MEDIUM EFFECTS FOR TERRESTRIAL AND ATMOSPHERIC NEUTRINO OSCILLATIONS

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Abstract

Matter effects in neutrino propagation translate into effective parameters for the oscillation and fake CP- and CPT-odd quantities, even in a scenario, such as $\Delta_{12} = 0$, where no genuine CP violation is present. This fact seems to impose severe restrictions on the determination of intrinsic parameters of the system from long-baseline experiments. We show, however, that the resonance in the effective mixing $\tilde{\theta}_{13}$ can be observed for a certain range of baselines. This provides a way to measure the vacuum mixing angle θ_{13} and the sign of Δm^2_{23} from atmospheric neutrinos, using a detector with energy resolution and charge discrimination.

February 2001

1 Introduction

Present evidence of neutrino oscillations in atmospheric neutrinos provides the range of values for the oscillation phase governed by Δm_{23}^2 to be [1]

$$1.5\ 10^{-3}\text{eV}^2 \leq \mid \Delta m_{23}^2 \mid \leq 5\ 10^{-3}\text{eV}^2, \tag{1}$$

and the neutrino mixing in the corresponding sector, i.e. θ_{23}, to be near maximal. On the contrary, the solution to the solar neutrino behaviour still presents several alternatives, the most favoured one [2] being the LAMSW solution for θ_{12} and Δm_{12}^2, that would allow, if connected by a non-vanishing θ_{13} to the other sector, the possibility of CP violation in leptonic physics. Up to now, θ_{13} has been bounded by CHOOZ [3] with a value

$$\sin^2 \theta_{13} \leq 0.05. \tag{2}$$

The studies around terrestrial long-baseline experiments and neutrino factories have precisely this objective in mind, the exploration of CP violation in the lepton case [4, 5, 6], a possibility open only for non-degenerate neutrino masses and non-vanishing mixings on the neutrino propagation. With this aim, the automatic self-inclusion of a new actor, the matter effect, is somehow seen as undesirable, as a background which could avoid the determination of the intrinsic properties of the neutrinos.

In this letter we will take a different attitude and show that, under appropriate conditions, matter effects bring the connecting mixing angle θ_{13} into the game, even when the sector $(1, 2)$ is irrelevant. In fact, we will consider terrestrial and atmospheric neutrino oscillations, with baselines such that, in a good approximation, the oscillating phase Δ_{12}, defined as $\Delta_{12} = \Delta m_{12}^2 L/(4E)$, can be neglected.

For $\Delta_{12} = 0$, the mixing in the sector $(1, 2)$ is inoperative and there is no room for **genuine** CP violation. However, the neutrino interaction with an asymmetric medium leads to (fake) CP-odd and CPT-odd non-vanishing quantities, which are by themselves a clean indicator of the effects to be searched for. These quantities, if non-vanishing, distinguish neutrinos from antineutrinos and their sign automatically indicates that of Δm_{23}^2.

General arguments [5] teach us that the difference in the survival probabilities for neutrinos and antineutrinos needs a CPT-odd origin, so this observable, particularly for ν_μ versus $\bar{\nu}_\mu$ where there are good prospects to distinguish the charge, will be of interest. There is a drawback, however, for this proposal, if the medium effect acts as a perturbative modification. The dominant ("allowed") vacuum oscillations would take place in the $(2, 3)$ sector and there would be no possibility for the small θ_{13} to show up. In this perturbative scenario, the alternative proposal has been to emphasize the "forbidden" appearance channel $\nu_e \rightarrow \nu_\mu$ [6, 7, 8] appropriate for neutrino factories and candidate to the search for genuine CP violation.

Another path can be to study whether the interaction of neutrinos with matter can generate an observable resonant situation in the effective mixing $\tilde{\theta}_{13}$, which naturally incorporates information on θ_{13}. The question then arises, of which the conditions of observability of such a resonance are, given the strict bounds on θ_{13}.

There seems to be a fatalistic result, a sort of no-go theorem, stating that the maximum in the effective mixing is compensated by a minimum in the oscillation and only the

product is measured. This cancelation seems to be also operative for the T-violating probabilities [9]. Although it is true that this compensation operates in many physical situations of practical interest, and we will confirm this, it cannot always be true, because the mixing is independent of the baseline whereas the oscillating phase depends linearly on it. We will study the conditions under which the resonant $\tilde{\theta}_{13}$ transition can be made visible in the disappearance channel, opening the door to an appreciable effect in the CPT-odd ν_μ survival probability and accessible in atmospheric neutrino experiments.

The plan of this paper is as follows. In Section 2 we develop the relevant amplitudes for 3-family neutrino oscillations in matter of constant density, in the approximation that $\Delta_{12} = 0$. A resonant behaviour appears for the effective mixing $\tilde{\theta}_{13}$ in matter with a width proportional to θ_{13}. Section 3 gives the behaviour with energies outside the resonance region. The interference pattern, for small θ_{13}, is shifted from the vacuum L/E behaviour to an additional term proportional to L and independent of E. The prospect is to determine θ_{13} from the forbidden channel probabilities $\nu_e \longrightarrow \nu_\mu$ and $\bar{\nu}_e \longrightarrow \bar{\nu}_\mu$. In Section 4 we study the conditions on L under which the resonance can be made visible: the disappearance channel $\nu_\mu \longrightarrow \nu_\mu$ then becomes the simplest and most spectacular one. The CPT-odd asymmetry is able not only to show the resonance effects but also to give information on both the magnitude of θ_{13} and the sign of Δm_{23}^2. From the resonance effect in both $\nu_\mu \longrightarrow \nu_\mu$ and $\nu_e \longrightarrow \nu_\mu$ channels, we estimate the charge asymmetry in atmospheric neutrinos. In Section 5 we summarize our conclusions.

2 Basics of three-neutrino oscillations in matter

The effective hamiltonian that describes the time evolution of neutrinos in matter can be written in the flavour basis as [10]

$$H = \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \Delta m_{12}^2 & \\ & & \Delta m_{13}^2 \end{pmatrix} U^+ + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}, \tag{3}$$

where $a = G\sqrt{2}N_e 2E$ represents matter effects from the effective potential of electron-neutrinos with electrons, and U is the flavour mixing matrix in vacuum (PDG representation [11]),

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{4}$$

For a baseline L, the evolution of neutrino states is given by

$$\nu(L) = S(L)\nu(0), \tag{5}$$

with

$$S(L) = e^{-iHL} \tag{6}$$

for constant matter density. The corresponding effective hamiltonian for antineutrinos is obtained by $U \to U^*$ and $a \to -a$.

If the hamiltonian can be separated into two pieces as $H = H_0 + H_1$, where H_0 can be exactly solved and H_1 can be treated in perturbation theory, then $S(L) = S_0(L) + S_1(L)$ and $S_0(L) = e^{-iH_0 L}$ gives the lowest order transition amplitudes.

For $\mid \Delta m_{12}^2 \mid \ll \mid \Delta m_{13}^2 \mid$, and a in a region of energies such that it is comparable to $\mid \Delta m_{13}^2 \mid$, we choose to solve exactly

$$
H_0 = \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & 0 & \\ & & \Delta m_{13}^2 \end{pmatrix} U^+ + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}
$$

$$
= \frac{1}{2E} \tilde{U} \begin{pmatrix} 0 & & \\ & \Delta \tilde{m}_2^2 & \\ & & \Delta \tilde{m}_3^2 \end{pmatrix} \tilde{U}^+, \tag{7}
$$

where $\Delta \tilde{m}^2$ describes the energy-level spacings in matter and $\tilde{U}$ the effective mixings. One realizes that the matter effect breaks the degeneracy. Therefore,

$$
S_0(L) = \tilde{U} \begin{pmatrix} 0 & & \\ & e^{-i\frac{\Delta \tilde{m}_2^2}{2E}L} & \\ & & e^{-i\frac{\Delta \tilde{m}_3^2}{2E}L} \end{pmatrix} \tilde{U}^+. \tag{8}
$$

The exact diagonalization of H_0 yields effective mass differences and mixings in matter, in the limit $\Delta m_{12} \to 0$, but with no approximation taken on a. We get the following result

$$
\tilde{U} = \begin{pmatrix} 0 & \frac{e^{-i\delta}}{n_2}(l_2 - c_{13}^2) & \frac{e^{-i\delta}}{n_3}(l_3 - c_{13}^2) \\ -c_{23} & \frac{1}{n_2}s_{23}s_{13}c_{13} & \frac{1}{n_3}s_{23}s_{13}c_{13} \\ s_{23} & \frac{1}{n_2}c_{23}s_{13}c_{13} & \frac{1}{n_3}c_{23}s_{13}c_{13} \end{pmatrix}, \tag{9}
$$

for the mixings in terms of the vacuum parameters, where we have defined

$$
n_2 = \sqrt{l_2^2 - 2l_2 c_{13}^2 + c_{13}^2}; \quad n_3 = \sqrt{l_3^2 - 2l_3 c_{13}^2 + c_{13}^2}, \tag{10}
$$

and

$$
l_2 \equiv \frac{\Delta \tilde{m}_2^2}{\Delta m_{13}^2} = \frac{1}{2}\left[1 + \alpha - \sqrt{1 + \alpha^2 - 2\alpha \cos(2\theta_{13})}\right];
$$

$$
l_3 \equiv \frac{\Delta \tilde{m}_3^2}{\Delta m_{13}^2} = \frac{1}{2}\left[1 + \alpha + \sqrt{1 + \alpha^2 - 2\alpha \cos(2\theta_{13})}\right], \tag{11}
$$

which give the effective mass differences. We have introduced the dimensionless parameter $\alpha \equiv \frac{a}{\Delta m_{13}^2}$ for the sake of simplicity. The ordering of levels in matter is (1,2,3) for the hierarchical case of $\Delta m_{13}^2 > 0$ and $\alpha < 1$.

The effective mixing matrix $\tilde{U}$ is independent of θ_{12} and δ, if the latter is appropriately rotated away. The matrix $\tilde{U}$ can be expressed also in the PDG form. To do so, we change

$$
\tilde{U} \to \tilde{U} \begin{pmatrix} 1 & & \\ & e^{i\delta} & \\ & & 1 \end{pmatrix}. \tag{12}
$$

3

By comparing with previous expressions, we may establish the correspondence between the effective mixing angles, $\tilde{\theta}_{ij}$, and the vacuum parameters, getting

$$
\begin{aligned}
\tilde{c}_{12} &= 0, & |\tilde{s}_{12}| &= 1, \\
\tilde{c}_{23} &= c_{23}, & \tilde{s}_{23} &= s_{23}, \\
\tilde{c}_{13} &= \frac{l_2 - c_{13}^2}{n_2}, & \tilde{s}_{13} &= \frac{l_3 - c_{13}^2}{n_3},
\end{aligned}
\tag{13}
$$

up to signs. The vanishing mixing in matter $\tilde{c}_{12} = 0$ is a consequence of the degeneracy $\Delta_{12} = 0$ in vacuum and says that the lowest mass eigenstate in matter contains no electron-neutrino flavour component. This result is the ingredient that avoids genuine CP violation in matter, even if one has three non-degenerate effective masses.

Transition amplitudes for $\nu_\alpha \rightarrow \nu_\beta$ in the $\Delta m_{12} = 0$ case are given by S_0 matrix elements, which can be written

$$
A(\alpha \rightarrow \beta; L) = S_0(L)_{\beta\alpha}
\tag{14}
$$

$$
= \delta_{\beta\alpha} + \tilde{U}_{\beta 2} \tilde{U}_{2\alpha}^{+} \left(e^{-i\frac{\Delta \tilde{m}_2^2}{2E}L} - 1 \right) + \tilde{U}_{\beta 3} \tilde{U}_{3\alpha}^{+} \left(e^{-i\frac{\Delta \tilde{m}_3^2}{2E}L} - 1 \right).
$$

From this expression we may calculate all probabilities,

$$
P\left(\nu_e \rightarrow \nu_e\right) = 1 - \sin^2(2\tilde{\theta}_{13}) \sin^2\left[\tilde{\Delta}_{13}\right]
\tag{15}
$$

$$
P\left(\nu_\mu \rightarrow \nu_e\right) = s_{23}^2 \sin^2(2\tilde{\theta}_{13}) \sin^2\left[\tilde{\Delta}_{13}\right]
\tag{16}
$$

$$
P\left(\nu_\tau \rightarrow \nu_e\right) = c_{23}^2 \sin^2(2\tilde{\theta}_{13}) \sin^2\left[\tilde{\Delta}_{13}\right]
\tag{17}
$$

$$
\begin{aligned}
P\left(\nu_\mu \rightarrow \nu_\mu\right) = {}& 1 - s_{23}^4 \sin^2(2\tilde{\theta}_{13}) \sin^2\left[\tilde{\Delta}_{13}\right] - 2s_{23}^2 c_{23}^2 \left\{ 1 - \cos\left[\Delta_{13}(1+\alpha)\right] \cos\left[\tilde{\Delta}_{13}\right] \right. \\
& \left. + \cos(2\tilde{\theta}_{13}) \sin\left[\Delta_{13}(1+\alpha)\right] \sin\left[\tilde{\Delta}_{13}\right] \right\}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
P\left(\nu_\tau \rightarrow \nu_\mu\right) = {}& s_{23}^2 c_{23}^2 \left\{ 2 - 2\cos\left[\Delta_{13}(1+\alpha)\right] \cos\left[\tilde{\Delta}_{13}\right] - \sin^2(2\tilde{\theta}_{13}) \sin^2\left[\tilde{\Delta}_{13}\right] \right. \\
& \left. + 2\cos(2\tilde{\theta}_{13}) \sin\left[\Delta_{13}(1+\alpha)\right] \sin\left[\tilde{\Delta}_{13}\right] \right\}
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
P\left(\nu_\tau \rightarrow \nu_\tau\right) = {}& 1 - c_{23}^4 \sin^2(2\tilde{\theta}_{13}) \sin^2\left[\tilde{\Delta}_{13}\right] - 2s_{23}^2 c_{23}^2 \left\{ 1 - \cos\left[\Delta_{13}(1+\alpha)\right] \cos\left[\tilde{\Delta}_{13}\right] \right. \\
& \left. + \cos(2\tilde{\theta}_{13}) \sin\left[\Delta_{13}(1+\alpha)\right] \sin\left[\tilde{\Delta}_{13}\right] \right\}
\end{aligned}
\tag{20}
$$

where

$$
\tilde{\Delta}_{13} \equiv \Delta_{13}\sqrt{1 + \alpha^2 - 2\alpha\cos(2\theta_{13})}, \quad \text{with} \quad \Delta_{13} \equiv \frac{\Delta m_{13}^2 L}{4E}.
\tag{21}
$$

and

$$
\sin^2(2\tilde{\theta}_{13}) = 4\frac{s_{13}^2 c_{13}^2}{1 + \alpha^2 - 2\alpha\cos(2\theta_{13})}.
\tag{22}
$$

The probabilities for time-reversal-conjugated transitions satisfy ($\alpha \neq \beta$)

$$
P\left(\nu_\beta \rightarrow \nu_\alpha\right) = P\left(\nu_\alpha \rightarrow \nu_\beta\right).
\tag{23}
$$

as they are even functions of the baseline L [5].

In order to get the corresponding expressions for antineutrinos, we must change $a \to -a$, i.e. $\alpha \to -\alpha$. The effect of such a change in the probabilities comes from the different relative sign between mass and matter terms in H_0. The same effect can be achieved by changing not the sign of a but that of Δm_{13}^2, i.e. by considering a different hierarchy for mass eigenstates in vacuum.

Now we analyse the resonance. Without taking any limit on s_{13}, Eq. (22) can be written as

$$\sin^2(2\tilde{\theta}_{13}) = \frac{4s_{13}^2 c_{13}^2}{(\alpha - \cos 2\theta_{13})^2 + 4s_{13}^2 c_{13}^2} \tag{24}$$

$$= \frac{4s_{13}^2 c_{13}^2 \left(\frac{\Delta m_{13}^2}{\tilde{a}}\right)^2}{\left(E - \cos 2\theta_{13}\frac{\Delta m_{13}^2}{\tilde{a}}\right)^2 + 4s_{13}^2 c_{13}^2 \left(\frac{\Delta m_{13}^2}{\tilde{a}}\right)^2},$$

where

$$\tilde{a} = 2\sqrt{2}G_F N_e. \tag{25}$$

From here we obtain the resonant energy, given by

$$E_R = \cos(2\theta_{13})\frac{\Delta m_{13}^2}{\tilde{a}}, \tag{26}$$

and the width

$$\Gamma = 2\sin(2\theta_{13})\frac{\Delta m_{13}^2}{\tilde{a}}. \tag{27}$$

If interpreted in terms of the variable α, the resonant parameters are given by

$$\alpha_R = \cos(2\theta_{13});$$
$$\Gamma_\alpha = 4s_{13}c_{13}. \tag{28}$$

It is important to stress that the resonant energy is not sensitive to s_{13}, for small θ_{13}, as it varies like the cosine, but is a measure of Δm_{13}^2. On the other hand, the resonance width depends linearly on θ_{13} and can be a useful tool to measure it.

An inspection of Eqs.(15)–(20) for the probabilities in different channels points out that there are contributions from both the imaginary part squared of the resonant amplitude, $\sin^2(2\tilde{\theta}_{13})$, and the interference with the real part, $\cos(2\tilde{\theta}_{13})$. On top of the resonance, $\sin^2(2\tilde{\theta}_{13}) = 1$, independent of s_{13}, and $\cos(2\tilde{\theta}_{13}) = 0$.

For $\Delta m_{13}^2 > 0$, the resonance appears only for neutrinos, whereas for $\Delta m_{13}^2 < 0$ it would show up only for antineutrinos.

3   Outside the resonance

When we are far from the resonance ($\alpha \simeq 1$), so that s_{13} is small with respect to $\alpha - 1$, up to quartic terms in s_{13} for the probabilities we can neglect the effects of the width,

$$\sqrt{1 + \alpha^2 - 2\alpha\cos(2\theta_{13})} \simeq |\alpha - 1| \tag{29}$$

and therefore the transition probabilities can be written as

$$P\left(\nu_e \to \nu_e\right) = 1 - \frac{4s_{13}^2}{(1-\alpha)^2}\sin^2\left[\Delta_{13}(1-\alpha)\right] \tag{30}$$

$$P\left(\nu_\mu \to \nu_e\right) = s_{23}^2\frac{4s_{13}^2}{(1-\alpha)^2}\sin^2\left[\Delta_{13}(1-\alpha)\right] \tag{31}$$

$$P\left(\nu_\tau \to \nu_e\right) = c_{23}^2\frac{4s_{13}^2}{(1-\alpha)^2}\sin^2\left[\Delta_{13}(1-\alpha)\right] \tag{32}$$

$$P\left(\nu_\mu \to \nu_\mu\right) = 1 - s_{23}^2\frac{4s_{13}^2}{(1-\alpha)^2}\sin^2\left[\Delta_{13}(1-\alpha)\right] - 4s_{23}^2c_{23}^2\sin^2\left[\Delta_{13}\right] \tag{33}$$

$$P\left(\nu_\tau \to \nu_\mu\right) = 4s_{23}^2c_{23}^2\sin^2\left[\Delta_{13}\right] - s_{23}^2c_{23}^2\frac{4s_{13}^2}{(1-\alpha)^2}\sin^2\left[\Delta_{13}(1-\alpha)\right] \tag{34}$$

$$P\left(\nu_\tau \to \nu_\tau\right) = 1 - c_{23}^4\frac{4s_{13}^2}{(1-\alpha)^2}\sin^2\left[\Delta_{13}(1-\alpha)\right] - 4s_{23}^2c_{23}^2\sin^2\left[\Delta_{13}\right]. \tag{35}$$

As was pointed out already in [8], the forbidden transition $P\left(\nu_\mu \to \nu_e\right)$ presents an interference pattern which, in addition to the vacuum $\Delta_{13} \sim L/E$ dependence of the oscillation phase, has an energy independent phase shift induced by matter as, $\Delta_{13}\alpha \sim L$, thus providing a possibility of quantifying matter effects. This phase shift is due to a purely quantum-mechanical effect with potentials and corresponds to an analogous to the Minkowski-rotated form of the Aharonov–Bohm experiment [12]. Instead of space interference, one has here flavour interference; the interferometer becomes the mixing matrix, the optical path difference the value of Δ_{13}, and the energy-independent effective potential $a/(2E)$.

   As widely recognized in the literature, the $\nu_e \to \nu_\mu$ transition probability provides a very good measurement of s_{13}. A very high flux is needed to be sensitive to its forbidden character. In the limit $\Delta_{13}(1-\alpha) \ll 1$, the $\sin^2\left[\Delta_{13}(1-\alpha)\right]$ factor is compensated by the $1/(1-\alpha)^2$ enhancement. Thus, matter effects would remain small for either small L/E, or α far away from the resonance, or both. Under these conditions, the (fake) CP-odd probability would be suppressed. On the contrary, once L/E is increased, the appearance neutrino probability is dramatically increased (for $\Delta m_{13}^2 > 0$) respect to that for propagation in vacuum, whereas the antineutrino probability becomes much smaller. To illustrate this feature quantitatively, we give in Figs. 1 and 2 the $\nu_e \to \nu_\mu$ probabilities for $L = 3000$ and 7000 km, respectively. (For $L = 700$ km the effects are not appreciable.) The three curves, dotted, solid and dashed, correspond in each figure to neutrino, vacuum and antineutrino probabilities. Notice that, besides the change in magnitude, there is a shift in the oscillation pattern, as imposed by an attractive or repulsive potential. In all these cases, we have taken $\Delta m_{23}^2 = 3\ 10^{-3}$ eV^2, $\tilde{a} = 2.8\ 10^{-13}$eV and $\theta_{13} = 0.23$. Under these conditions the mixing resonance has parameters $E_R = 9.6$ GeV with half-width $\Gamma/2 = 4.7$ GeV. An inspection of these results shows no special role of the resonance, except for $L = 7000$ km (see next section). In fact, important matter effects appear at $L = 3000$ km for both neutrinos and antineutrinos, indicating that they are non-resonant.

   Contrary to the forbidden channel $\nu_e \to \nu_\mu$ (or $\nu_e \to \nu_\tau$ or the dissapearance $\nu_e \not\to \nu_e$), the allowed channel $\nu_\mu \to \nu_\tau$ or the survival probability $\nu_\mu \to \nu_\mu$ have contributions coming from the interference with the real part $\cos(2\tilde{\theta}_{13})$ of the amplitude, besides the imaginary

part squared, $\sin^2(2\tilde{\theta}_{13})$. Outside the resonance, the first contribution dominates and all the physics is controlled by the (2,3) sector, without any room for appreciable matter effects. Needless to say, s_{13} plays no role in these cases. It remains to be seen whether there are new features around the resonance, which could allow s_{13} to show up.

4 On the observability of the resonance

The main difficulty for the observation of resonant effects in the "connecting" mixing $\tilde{\theta}_{13}$ comes from the product compensation, in measurable quantities, of the effective mixing resonance and the oscillation factor. Therefore, to induce appreciable effects of the resonance we have to impose an overlap of the mixing peak with a maximum of the oscillation factor. If this is possible, is a way of escaping the washing out effect that was thought to be unavoidable.

The maximal mixing, $\sin^2(2\tilde{\theta}_{13}) = 1$, is reached on top of the resonance, corresponding to a value of α equal to α_R, Eq.(28). On the other hand, the first maximum of the oscillation term is reached when the oscillation phase, $\Delta_{13}\sqrt{1 + \alpha^2 - 2\alpha\cos(2\theta_{13})}$, takes the value $\pi/2$, that corresponds to a value $\alpha_{\max}$. By imposing the condition that both maxima coincide, i.e. that $\alpha_R = \alpha_{\max}$, we obtain the baseline that maximizes the observability of the resonant effect,

$$L_{\max} = \frac{2\pi}{\tilde{a}\tan(2\theta_{13})}. \tag{36}$$

Notice that $L_{\max}$ is independent of Δm_{13}^2, which determines the resonant energy, and it is inverse to θ_{13}. The condition to avoid the cancellation of the resonant effect by the oscillation does not need to be so restrictive. One could allow a separation between the two maxima of one resonance half-width, given in terms of α by $\Gamma_\alpha/2$, Eq.(28), and still expect to observe the overlap. The corresponding baseline for this less restrictive condition would be

$$L_{\min} \simeq \frac{L_{\max}}{\sqrt{2}}. \tag{37}$$

For $\tilde{a}$ and θ_{13} as taken in the previous section, $L = 7000$ km approximately satisfies this condition.

In Fig. 3 we give the survival probabilities $\nu_\mu \to \nu_\mu$ for $L = 7000$ km. Again the three curves in each figure correspond to neutrino, vacuum and antineutrino probabilities. For the short baselines, the physics is dominated by the (2,3) sector without any appreciable matter effect even at $L = 3000$ km, but there appears a spectacular change of regime for $L = 7000$ km, in which the resonance becomes apparent. In this way, matter effects (through the resonance) are only important in one channel: neutrinos (antineutrinos) for $\Delta m_{23}^2 > 0 \ (< 0)$.

An impressive plateau around the resonance is the signal expected for the CPT-odd asymmetry, with a sign opposite to that of Δm_{23}^2. We give in Fig. 4 the (fake) CPT-odd asymmetry for the discussed case of $\theta_{13} = 0.23$ and for $\theta_{13} = 0.15$. We conclude that, for appropriate L, the muon-neutrino survival probability is sensitive to the resonance effect

in matter and its CPT-odd asymmetry provides a measure of the connecting mixing θ_{13} in vacuum.

To quantify the implication of such an indicator for the zenith angle effect in atmospheric neutrino oscillations, we proceed in the following way. We convolute both, the ν_μ survival probability (Fig. 3) and the ν_μ appearance probability from ν_e (Fig. 2) with the corresponding ν_μ and ν_e atmospheric fluxes [13] and with the ν_μ cross section in matter [11]. Similarly for antineutrinos. Notice that the difference of behaviour for ν_μ and $\bar{\nu}_\mu$ comes now from both, the matter effect and the convolution. The observed muon charge asymmetry is plotted in Fig. 5 as a function of energy in the interesting resonance region. Our results are obtained for $L \sim 7000$ km, within a zenith angle resolution of 5°. We conclude that, even for this minimum value of L in the sense of Eq.(37), the four values of the set $(\theta_{13}, \pm \mid \Delta m_{23}^2 \mid)$ can be distinguished. Needless to say, one can amplify the effect going to values of L closer to $L_{\max}$ (Eq.(36)), leading to a better sensitivity for smaller values of θ_{13}.

5 Conclusions

In this paper we have explored the medium effects in neutrino oscillations for baselines appropriate to terrestrial or atmospheric neutrinos. The analysis has been made in the approximation $\Delta_{12} = 0$ and it has been mainly applied to both the forbidden appearance channel $\nu_e \rightarrow \nu_\mu$ and the survival probability for $\nu_\mu \rightarrow \nu_\mu$. The manifestation of the matter effects has been presented in terms of the fake CP-odd and CPT-odd asymmetries, respectively. These observables are sensitive to the connecting mixing angle θ_{13} in magnitude and to the sign of Δm_{23}^2.

We have analysed the change of regime in going from a short baseline of 700 km to a long baseline of 7000 km. For the latter, we are entering into a manifestation of the resonance present in the effective mixing $\sin^2(2\tilde{\theta}_{13})$.

The forbidden appearance $\nu_e \rightarrow \nu_\mu$ probabilities, which are very sensitive to s_{13}, show already at $L = 3000$ km very important matter effects, which are non-resonant, with phase shifts of opposite sign for neutrinos and antineutrinos. The CP-even probability, relevant for detectors without charge discrimination, still sees appreciable matter effects, more apparent in oscillation phase shifts than in magnitude. Of course, the sensitivity to s_{13}^2 in magnitude is there, and a variation in s_{13} does not affect the oscillation pattern. At $L = 7000$ km, matter effects become resonance-dominated and affect neutrinos both in magnitude and phase (for $\Delta m_{23}^2 > 0$).

Contrary to the transition in regime discussed for $\nu_e \rightarrow \nu_\mu$, the disappearance channel $\nu_\mu \rightarrow \nu_\mu$ only sees matter effects from a baseline above $L \sim 7000$ km, i.e. when the resonance shows up. Even at $L = 3000$ km, one cannot induce appreciable medium effects. This is understood: outside the resonance, the physics is here dominated by the "allowed" sector (2,3), which is not sensitive to interactions with matter. Once the resonance in $\sin^2(2\tilde{\theta}_{13})$ operates, medium effects appreciably modify the magnitude for the neutrino channel but not for antineutrinos, where no resonance appears for $\Delta m_{23}^2 > 0$. The corresponding CPT-odd asymmetry, shown in Fig. 4, is sensitive to the connecting mixing s_{13} in its magnitude and its sign distinguishes the sign of Δm_{23}^2.

The calculated muon charge asymmetry originated from both $\nu_\mu \to \nu_\mu$ and $\nu_e \to \nu_\mu$ shows important effects in the resonance region. Fig. 5 shows that we are able to distinguish the values of θ_{13}, and the sign of Δm_{13}^2. Notice that this sign is not automatically translated into the sign of the asymmetry, as it was in Fig. 4. We have estimated that a 10 kT detector with energy resolution and charge discrimination can reach a few percent accuracy in the measured asymmetry in one year of data taking.

Acknowledgements

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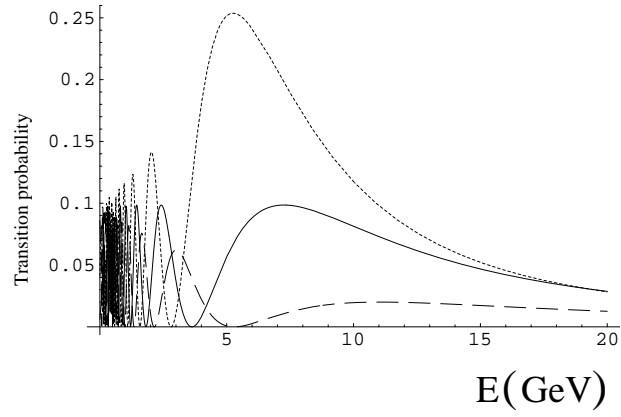[13] See, e.g.,T.K. Gaisser, Nucl.Phys.Proc.Suppl. **87**, 145 (2000) and references therein.

Figure 1: $\nu_e \to \nu_\mu$ transition probability for neutrinos (dotted), vacuum (solid) and antineutrinos (dashed) for a baseline $L = 3000$ km.
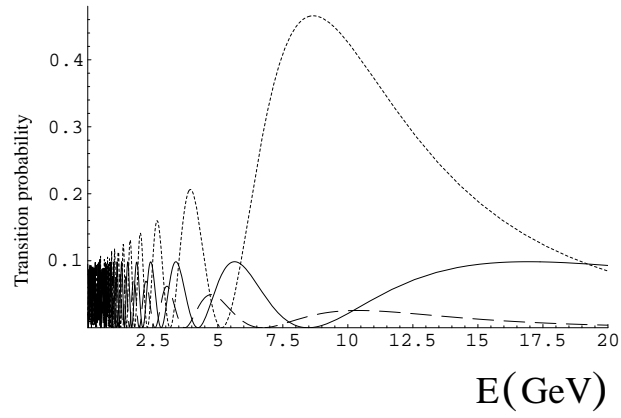

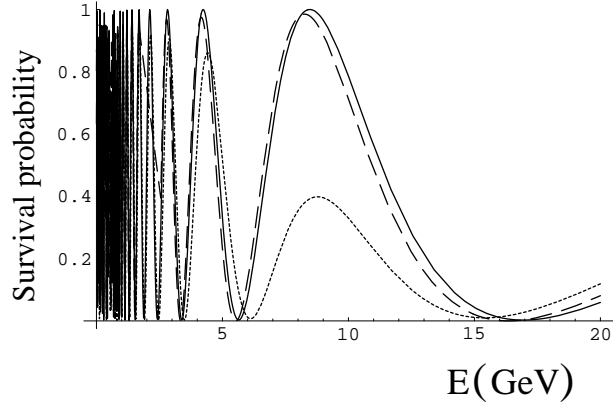Figure 2: Same as Fig. 1 for $L = 7000$ km.

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Figure 3: Muon neutrino survival probability for neutrinos (dotted), vacuum (solid) and antineutrinos (dashed) for a baseline $L = 7000$ km.
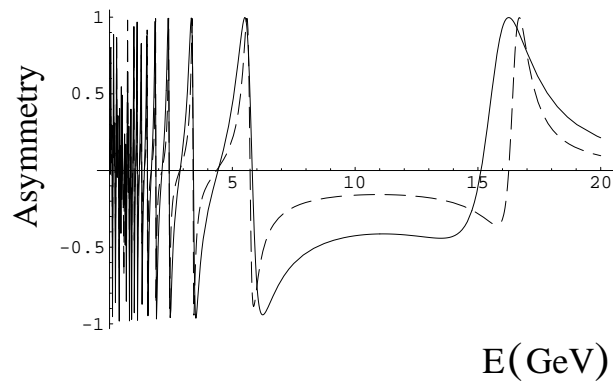

Figure 4: (Fake) CPT-odd asymmetry for muon neutrinos and $L = 7000$ km. Solid line for $\theta_{13} = 0.23$, dashed line for $\theta_{13} = 0.15$.
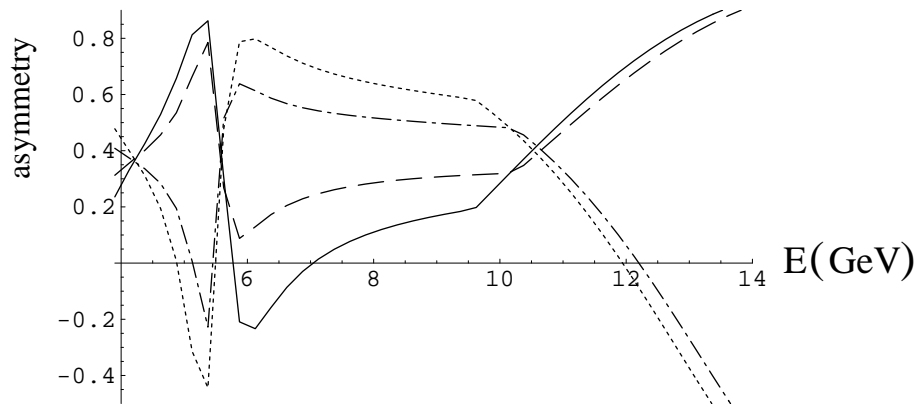
Figure 5: Muon charge asymmetry for $\theta_{13} = 0.23$ and $\text{sign}(\Delta m_{23}^2) = +$ (solid), $\theta_{13} = 0.15$ and $\text{sign}(\Delta m_{23}^2) = +$ (dashed), $\theta_{13} = 0.23$ and $\text{sign}(\Delta m_{23}^2) = -$ (dotted) and $\theta_{13} = 0.15$ and $\text{sign}(\Delta m_{23}^2) = -$ (dash-dotted)