lengthy discussion:

Basic bosonic grafts	Basic fermion grafts	Blind directions $\not\subset \{\mathcal{O}_i\}$
$\mathcal{O}_{WB} \equiv \Phi^\dagger \vec{\sigma} \Phi \, \vec{W}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{e\mu} \equiv \vec{J}_\rho(L_e)\, \vec{J}^\rho(L_\mu)$	$\mathcal{O}_W \equiv (1/3!)\ \vec{W}_\mu^{\ \nu} \times \vec{W}_\nu^{\ \lambda} \cdot \vec{W}_\lambda^{\ \mu}$
$\mathcal{O}_\Phi \equiv J_\rho(\Phi)\, J^\rho(\Phi)$	$\mathcal{O}_e \equiv \vec{J}_\rho(\Phi)\, \vec{J}^\rho(L_e)$	$\mathcal{O}_{B\Phi} \equiv i\, B^{\mu\nu}\, (D_\mu \Phi)^\dagger\, D_\nu \Phi$
$\mathcal{O}_{DW} \equiv [D^\rho \vec{W}_{\mu\nu}]^\dagger\, [D_\rho \vec{W}^{\mu\nu}]$	$\mathcal{O}_\mu \equiv \vec{J}_\rho(\Phi)\, \vec{J}^\rho(L_\mu)$	$\mathcal{O}_{W\Phi} \equiv i\, \vec{W}^{\mu\nu}\, (D_\mu \Phi)^\dagger\, \vec{\sigma}\, D_\nu \Phi$

Table I: Elements of the $d = 6$ basis $\{\mathcal{O}_i\}$; and blind directions.

The different types of operators to be considered and sieved from the complete $d = 6$ ensemble [13, 14] are:

ℵ The obvious, constructed exclusively with vector fields, such as:

$$\mathcal{O}_W \equiv \frac{1}{3!}\ \vec{W}_\mu^{\ \nu} \times \vec{W}_\nu^{\ \lambda} \cdot \vec{W}_\lambda^{\ \mu}, \tag{29}$$

that describes purely transverse vertices of three or more vector bosons, and has no tree-level effects on currently measurable observables (but for $W\gamma$ production at hadron colliders, not a process to be soon measured with competitive precision).

♠ Operators built of scalar and vector fields, such as:

$$\mathcal{O}_{WB} \equiv \Phi^\dagger \vec{\sigma} \Phi \, \vec{W}_{\mu\nu} B^{\mu\nu}, \tag{30}$$

and

$$\mathcal{O}_\Phi \equiv J_\rho(\Phi)\, J^\rho(\Phi), \tag{31}$$

which (with the neutral component of Φ written in a physical gauge as $[H_0 + v]/\sqrt{2}$) describe vertices involving two or three gauge bosons. Via its two-boson components, $\mathcal{O}_{WB}$ has tree-level effects on empirically tested relations, such as the standard-model liaisons (6) involving M_W and M_Z. Even though it contains no more than two gauge bosons, a non-vanishing $\mathcal{O}_\Phi$ has "indirect" effects on LEP-2 observables. The reason is that, in defining the parameters of the standard Lagrangian, one is advised to input and keep fixed the best known relevant quantities (α, G_F and M_Z) at their measured values. Some of the effects of $\mathcal{O}_\Phi$ are moved to the triple-gauge couplings by this finite renormalization.

◊ Innocuous operators, like

$$\mathcal{O}_{WW} \equiv \Phi^\dagger\Phi\, \vec{W}_{\mu\nu}\vec{W}^{\mu\nu}, \tag{32}$$

$$\mathcal{O}_{BB} \equiv \Phi^\dagger\Phi\, B_{\mu\nu}B^{\mu\nu}, \tag{33}$$

$$\mathcal{O}_{\Phi\Phi} \equiv \Phi^\dagger\Phi\, (D_\mu\Phi)^\dagger D^\mu\Phi, \tag{34}$$

$$\mathcal{O}_{\widetilde{\Phi\Phi}} \equiv \Phi^\dagger\Phi\; \Phi^\dagger D^2\Phi, \tag{35}$$

whose effects not involving external scalar fields can be totally eliminated from $\mathcal{L}_{eff}$ by ineffectual field and coupling constant redefinitions⋆ .

♡ Stealthy "fermionic" grafts, constructs that involve fermionic fields, and whose non-standard effects on the properties of vector bosons occur only indirectly. Much as the operator $\mathcal{O}_\Phi$ previously discussed, fermionic operators such as

$$\mathcal{O}_{e\mu} \equiv \vec{J}_\rho(L_e)\, \vec{J}^\rho(L_\mu) \tag{36}$$

(that affects the μ-decay process that defines G_F) have indirect effects on the triple gauge-boson vertices. Two more operators of this same type are:

$$\mathcal{O}_l \equiv \vec{J}_\rho(\Phi)\, \vec{J}^\rho(L_l), \tag{37}$$

with $l = e, \mu$. We shall see that $\mathcal{O}_{e\mu}$, $\mathcal{O}_e$ and $\mathcal{O}_\mu$, that are constructed with "$V - A$" currents, are related to non-fermionic operators via the equations of motion. On a different footing is the "$S + P$" graft

$$\mathcal{O}_{\widetilde{e\mu}} \equiv \left\{\bar{L}_e e\right\}\left\{\bar{\mu} L_\mu\right\} + \text{h.c.}, \tag{38}$$

whose effects on LEP observables we shall find to be negligible. We have not assumed universality between similar operators constructed with different flavours, nor listed operators such as $\mathcal{O}_{e\tau}$, defined in analogy with $\mathcal{O}_{e\mu}$ in (36). This discrimination is justified by a circumstantial nuance. If the $\tau \to e$ decay were better measured than μ decay, the former would serve to define G_F, and be used as one of the inputs in the most precise standard prediction of the triple gauge boson couplings. The roles of $\mathcal{O}_{e\tau}$ and $\mathcal{O}_{e\mu}$ would be interchanged.

⋆ Consider $\mathcal{L}_{eff} = \mathcal{L}_0 + \mathcal{L}_{WW}$ below the scale v. Let $\bar{\epsilon} = \alpha_{WW} v^2/\Lambda^2$. The substitutions $\vec{W}_\mu \to (1-\bar{\epsilon})\vec{W}_\mu$, $g \to (1+\bar{\epsilon})g$, with g the SU(2) coupling, eliminate the quoted effects of $\mathcal{O}_{WW}$. The fate of $\mathcal{O}_{BB}$, $\mathcal{O}_{\Phi\Phi}$ and $\mathcal{O}_{\widetilde{\Phi\Phi}}$ is entirely analogous. Similar redefinitions would do away with the couplings of only vector bosons induced by loops involving also scalars.

♣ "Conspiratorial" operators, like

$$\mathcal{O}_{\Phi f} \equiv J^{\rho}(\Phi)\, J_{\rho}(f) \tag{39}$$

and $\mathcal{O}_{e\tau}$ and its sibs. These grafts would affect the relation† between the standard $e^+e^- \to f\bar{f}$ cross sections in the Z-pole region and their measured values, from which the comparison to the standard $Zf\bar{f}$ couplings is made. The distinction between stealthy and conspiratorial operators is, once again, a matter of current experimental precision: if the Z-widths and asymmetries were better measured than the Z-mass is, roles would interchange. Conspiratorial grafts could only be relevant if they happened to accidentally cancel the contributions of the operators previously listed to the observables used to constrain their coefficients. We assume no systematic accidental cancellations and consequently ignore these grafts, though bounds on them of the same type and general magnitude as the ones to be derived for stealthy operators could readily be obtained.

For our specific goal of constraining non-standard deviations in the trilinear gauge couplings from the available experimental information, the ensemble of $d = 6$ operators (29) to (31), (36) to (37), is complete, though not yet optimized. One may even get the impression that the list we just quoted is unduly over-complete, since fermionic grafts such as $\mathcal{O}_{e\mu}$ appear to have little to do with gauge bosons. It is tempting to dispose of these operators by assuming that they are generated by the meta-theory at some negligible higher order of a perturbative expansion. We do not take the liberty of making this kind of prejudice-laden assumption, since our main aim is to analyze in full generality the LEP-2 prospects regarding TGVs. We proceed to show that one stands to learn something useful by studying the relations that the equations of motion imply between fermionic and gauge operators, before a decision is made on which ones to "privilege" as elements of a basis.

Fermionic and pure-gauge operators are mixed by the equations of motion that one may use to change operator basis. To be specific, introduce a $d = 6$ graft that we have not explicitly discussed so far:

$$\mathcal{O}_{DW} \equiv \left[D^{\rho}\vec{W}_{\mu\nu}\right]^{\dagger} \left[D_{\rho}\vec{W}^{\mu\nu}\right] \tag{40}$$

† Because of their left-right chiralities, electroweak magnetic moment operators, $\bar{f}\,\sigma^{\mu\nu}\,L_f\,\Phi^{\dagger}\,B_{\mu\nu}$, and electroweak charge form-factors, $\bar{f}\,(D_{\rho}\,L_f)\,D^{\rho}\Phi$, only contribute, in the limit of vanishing fermion masses, to second order in α_i/Λ^2.

and consider the equation of motion

$$D^\mu \vec{W}_{\mu\nu} = g\,\vec{J}_\nu(\Phi) + g \sum_{f=l,q} \vec{J}_\nu(L_f), \tag{41}$$

with $\vec{J}$ the weak-isospin currents. Recall the definition (29) of $\mathcal{O}_W$ and the identity

$$12\,g\,\mathcal{O}_W \equiv \left[D^\rho \vec{W}_{\mu\nu}\right]^\dagger \left[D_\rho \vec{W}^{\mu\nu}\right] - 2\left[D^\rho \vec{W}_{\rho\nu}\right]^\dagger \left[D_\mu \vec{W}^{\mu\nu}\right] = \mathcal{O}_{DW} - 2\,g^2\,\vec{J}_\nu^\dagger \vec{J}^\nu \tag{42}$$

to learn that $\mathcal{O}_W$ is a specific linear combination of the fermionic objects $\mathcal{O}_{e\mu}$ and $\mathcal{O}_l$ (plus other fermionic* operators appearing in $\vec{J}_\nu^\dagger \vec{J}^\nu$) and $\mathcal{O}_{DW}$. Single out that part of $\mathcal{L}_{eff}$ in (24) and (25) that contains the operators related by (42). In an over-abundant basis:

$$\Lambda^2 \sum_k \mathcal{L}_k = \alpha_W\,\mathcal{O}_W + \alpha_{DW}\,\mathcal{O}_{DW} + \alpha_{e\mu}\,\mathcal{O}_{e\mu} + \alpha_e\,\mathcal{O}_e + \alpha_\mu\,\mathcal{O}_\mu + \cdots \tag{43}$$

with the dots referring to, among others, the rest of the fermionic grafts in $\vec{J}_\nu^\dagger \vec{J}^\nu$. All the operators listed above, but $\mathcal{O}_W$ (that describes vertices with three or more vector bosons) have tree-level effects on the relations between contemporarily well measured observables, and their coefficients can be tightly bound. But the presence of $\mathcal{O}_W$ in the inventory may seem superfluous, since it can be eliminated in favour of the others by use of (42). One is tempted to conclude that finding limits on the coefficients of the operators that span a complete basis constitutes the complete job of limiting the effects of the meta-theory. Not quite so. The equations of motion as reflected in (42) also mean that a particular "direction" ($\mathcal{O}_W$) in the space spanned by the rest of the operators in (43) cannot at present be bound from tree-level effects. We have called these "directions" in $d = 6$ operator space "blind" directions. There is nothing fundamental in the distinction between blind and other directions, which will only last for as long the post-LEP-1 and pre-LEP-2 interregnum. Had LEP-2 been run prior to LEP-1, as it would if the Z-mass were larger than twice the W-mass, the directions we call blind would have been born sighted.

* For processes not involving real scalars, the operator $\vec{J}_\nu^\dagger(\Phi)\vec{J}^\nu(\Phi)$ can be eliminated by innocuous redefinitions.

The hypercharge sibling of $\mathcal{O}_{DW}$ (40), the operator [15]

$$\mathcal{O}_{\partial B} \equiv [\partial^\rho B_{\mu\nu}]\,[\partial_\rho B^{\mu\nu}] \tag{44}$$

is, in our basis, truly superfluous. The equations of motion result in

$$\mathcal{O}_{\partial B} = 2\,g'^2\, J_\nu^\dagger J^\nu \tag{45}$$

with J the total hypercharge current. The operators of the form $J_\rho(f)J^\rho(f')$ and $J_\rho(f)J^\rho(\Phi)$ on the right-hand side of (45) belong to the category that we have defined as "conspiratorial" and the $J_\rho(\Phi)J^\rho(\Phi)$ remainder is simply our $\mathcal{O}_\Phi$ (31).

There is a moral to our lengthy discussion of the equations of motion. It is wiser, given the current experimental situation, to use a basis containing $\mathcal{O}_{DW}$ and fermionic operators rather than a basis in which $\mathcal{O}_W$ (which cannot now be bound at tree level) substitutes for $\mathcal{O}_{DW}$. The reason is that it is difficult to imagine a meta-theory in which $\alpha_W = O(g\alpha_{DW})$ is not satisfied. The tree-level limit on α_{DW} can be used to guess a limit on α_W. But evil may the Lord be, in which case extra toil is mandatory. We shall see that the low energy data are restrictive enough to place, via one-loop quantum corrections, significant limits even on operators pointing in blind directions, of which $\mathcal{O}_W$ is an example.

There are only two other $d = 6$ blind directions:

$$\mathcal{O}_{B\Phi} \equiv i\,B^{\mu\nu}\,(D_\mu\Phi)^\dagger\, D_\nu\Phi\ , \tag{46}$$

$$\mathcal{O}_{W\Phi} \equiv i\,\vec{W}^{\mu\nu}\,(D_\mu\Phi)^\dagger\; \vec{\sigma}\; D_\nu\Phi\, , \tag{47}$$

that are intimately related to operators naturally arising in non-linear realizations of the SM, and to whose discussion we return in Chapter 5. Suffice it to say here that the equations of motion result in:

$$4\,\mathcal{O}_{B\Phi} = g\;\mathcal{O}_{WB} + g'\left[\;\mathcal{O}_{BB} - 4\,\mathcal{O}_\Phi - 4\,J^\nu(\Phi)\sum_f J_\nu(f)\right]\ , \tag{48}$$

$$4\,\mathcal{O}_{W\Phi} = g'\;\mathcal{O}_{WB} + g\left[2\,\mathcal{O}_{\widetilde{\Phi\Phi}} - 2\,\mathcal{O}_{\Phi\Phi} - 4\,\vec{J}^\nu(\Phi)\sum_f \vec{J}_\nu(f)\right]\ . \tag{49}$$

In computing in subsequent chapters corrections to various observables to first order in any of the grafts we have discussed, one may insert in the relevant Feynman diagrams either the left- or the right-hand side of any of the equations of motion we exploit. As a check of the algebra, we have performed every calculation in an ambidextrous way.

To conclude, the most useful basis of $d = 6$ operators is specific to the problem being solved. In our goal of constraining the possible three-vector-boson couplings, we are led to introduce a complete basis whose operators affect at tree level the relations dictated by the minimal standard model between precisely measured observables. One such basis consists of three "bosonic" and three "fermionic" operators:

$$\{\mathcal{O}_i\} \equiv \{\mathcal{O}_{DW}, \mathcal{O}_{WB}, \mathcal{O}_{\Phi}; \mathcal{O}_e, \mathcal{O}_{\mu}, \mathcal{O}_{e\mu}\} \tag{50}$$

announced in Table I, plus the operator $\mathcal{O}_{\widetilde{e\mu}}$, that we shall find to play no significant role.

3.2 EFFECTS OF $\mathcal{O}_{WB}$.

We proceed to discuss the theory and phenomenology of the effects induced by the addition to the standard Lagrangian, as in (24) and (25), of the $d = 6$ operators in the basis $\{\mathcal{O}_i\}$ of (50) and Table I, augmented by $\mathcal{O}_{\widetilde{e\mu}}$. All the effects not including real scalar fields vanish as v^2/Λ^2, and it is convenient to introduce the quantities

$$\epsilon_i \equiv \alpha_i \, \frac{v^2}{\Lambda^2} \,. \tag{51}$$

Since the $d = 6$ operator with the most fertile set of effects is $\mathcal{O}_{WB}$, defined in (30), we use it as a detailed example, while for the remaining operators we simply concentrate on results. The $\mathcal{O}_{WB}$ graft has been often discussed in a "linear" context [13 to 15]; its effects in a "chiral" realm have also been studied either explicitly in [8, 9, 11, 33, 35 to 39], or more obliquely in [5, 40 to 43].

The addition of $\mathcal{L}_{WB}$ to $\mathcal{L}_0$ induces tree-level departures from the standard relations between couplings and intermediate vector boson masses. To analyze them, follow custom in shifting the neutral component of Φ in $\mathcal{L}_0 + \mathcal{L}_{WB}$ to $(H_0 + v)/\sqrt{2}$, with H_0 the physical scalar doublet(s). $\mathcal{L}_{WB}$ modifies the relations between the parameters e_0 and $M_{Z,0}$ in the standard $\mathcal{L}_0$ and the observable input values of α and M_Z by the amounts $\Delta\alpha/\alpha \simeq -2\,s\,c\,\epsilon_{WB}$ and $\Delta M_Z/M_Z \simeq s\,c\,\epsilon_{WB}$, with $s \equiv \sin\theta$, $c \equiv \cos\theta$ for short. Naturally, these finite renormalizations are to be reabsorbed into the definition of the physical parameters, whereby their effects resurface elsewhere in the full Lagrangian. We employ the notation Δ for shifts in parameters in intermediate stages of a calculation, reserving in what follows the symbol δ for deviations from the standard results on predicted observables. The diagonalization of the neutral-boson mass matrix in terms of canonically normalized fields, with α, G_F and M_Z kept fixed as input empirical parameters, results in a Lagrangian identical to that of the standard model in all respects, but six:

ℵ) The triple gauge vertices of interest are modified. The coefficients introduced in (20) and (21) acquire the extra contributions

$$\delta g_Z \simeq \frac{-1}{\cos 2\theta}\,\epsilon_{WB} \qquad \delta\kappa_\gamma \simeq \frac{c}{s}\,\epsilon_{WB} \qquad \delta\kappa_Z \simeq -\frac{s}{c}\,\epsilon_{WB}, \tag{52}$$

with the $\simeq$ sign referring here (and below) to a result to first order in ϵ_{WB}. The explicit relations in (52) between δg_Z, $\delta\kappa_\gamma$ and $\delta\kappa_Z$ are a consequence of gauge invariance. Overlooking this kind of constraint makes the theory hard to defend and the phenomenology more than suspect, a point we shall return to. The TGV deviations in (52) cannot be arbitrarily large, for this would upset the success of the standard model concerning the next items.

♣) The W mass prediction of eq. (6) acquires a correction

$$\delta M_W \simeq -2\,\tilde{n}\,\frac{s}{c}\,\epsilon_{WB}\,M_Z\,, \tag{53}$$

with

$$\tilde{n} \equiv \frac{1}{2}\,\frac{c^3}{\cos 2\theta} \simeq 0.607 \tag{54}$$

an ubiquitous factor in the modifications of M_W by various grafts.

◇) The coefficients potentially modifying the Z partial widths, as in (13), receive the contributions:

$$\delta\gamma_f \simeq 0\,, \tag{55}$$

$$\delta\kappa_f \simeq \frac{c}{s\,\cos 2\theta}\,\epsilon_{WB}. \tag{56}$$

♡) The leptonic axial couplings to Z's stay put:

$$\frac{\delta\bar{g}_l^A}{\bar{g}_l^A} \simeq 0\,, \tag{57}$$

as implied by (14) and (55).

♠) The ratio of neutrino cross sections of eq.(7) is shifted by:

$$\frac{\delta R_\nu}{R_\nu} \simeq -\breve{n}\,\epsilon_{WB}, \tag{58}$$

with

$$\breve{n} \equiv \frac{1-\frac{40}{27}s^2}{\frac{1}{2}-s^2+\frac{20}{27}s^4}\,\frac{s\,c}{\cos 2\theta} \simeq 1.517 \tag{59}$$

to occur repeatedly in the modifications of R_ν by various operators.

⋈) New interactions of $O(\epsilon_{WB})$ involving one or two H_0 fields (and two or three vector bosons) appear as additional predictions of an effective interaction proportional to $\mathcal{O}_{WB}$. Once again, all of their coefficients are determined, as a consequence of gauge invariance. The form of these interactions:

$$\mathcal{L}^H_{WB} = -\frac{\epsilon_{WB}}{2}\left[\frac{2\,H_0}{v}+\frac{H_0^2}{v^2}\right]\left\{\begin{array}{l} s\,c\,(F_{\mu\nu}F^{\mu\nu}-Z_{\mu\nu}Z^{\mu\nu}) \\ +\cos(2\,\theta)\,F_{\mu\nu}Z^{\mu\nu} \\ +\,2\,i\,g\,W^\dagger_\mu W_\nu\,(c\,F^{\mu\nu}-s\,Z^{\mu\nu}) \end{array}\right\} \tag{60}$$

is suggestive of all kinds of interesting novel decays, and of modifications of some standard ones.

3.3 Current limits on the effects of $\mathcal{O}_{WB}$.

The unworldly agreement between experiment and the standard model implies limits on ϵ_{WB}, the coefficient of a putative novel interaction characterized by $\mathcal{O}_{WB}$. To illustrate these constraints we assume no accidental cancellation between the effects of $\mathcal{L}_{WB}$ and the other $d = 6$ effective interactions relevant to our analysis. The limits on ϵ_{WB} depend sensitively on m_t; their weak dependence on M_H we combined linearly with the experimental uncertainties. As a result, we can present our constraints on ϵ_{WB} as an allowed domain in the plane (m_t, ϵ_{WB}). This is done in Figure [1] wherein the $2\,\sigma$ limits from the individual observables are depicted, as well as the combined domain (interior to the $\chi^2 = \chi^2(\text{min}) + 4$ contour) whose projection on either the ϵ_{WB} or m_t axis is the corresponding single-variable 95.5% ($\Leftrightarrow 2\sigma$) confidence-level interval.

The different limits are obtained by comparing the current experimental values of (19) with the predictions of the standard model, modified by the $O(\epsilon_{WB})$ corrections discussed in the previous section.

One can deduce from Fig.[1] that, whatever the value of m_t turns out to be,

$$-0.008 < \epsilon_{WB} < 0.010 \ (2\sigma). \tag{61}$$

Given this constraint, can the effects of the non-standard vector boson couplings (52) induced by $\mathcal{L}_{WB}$ be *directly* observed in W-pair production at LEP-2? The answer is a sorrowful "niet", as we proceed to demonstrate.

3.4 Unobservability of $\mathcal{O}_{WB}$ at LEP-2.

The theoretical expression for the $e^+e^- \to W^+W^-$ cross section implied by $\mathcal{L}_0 + \mathcal{L}_{WB}$ need not be reproduced here, since it has been computed and reported in the literature [44] for generic values of, among other quantities, κ_γ, κ_Z, M_Z, M_W, $\bar{g}_e^V$ and $\bar{g}_e^A$, and we have checked the results to be correct. The corresponding total W-pair production cross section, σ_T, is well behaved at high energy and deviates little from the standard one, ***provided*** that the non-standard effects are all linked to each other by gauge invariance, and consequently expressible in terms of a single non-standard number, ϵ_{WB}, as in (52), (53), (55), and (56). This and/or other consequences of gauge invariance, in this and/or other closely related contexts, have occasionally been overlooked in previous work [44 to 47] and have been discussed in [48, 49]. We illustrate the point by way of a first example in Figure [2], wherein σ_T is plotted for the standard model and compared with the correct prediction for $\epsilon_{WB} = 0.04$. To demonstrate the incidence of gauge invariance, we also include results with $\delta\bar{g}_e^V$ artificially set to zero, but with all other non-standard modifications correctly related to each other, for $\epsilon_{WB} = 0.04$. The exercise is repeated with $\delta\bar{g}_e^V = 0$ substituted for $\delta\kappa_\gamma = 0$. Notice the brutal reduction that the constraint of gauge invariance imposes on the novel effects.

A very sensitive direct test of the presence of $\mathcal{L}_{WB}$ concerns the differential cross section $d\sigma/d\cos\theta_+$, with θ_+, for definiteness, the e^+W^+ scattering angle. These softwary days, nothing short of Monte-Carlo simulations suffices to convey a message. We have dutifully generated the rather generous statistics of 10^4 W-pairs at $\sqrt{s} = 200$ GeV with various assumed values of ϵ_{WB}. As an example, the $\cos\theta_+$ distribution (normalized to the standard expectation, and denoted R) is plotted in Figure [3] for $\epsilon_{WB} = 0.04$, 40 bins in the $\cos\theta_+$ variable, and purely statistical error bars. We have performed χ^2 tests of the significance of deviations from the standard prediction for the shape of this differential cross-section at various assumed values of ϵ_{WB}. The result, shown in Figure [4], indicates that values of ϵ_{WB} greater than 0.023 or smaller than -0.027 would be distinguishable from zero at the 95.5% ($\Leftrightarrow 2\sigma$) level. Also shown in the figure as a vertical band is the domain (61) of ϵ_{WB} values currently allowed by the lower energy data. The moral is that more than a fourfold increase in the assumed number of W-pairs (10^4), as well as negligible systematic errors, would be necessary for LEP-2 to compete with LEP-1 in detecting an interaction of the $\mathcal{L}_{WB}$ ilk. Yet another quadrupling of the statistics would be called for in reaching a decent "discovery level": that which may produce a result on ϵ_{WB} 4σ-away from zero.

The only hope for LEP-2 to (indirectly) detect a non-vanishing ϵ_{WB} (or reduce its LEP-1 limits) relies on an improvement of the determination of M_W. LEP-2 may [2] measure M_W to a little less than 0.2% accuracy, reducing the error in M_W/M_Z to a similar level from its current 0.6%. But inspection of Fig.[1] shows that even this improvement is marginal: the LEP-1 results are indeed very restrictive.

3.5 Effects of $\mathcal{O}_{WB}$ on scalars.

To complete the tree-level discussion of a non-vanishing $\mathcal{L}_{WB}$, we comment on some of the effects implied by the couplings (60) that involve scalars. This is the only point at which, for the sake of discussion, we commit ourselves to a single isodoublet Higgs particle. Of the many vertices described by (60) the most interesting are the three-particle ones that are not present at tree level in the standard model: $ZH_0\gamma$ and $H_0\gamma\gamma$. In the corresponding amplitudes a non-vanishing ϵ_{WB} only has to compete with a standard $O(\alpha)$ effect.

To discuss the $ZH_0\gamma$ vertex, we choose the case $M_H < M_Z$. Let $A = A_0 + A_{WB}$ be the amplitude for $Z \to H_0\gamma$ decay. The corresponding width is:

$$\Gamma(Z \to H_0\gamma) = |A|^2 \, \frac{E_\gamma^3}{12\,\pi} \cdot \tag{62}$$

The standard A_0 is dominated by the triangle graph with intermediate W's and is numerically equal [50] to:

$$A_0 \simeq -\frac{e\,\alpha}{4\,\pi\,\sin^2\theta\, M_W}\left[4.56 + 0.25\left(\frac{M_H}{M_W}\right)^2\right], \tag{63}$$

practically independent of M_H for $M_H < M_Z$. With the same normalization

$$A_{WB} \simeq -\frac{2}{v}\,\cos(2\,\theta)\,\epsilon_{WB}. \tag{64}$$

The ratio $\Gamma(Z \to H_0\gamma)/\Gamma_0(Z \to H_0\gamma)$ varies from ~ 0 to ~ 3.7 in the interval $|\epsilon_{WB}| < 0.01$, indicating a strong sensitivity to the new physics. The trouble is that current bounds on the branching ratio B for $Z \to H_0\gamma$ are not overly restrictive. From the L3 result [51] $B < 10^{-3}$ (for $30 < M_H < 86$ GeV) we can only extract from the above expressions rather weak constraints on ϵ_{WB}. To give a couple of examples, for $M_H = 40$ GeV ($M_H = M_W$) we obtain $|\epsilon_{WB}| < 0.3\,(2.3)$, neither of which is competitive with (61).

The partial width for $H_0 \to \gamma\gamma$ is

$$\Gamma(H_0 \to \gamma\gamma) = |A^{\gamma\gamma}|^2 \, \frac{\alpha^2 M_H^3}{16\,\pi^3\,v^2}\,, \tag{65}$$

with $A^{\gamma\gamma} = A_0^{\gamma\gamma} + A_{WB}^{\gamma\gamma}$. The standard triangle graphs with intermediate fermions and W's result [52] in $A_0^{\gamma\gamma} \sim 1$, while, with the same normalization, $A_{WB}^{\gamma\gamma} \simeq +2\,s\,c\,\pi\epsilon_{WB}/\alpha$. The ratio $\Gamma(H_0 \to \gamma\gamma)/\Gamma_0(H_0 \to \gamma\gamma)$ swings from ~ 5.6 to ~ 19 in the interval $-0.01 < \epsilon_{WB} < 0.01$, vanishing on its way. Again, this fine sensitivity to the new physics is beneaped by the fact that the $H_0 \to \gamma\gamma$ channel is not easily accessible. We can interpret the Opal limit [53] $B_{\gamma\gamma\gamma} < 6.6\,10^{-5}$ on the $Z \to \gamma\gamma\gamma$ branching ratio as a limit on the chain $Z \to \gamma H_0(\to \gamma\gamma)$, and use the preceding information to obtain the non-competitive limits $|\epsilon_{WB}| < 0.18\,(0.3)$, again in the examples $M_H = 40$ GeV ($M_H = M_W$).

Should H_0 be found at future proton colliders to have a mass in the interval $M_Z < M_H < 2\,M_W$, the branching ratios for $H_0 \to Z\gamma$, $\gamma\gamma$ may be measurable. The resulting information on ϵ_{WB} would be very precise.

3.6 Limits and observability of operators other than $\mathcal{O}_{WB}$.

All $d = 6$ operators belonging to the basis of (50) and Table I can be studied in a manner entirely analogous to the discussion of $\mathcal{O}_{WB}$ in the previous three sections. Their effects on the input quantities of our Z-scheme, encountered in intermediate steps of the calculations, are:

$$\begin{aligned} \frac{\Delta G_F}{G_F} &\simeq \frac{1}{4}\,\left[\epsilon_e + \epsilon_\mu - \epsilon_{e\mu}\right]\,, \\ \frac{\Delta\alpha}{\alpha} &\simeq -2\,s\,c\,\epsilon_{WB}\,, \\ \frac{\Delta M_Z}{M_Z} &\simeq s\,c\,\epsilon_{WB} + \frac{1}{4}\,\epsilon_\Phi + \frac{g^2}{2}\,\epsilon_{DW}\,, \end{aligned} \tag{66}$$

with the various ϵ's defined as in (51). The incidence of these grafts on predicted quantities: the ZWW and γWW couplings defined in (20) to (23), as well as on the current observables discussed in Section 2 (the W mass, the parameters $\delta\gamma$ and $\delta\kappa$

describing the Z partial widths as in (15) and (16) and axial couplings as in (14), and the ratio R_ν of neutral to charged current neutrino-scattering cross-sections) can be gathered in matrix form as:

$$\begin{pmatrix} \cos 2\theta\, \delta g_Z \\ \delta\kappa_\gamma \\ \delta\kappa_Z \\ \lambda \\ \delta M_W/\tilde{n}\, M_Z \\ \delta\gamma_{e,\nu_e} \\ \delta\gamma_{\mu,\nu_\mu} \\ \delta\gamma_q \\ \cos 2\theta\, \delta\kappa_{e,\nu_e} \\ \cos 2\theta\, \delta\kappa_{\mu,\nu_\mu} \\ \cos 2\theta\, \delta\kappa_q \\ \delta R_\nu/\breve{n}\, R_\nu \end{pmatrix} \simeq \begin{pmatrix} -1 & -\frac{1}{t} & -\frac{1}{2t} & -\frac{1}{2t} & -\frac{1}{2t} & \frac{1}{2t} \\ \frac{1}{t} & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ -2t & -2 & -t^2 & -t^2 & -t^2 & t^2 \\ 0 & -2 & 1 & 1 & -1 & 1 \\ 0 & -2 & 1 & -1 & 1 & 1 \\ 0 & -2 & 1 & -1 & -1 & 1 \\ \frac{1}{t} & 2c^2 & s^2 & s^2 & c^2 & -c^2 \\ \frac{1}{t} & 2c^2 & s^2 & c^2 & s^2 & -c^2 \\ \frac{1}{t} & 2c^2 & s^2 & c^2 & c^2 & -c^2 \\ -1 & -2[\frac{2}{\breve{n}} + sc] & -sc & -sc & -sc & sc \end{pmatrix} \times \begin{pmatrix} \epsilon_{WB} \\ \frac{1}{4}\epsilon_\Phi \\ g^2\, \epsilon_{DW} \\ \frac{1}{4}\epsilon_e \\ \frac{1}{4}\epsilon_\mu \\ \frac{1}{4}\epsilon_{e\mu} \end{pmatrix} \qquad (67)$$

where $\tilde{n}$ and $\breve{n}$ are as in (54) and (59), and $t = \tan\theta$.

All results reported in (67) are q^2-independent, with the exception of the $\mathcal{O}_{DW}$ contributions to the various $\delta\gamma_f$ and $\delta\kappa_f$, that are therein given for $q^2 = M_Z^2$. Their general expression:

$$\begin{aligned} \delta\gamma_f &\simeq g^2\, \epsilon_{DW}\, \frac{q^2}{M_Z^2}, \\ \delta\kappa_f &\simeq g^2\, \epsilon_{DW} \left[\frac{c^2}{\cos 2\theta} - \frac{q^2}{M_Z^2}\right] \end{aligned} \qquad (68)$$

reflects the higher-derivative character of the operator $\mathcal{O}_{DW}$. Strictly speaking, the approximation of constant Γ_f's (as in the "most general" expression (8) used to fit the data) is invalid for the P-wave decays mediated by $\mathcal{O}_{DW}$ or a weak magnetic-moment coupling. One cannot trivially "undo" this approximation, since its effects are interwoven with those of QED radiative corrections. But the measurements rely on data for which $|q^2 - M_Z^2|$ is a small fraction of M_Z^2, for which the energy dependence is a correspondingly small correction: while waiting for a less standard analysis of the data we may simply employ the fixed energy result of (67). As the measurements improve, it would be interesting to search for a q^2-dependence of the "widths" in (8).

Much as for $\mathcal{O}_{WB}$, we can assume no accidental cancellations between the contributions of the various operators in the basis $\{\mathcal{O}_i\}$ of (50) and Table I to the different observables and extract from existing experiments the combined allowed domains in the (m_t, ϵ_i) planes, whose projections are the 95.5%-confidence ($\Leftrightarrow 2\sigma$) intervals on the individual variables. These results are gathered in Figure [5]. The operators $\mathcal{O}_e$ and $\mathcal{O}_\mu$ are flavour-sensitive and in constraining their coefficients we have used the experimental inputs Γ_e, Γ_μ, $\bar{g}_e^A$ and $\bar{g}_\mu^A$ in (19). For the remaining grafts we use Γ_l and $\bar{g}_l^A$, whose values, also in (19), are extracted from the data assuming lepton universality. The results shown in Fig.[5] are akin to the results on $\mathcal{O}_{WB}$, with one exception, concerning the operator $\mathcal{O}_\Phi$ defined in (31). The allowed domain in the (m_t, ϵ_Φ) plane extends to an unusually large value* of m_t (~ 345 GeV). The upper limit on ϵ_Φ turns out to be significantly larger than that of the coefficients of the other grafts, but can only be saturated if m_t turns out to be very large, as can be seen in Fig.[5b].

From the various plots in Fig.[5], one can read that whatever the value of m_t turns out to be:

$$\begin{aligned}
&-0.008 < \epsilon_{WB} < 0.010 \\
&-0.008 < \epsilon_\Phi < 0.053 \\
&-0.024 < \epsilon_{DW} < 0.014 \\
&-0.054 < \epsilon_{e\mu} < 0.019 \\
&-0.035 < \epsilon_e < 0.018 \\
&-0.025 < \epsilon_\mu < 0.030
\end{aligned} \tag{69}$$

all at the 2σ level. Limits on the four-fermion operator $\mathcal{O}_{e\mu}$ have been previously discussed in [16].

We extend in Figure [6] to other operators the exercise of Section 3.4 and Fig.[4] concerning $\mathcal{O}_{WB}$. That is, we compare the detectability level of various operators in W-pair production at LEP-2 with the corresponding lower-energy constraints. The figure indicates that the observability of these non-standard effects at LEP-2 is excluded for all operators, with the borderline exception of $\mathcal{O}_\Phi$. By "borderline" we mean the following: Fig.[6b] indicates that while the current 2σ-limit on the observability of ϵ_Φ at LEP-2 could be improved with the assumed statistics (10^4 W-pairs), an honest-to-goodness LEP-2 discovery potential (a measurement 4σ away from zero, or $\chi^2(40\,\mathrm{d.o.f.}) \simeq 90$) lies beyond the present LEP-1 limit.

Perhaps noticeable is the fact that we have not included in the preceding discussion the operator $\mathcal{O}_{\widetilde{e\mu}}$ of (38). The coefficient of this operator is limited by the known $V-A$ structure of muon decay to the level $\epsilon_{\widetilde{e\mu}} < 0.14$ [54, 14]. In the limit of negligible fermion

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* In our calculations the lowest order vacuum insertions of loops containing t's have been iterated in the usual manner, but corrections to them with inner scalars have not been included. At $m_t = 345$ GeV, the top Yukawa coupling is such that $y_t^2/4\pi \sim 0.2$, so that these corrections are not expected to be overwhelming.

masses, $\mathcal{L}_{\widetilde{e\mu}}$ contributes only to second order in $\epsilon_{\widetilde{e\mu}}$ to the observables of interest, given that its "chirality" is opposite to that of the standard currents occurring in the amplitudes that could interfere with it. Its effects are negligible at the attainable precision of the relevant experiments.

Current limits on the coefficients of the operators discussed in this section from the non-observation of fundamental scalars are, like in the case of $\mathcal{O}_{WB}$, relatively weak; we do not discuss them.

The individual limits on each ϵ_i in (69) have been obtained setting all ϵ_j, $j \neq i$ to zero, that is assuming no accidental cancellations. In principle we might have considered the limiting hypersurfaces in the full (ϵ_l, m_t), $l = 1, 6$ space and projected then onto the individual (ϵ_i, m_t) planes. Even harder to visualize would be the exercise of extending this procedure to the inclusion of other operators, such as $\mathcal{O}_{eT}$, that we have argued not to deserve the honour of belonging to our complete basis. The result of the fully general exercise would be disappointing indeed: the limits on the individual ϵ_i would simply disappear! We would have "experimentally" discovered the existence of "blind directions", unnaturally contrived linear combinations of the tree grafts to which current observables are insensitive at tree level. Even though there is no known reason why a meta-theory would "point" exclusively in such directions, it is interesting to explore how the current information can be used to "cut" them, as one considers quantum effects at the one-loop level.

3.7 Blind directions: the example of $\mathcal{O}_W$.

Available experimental results are not sensitive at tree level to triple gauge vertices. We have seen that the operators $\mathcal{O}_W$ (29), $\mathcal{O}_{B\Phi}$ (46) and $\mathcal{O}_{W\Phi}$ (47), that describe TGVs, can be written as linear combinations (42), (48) and (49) of operators that are tightly constrained at tree level from current data. As a test of the entries in (67), we have explicitly checked that these particular "directions" give vanishing tree-level contributions to all the observables in (67). These linear combinations are "blind directions" in $d = 6$ operator space, their coefficients in $\mathcal{L}_{eff}$ can only be bound via one-loop quantum effects on current observables; the bounds are bound to be weaker than the tree-level ones. The question arises: are the constraints on the blind-direction coefficients α_W, $\alpha_{B\Phi}$ and $\alpha_{W\Phi}$ loose enough to allow for a window of opportunity at LEP-2? We have analyzed this question in full detail for the case of $\mathcal{O}_W$, and we proceed to demonstrate that its answer is marginally affirmative.

None of the parameters of the standard $\mathcal{L}_0$ are modified at tree level by the appendage of a term $\mathcal{L}_W$ proportional to $\mathcal{O}_W$. The only effect is an additional interaction of the non-standard form (22), plus the accompanying extra multiple-vector-boson vertices fully determined by the gauge-invariant nature of eq.(29) [48]. In terms of $\epsilon_W \equiv \alpha_W\, v^2/\Lambda^2$, the coefficients in (22) are

$$\lambda_\gamma = \lambda_Z = \lambda = \frac{e}{4\,\sin\theta}\,\epsilon_W\,. \tag{70}$$

$\mathcal{L}_W$ is a higher-derivative interaction and the vertices it describes vanish as any of the external boson momenta tend to zero. Consequently, its addition does not affect, even beyond tree level, many of the standard relations between the standard-model parameters and low-energy observables. An example of an unmodified relation concerns the input Fermi constant G_F: all diagrams in Figure [7] vanish for $q^2 \ll M_W^2$. To limit the coefficient of $\mathcal{O}_W$ we must look elsewhere.

Quantum loops involving $\mathcal{L}_W$ are gauge-independent. They result in apparent quadratic divergences that must and do cancel once observables are expressed in terms of other observables (in a more old-fashioned mood, one could also renormalize these divergences right away with a counterterm, prior to the establishment of contact with the actual physics). To specify our intermediate results we choose the search for $n = 2, 4$ poles in dimensional regularization, and trade their respective residues, in the usual way, for the coefficients of Λ^2 and $\ln\Lambda$ cut-off dependences, with Λ set to coincide with the characteristic scale of the new dynamics. With these substitutions our final results are meaningful up to (but not including) "constant" terms: for not-very large a value of $\ln(\Lambda^2/M_W^2)$, these terms may affect the forthcoming bounds on ϵ_W by a factor which, barring cancellations in all relevant observables, should be of $O(1)$. "Conspiratorial" grafts such as (39) can also be viewed here as counterterms and also compete with the constant terms with different coefficients in the different observables. We neglect conspiratorial and "constant" contributions on grounds of the assumption of no pervasive accidental cancellations.

Two of the input quantities of our Z-scheme, M_Z and α, are affected by the grafting of $\mathcal{L}_W$. The shifts induced by the diagrams of Figure [8] and Figure [9]:

$$\frac{\Delta\alpha}{\alpha} \simeq \left.\frac{\Pi_{\gamma\gamma}}{q^2}\right|_{q^2=0} \simeq \frac{3\,\alpha\,\lambda}{2\,\pi}\left[-\frac{\Lambda^2}{M_W^2} + 2\,\ln\left(\frac{\Lambda^2}{M_W^2}\right)\right], \tag{71}$$

$$\frac{\Delta M_Z^2}{M_Z^2} \simeq \left.\frac{\Pi_{ZZ}}{M_Z^2}\right|_{q^2=M_Z^2} \simeq \frac{3\,\alpha\,\lambda}{2\,\pi\,s^2}\left[-\frac{\Lambda^2}{M_Z^2} + \left(2\,c^2 - \frac{1}{3}\right)\ln\left(\frac{\Lambda^2}{M_W^2}\right)\right], \tag{72}$$

have the announced apparent quadratic divergences that will disappear as a predicted quantity is expressed in terms of the physical input parameters* . We employ again the symbol Δ for corrections to parameters occuring at intermediate stages of a calculation, reserving δ, as before, to describe the shift away from the standard model of a predicted quantity.

The diagrams of Fig.[9] also describe the leading effect of $\mathcal{L}_W$ in the WW two-point function. At $q^2 = M_W^2$:

$$\left.\frac{\Pi_{WW}}{M_W^2}\right|_{q^2=M_W^2} \simeq \frac{3\,\alpha\,\lambda}{2\,\pi\,s^2}\left[-\frac{\Lambda^2}{M_W^2}+\frac{5}{3}\ln\left(\frac{\Lambda^2}{M_W^2}\right)\right]. \tag{73}$$

The shift on the predicted value of M_W, expressed in terms of the input parameters α, G_F and M_Z (or equivalently α and the standard values of $\sin\theta$ and M_W) is:

$$\delta M_W \simeq 4\,\tilde{n}\tan^2\theta\; M_Z\,\delta_W, \tag{74}$$

with $\tilde{n}$ as in (54) and

$$\delta_W \equiv \frac{\alpha\,\lambda}{8\,\pi\,s^2}\,\ln\left(\frac{\Lambda^2}{M_W^2}\right), \tag{75}$$

a result with no quadratic cut-off dependence.

The grafting of $\mathcal{L}_W$ onto $\mathcal{L}_0$ also modifies at one loop the description of e^+e^- annihilation in the region of the Z resonance. To first order in ϵ_W (or λ), the effects solely arise from the three-boson coupling of (22). To describe the $e^+e^- \to f\bar{f}$ cross section we must add to the standard one-loop result the effects of the diagrams in Figure [10]. The resulting one-particle irreducible vacuum insertions $i\Pi_{ZZ}$ and $i\Pi_{\gamma\gamma}$ of Fig.[10a] are to be iterated and "put in the denominator" in the usual fashion, to result in dressed γ and Z propagators. The vertex corrections of Fig.[10b,d] and the $Z\gamma$ mixing insertion $i\Pi_{Z\gamma}$, whose non-standard piece is that of Figure [10c], are most conveniently kept "in the numerator" and interpreted as contributions to the $Z \to e^+e^-$ (or $f\bar{f}$) amplitude, as in the standard case [55]. The imaginary part of $i\Pi_{Z\gamma}$ can be dropped in practice, since it departs from the standard result (which is by itself negligible) by an $O(g\,\lambda)$ correction.

The above exercise results in modifications of the predicted Z-widths occurring in the cross-section of eq.(8) that can, once again, be expressed in the form of eq.(13).

$\star$ Even though α is defined at $q^2 \to 0$ and the diagrams of Fig.[8a,b] vanish in this limit, the extra photon propagator in the diagram of Fig.[8c] cancels the leading q^2 behaviour of the vacuum-insertion correction $i\Pi_{\gamma\gamma}$.

Let $\uparrow$ refer to neutrinos and $Q = 2/3$ quarks, and $\downarrow$ to their iso-partners. The results are:

$$\begin{aligned}
&\delta\gamma_{\uparrow} \simeq 0\,, \\
&\delta\gamma_{\downarrow} \simeq -8\,\delta_W\,, \\
&\delta\kappa_{\uparrow} \simeq -\left(2 + 4\left[\frac{c^2}{\cos 2\theta} - \frac{q^2}{M_Z^2}\right]\right)\delta_W\,, \\
&\delta\kappa_{\downarrow} \simeq \left(2 - 4\left[\frac{c^2}{\cos 2\theta} - \frac{q^2}{M_Z^2}\right]\right)\delta_W\,.
\end{aligned} \tag{76}$$

For the interesting cases $f = l^-$ (a given charged lepton) and $f = h$ (the sum over all quarks but t) the above expressions lead, at $q^2 = M_Z^2$, to the numerical results:

$$\frac{\delta\Gamma_l}{\Gamma_l} \simeq -8.13\,\delta_W\,, \tag{77}$$

$$\frac{\delta\Gamma_h}{\Gamma_h} \simeq -4.85\,\delta_W\,. \tag{78}$$

Vertex corrections involving $\mathcal{O}_W$, such as the one depicted in Fig.[10], distinguish upper and lower flavours, so that (78) is not of the form (16), valid for all the other operators of the linear basis (50) that we have previously discussed.

The effect of $\mathcal{O}_W$ on the axial coupling of Z's to charged leptons and on the ratio R_ν of neutrino cross sections are:

$$\frac{\delta\bar{g}_l^A}{\bar{g}_l^A} \simeq -4\,\delta_W\,, \tag{79}$$

$$\frac{\delta R_\nu}{R_\nu} \simeq -1.96\,\delta_W\,. \tag{80}$$

Finally, at $q^2 \neq 0$, the presence of $\mathcal{L}_W$ makes the right- and left-handed couplings of photons to fermions differ. The right-handed couplings of the photon stay put ($Q_f^R = Q_{f0}^R$) but the left-handed "charge" acquires a form-factor q^2-dependence:

$$Q_f^L = Q_{f0}^L + \frac{q^2}{c^2\,M_Z^2}\,\delta_W\,. \tag{81}$$

These expressions are to be used in the $Z - \gamma$ interference (17) and purely photonic terms (18) of the $e^+e^- \to f\bar{f}$ cross section (8).

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The Self-Couplings of Vector Bosons: does LEP-1 obviate LEP-2?

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ERRATUM

We have noticed an error affecting the formulas occuring in one page of our manuscript of the above title. The corrected page is enclosed. These errors imply minor modifications in Figs. [11] and [14], that will appear corrected in an eventual published version. No conclusions are affected by these changes.

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The results are:

$$\begin{aligned}\delta\gamma_f &\simeq -8\,\frac{q^2}{M_Z^2}\,\delta_W\,,\\ \delta\kappa_f &\simeq -\left[4\,\frac{c^2}{\cos 2\theta}-6\,\frac{q^2}{M_Z^2}\right]\delta_W\,,\end{aligned}\tag{76}$$

For the interesting cases $f = l^-$ (a given charged lepton) and $f = h$ (the sum over all quarks but t) the above expressions lead, at $q^2 = M_Z^2$, to the numerical results:

$$\frac{\delta\Gamma_l}{\Gamma_l} \simeq -8.13\,\delta_W\,,\tag{77}$$

$$\frac{\delta\Gamma_h}{\Gamma_h} \simeq -8.17\,\delta_W\,.\tag{78}$$

The effect of $\mathcal{O}_W$ on the axial coupling of Z's to charged leptons and on the ratio R_ν of neutrino cross sections are:

$$\frac{\delta\bar{g}_l^A}{\bar{g}_l^A} \simeq -4\,\delta_W\,,\tag{79}$$

$$\frac{\delta R_\nu}{R_\nu} \simeq 2.4\,\delta_W\,.\tag{80}$$

Finally, at $q^2 \neq 0$, the presence of $\mathcal{L}_W$ makes the right- and left-handed couplings of photons to fermions differ. In the convention in which the right-handed couplings of the photon stay put ($Q_f^R = Q_{f0}^R$), the left-handed "charge" and the fine structure constant acquire form-factor q^2-dependences:

$$\begin{aligned}Q_f^L &\simeq Q_{f0}^L - 2\,T_3\,\frac{q^2}{c^2\,M_Z^2}\,\delta_W\,,\\ \frac{\delta\alpha}{\alpha} &\simeq -4\,\tan^2\theta\,\frac{q^2}{M_Z^2}\,\delta_W\,.\end{aligned}\tag{81}$$

These expressions are to be used in the $Z-\gamma$ interference (17) and purely photonic terms (18) of the $e^+e^- \to f\bar{f}$ cross section (8).

We are indebted to Alain Blondel for asking the right question on this point, and to Andy Cohen for helpful discussions.

At this point, several comments are in order. With the phase-space factors having been explicitly taken into account, the widths in the standard interpretation of (8) are constants. The energy dependence of the widths implied by (76) stems from the higher-derivative character of $\mathcal{L}_W$ and of the corresponding three vector-boson coupling (22), a situation we had already encountered in the case of the $\mathcal{O}_{DW}$ results in (68). In the modified form (81) of the γ coupling to fermions, the new term solely arises from the vertex correction of Figure [10d]. But in the modified couplings of (76) and the corresponding Z-widths (77) and (78) vertex corrections do not give the complete answer. The reason is that the imaginary part of the Z propagator gets rescaled as its real part is renormalized to $s - M_Z^2$, an act that can be interpreted as wave function renormalization, or as an ingredient in the proper definition of a running coupling. The authors of a recent paper on this subject [56] overlook the minor point of the q^2-dependence, as well as the effects of propagator rescaling* , triggering the detail of our discussion.

Much as for other operators, the agreement between experiment and standard expectations can be cast into an allowed domain in the plane spanned by m_t and the parameter δ_W of eq.(75), that describes the quantum corrections induced by $\mathcal{O}_W$. The results are shown in Figure [11], from which one can read that

$$-0.0007 < \delta_W < 0.0022 \;\; (2\sigma). \tag{82}$$

The operators $\mathcal{O}_i$ that we have analyzed in previous sections have tree-level effects on LEP-2 and on lower energy observables, all linear in $\epsilon_i = \alpha_i v^2/\Lambda^2$, with α_i their coefficients in $\mathcal{L}_{eff}$. Comparison of present bounds with the LEP-2 potential is straightforward. The case of a blind direction such as $\mathcal{O}_W$ is somewhat different. Its incidence on W-pair production is linear in ϵ_W, or the related quantity λ of eq.(70), but its lower-energy effects and bounds arise from quantum corrections linear in the quantity $\delta_W \propto \alpha\lambda \ln(\Lambda^2/M_W^2)$ defined in (75). To extract a limit on λ from (82) we must choose a scale Λ to specify a value for the logarithm. The very minimum consistent new physics scale is $\Lambda = v$, and it will be our most conservative choice. For no compelling reason, we also give explicit results for $\Lambda = 1$ TeV:

$$\begin{aligned} &-0.23\,(-0.10) < \lambda < 0.71\,(0.32) \quad [\Lambda = v \;(1 \text{ TeV})], \\ &-1.4\,(-0.62) < \epsilon_W < 4.2\,(2.0) \quad [\Lambda = v \;(1 \text{ TeV})] \end{aligned} \tag{83}$$

all at the 2σ level, and once again independent of m_t. Not surprisingly, since they

$\star$ On grounds of custodial symmetry [57], that can be imposed on non-gauge models with W-dominance, the authors of [56] correctly impose $\lambda_\gamma = \lambda_Z$, a constraint that actually follows from the deeper and exact principle of gauge invariance.

spring from a radiative correction, the constraints on ϵ_W are much weaker than the tree-level ones on the coefficients of the operators previously discussed. Looking back at Fig.[11] one notices that the rather large 2σ upper limits on λ or ϵ_W correspond to large values of $m_t \sim 200$ GeV, a behaviour not unlike the one we found in the case of the $\mathcal{O}_\Phi$ graft. For $m_t \sim 100$ GeV, the upper limits on each of the quantities in (82) and (83) are similar in absolute value to the corresponding lower limits quoted in these expressions.

Can the effects of the non-standard vector couplings (22) induced by $\mathcal{L}_W$ be *directly* observed in W-pair production at LEP-2? We proceed to show that if the top quark happens to be very heavy, and the scale Λ is not significantly bigger than 1 TeV, the current limit (83) leaves ajar a rather narrow window of hope.

The theoretical expression for the $e^+e^- \to W^+W^-$ angular distribution implied by $\mathcal{L}_0 + \mathcal{L}_W$ has been reported in the literature [58] for generic values of λ_γ and λ_Z, but we disagree with the results. Other calculations, that we have partially checked, have been published [59] for polarized beams, but they are a little cumbersome to handle. We do therefore explicitly report on our results, that we would be loath to present other than in the gauge invariant case of only one λ, as in (23) and (70). Upon grafting $\mathcal{L}_W$, the differential cross section $d\sigma/d\cos\theta_+$ acquires an additive term:

$$\frac{\delta\, d\sigma}{d\cos\theta_+} = \lambda\,\frac{\pi\alpha^2}{s}\sqrt{1-\frac{4M_W^2}{s}}\left\{\begin{array}{l}\left(b^2\left[\left(\bar{g}_e^A\right)^2+\left(\bar{g}_e^V\right)^2\right]-2\,b\,\bar{g}_e^V+1\right)\;A\\[4pt] +\dfrac{1}{2\sin^2\theta}\left(b\left[\bar{g}_e^A+\bar{g}_e^V\right]-1\right)\;I\end{array}\right\} \tag{84}$$

where

$$\begin{aligned} b &\equiv \frac{1}{2\sin^2\theta}\,\frac{s}{s-M_Z^2}\,,\\ A &\equiv \frac{s}{M_W^2}-4+\lambda\left[\frac{s}{4M_W^2}-1+\left(\frac{u\,t}{M_W^2}-1\right)\left(\frac{1}{2}-\frac{M_W^2}{2s}\right)\right],\\ I &\equiv \frac{s}{2M_W^2}-1+\frac{M_W^2}{t} \end{aligned} \tag{85}$$

and the notation of Mandelstam variables has been used.

Given the higher-derivative character of $\mathcal{L}_W$, the total W-pair production cross section $\sigma_{\rm TOT}$ is not well behaved at high energy and considerably deviates from the standard result for $\lambda \neq 0$, a behaviour illustrated in Figure [12].

The shape of the differential cross section (84) should be less sensitive than $\sigma_{\rm TOT}$ to systematic errors. We have Monte-Carlo-generated 10^4 W-pairs at $\sqrt{s} = 200$ GeV with various assumed values of λ. The $\cos\theta_+$ distribution (normalized to the standard expectation and called R) is plotted in Figure [13] for $\lambda = 0.1$, and purely statistical error bars. We have performed χ^2 tests of the significance of deviations from the standard prediction for the shape of this differential cross-section at various assumed values of λ. The result, shown in Figure [14], indicates that values of $|\lambda|$ greater than 0.08 would be distinguishable from zero at LEP-2 at the 95.5% ($\Leftrightarrow 2\sigma$) level. Also shown in the same figure are values currently allowed by the lower energy data for two fixed values of m_t.

The moral is that even blind directions are relatively well constrained via their quantum effects. In the $\mathcal{O}_W$ case, Fig.[14] indicates that if the top quark turns out to be relatively light, there is little hope for LEP-2 to have a sensitivity superior to that of LEP-1, while if m_t is relatively large ($O(200)$ GeV) a window of opportunity appears to open up. But for this window to materialize, a rather mysterious mechanism must be at work for the cunning meta-theory not to generate the other grafts of our basis (50) at a much more accessible level.

3.8 THE QUESTION OF "FORM FACTORS".

We have extracted from experiment the present limits on the coefficients of the "bosonic" operators $\mathcal{O}_{DW}$, $\mathcal{O}_{WB}$ and $\mathcal{O}_\Phi$ and of the "fermionic" operators $\mathcal{O}_e$, $\mathcal{O}_\mu$, $\mathcal{O}_{e\mu}$, an ensemble that constitutes (for our repeatedly stated main purpose) a complete $d = 6$ basis. Ditto for the blind-direction graft $\mathcal{O}_W$. It is appropriate to rediscuss at this point the question of $d > 6$ gauge-invariant effective operators, starting with the $d = 8$ ones.

The observable effects of $d = 8$ operators below the scale v are of the same nominal size as the second-order effects of the $d = 6$ operators we have discussed: v^4/Λ^4, in the worst-case scenario wherein the dimensionless coefficients in front of these corrections are $O(1)$. The only $d = 8$ operators that are relevant to the two- and three-point gauge-boson functions of interest to us are those that have the same number of gauge fields as the operators we have discussed, but an extra couple of scalar fields or $SU(2)\otimes U(1)$ covariant derivatives. Such $d = 8$ operators are of two types: those that do contain combinations of fields that are allowed by the gauge symmetry at the $d = 6$ level and those that do not*. The effects on observables linear in the coefficients of the first type

$\star$ We shall face operators of the second ilk, such as $\mathcal{L}_8$, in a subsequent chapter on chiral realizations of physics beyond the SM, wherein they arise naturally.

of $d = 8$ operators, or quadratic in those of the $d = 6$ operators we have discussed, can be thought of as "form factors" that multiply our leading order results. The question is the extent to which these form factors may deviate from unity.

In the case of the operators in the $d = 6$ basis of (50) and Table I, and their look-alike $d = 8$ siblings, the situation seems to be entirely clear. The limits on the various ϵ_i are all significantly smaller than unity, indicating that quadratic effects should be negligible, unless a fine-tuned cancellation occurred between the first-order corrections and the nominally smaller second-order ones.

In the case of the blind direction $\mathcal{O}_W$ we cannot sport a similar degree of confidence. A future theory may be so wayward as to generate a non-vanishing $\mathcal{L}_W$ and vanishing or negligible $\mathcal{L}_i$ ($i \neq W$), not letting us fall into the temptation of expecting $\epsilon_W = O(\epsilon_i)$. Our m_t-independent limits on ϵ_W in (83) are of order unity, so that significant form factors may multiply our $O(\epsilon_W)$ results. They might all be considerably smaller than unity. In the unlikely case that all these factors play in the "right direction", the search at LEP-2 for an anomalous coupling of the form (22) may be simpler than we said. Good luck to the blind directions!

4. Gauge invariance versus quantum suicide.

We have assumed throughout the preceeding chapters that the "meta-theory" that may one day supersede the standard model respects its pristine $SU(2) \otimes U(1)$ local symmetry. This is more than aesthetics, for alternatives [44 to 47] face severe problems: quantum non-decoupling, quantum non-uniformity and quantum suicide.

The "linear new-physics" additions, $\mathcal{L}_j$, in the Lagrangian of eqs.(24) and (25) vanish as $\Lambda \to \infty$, leaving no trace whatsoever: all non-standard tree-level effects "decouple". Consistency of the effective Lagrangian approach requires that the quantum corrections involving $\mathcal{L}_j$ also decouple. An observable effect that at tree level is $\sim \alpha_j/\Lambda^2$ simply ***cannot*** develop at one loop a multiplicative correction [1+ O(Λ^2)], that would leave a non-decoupling trace of the high energy scale, no matter how high. Such a non-uniformity of the $\hbar \to 0$ and $\Lambda \to \infty$ limits would mean that the effective Lagrangian is subject to quantum suicide, destroying its own very meaning at the one-loop level†.

We have seen that deviations from the standard TGVs of eqs.(20) and (21) are subject to constraints imposed by gauge invariance, such as those of (52) in the case of

† Recall that loop effects in Fermi's weak theory do decouple, while some early attempts to limit deviations from QED at e^+e^- colliders did not, till the full implications of $U_\gamma(1)$ gauge invariance were imposed on their possible forms [1].

$\mathcal{O}_{WB}$ modifications, or those embedded in (67) for other operators. New-physics effects inducing TGVs of entirely non-standard looks, of which (22) is the example concerning $\mathcal{O}_W$, are also restricted by the local symmetry, as in (70). We have illustrated in Fig.[2] the brutal phenomenological impact of a cavalier slight of gauge invariance. To quote Kroll [1] on the historical QED analogue to such misdemeanors "As long as one wishes to do no more than parametrize the degree to which the theory has been experimentally confirmed there is no special penalty associated with ignoring the general requirements...[but]...one usually pays for violation of the general theorems by finding that the most striking predictions cannot be believed".

This detailed Chapter would seem superfluous, were it not that the literature abounds in estimates of the LEP-2 "new physics reach" and in calculations of one-loop corrections involving anomalous vector-boson couplings that do not respect the gauge-symmetry constraints. As an example, the one-loop corrections of [46] regarding $\mathcal{O}_W$ and $\mathcal{O}_{WB}$ diverge as Λ^4, and attempts have been made to tame them to a mere Λ^2 behaviour in subsequent work [60]. ***Abusus non tollit usum***, these calculations have sometimes been used to establish upper limits on anomalous couplings (that may as well be read as upper [!] limits on Λ) and to gauge the physics reach of future machines [61]. In Section 3.7 we have seen en passant an example of how gauge invariance restores the decoupling nature of the non-standard quantum corrections: the shift in the predicted W-mass induced by $\mathcal{O}_W$, as in (74) (75), has the promised and expected decoupling properties, since $\lambda \propto 1/\Lambda^2$; the quadratic divergences in the intermediate steps (71), (72) and (73) have cancelled as a consequence of the condition $\lambda_\gamma = \lambda_Z$ (70) imposed by gauge invariance.

A devil's advocate might argue* along the following line. Optimistically assume that the quartic divergences can be reduced to quadratic ones [60]. For $\lambda_\gamma \neq \lambda_Z$ the quadratic and logarithmic divergences in α and M_Z can be reabsorbed into the input quantities, resulting, for instance, in an asymptotically constant non-decoupling $O(g^3 \epsilon_W \Lambda^2)$ shift in the prediction for M_W^2. The result may happen not to disagree, within errors, with the current standard tests. Since the $SU(2)\otimes U(1)$ gauge invariance is not numerically tested, in this sense, beyond a fraction of a per-cent, one has the right to challenge it below this level of precision. Trouble with this line of thought occurs as one realizes that the unmodified G_F and the modified quantities α, M_Z and M_W, into which the "new physics" effects have been reabsorbed are no longer related by gauge invariance as they would be in the standard model at tree level. The new $d \leq 4$ Lagrangian is not predictive, in a manner that can no longer be cured by sending Λ (or

* We are indebted to Gordi Kane for discussions on this point.

yet another regulator) to infinity. As one uses this Lagrangian to correct to one loop the standard relation between M_Z and M_W, or the prediction for a Z partial width, the results diverge badly enough to make the corrections entirely arbitrary. We consider the devil's argument to be phenomenologically iffy and theoretically self-destructive. The temptation to "assume" gauge invariance, deal with consistent quantum theories, and skirt all this trouble, is strong indeed.

As an extra check of decoupling in a gauge-symmetry abiding framework, we have computed the full one-loop contribution to δM_W linear in ϵ_{WB}. This parallels the discussion in Section 3.7 concerning $\mathcal{O}_W$, but for the fact that $\mathcal{O}_{WB}$ also contributes at tree level. The quartic (Λ^4) divergencies cancel in unitary gauge and do not occur in any single diagram in a covariant gauge. Quadratic divergencies are only associated to vacuum-polarization corrections; e.g., for Π_{WW}, to the diagrams of Fig.[9], with the heavy dot now representing the $\mathcal{O}_{WB}$ insertion. In dimensional regularization in 't Hooft-Feynman gauge, and up to an overall factor $\epsilon_{WB}\, g^2\, g_{\mu\nu}\, (sc/4)\Gamma(\epsilon/2)$, the $V = \gamma, Z$ loop contributions to δM_W in Fig.[9a] are $\pm 3\, \cos(2\theta)$, respectively, while those of Fig.[9b] are $\mp 2\, s^2$. The cancellations in the total effect clearly stem from the constraints imposed by gauge invariance. The one-loop effects boil down to a multiplicative correction to (53) of the form $[1 + O([e/16\,\pi^2]\, ln\, \Lambda/M_{W,H})]$, a consistent and decoupling overall result.

5. STRONGLY INTERACTING LONGITUDINAL VECTOR BOSONS.

We have discussed so far possible extensions of a linearly realized standard model, whose effects on current observables would vanish as their typical energy scale, Λ, is made much larger than the standard scale v. Their impact on "low" energy physics can be organized as a Taylor series in v^2/Λ^2, with the leading term described by the (mass dimensionality) $d = 6$ operators we have studied in detail. Hypothetical theories of the technicolour variety or a standard model with a strongly interacting scalar sector do not fit into that picture. The acts of obliterating the physical scalar field Φ or of sending its mass to infinity do not "commute" with the naive-dimension expansion, since Φ is no longer there to construct gauge-invariant effective operators. Moreover, the characteristic scale Λ of these models is of order v, and a power series in v^2/Λ^2 is not that useful a tool. In this sense, chiral realizations are the only known sensible example of "form-factor" effects not to be discarded as in Section 3.8. A systematic analysis proceeds in this case along the lines of the chiral non-linear σ-model and was first discussed in detail for the standard gauge group in [8, 9, 11].

Following a beaten path, assume the interactions responsible for the generation of intermediate vector boson masses to be endowed ab initio with an $SU(2)_L \otimes SU(2)_C$ symmetry, with $SU(2)_C$ the accidental custodial symmetry [57] of the standard potential of scalar doublets. Introduce the consuetudinary matrix U describing the longitudinal gauge-boson degrees of freedom, subject to the non-linear unitarity constraint $U^\dagger U = UU^\dagger = 1$ and transforming as a (2,2) of $SU(2)_L \otimes SU(2)_C$. The $U(1)$ piece of the covariant derivative, $\mathcal{D}$ on U

$$\mathcal{D}_\mu U \equiv \partial_\mu U + i\, g\, \frac{\vec{\sigma}}{2} \vec{W}_\mu\, U - i\, g'\, U\, \frac{\sigma_3}{2} B_\mu \tag{86}$$

has an extra σ_3 factor, relative to (27), reflecting the opposite hypercharges of the two columns of U. Let $W_{\mu\nu} \equiv \vec{W}_{\mu\nu}\vec{\sigma}/2$, $\dot{V}_\mu \equiv (D_\mu U)U^\dagger$ and $T \equiv U\sigma_3 U^\dagger$ in what follows.

Armed with the above notational artillery, one may proceed to build up a power series describing the most general Lagrangian compatible with the symmetries of the theory, along lines akin to those of the extensions of the linear version of the standard model. The technique is also similar, but for the fact that the U field, unlike the linear Φ field, has no mass dimension. The power series in "chiral" dimensions, d_χ, is a Taylor expansion in terms of operators constructed with the objects U, D^μ/Λ, $W^{\mu\nu}/\Lambda^2$ and $B^{\mu\nu}/\Lambda^2$, as recently reviewed in [6].

In chiral notation the pure-gauge sector of the standard Lagrangian is:

$$\mathcal{L}_\chi = \frac{v^2}{2}\,\mathrm{Tr}\left\{\mathcal{D}_\mu U (\mathcal{D}^\mu U)^\dagger\right\} + \frac{1}{2}\,\mathrm{Tr}\left\{W_{\mu\nu}W^{\mu\nu}\right\} - \frac{1}{4}\, B_{\mu\nu}B^{\mu\nu} \tag{87}$$

with $d_\chi = 2$ for the first term, $d_\chi = 4$ for the others. There is another $d_\chi = 2$ effective operator and thirteen other $d_\chi = 4$ ones that respect the standard $SU(2) \times U(1)$ symmetry [9], with inclusion of the custodial-breaking terms that the quantum corrections inevitably induce. The fourteen extra terms are to be added with not precisely calculable weights to (87), to obtain an effective Lagrangian that is complete and unprejudiced up to $d_\chi = 4$. This unseemingly long list of operators is reduced to a total of six as we proceed to concentrate on those describing two- and three-point functions, and to use the equations of motion to eliminate redundant terms.

The equations of motion⋆ emanating from $\mathcal{L}_\chi$, augmented by the standard cou-

⋆ We are indebted to Luciano Maiani for discussions on this point.

plings to "external" fermionic currents, are:

$$\begin{aligned} D_\mu W^{\mu\nu} &= i\,g\,\frac{v^2}{4}\dot{V}^\nu + g\,\frac{\vec{\sigma}}{2}\,\vec{J}(f)\,, \\ \partial_\mu B^{\mu\nu} &= -i\,g'\,\frac{v^2}{4}\,\mathrm{Tr}\left\{T\dot{V}^\nu\right\} + g'\,J(f)\,, \\ \mathcal{D}_\mu \dot{V}^\mu &= \mathrm{O}(m_f/v)\,, \end{aligned} \tag{88}$$

where $\vec{J}(f)$ and $J(f)$ are once again the fermionic isospin and hypercharge currents. Of the above equations, we need use only the last, whose r.h.s. can be safely neglected to the precision of the LEP experiments of interest here† .

Of the fourteen operators with $d_\chi \leq 4$ to be added to $\mathcal{L}_\chi$, three describe vertices of no less than two gauge bosons, and correspond to the effective interactions:

$$\mathcal{L}_1 = gg'\,\beta_1\,B^{\mu\nu}\mathrm{Tr}\left\{TW_{\mu\nu}\right\}\ , \tag{89}$$

$$\mathcal{L}_1' = g'^2\,\beta_1'\,v^2\left[\mathrm{Tr}\left\{T\dot{V}_\mu\right\}\right]^2\ , \tag{90}$$

$$\mathcal{L}_8 = g^2\,\beta_8\left[\mathrm{Tr}\left\{TW_{\mu\nu}\right\}\right]^2\ , \tag{91}$$

Yet another threesome relates to interactions of no less than three gauge bosons:

$$\mathcal{L}_2 = ig'\,\beta_2\,B^{\mu\nu}\mathrm{Tr}\left\{T[\dot{V}_\mu,\,\dot{V}_\nu]\right\}\ , \tag{92}$$

$$\mathcal{L}_3 = ig\,\beta_3\mathrm{Tr}\left\{W^{\mu\nu}[\dot{V}_\mu,\,\dot{V}_\nu]\right\}\ , \tag{93}$$

$$\mathcal{L}_9 = ig\,\beta_9\mathrm{Tr}\left\{TW^{\mu\nu}\right\}\mathrm{Tr}\left\{T[\dot{V}_\mu,\,\dot{V}_\nu]\right\}\ , \tag{94}$$

while of the remaining eight operators of the complete $d_\chi \leq 4$ set of chiral grafts, five describe vertices of four or more gauge bosons that are irrelevant to the study of TGVs, and the last three are made redundant by the equations of motion. The seemingly capricious numbering of the six operators in (89) to (94) follows [9, 11]. A different numbering, rooted in the work of Gasser and Leutwyler [33] has recently flourished in the literature* . The powers of g and g' in (89) to (94) are those dictated by chiral perturbation theory, which also indicates that all β's are of order $1/16\pi^2$. Of the six $d_\chi \leq 4$ effective interactions relevant to our analysis of TGV's, three ($\mathcal{L}_1$, $\mathcal{L}_2$ and $\mathcal{L}_3$) preserve the custodial symmetry, while the rest ($\mathcal{L}_1'$, $\mathcal{L}_8$, $\mathcal{L}_9$) do not.

† The effects m_b on Γ_b will be explicitly taken into account.

* For the most often discussed objects, the translation is $\beta_1 \leftrightarrow L_{10}(\mathrm{GL})$ and $\beta_2 + 2\,\beta_3 \leftrightarrow -L_9(\mathrm{GL})$.

The connection between the chiral expansion in terms of the above $\mathcal{L}_i$ and our previous field-dimensionality expansion for a linearly-realized SM is transparent in the realization in which U in expressed in terms of Φ as:

$$U \Rightarrow \frac{\sqrt{2}}{v} \begin{pmatrix} \Phi_0^* & \Phi_+ \\ -\Phi_+^* & \Phi_0 \end{pmatrix} . \tag{95}$$

Two chiral operators correspond to objects (30), (31) that we have studied in appalling detail:

$$\begin{aligned} \mathcal{L}_1 &\Rightarrow -\frac{2\,gg'\,\beta_1}{v^2}\,\mathcal{O}_{WB}\big|_{(\Phi\to v)}\,, \\ \mathcal{L}_1' &\Rightarrow -\frac{16\,g'^2\,\beta_1'}{v^2}\,\mathcal{O}_{\Phi}\big|_{(\Phi\to v)}\,, \end{aligned} \tag{96}$$

where the rightmost subscripts indicate the substitution of Φ by the expectation value of its neutral component. The symbol "$\Rightarrow$" for "corresponds to" means that the analysis in our framework of the effects of the operator on the left-hand side is identical in practice‡ to that of the (scaled) operator on the right-hand side. We shall call d_{eff} the naive dimensionality of the smallest-dimension linear graft to which a given chiral graft corresponds to. Thus $d_{eff}(\mathcal{L}_1,\,\mathcal{L}_1') = 6$.

Two other chiral grafts also have $d_{eff} = 6$:

$$\begin{aligned} \mathcal{L}_2 &\Rightarrow \frac{8\,g'\,\beta_2}{v^2}\,\mathcal{O}_{B\Phi}\big|_{(\Phi\to v)}\,, \\ \mathcal{L}_3 &\Rightarrow \frac{8\,g\,\beta_3}{v^2}\,\mathcal{O}_{W\Phi}\big|_{(\Phi\to v)}\,, \end{aligned} \tag{97}$$

and correspond to the blind directions♮ defined in (46), (47). The equations of motion relate these operators, as in (48) and (49), to the elements of the $d = 6$ linear basis of (50) and Table I.

The two remaining chiral operators translate into $d_{eff} = 8$ newcomers:

$$\begin{aligned} \mathcal{L}_8 &\Rightarrow \frac{16\,g^2\,\beta_8}{v^4}\,(\Phi^\dagger W_{\mu\nu}\Phi)\,(\Phi^\dagger W^{\mu\nu}\Phi)\big|_{(\Phi\to v)}\,, \\ \mathcal{L}_9 &\Rightarrow -\frac{32\,i\,g\,\beta_9}{v^4}\,(\Phi^\dagger W^{\mu\nu}\Phi)\,(D_\mu\Phi)^\dagger D_\nu\Phi\big|_{(\Phi\to v)}\,, \end{aligned} \tag{98}$$

of which the second one is a blind direction.

‡ A techni-theory is supposed to mock up the regulator role that a heavy standard scalar plays. In limiting chiral grafts, by comparing their incidence to the radiatively corrected standard results, we ought to have used an M_H-uncertainty with M_H in the few TeV range [40] in which the standard quartic couplings are strong. The differences with our generous $M_H = 40$ GeV to 1 TeV range are too tiny to be relevant.

♮ We thank E. de Rafael and D. Espriu for pointing us in these blind directions.

To sum up, a $d_\chi \leq 4$ chiral basis complete for the purposes of analyzing TGVs is

Graft type	Tree grafts			Blind dirs.		
Graft name	$\mathcal{L}_1$	$\mathcal{L}_1'$	$\mathcal{L}_8$	$\mathcal{L}_2$	$\mathcal{L}_3$	$\mathcal{L}_9$
d_{eff}	6	6	8	6	6	8
d_χ	4	2	4	4	4	4
Custodial	Yes	No	No	Yes	Yes	No

Table II: Elements of the $d_\chi \leq 4$ chiral basis.

Once again, there would be the possible choice of including or not including fermionic operators as elements of a chiral basis, and to trade the blind-direction grafts for fermionic operators whose effects are currently measurable at tree-level. In a chiral realization, however, one may defensibly argue that effective fermionic operators are generated at negligible orders of perturbation theory (implicitly these operators must be assumed not to vanish exactly at all momentum scales, lest extra false relations between the elements of the chiral basis ensue from the equations of motion). The exclusion of fermion operators and the consequent inclusion of blind directions in the chiral basis is a fairly natural choice, and not an unnecessarily impractical one, as it would be in the linear case.

We have already explicitly analized$^\diamond$ the current bounds and future detectability of the effective interactions described by $\mathcal{L}_1$ and $\mathcal{L}_1'$. With use of (51) and (96) the 2σ-bounds in (61) and (69) on ϵ_{WB} and ϵ_Φ translate into:

$$\begin{aligned} -3.6 < 16\,\pi^2\,\beta_1 < 2.9\,, \\ -4.5 < 16\,\pi^2\,\beta_1' < 0.68\,. \end{aligned} \tag{99}$$

We have seen that these bounds are very unlikely to be improved by a search for the effects of $\mathcal{O}_{WB}$ or $\mathcal{O}_\Phi$ at LEP-2.

The effects of the $d_{eff} = 8$ chiral graft $\mathcal{L}_8$ of (98) are simple to discuss, along the same lines as those of the other ($d = 6$) tree-grafts we have investigated. Of the input

$\diamond$ In studying the effects of chiral grafts and comparing them with experiment, one ought to correct the tree-level standard predictions with use of the $d_\chi = 2$ term in $\mathcal{L}_\chi$, as opposed to using the standard one-loop corrections. This point of principle is moot in practice [38]: differences are unmeasurably small.

quantities of a Z-scheme the addition of $\mathcal{L}_8$ only affects M_Z:

$$\Delta M_Z \simeq \frac{8\,\pi\,\alpha}{\tan^2\theta}\, M_Z\, \beta_8\,, \tag{100}$$

and of all the observables in (19), only the prediction for M_W is modified:

$$\delta M_W \simeq -\,2\,g^2\,c\,M_Z\,\beta_8\,. \tag{101}$$

The $2\,\sigma$ constraints on the (m_t, β_8) plane are shown in Figure [15a], from which one can read the m_t-independent $2\,\sigma$ limits:

$$-1.7 < 16\,\pi^2\,\beta_8 < 1.6\,. \tag{102}$$

$\mathcal{L}_8$ affects the quantities describing TGVs by the amounts:

$$\begin{aligned} \delta g_Z &= \delta g_\gamma \simeq 0\,, \\ \delta\kappa_\gamma &= \delta\kappa_Z \simeq -4\,g^2\,\beta_8\,, \\ \lambda &\simeq 0\,. \end{aligned} \tag{103}$$

The results of a χ^2 test of the sensitivity at LEP-2 of the angular distribution for W-pair production to an $\mathcal{L}_8$ graft are shown in Fig.[15b]. The current limit on β_8 and the optimistic LEP-2 sensitivity are seen to be comparable.

Need we explicitly and laboriously analyze the current bounds and future detectability of the effective interactions described by $\mathcal{L}_2$, $\mathcal{L}_3$ and $\mathcal{L}_9$? These operators are, in precisely the same sense as $\mathcal{O}_W$, blind directions. But the expectation in models describing a strongly interacting longitudinal vector boson sector is $\beta_{2,3} \sim \beta_1$, $\beta_9 \sim \beta_8$ for the custodial preserving and violating grafts, respectively. The possibility that β_1, β_1' and β_8 be suppressed by a large factor relative to their blind-direction partners would be a rather mysterious dynamical accident. If this accident does not occur, there is meagre hope for currently derivable bounds on the blind directions to compete with the limits in (99) and (102). Ditto for attempts to measure the effects of $\mathcal{L}_{2,3,9}$ at LEP-2. We have therefore skipped the toil of computing the quantum effects of the chiral blind-direction grafts on current observables• .

To conclude, the current limits on β_1 (or ϵ_{WB}), β_1' (or ϵ_Φ) and β_8 are unlikely to be superseded by any other manifestation of a non-linear standard model accessible to LEP-1 or LEP-2. The discovery of a strongly interacting longitudinal gauge sector, should it be the choice of nature, must await for data from higher-energy and/or higher-luminosity machines.

• This extra effort is made in a forthcoming article [62].

6. Related Work.

The recently gathered very precise body of data has triggered a swift response from theorists. Not surprisingly, this has resulted in considerable overlap and an orgy of notations. It may thus be useful to comment briefly on the concomitant literature, without the untimely pretence of reviewing the subject.

Work in the linear realm, related to ours but concerning two-point functions (as opposed to these plus their relations to TGVs) is reported in [15].

Considerable effort has been devoted to "general" characterizations of possible deviations from the SM, concentrated mainly on what are mystifyingly called "oblique" [19] corrections: those concerning vector-boson two-point functions. A seminal and extensive work in this direction is that in [5]. To describe oblique effects, the authors introduce three functions $\Delta_{\rho,+,3}(q^2)$ that are linear combinations of the above-mentioned two-point functions.

In studying non-decoupling theories and in particular technimodels, it is pertinent [40] to concentrate on oblique corrections, to set $\Delta_3 = \Delta_+$, to expand only to first order in q^2 and to reduce the three functions of [5] to two mere numbers (S and T):

$$\begin{aligned} \Delta_3 \Rightarrow \alpha\, S &\simeq -\,4\,e^2\,\frac{d}{d\,q^2}\,\left[\Pi_{30}(q^2)\right]\big|_{q^2=0}\,, \\ \Delta_\rho \Rightarrow \alpha\, T &\simeq \frac{e^2}{s^2\,M_W^2}\,\left[\Pi_{11}(0)-\Pi_{33}(0)\right]\,. \end{aligned} \tag{104}$$

A third quantity U arises when $\Delta_3 = \Delta_+$ is not imposed:

$$\Delta_+ \Rightarrow \alpha\, U \simeq 4\,e^2\,\frac{d}{d\,q^2}\,\left[\Pi_{11}(q^2)-\Pi_{33}(q^2)\right]\big|_{q^2=0}\,. \tag{105}$$

These quantities, the completely equivalent ones $\epsilon_{1,2,3}$ introduced in [43], and the $q^2 \neq 0$ ones $h_{V,AZ,AW}$ of [41] have become part of the current lore on potential departures from the standard model.

Only a few of the operators we have discussed contribute to the oblique objects S,T,U. Contributions to S and T start with operators with $d_{eff} = 6$ ($\alpha\, S = 4\,s\,c\,\epsilon_{WB}$; $\alpha\, T = -(1/2)\,\epsilon_\Phi$), while those to U start at $d_{eff} = 8$ ($\alpha\, U = -16\,e^2\,\beta_8$). To constrain the technimodels that inspired them, these oblique results are adequate. To give but one example, our m_t-independent limit (61) translates into $S < 2.24$ ($2\,\sigma$) for a positive S at $T = 0$. This result and the analogous one on T (at $S = 0$) are in fair agreement with the earlier ones of [40] and others. To the current limits on technicolour we have nothing to add, but our bounds on U (or β_8) and our usual warning that LEP-2 does not offer a greater hope than LEP-1 does.

An important remark, phrased in the language of S and T, is that of [63], where the role of the very precise atomic parity-violation experiments [22] is studied. To reach our conclusions on the observability of TGVs at LEP-2, we did not use the further constraints implied by these measurements.

Contemporaneous with [40] are the related contributions [35 to 37], in which the effective field theory language is fully exploited, as in [38]. The advantages of so doing are emphasized in [6].

Part of the literature subsequent to the introduction of S and T concerned the various possibilities to undo the approximations of [40] in attempts to describe a more ample domain of meta-theories [41 to 64]. In a recent example, the authors of [64] discuss the possibility of redefining their ϵ_i parameters [43] as algebraic combinations of measured quantities, from which the m_t, M_H-dependent electroweak corrections are not subtracted; and of lifting various hypotheses, such as lepton or flavour universality, to obtain various sets of ϵ_i's. This is certainly a long way towards model independence, only one step away (undoing the algebraic combinations) from what is indisputably the most model-independent characterization of the unknown. Its advantage is a systematization of the sensitivity to various hypotheses or free inputs, such as m_t and M_H [65, 38]. The new ϵ_i [64] differ from the old [43], and various of the non-oblique operators we have discussed ($\mathcal{O}_e$, $\mathcal{O}_\mu$, $\mathcal{O}_{e\mu}$) contribute to the new and not to the old ϵ_i's. Our description of new physics is less general (and more specific) than the most general one of [64] only inasmuch as we truncate the Taylor expansions in naive or chiral dimensions of an effective-Lagrangian description. It is the power of this latter approach that allows one to establish well controlled connections between current observables at different momentum scales and with future observables, such as TGVs.

All of the above-mentioned literature concerns the consistency and implications of low-energy and LEP-1 data, while we have put emphasis on the impact of these data on the triple gauge-vertices to be studied at LEP-2, and on their general analysis in the linear and chiral realms. The $e^+e^- \to W^+W^-$ cross-section modifications (that we argued to be unmeasurably small at LEP-2) are studied with focus on a custodial technicoloured context in [66], and in forthcoming work [67].

We have repeatedly enough expressed our view of considerations wherein the consequences of $SU(2)\otimes U(1)$ gauge invariance are not treated with the respectful veneration they fully deserve. We do not redo it here.

7. Conclusions

The foundations of the standard model are a quarter of a century old. The last surprise in its realm –the existence of a third generation– occurred no less than three lustra ago. As the data fatidically narrow down on the standard predictions, it no longer seems to be a mere accident that the model shares its denomination with the Bureau of Standards. Under these circumstances, it behooves one to gauge the extent to which the present success narrows down the space for theoretical and experimental departures from our current credo. This we have done in the context of what is undoubtedly the main numerically-precise prediction of the model yet to be directly tested: the structure and strength of the triple gauge-boson vertices. Though our concrete framework concerned the $e^+e^- \to W^+W^-$ process at LEP-2, it is clear that our operator analysis can be directly applied to the study of the potential of other e^+e^- machines or readily extended to that of other colliding particles.

We have distinguished between the two main perceived avenues for plausible departures from the original (entirely perturbative) version of the standard model. The first is the possibility that the mechanism of spontaneous symmetry breakdown occur "as in the book", with one or various relatively light, "elementary" and weakly-interacting scalars to be unveiled by experiment in the foreseeable future; the new physics is, in a sense, an ado. The second avenue is the appealing, though theoretically less well-architectured, ansatz that the mechanism of mass generation involves some form of strongly coupled dynamics, wherein the elementary scalars may be an unnecessary or meaningless ingredient. Clearly, the search for these objects remains the main known task of current and future machines.

The two non-trivial types of "meta-theories" can be characterized at energies below their own intrinsic scale by effective non-renormalizable interactions between the standard fields. These are to be systematically organized in a "Taylor" expansion and grafted onto the original Lagrangian in a search for a comprehensive description of the "low" energy impact of the putative future dynamics. The procedure –to the extent that accidental cancellations between terms of the same or different orders appear to be unlikely– is a complete procedure, provided all the terms relevant to a given order are duly taken into account. We have improved previous analysis by explicitly considering all relevant operators and by studying in unprecedented detail the role that the equations of motion play in this context. We have taken adequate care of experimental and theoretical uncertainties, in particular those associated with the unknown value of the top quark mass. In a sense, we have also "depleted" other previous theoretical efforts by insisting ad nauseam on the impact and inevitability of the "assumption" of

$SU(2) \otimes U(1)$ gauge invariance.

In a nut-shell, we find that the current body of low-energy data suffices to place serious doubt on the possibility of observing at LEP-2 a non-standard strength or form of the triple gauge vertices. The gauge nature of the theory not only determines what these couplings ought to be in the standard model, but precludes the existence of observable deviations in its theoretically sensible extensions.

The previous paragraph is not a "theorem", for fine-tuned low energy cancellations (in half a dozen grafts and observables!) of wild higher energy behaviours cannot be ruled out by arguments other than common sense. Moreover, we find that one of the effective interactions reflecting the "new physics" ($\mathcal{L}_\Phi$) produces tree-level effects whose bounds from current data (if the t quark mass happens to be in the 300 GeV ballpark) could be a wee bit improved at LEP-2. But, even in this case, a true (4σ) discovery potential is excluded. Finally, there are "blind directions" in the operator space describing the new physics ($\mathcal{L}_W$, $\mathcal{L}_{B\Phi}$ and $\mathcal{L}_{W\Phi}$ and the chiral graft $\mathcal{L}_9$) to which the current data are only sensitive at the level of quantum corrections, as discussed in detail for the case of $\mathcal{L}_W$. Though our phenomenological information on these "directions" may be improved (just a little) by observations of W-pair production at LEP-2, it takes an act of faith to foresee that the hypothetical theory beyond the standard model is perverse enough to generate these effects at observable levels, while skipping the strictures from all other currently much better constrained "directions". The window of opportunity is narrow indeed. This is not saying that LEP-2 is useless, for an improved determination of M_W would be welcome, and none of our considerations precludes the existence of, say, a relatively light new heavy lepton or elementary scalar. But our arguments do indeed advocate in favour of higher-luminosity e^+e^- colliders, both at $\sqrt{s} \sim M_Z$ and at larger energies.

The substance of our conclusions is in a sense quite simple: all we are saying is that the standard theory describes the data so well from low energy up to M_Z, and the possible deviations are so constrained by gauge invariance, that there is very little hope for a doubling of the energy, in the search for subtle effects, to compensate for the anticipated decrease in statistics. But there is no substitute for a direct measurement, and one* may say that a deviation in the W-pair production process that significantly violates our bounds would constitute a major discovery, suggesting an unfathomable dynamical accident, or an entirely new type of hidden symmetry. The value of these apparently trivial assertions increases with the realization that, in substantiating them, we have rather systematically characterized the low energy effects of possible new

* We are indebted to Lev Okun for his insistence that the bottle may be perceived as being half-full.

dynamics, and placed useful and defensible limits on the coefficients of all the best-constrained effective operators characterizing the long-awaited manna[†] .

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† Debatably, the etymology of this figurative spiritual food is "*mān hū*", Aramean for "what the hell is this?".

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FIGURE CAPTIONS

Fig. 1. Allowed 2σ limits in the (ϵ_{WB}, m_t) plane from the different individual observables. The symbols specified here for the various constraints will recur in subsequent figures concerning the coefficients of other operators. The dotted region combines the limits from all observables and is the interior of the contour whose projections are the 95.5% confidence-level ($\Leftrightarrow 2\sigma$) segments for the individual variables ϵ_{WB} and m_t.

Fig. 2. The total $e^+e^- \to W^+W^-$ cross section as a function of the c.m.s. electron energy in units of the W mass. A gauge-invariant correction $\epsilon_{WB} = 0.04$ hardly affects the standard result, while non-gauge-invariant modifications of similar magnitude, described in the text, would have considerable effects.

Fig. 3. The ratio R of corrected to standard $e^+e^- \to W^+W^-$ differential cross sections $d\sigma/d\cos\theta_+$ for 10^4 generated W-pairs, $\epsilon_{WB} = 0.04$ and $\sqrt{s} = 200$ GeV; as a function of θ_+, the e^+W^+ scattering angle.

Fig. 4. χ^2 test of the significance of the effect of $\epsilon_{WB} \neq 0$ on $d\sigma/d\cos\theta_+$. The horizontal line shows the LEP-2 95.5% ($\Leftrightarrow 2\sigma$) sensitivity for 10^4 produced W-pairs at $\sqrt{s} = 200$ GeV, the projections along the vertical arrows delimit the interval of ϵ_{WB} inside which a LEP-2 measurement would test the hypothesis $\epsilon_{WB} \neq 0$ with less than 2σ significance. The vertical band encompasses the values of ϵ_{WB} currently allowed by the lower-energy tests.

Fig. 5. Individual 2σ constraints and allowed combined contours in the (ϵ_i, m_t) planes. The definition of the dotted domains is as in Fig.[1], and so is the notation for the various experimental inputs, but for the operators $\mathcal{O}_e$ and $\mathcal{O}_\mu$ in (e,f) that are not flavour-insensitive. In the corresponding graphs the crossed, dashed-dotted, open-circled and dashed lines correspond to Γ_e, Γ_μ, $\bar{g}_e^A$ and $\bar{g}_\mu^A$, respectively.

Fig. 6. χ^2 tests of the significance of the effects of various modifications of the standard prediction for the W-pair production differential cross section $d\sigma/d\cos\theta_+$, in terms of the coefficients ϵ_i of the various grafts in the complete $d = 6$ basis of (50) and Table I. The horizontal lines, vertical arrows and dotted domains have the same meaning as in Fig.[4].

Fig. 7. One-loop corrections to G_F with one $\mathcal{O}_W$ insertion (represented by a heavy dot) with $V = Z, \gamma$.

Fig. 8. One-loop corrections to α with one $\mathcal{O}_W$ insertion (represented by a heavy dot) with $V = Z, \gamma$. (a) Diagrams that do not modify α. (b) Diagrams that do.

Fig. 9. One-loop corrections to vector boson two-point functions with one $\mathcal{O}_W$ insertion (represented by a heavy dot) with $V = Z, \gamma$. (b) vanishes identically.

Fig. 10. Diagrams relevant to the e^+e^- cross section around the Z peak, as modified by an $\mathcal{O}_W$ insertion, represented by the heavy dots. The dashed bubble in (a) defines dressed $V = Z, \gamma$ propagators, and those in (b+c) and (d) define the corresponding $f\bar{f}$ vertices. (e) describes the complete e^+e^- amplitude. The four-vector tadpoles have not been drawn: their contribution vanishes.

Fig. 11. Individual $2\,\sigma$ constraints and allowed combined contours in the (δ_W, m_t) plane. The definition of the dotted domain and the notation for the various experimental constraints are as in Fig.[1].

Fig. 12. The total $e^+e^- \to W^+W^-$ cross section as a function of the c.m.s. electron energy in units of the W mass, for various values of λ, defined in (70).

Fig. 13. The ratio R of corrected to standard $e^+e^- \to W^+W^-$ differential cross sections $d\sigma/d\cos\theta_+$ for $\lambda = 0.1$, $\sqrt{s} = 200$ GeV, and 10^4 generated W-pairs; as a function of θ_+, the e^+W^+ scattering angle.

Fig. 14. χ^2 test of the significance of an $\mathcal{O}_W$ insertion, $\lambda \neq 0$, on the W pair production cross section $d\sigma/d\cos\theta_+$. The horizontal line shows the estimated LEP-2 95.5% ($\Leftrightarrow 2\sigma$) sensitivity, the vertical arrows are also as in Fig.[4], the dotted and dashed intervals encompass the limits from current data, for $\Lambda = 1$ TeV and $m_t = 100, 210$ GeV, respectively.

Fig. 15. (a) Individual $2\,\sigma$ constraints and allowed combined contour in the (m_t, β_8) plane. The definition of the dotted domain and the notation for the various experimental inputs are as in Fig.[1]. (b) χ^2 test of the significance of the effect of $\beta_8 \neq 0$ on $d\sigma/d\cos\theta_+$. The horizontal line shows the LEP-2 95.5% ($\Leftrightarrow 2\sigma$) sensitivity for 10^4 produced W-pairs at $\sqrt{s} = 200$ GeV, the vertical arrows are also as in Fig.[4]. The vertical band encompasses the values of β_8 currently allowed by the lower-energy tests.

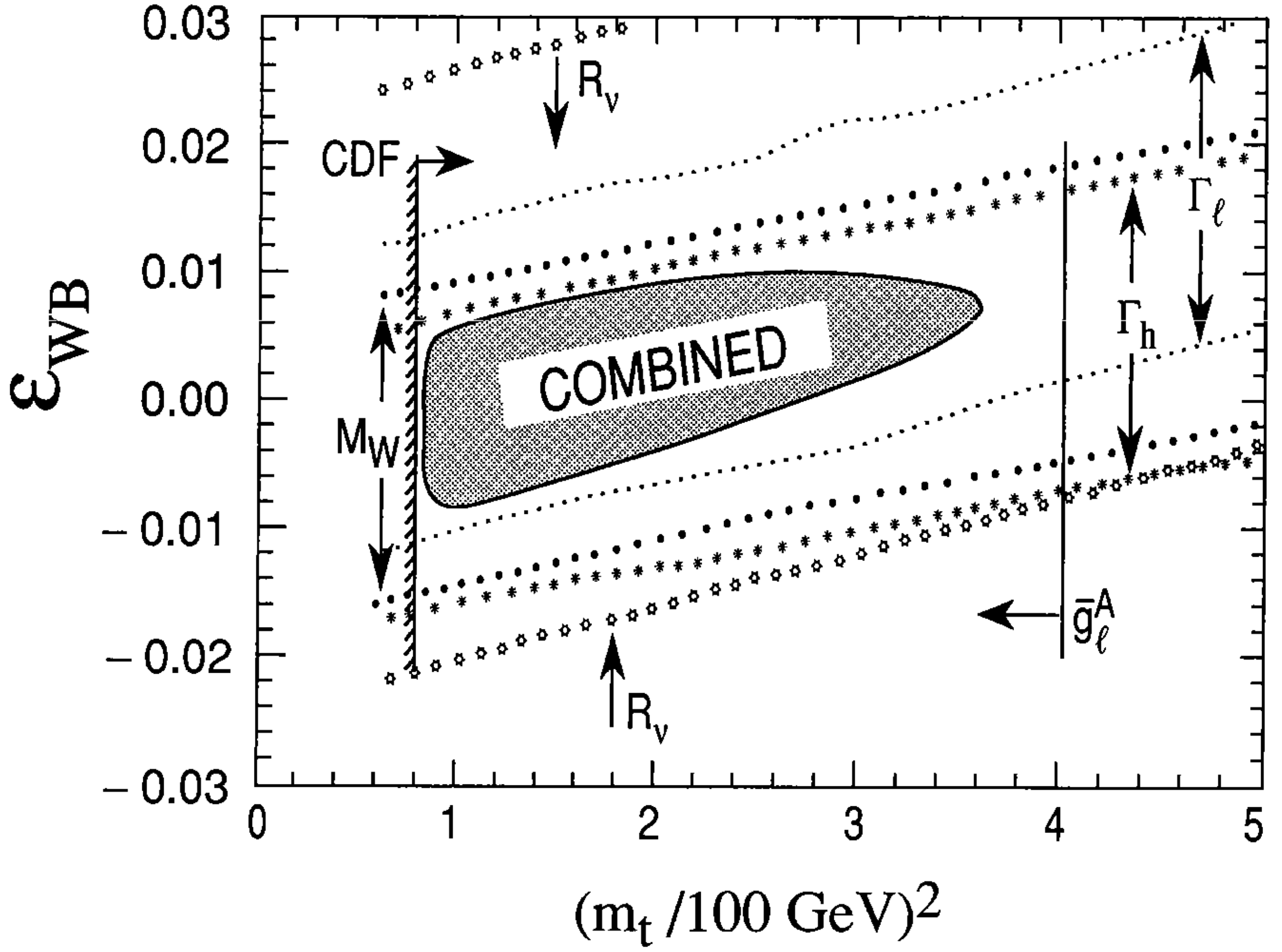
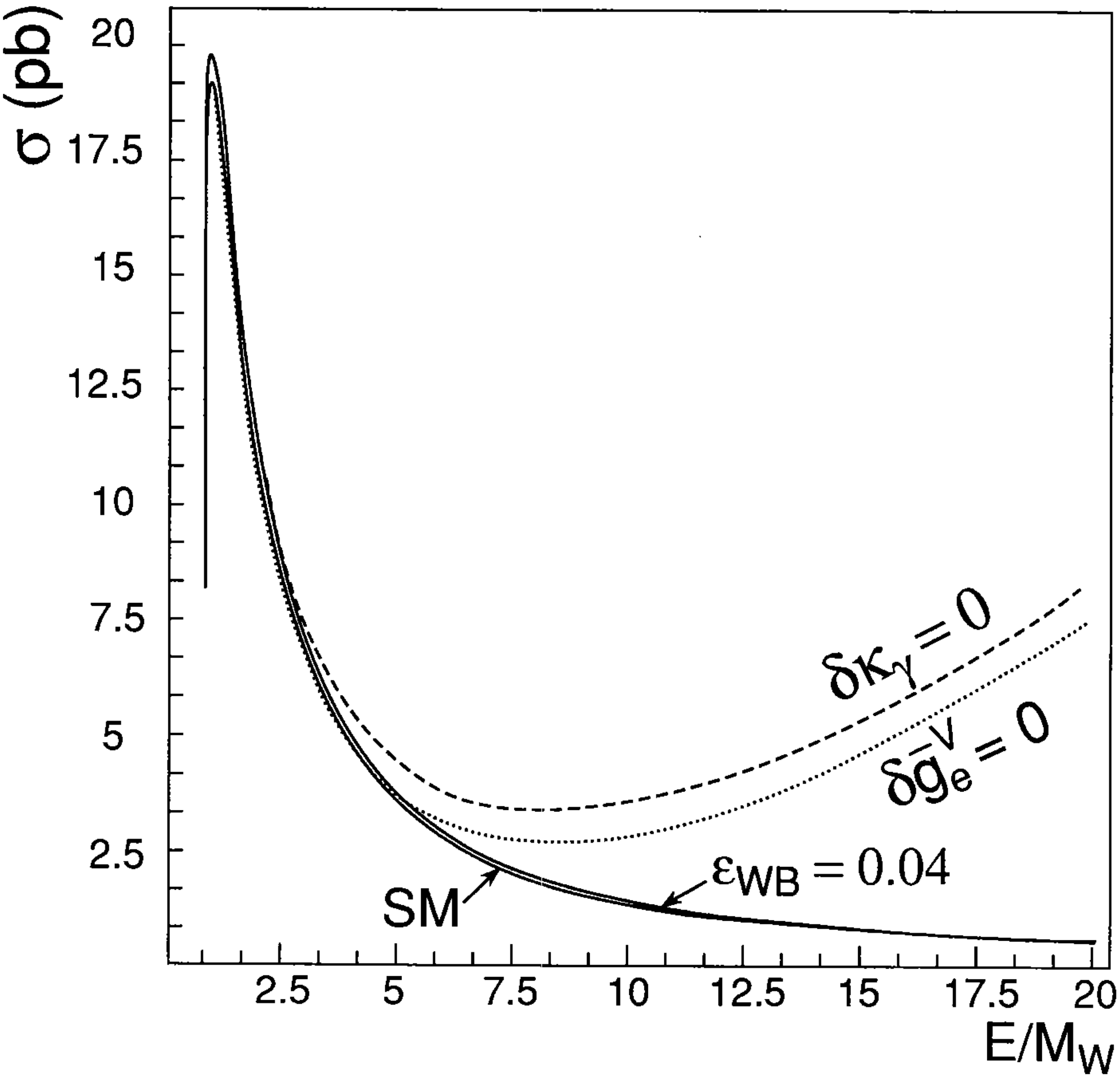

0.03
0.02
0.01
0.00
– 0.01
– 0.02
– 0.03
ε_{WB}
0
1
2
3
4
5
$(m_t /100\ GeV)^2$
CDF
R_ν
R_ν
M_W
COMBINED
Γ_ℓ
Γ_h
$\bar{g}_\ell^A$

Fig. 1



σ (pb)
20
17.5
15
12.5
10
7.5
5
2.5
2.5
5
7.5
10
12.5
15
17.5
20
E/M_W
δκ_γ = 0
δḡ_e^V = 0
ε_WB = 0.04
SM

Fig. 2

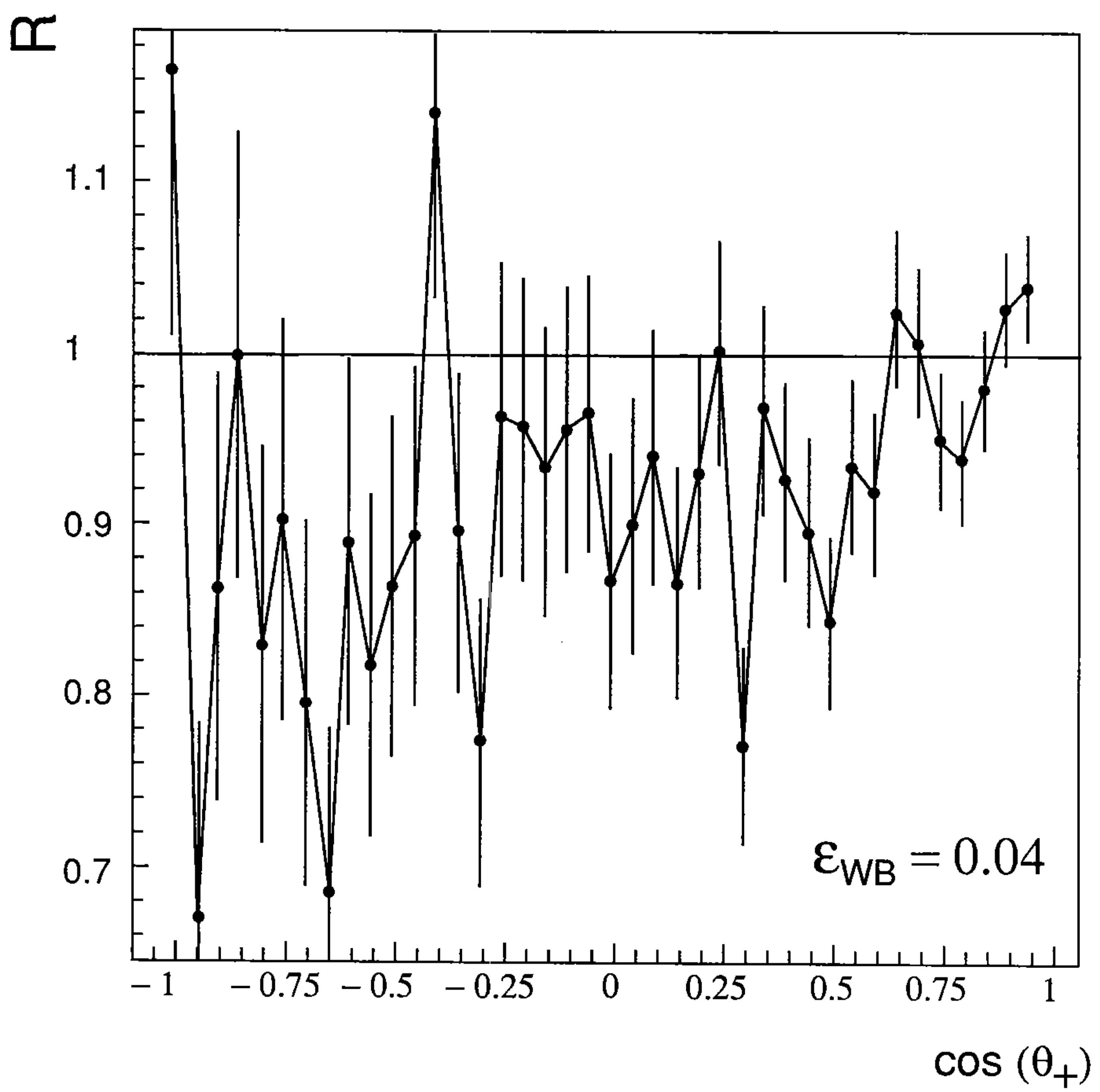


Fig. 3

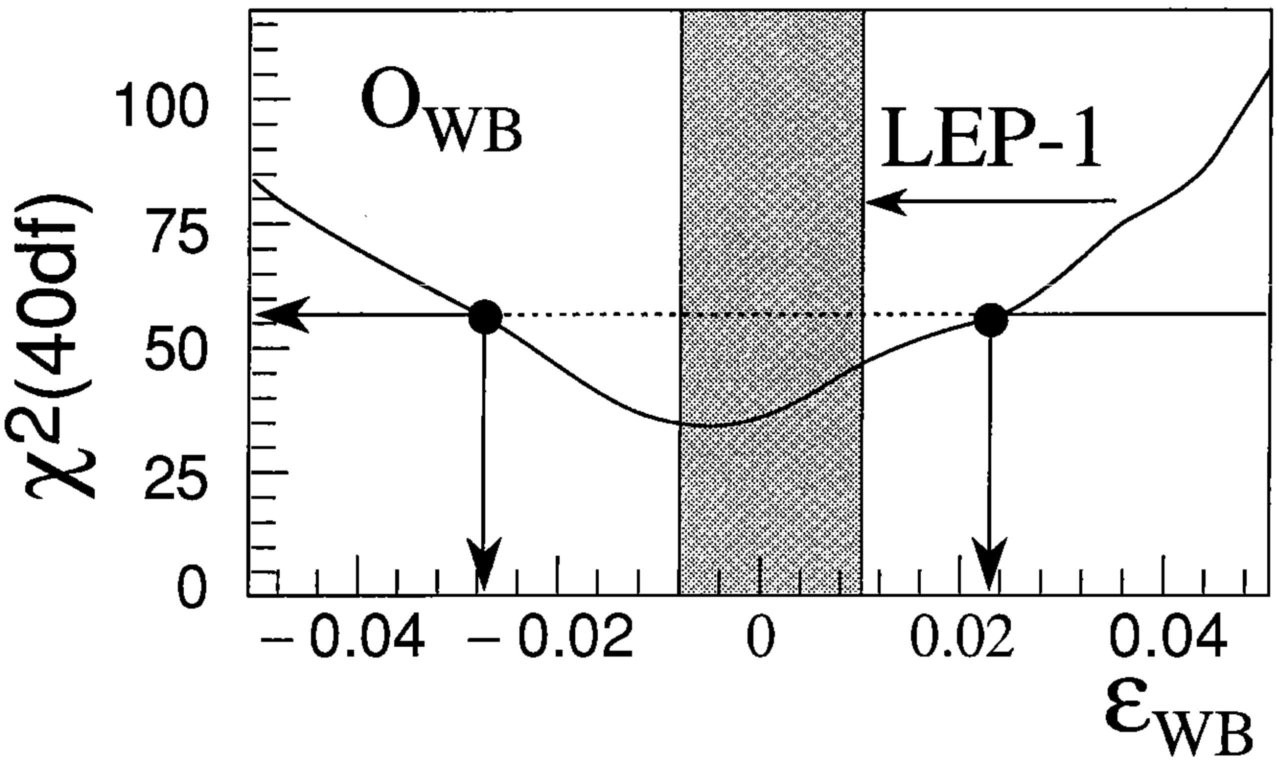
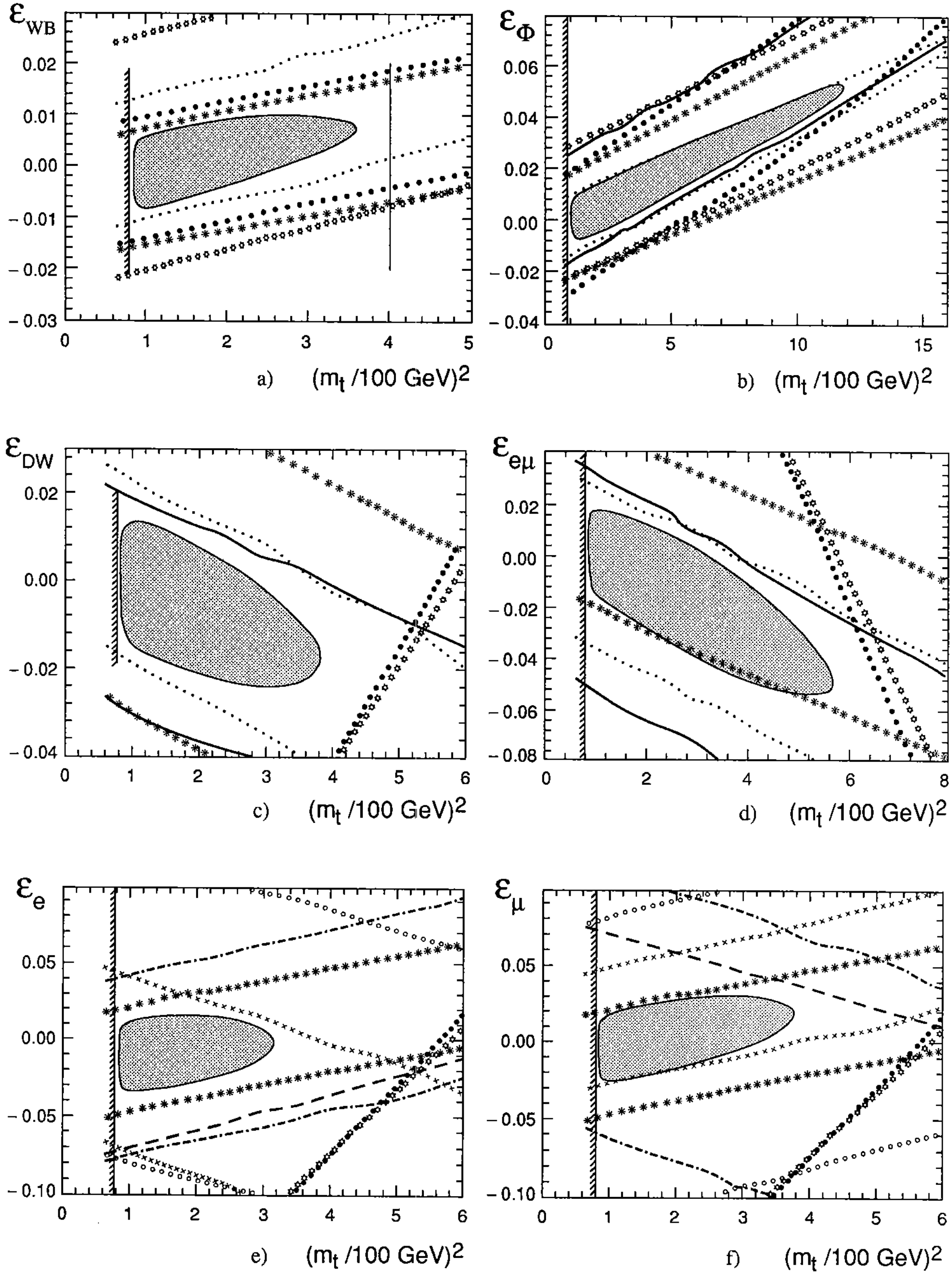
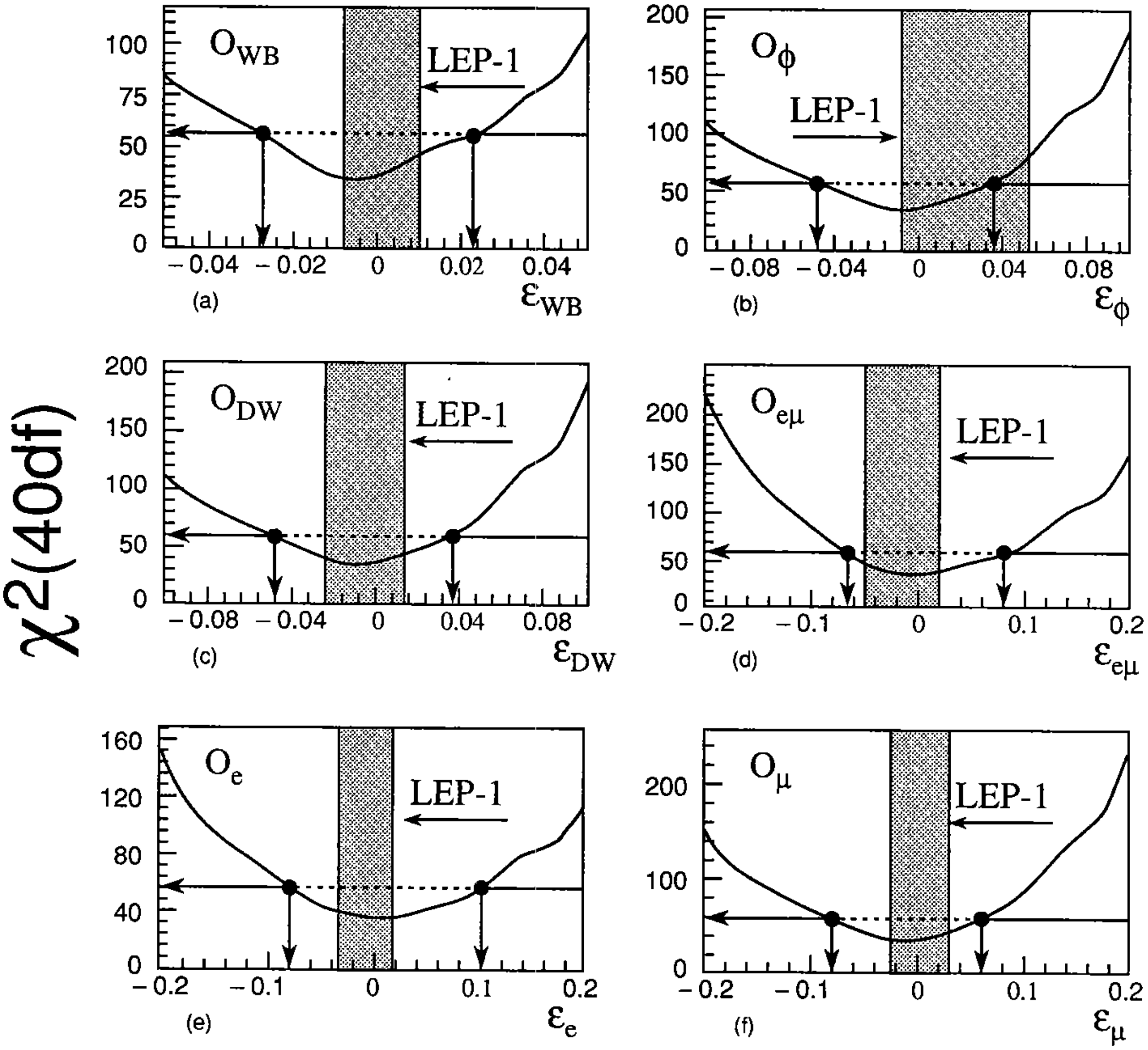

O_{WB}
LEP-1
$\chi^2(40df)$
100
75
50
25
0
− 0.04
− 0.02
0
0.02
0.04
ε_{WB}

Fig. 4



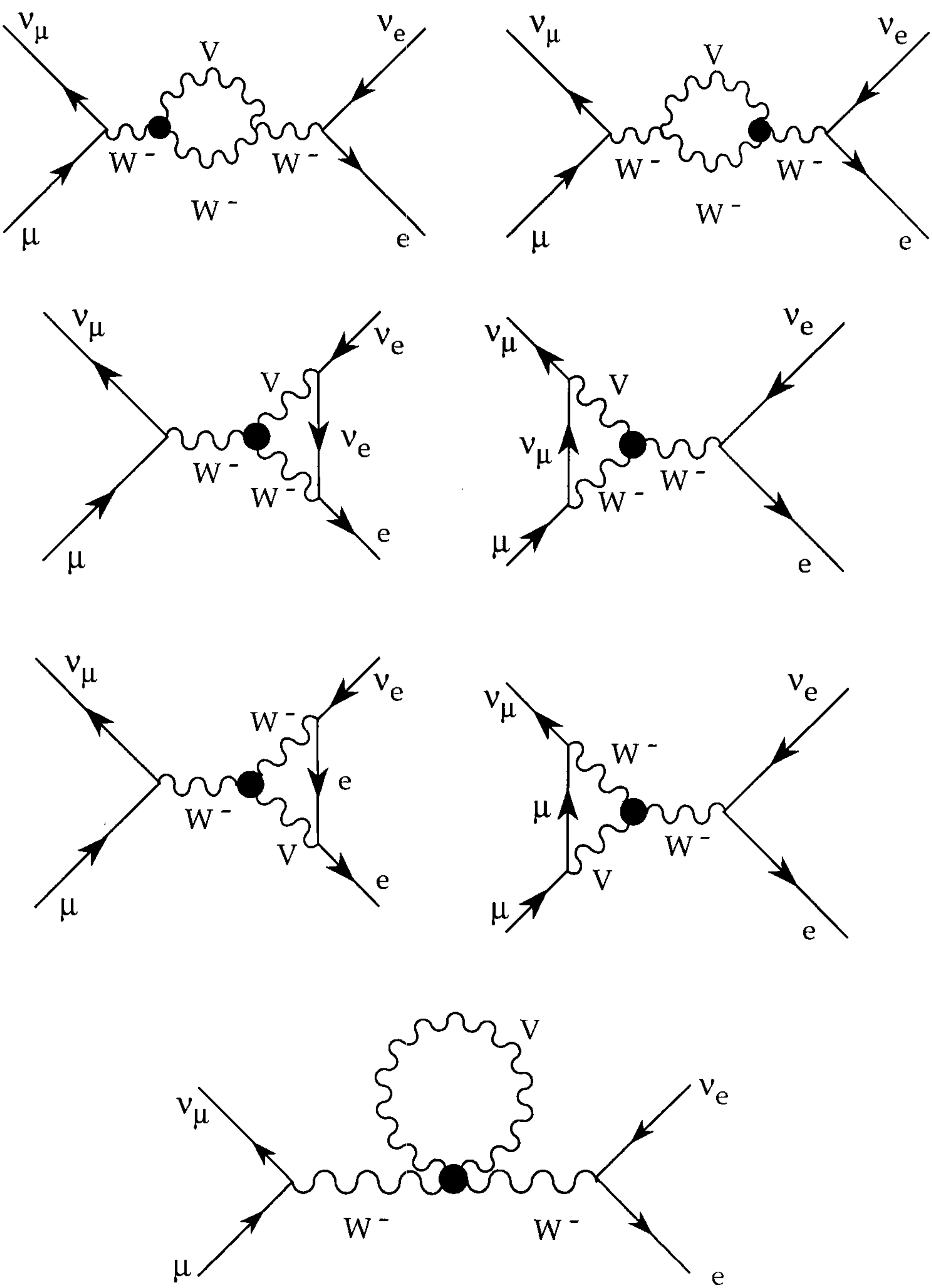
ε_{WB}
a) $(m_t/100\ GeV)^2$
ε_{Φ}
b) $(m_t/100\ GeV)^2$
ε_{DW}
c) $(m_t/100\ GeV)^2$
$\varepsilon_{e\mu}$
d) $(m_t/100\ GeV)^2$
ε_{e}
e) $(m_t/100\ GeV)^2$
ε_{μ}
f) $(m_t/100\ GeV)^2$

Fig. 5



χ2(40df)
O_WB
LEP-1
(a)
ε_WB
O_φ
LEP-1
(b)
ε_φ
O_DW
LEP-1
(c)
ε_DW
O_eμ
LEP-1
(d)
ε_eμ
O_e
LEP-1
(e)
ε_e
O_μ
LEP-1
(f)
ε_μ

Fig. 6



ν_μ
ν_e
V
W^-
W^-
W^-
μ
e
ν_μ
ν_e
V
W^-
W^-
W^-
μ
e
ν_μ
ν_e
V
ν_e
W^-
W^-
e
μ
ν_μ
ν_e
V
ν_μ
W^-
W^-
μ
e
ν_μ
ν_e
W^-
e
W^-
V
e
μ
ν_μ
ν_e
W^-
μ
W^-
V
μ
e
V
ν_μ
ν_e
W^-
W^-
μ
e

Fig. 7

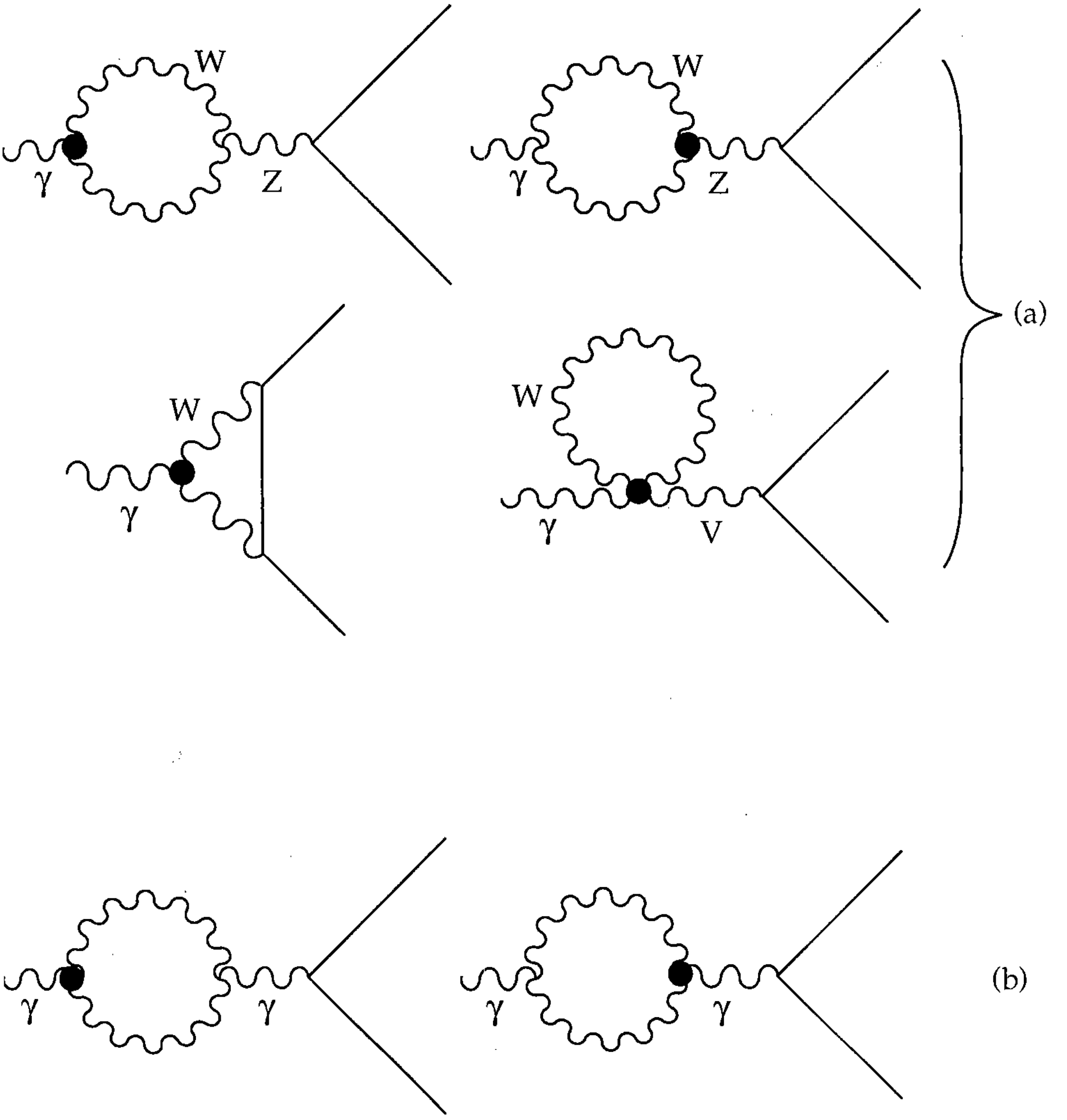


Fig. 8

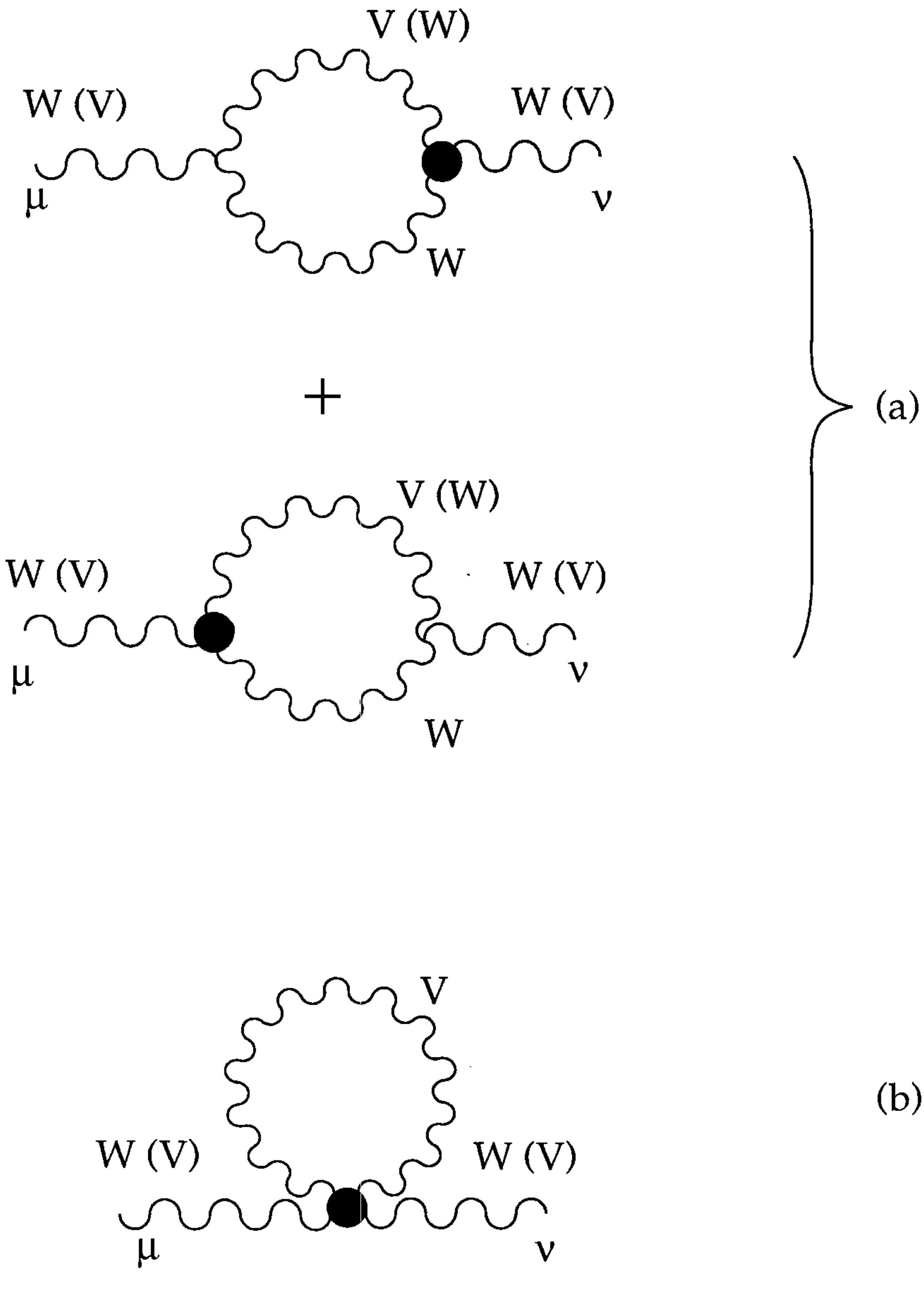


Fig. 9

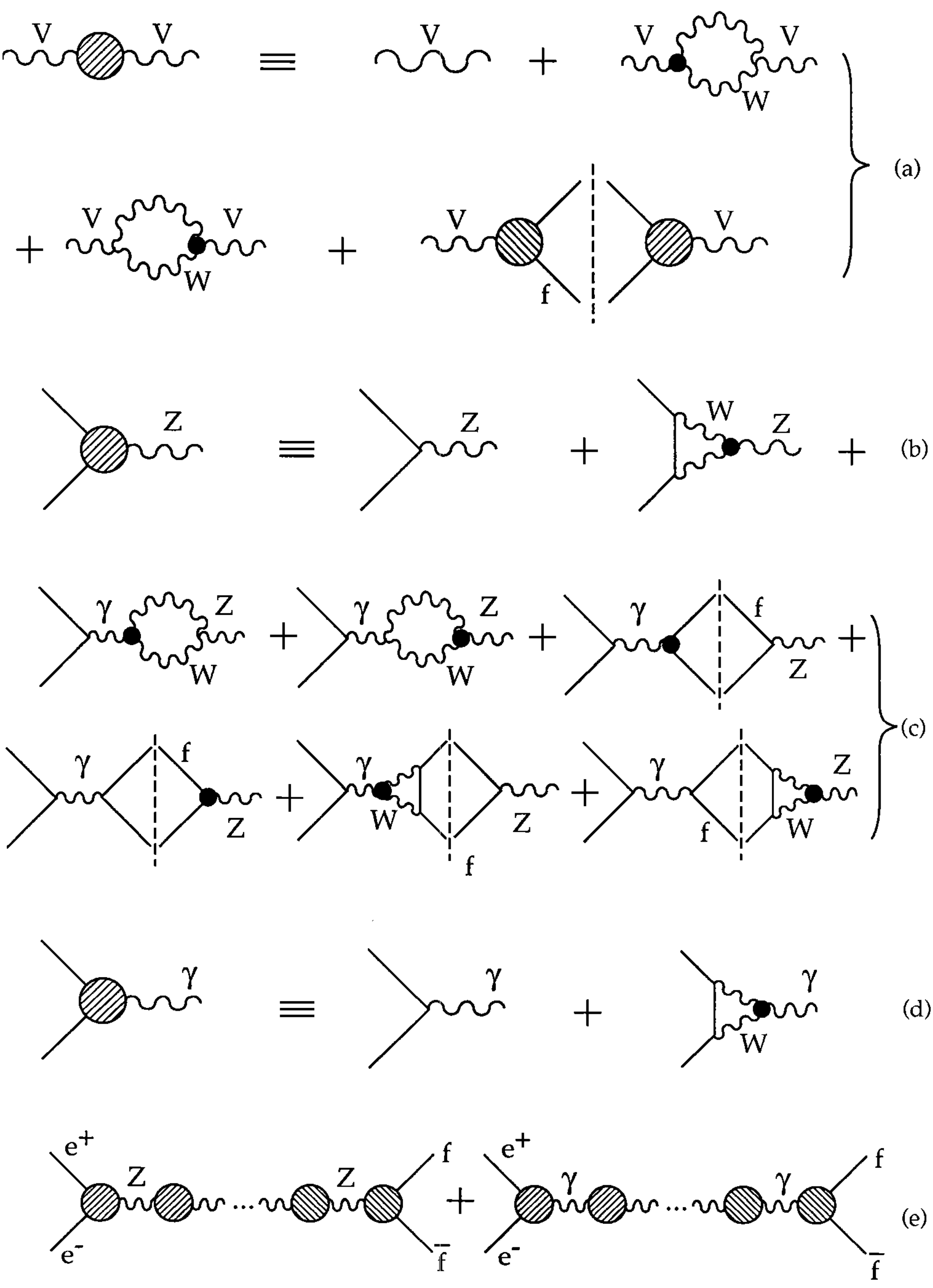

V
V
V
V
V
W
(a)
V
V
W
V
f
V
Z
Z
W
Z
(b)
γ
Z
W
γ
Z
W
γ
f
Z
f
γ
Z
γ
W
Z
f
γ
Z
f
W
(c)
γ
γ
γ
W
(d)
e+
e-
Z
Z
f
f̄
γ
γ
e+
e-
f
f̄
(e)

Fig. 10

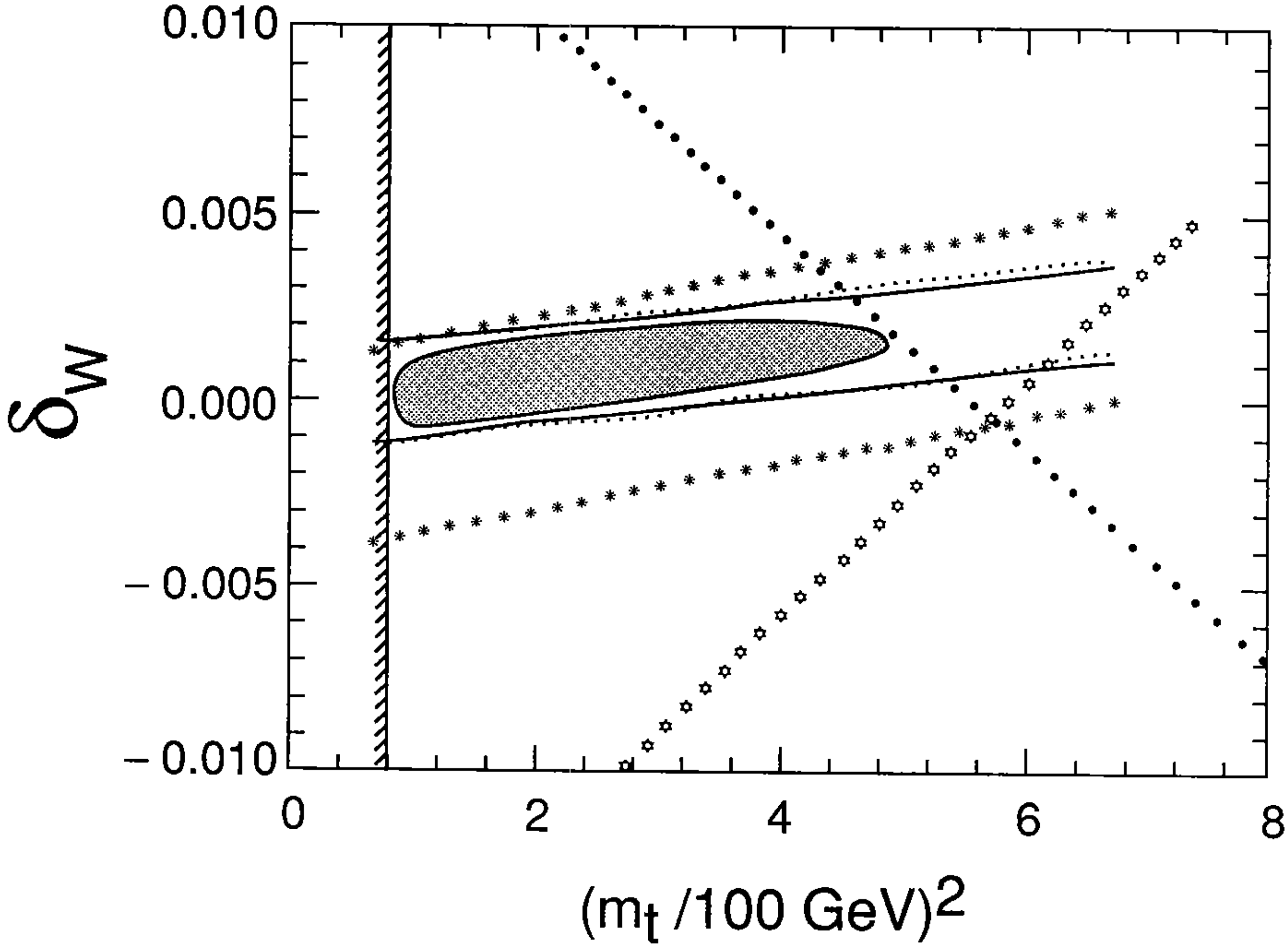


Fig. 11

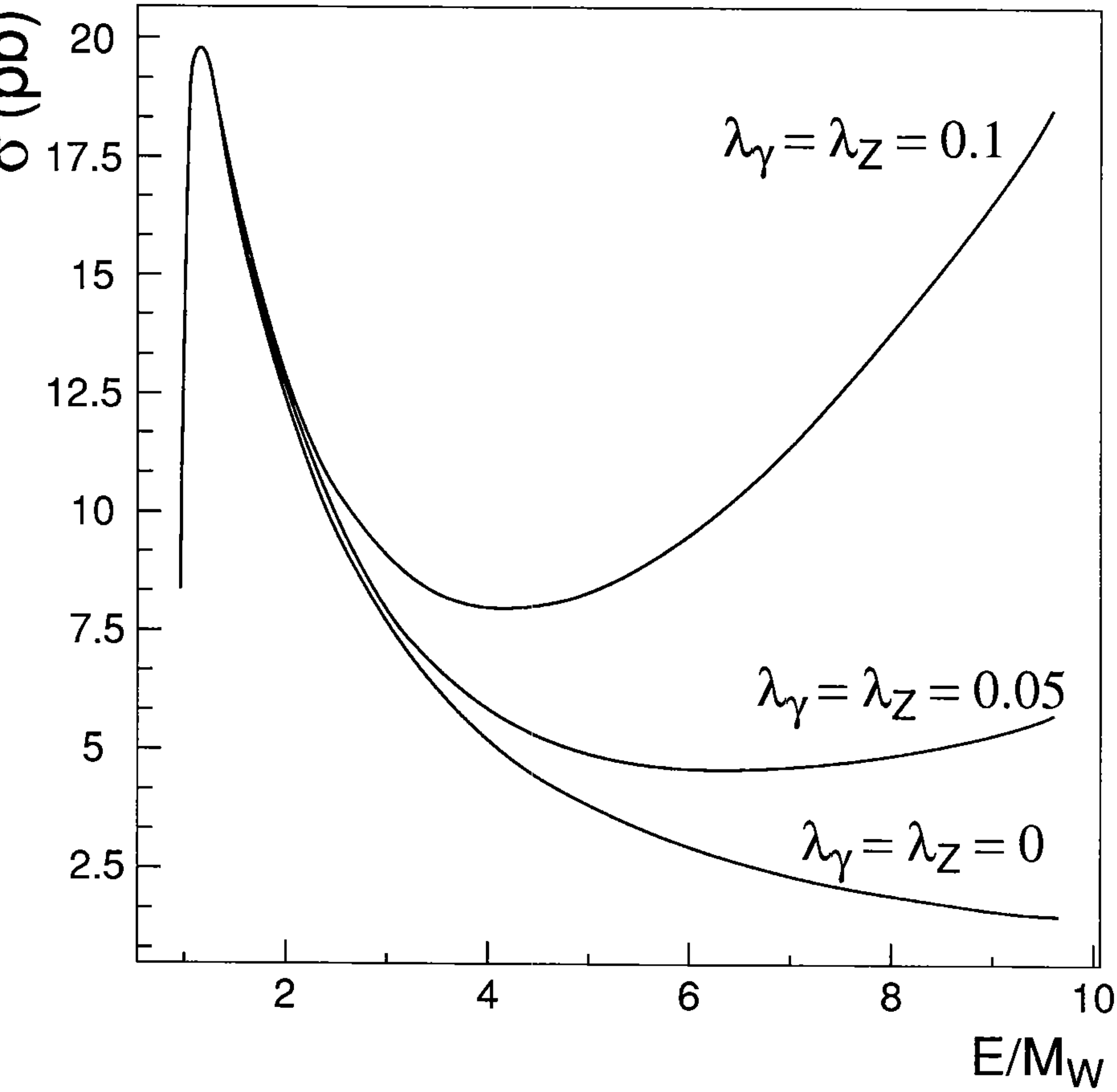


Fig. 12

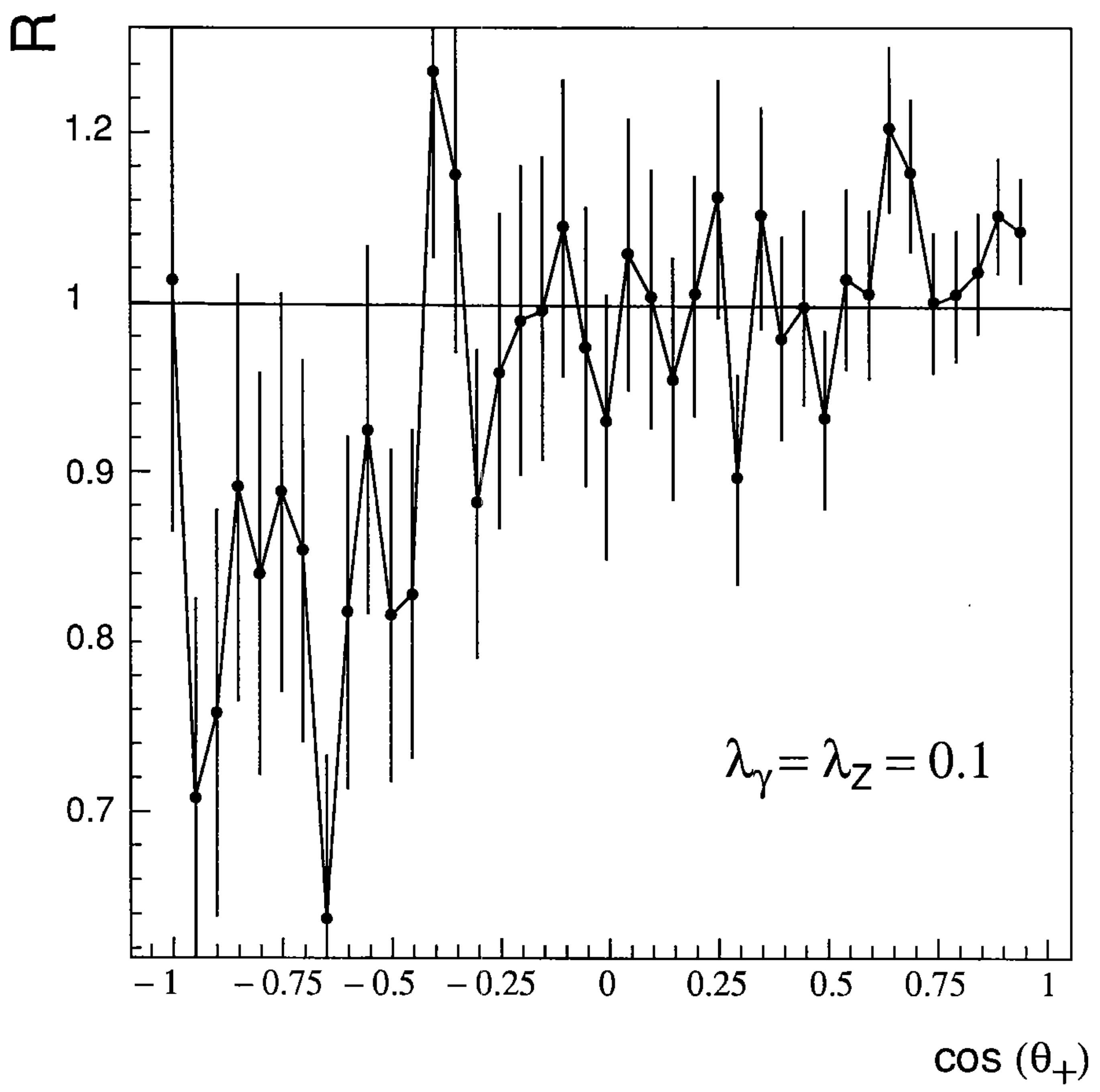
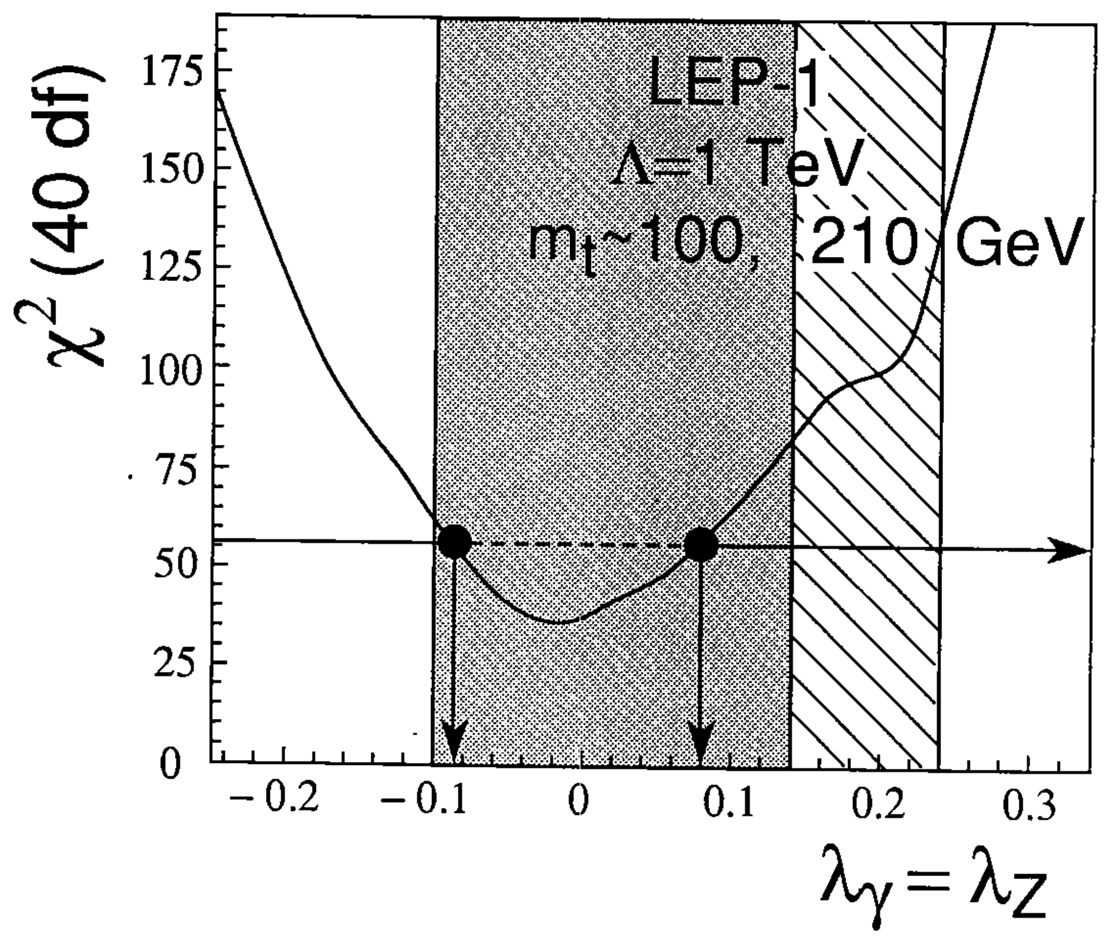

R
1.2
1
0.9
0.8
0.7
$\lambda_\gamma = \lambda_Z = 0.1$
− 1
− 0.75
− 0.5
− 0.25
0
0.25
0.5
0.75
1
cos (θ_+)

Fig. 13



LEP-1
Λ=1 TeV
m_t~100, 210 GeV
χ² (40 df)
175
150
125
100
75
50
25
0
− 0.2
− 0.1
0
0.1
0.2
0.3
λγ = λZ

Fig. 14

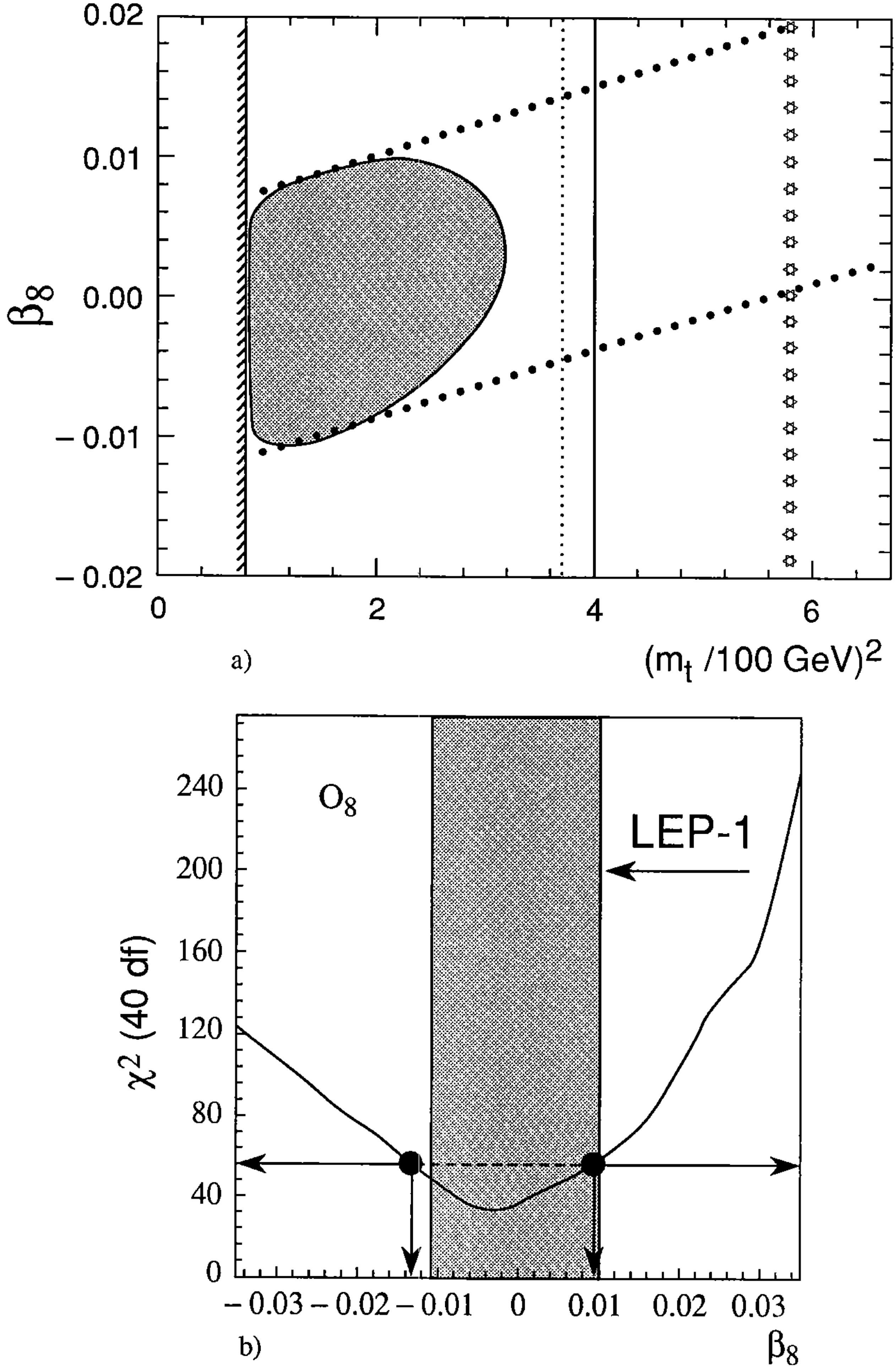


Fig. 15

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