Decoherent Neutrino Mixing, Dark Energy and Matter-Antimatter Asymmetry

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A CPT violating decoherence scenario can easily account for all the experimental evidence in the neutrino sector including LSND. In this work it is argued that this framework can also accommodate the Dark Energy content of the Universe, as well as the observed matter-antimatter asymmetry.

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In a previous work[1], henceforth referred to as I, we have discussed a phenomenological way of accounting for the LSND results [2] on evidence for antineutrino oscillations $\overline{\nu}_e \to \overline{\nu}_\mu$, but with lack of corresponding evidence in the neutrino sector, by means of invoking CPT Violating (CPTV) decoherence, due to quantum gravity. Indeed, quantum decoherence in matter propagation occurs when the matter subsystem interacts with an 'environment' [3], according to the rules of open-system quantum mechanics. At a fundamental level, such a decoherence may be the result of propagation of matter in quantum gravity space-time backgrounds with 'fuzzy' properties, which may be responsible for violation of CPT symmetry[4] in a way not necessarily related to mass differences between particles and antiparticles. As demonstrated in I, it is possible to fit all the available neutrino data, including the results from LSND and Karmen[5] experiments, not by enlarging the neutrino sector or implementing CPTV mass spectra for neutrinos, but by invoking a CPTV difference in the decoherence parameters between particle and antiparticle sectors in three generation neutrino models (we refer the reader to the original work for technical details). From this point of view, then, the LSND result would evidence CPT violation in the sense of different decohering interactions between particle and antiparticle sectors, while the mass differences (and widths) between the two sectors remain the same. From I it became clear that both mixing and decoherence, the latter in the antineutrino sector only, were necessary to account for all the available experimental information, including LSND and Karmen results[2, 5]. Mixing, in the sense of non trivial mass differences between energy eigenstates, was important, since pure decoherence, that is absence of any mass terms in the Hamiltonian, was not sufficient to fit the data. However, this does not mean that the mass terms are necessarily of conventional origin. As stressed in I, the Hamiltonian appearing in the decoherent evolution should be viewed as an "effective" one, receiving possible contributions from the environment as well. In this sense, one cannot exclude the possibility that some contribution to the neutrino masses have a quantum-decoherence origin, as a result of interactions with the foam, as happens for example when neutrinos interact with matter and the mass differences get modified (and mass degeneracies lifted) as a result of the interaction. Such an effect will disentangle neutrino masses from standard electroweak symmetry breaking scenaria. As we shall discuss below, this is an important feature that will allow us to associate the recently claimed amount of dark energy in the Universe by means of astrophysical observations[6, 7] to our decoherent neutrino mass differences.

The (energy depending) decoherence parameters needed to account for all the experimental information, $\gamma_1 \sim 10^{-18} \cdot E$, $\gamma_3 \sim 10^{-24}/E$, found in our sample point in I, call for an explanation within a consistent theoretical framework. To this end, the reader should first observe that, for energies of a few GeV, which are typical of the pertinent experiments, such values are not far from $\gamma_j \sim \Delta m^2_{ij}$. If our conclusions survive the next round of experiments, and therefore if MiniBOONE experiment [8] confirms previous LSND claims, then this may be a significant result. One would be tempted [1] to conclude that if the above estimate holds, this would probably mean that the neutrino mass differences might be due to quantum gravity decoherence. Theoretically it is still unknown how neutrinos acquire a mass, or what kind of mass (Majorana or Dirac) they possess. Thus, if the above model turns out to be right we might then have, for the first time in low energy physics, an indication of a direct detection of a quantum gravity effect, which disguised itself as an induced decohering neutrino mass difference. Notice that in our model only antineutrinos have non-trivial decoherence parameters, while the corresponding quantities in the neutrino sector vanish. This implies that there may be a single cause for mass differences, the decoherence in antineutrino sector, compatible with common mass differences in both sectors.

In what follows we will make this assumption, namely that decoherence effects, due to interactions with the foam, contribute to the Hamiltonian terms in the evolution of the neutrino density matrix, and result in neutrino mass differences in much the same way as the celebrated MSW effect[9], responsible for a neutrino mass splitting due to interactions with a medium. Indeed, when neutrinos travel through matter, the neutral current contribution to this interaction, proportional to -$G_F n_n/\sqrt{2}$, with G_F Fermi's weak interaction constant, and n_n the neutron density in the medium, is present for *both* ν_e and ν_μ (in a two flavour scenario), while the charged current contribution, given by $\sqrt{2}G_F n_e$, with n_e the medium's electronic density, is present only for ν_e. The flavour eigenstates $\nu_{e,\mu}$ can then be expressed in terms of fields $\tilde{\nu}_{1,2}$ with definite masses $\tilde{m}_{1,2}$ respectively, with a mixing angle $\tilde{\theta}$, the tilde notation indicating the effects of matter. The tilded quantities are diagonalised with respect to the Hamiltonian of ν_e,ν_μ in the presence of non-trivial matter media, and one can find the following relations between vacuum (untilded) and medium parameters[9] $\sin^2 2\tilde{\theta} \simeq \sin^2 2\theta \left(\frac{\Delta m^2}{\Delta \tilde{m}^2}\right)$, with $\Delta\tilde{m}^2 = \sqrt{(D - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2 \sin 2\theta)^2}, D = 2\sqrt{2}G_F n_e k$. From this we observe that the medium-induced effects in the mass splittings are proportional to the electronic density of the medium and in fact, even if the neutrinos would have been mass degenerate in vacuum, such a degeneracy would be lifted by a medium.

To get a qualitative idea of what might happen with the foam, one imagines a similar mixing for neutrinos, as a result of their interaction with a quantum-gravity decohering foam situation. As a result, there are *gravitationally-induced effective masses* for neutrinos, due to flavour dependent interactions of the foam, which are in principle allowed in quantum gravity. In analogy (but we stress that this is only an analogy) with the MSW effect, the gravitationally-induced mass-splitting effects are expected now to be proportional to $G_N n_{\rm bh} k$, where $G_N = 1/M_P^2$ is Newton's constant, $M_P \sim 10^{19}$ GeV is the quantum gravity scale, and n_{bh} is a "foam" density of appropriate space time defects (such as Planck size black holes *etc.*), whose interaction with the neutrinos discriminates between flavours, in an analogous way to the matter effect. Neutrinos, being electrically neutral can indeed interact non-trivially with a space time foam, and change flavour as a result of such interactions, since such processes are allowed by quantum gravity. On the other hand, due to electric charge conservation of microscopic black holes, quarks and charged leptons, cannot interact non-trivially with the foam. In this spirit, one can imagine a microscopic charged black-hole/anti-black-hole pair being created by the foam vacuum. Evaporation of these black holes (probably at a slower rate than their neutral counterparts, due to their near extremality [10]) can produce preferentially e^+e^- pairs (lighter than muons), of which the positrons, say, are absorbed into the microscopic event horizons of the evaporating charged anti-black hole. This leaves us with a stochastically fluctuating (about a mean value) electron (or more general charge) density, $n^c_{\rm bh}(r)$, induced by the gravitational foam, $\langle n^c_{\rm bh}(r)\rangle = n_0 \neq 0$, $\langle n^c_{\rm bh}(r) n^c_{\rm bh}(r')\rangle \neq 0$, which, in analogy with the electrons of the MSW effect in a stochastically fluctuating medium[11], can interact non-trivially only with ν_e but not with the ν_μ, in contrast to neutral black holes which can interact with all types of neutrinos[12]. We assume, of course, that the contributions to the vacuum energy that may result from such emission and absorption processes by the black holes in the foamy vacuum are well within the known limits. For instance, one may envisage supersymmetric/superstring models of space-time foam, where such contributions may be vanishingly small[13]. The mean value (macroscopic) part, n_0, of $n^c_{\rm bh}(r)$, assumed time independent, will contribute to the Hamiltonian part of the evolution of the neutrino density matrix, ρ. In analogy with the (stochastic) MSW effect[9, 11], this part yields space-time foam-induced mass-squared splittings for neutrinos:

$$\langle \Delta m^2_{\rm foam}\rangle \propto G_N \langle n^c_{\rm bh}(r)\rangle k \qquad (1)$$

with non trivial quantum fluctuations (k is the neutrino momentum scale). To ensure a constant neutrino mass one may consider the case where $\langle n^c_{\rm bh}(r)\rangle$, which expresses the average number of virtual particles emitted from the foam with which the neutrino interacts, is inversely proportional to the (neutrino) momentum. This is reasonable, since the faster the neutrino, the less the available time to interact with the foam, and hence the smaller the number of foam particles it interacts with. Such flavour-violating foam effects would also contribute to decoherence through the quantum fluctuations of the foam-medium density[11, 12], by means of induced non-Hamiltonian terms in the density-matrix evolution. Such effects assume a double commutator structure[11, 12, 14] and are due to *both*, the fluctuating parts of the foam density, as well as the effects of the mixing (1) on the vaccum energy. Indeed, as we shall show below, neutrino flavour mixing leads to a non-trivial contribution to the vacuum energy, in a non-perturbative way suggested in [15]. Hence, such effects are necessarily CPT violating[16], in the sense of entailing an evolution of an initially pure neutrino quantum state to a mixed one due to the presence of the Hubble horizon associated with the non zero cosmological constant, which prevents pure asymptotic states from being well defined. In that case, CPT is violated in its strong form, that is CPT is not a well-defined operator, according to the theorem of [17].

For convenience we shall discuss explicitly the two-generation case. The arguments can be extended to three generations, at the expense of an increase in mathematical complexity, but will not affect qualitatively the conclusions drawn from the two-generation case. The arguments are based on the observation[18] that in quantum field theory, which by definition requires an infinite volume limit, in contrast to quantum mechanical treatment of fixed volume[19], the neutrino *flavour* states are *orthogonal* to the *energy* eigenstates, and moreover they define two inequivalent vacua related to each other by a *non unitary* transformation $G^{-1}(\theta, t)$: $|0(t)\rangle_f = G^{-1}_\theta(t)|0(t)\rangle_m$, where θ is the mixing angle, t is the time, and the suffix f(m) denotes flavour(energy) eigenstates respectively, and $G^{-1}_\theta(t) \neq G^\dagger_\theta(t)$ is a non-unitary operator expressed in terms of energy-eigenstate neutrino free fields $\nu_{1,2}$[15]: $G_\theta(t) = \exp\left(\theta \int d^3x [\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)]\right)$. A rigorous mathematical analysis of this problem has also appeared in [20]. As a result of the non unitarity of $G^{-1}_\theta(t)$, there is a Bogolubov transformation[18] connecting the creation and annihilation operator coefficients appearing in the expansion of the appropriate neutrino fields of the energy or flavour eigenstates. Of the two Bogolubov coefficients appearing in the treatment, we shall concentrate on $V_{\vec{k}} = |V_{\vec{k}}|e^{i(\omega_{k,1}+\omega_{k,2})t}$, with $\omega_{k,i} = \sqrt{k^2+m_i^2}$, the (positive) energy of the neutrino energy eigenstate $i = 1,2$ with mass m_i. This function is related to the condensate content of the flavour vacuum, in the sense of appearing in the expression of an appropriate non-zero number operator of the flavour vacuum[15, 20]: ${}_f\langle 0|\alpha^{r\dagger}_{\vec{k},i}\alpha^{r}_{\vec{k},i}|0\rangle_f = {}_f\langle 0|\beta^{r\dagger}_{\vec{k},i}\beta^{r}_{\vec{k},i}|0\rangle_f = \sin^2\theta|V_{\vec{k}}|^2$ in the two-generation scenario [18]. $|V_{\vec{k}}|$ has the property of vanishing for $m_1 = m_2$, it has a maximum at the momentum scale $k^2 = m_1 m_2$, and for $k \gg \sqrt{m_1 m_2}$ it goes to zero as:

$$|V_{\vec{k}}|^2 \sim \frac{(m_1-m_2)^2}{4|\vec{k}|^2}, \quad k \equiv |\vec{k}| \gg \sqrt{m_1 m_2} \qquad (2)$$

The analysis of [15] argued that the flavour vacuum $|0\rangle$, is the correct one to be used in the calculation of the average vacuum energy, since otherwise the probability is not conserved[21]. The energy-momentum tensor of a Dirac fermion field in the Robertson-Walker space-time background can be calculated straightforwardly in this formalism. The flavour-vacuum average value of its temporal T_{00} component, which yields the required contribution to the vacuum energy due to neutrino mixing, is[15]:

$$\begin{aligned}
&{}_f\langle 0|T_{00}|0\rangle_f = \langle\rho_{\rm vac}^{\nu-{\rm mix}}\rangle\eta_{00} \\
&= \sum_{i,r}\int d^3k\,\omega_{k,i}\left({}_f\langle 0|\alpha^{r\dagger}_{\vec{k},i}\alpha^{r}_{\vec{k},i}|0\rangle_f + {}_f\langle 0|\beta^{r\dagger}_{\vec{k},i}\beta^{r}_{\vec{k},i}|0\rangle_f\right) = \\
&8\sin^2\theta\int_0^K d^3k(\omega_{k,1}+\omega_{k,2})|V_{\vec{k}}|^2. \qquad (3)
\end{aligned}$$

where $\eta_{00} = 1$ in a Robertson-Walker (cosmological) metric background. The momentum integral in (3) is cut-off from above at a certain scale, K relevant to the physics of neutrino mixing. In conventional approaches, where the mass generation of neutrino occurs at the electroweak phase transition, this cutoff scale can be put on the electroweak scale $K \sim 100$ GeV, but this yields unacceptably large contributions to the vacuum energy. An alternative scale has been suggested in [15], namely $K \sim \sqrt{m_1 m_2}$ as the characteristic scale for the mixing. In this way these authors obtained a phenomenologically acceptable value for $\langle\rho_{\rm vac}^{\nu-{\rm mix}}\rangle$.

In our case we shall use a different cutoff scale, which allows for some analytic estimates of (3) to be derived, as being mathematically consistent with the asymptotic form of (2), which is valid in a regime of momenta $k \gg \sqrt{m_1 m_2}$. This cutoff scale is simply given by the sum of the two neutrino masses, $K \equiv k_0 = m_1 + m_2$, is compatible with our decoherence-induced mass difference scenario, and also allows for a mathematically consistent analytic estimate of the neutrino-mixing contribution to the vacuum energy in this framework. For hierarchical neutrino models, for which $m_1 \gg m_2$, we have that $k_0 \gg \sqrt{m_1 m_2}$, and thus, if we assume that the modes near the cutoff contribute most to the vacuum energy (3), which is clearly supported by the otherwise divergent nature of the momentum integration, and take into account the asymptotic properties of the function $V_{\vec{k}}$, which are safely valid in this case, we obtain:

$$\begin{aligned}
&\langle\rho_{\rm vac}^{\nu-{\rm mix}}\rangle \sim 8\pi\sin^2\theta(m_1-m_2)^2(m_1+m_2)^2 \times \\
&\left(\sqrt{2}+1+\mathcal{O}(\frac{m_2^2}{m_1^2})\right) \propto \sin^2\theta(\Delta m^2)^2 \qquad (4)
\end{aligned}$$

in the limit $m_2 \ll m_1$. For the (1,2) sector, the corresponding Δm^2 is given by the solar neutrino data and is estimated to be $\Delta m^2_{12} \simeq 10^{-5}$ eV^2, resulting in a contribution of the right order. This dependence of the cosmological constant on the square of the neutrino mass-squared difference has been conjectured in I, and was "derived" here following the flavour/mixing quantum field theoretic treatment of [15]. In this way the cosmological constant Λ is elegantly expressed in terms of the smallest (infrared, Δm^2) and the largest (ultraviolet, M_P^2) Lorentz-invariant mass scales available. The choice of the cutoff $k_0 \sim m_1 + m_2$ is consistent with our conjecture on the decoherence origin of the neutrino mass difference, due to interaction with the foam medium (1). Indeed, for momenta $k \sim m_1 + m_2$, which have been argued above to be the dominant contribution to the dark energy component (4), the induced mass splittings become $\Delta m^2_{\rm foam} \sim G_N\langle n^c_{\rm bh}\rangle(m_1+m_2)$ from which $m_1 - m_2 \sim \langle n^c_{\rm bh}\rangle/M_P^2$. If we assume there are $\mathcal{N}_c$ charged foam-induced objects per Planck volume, $V_P \sim M_P^{-3}$ then, $\mathcal{N}_{c,{\rm max}} \sim m_1 - m_2/M_P$. For realistic neutrino mass values this is very small, indicating that in such scenaria, a tiny amount of black holes in the foamy vacuum suffices to produce observable effects in neutrino physics. It goes without saying of course that none of the above statements should be considered as a rigorous derivation. Nevertheless, we think that the above arguments are non trivial and we believe they may be related with an actual theory of quantum gravity. Notice that the above way of deriving the neutrino-mixing contribution to the dark energy is independent of the usual perturbative loop arguments, and, in this sense, the result (4) should be considered as exact (non perturbative), if true.

Some important remarks are now in order. First of all, our choice of cutoff scale was such that the resulting contribution to the cosmological constant depends on the neutrino mass-squared differences and not on the absolute mass, and hence it is independent of any zero-point energy, in agreement with energy-driven decoherence models [14]. For us, it is curved space physics that is responsible for lifting the mass degeneracy of neutrino mass eigenstates and create the "flavour" problem. This is an important point, which may serve as motivation (not proof) behind such a cutoff "choice", which we conjecture is a physical "necessity". We have argued above that such a cutoff "choice" is a natural one from the point of view of quantum-gravity decoherence-induced mass differences. Detailed models of this fall way beyond the purposes of this brief note. Nevertheless, we believe that the above-demonstrated self-consistency of this cutoff choice within the remit of our toy model of space time foam is intellectually challenging and encouraging for further studies of this important issue.

It should be noted at this stage that our considerations above are based on the suggestion (which is not beyond doubt) of ref. [18] on a Fock-like quantisation of the flavour space. There is still controversy in the literature regarding the physical meaning of such quantum flavour states [22], in particular it has been argued that, although such states are mathematically elegant and correct constructions, nevertheless they lead to no observable consequences. However, in view of the results of [15] and of the present work, such an argument may not be correct, since the mass-squared difference contribution to the cosmological constant is an observable (global) consequence of the Fock-like flavour space quantisation. The presence of a time independent cosmological constant (4) in the flavour vacuum, which notably is not present if one uses instead the mass eigenstate vacuum, implies an asymptotic future event horizon for the emerging de-Sitter Universe. The flat-space time arguments of [22] for the flavour space field theory cannot then be applied, at least naively, and the problem of quantisation of the Fock-like flavour space is equivalent to the (still elusive) quantisation of field theories in (curved) de-Sitter space times.

In such a case one cannot define properly asymptotic states, and hence a scattering matrix. This will lead to decoherence, in the sense of a modified temporal evolution for matter states. For instance, string theory considerations[16] suggest that the temporal evolution of the matter density matrix ρ in such a de-Sitter Universe, will be decoherent: $\partial_t \rho = i[\rho, H] + :\Lambda g_{\mu\nu}[g^{\mu\nu}, \rho]:$, where Λ is the cosmological constant, given in our case by (4), and $: \ldots :$ denotes quantum ordering. Notice that the decoherent non-Hamiltonian term is proportional to (a quantum version of) the conformal anomaly (trace of the stress tensor) of the de-Sitter universe. For anti-symmetric ordering, one obtains a double commutator structure $[g_{\mu\nu}, [g^{\mu\nu}, \rho]]$, which when considered between, say, energy eigenstates yields variances of the metric field $(\Delta g_{\mu\nu})^2$, expressing quantum fluctuations of the space time geometry, as a result of the interactions of neutrinos with the foam. The terms proportional to Λ lead in general to a decoherent evolution of a pure quantum mechanical state to a mixed one. According to the general arguments of [17], then, one should expect in this case a strong form of CPT Violation, in the sense that the CPT operator is not well defined. This may lead to different decoherent parameters eventually between particles and antiparticles, reflecting the different ways of interaction with the foam between the two sectors. In other words, it is possible that the variance $(\overline{\Delta} g_{\mu\nu})^2$ in the antiparticle sector is much larger than the corresponding one in the particle sector. See however [12] for the suppression of this second effect in our case, where the foam-density-fluctuation terms may be held responsible for the leading contributions to decoherence. Nevertheless, due to the presence of the Λ term there will be a mixed state CPTV description.

We now notice that, in the case of (anti)neutrinos passing through stochastic media, including space time foam, there are additional contributions to decoherence, which may offer a natural explanation of the decoherence parameters of I. An important source of decoherence in such media is due to the uncertainties in the energy E and/or the oscillation length L of the (anti)neutrino beam. In fact, it can be shown [23] that if one averages the standard oscillation probabilities $P_{\nu_\alpha \to \nu_\beta}$ over Gaussian distributions for E and/or L with a variance σ^2, the result is equivalent to neutrino decoherence models, in the sense of the time dependent profile of the associated probability being identical to that of a completely-positive decoherence model. One finds for n flavours [23],

$$\langle P_{\alpha\beta} \rangle = \delta_{\alpha\beta} - 2 \sum_{a=1}^{n} \sum_{\beta=1, a<b}^{n} \mathrm{Re}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b}\right) \times \left(1 - \cos(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2}\right) - 2 \sum_{a=1}^{n} \sum_{b=1, a<b}^{n} \mathrm{Im}\left(U^*_{\alpha a} U_{\beta a} U_{\alpha b} U^*_{\beta b}\right) \times \sin(2\ell \Delta m^2_{ab}) e^{-2\sigma^2 (\Delta m^2_{ab})^2} \tag{5}$$

where U is the mixing matrix $\ell \equiv \langle x \rangle$, $\sigma = \sqrt{\langle (x - \langle x \rangle)^2} \equiv (L/4E) r$, and $x = L/4E$. The form is identical to that of decoherence, as becomes evident by noting that the exponential damping factors can be written in the form $e^{-\gamma_j L}$ with $t = L$ $(c = 1)$, and decoherence parameters γ_j of order: $2\sigma_j^2 (\Delta m^2)^2 = \gamma_j L$, from which $\gamma_j = \frac{(\Delta m^2)^2}{8E^2} L r_j^2$. There are various scenaria that restrict the order of σ. In general, the acceptable bounds on σ may be divided in two major categories, depending on the form of the uncertainties [23]: $\sigma_j \simeq \Delta x \simeq \Delta_j \frac{L}{4E} \le \frac{\langle L \rangle}{4\langle E \rangle} \left(\frac{\Delta_j L}{\langle L \rangle} + \frac{\Delta_j E}{\langle E \rangle} \right)$, or $\sigma_j \le \frac{\langle L \rangle}{4\langle E \rangle} \left([\frac{\Delta_j L}{\langle L \rangle}]^2 + [\frac{\Delta_j E}{\langle E \rangle}]^2 \right)^{1/2}$. In three generation models the values of the length and energy uncertainties may vary between flavours, and also between neutrinos and antineutrinos, as a result of the intrinsic CPT violation, hence the subscript j in the above formulae (for antiparticle sectors it is understood that $j \to \overline{j}$). From the above considerations it becomes clear that, for $L \sim 2E/\Delta m^2$, which is characteristic for oscillations, one has decoherence parameters $\gamma_j \sim (\Delta m^2/E) r_j^2$. It is interesting to estimate first the order of decoherence induced by conventional physics, for instance decoherence induced by uncertainties in the measured energy of the beam due to experimental limitations. For long base line, atmospheric or cosmic neutrino experiments, where $\Delta L/L$ is negligible, and $\Delta E/E \sim 1$ such decoherence parameters are found at most of order $\gamma \sim 10^{-24}$ GeV, for the relevant range of energies, and they diminish with energy, vanishing formally when $E \to \infty$, which seems to be a general feature of conventional matter-induced decoherence effects [23].

To obtain the decoherence parameters of the best-fit model of I it suffices to choose for the antineutrino sector $r_{\overline{3}} = r_{\overline{8}} \sim \Delta E/E \sim 1$, and $r^2_{\overline{1}} = r^2_{\overline{2}} \sim 10^{-18} \cdot E^2/\Delta m^2$. As seen above, the decoherence parameters exhibiting a $1/E$ energy dependence could be attributed to conventional energy uncertainties occuring in the beam of the (anti)neutrinos. However, the parameters proportional to E, if true, may be attributed to exotic physics.

The fact that r_j in general receives contributions from both length and energy uncertainties provides a natural explanation for the different energy dependence of the decoherence parameter of the model of I in the antiparticle sector. Indeed, having identified $r_{\overline{3}} = r_{\overline{8}}$ as decoherence induced by 'conventional-looking' energy uncertainties in the antineutrino sector, it is natural to assume that the $\gamma_3 = \gamma_8 \propto E$ decoherence is due to genuine quantum gravity effects, increasing with energy, which are associated with metric tensor quantum fluctuations. This is achieved provided we assume that $r^2_{\overline{3}} = r^2_{\overline{8}} \sim (\Delta L/L)^2$, i.e. these decoherence coefficients are predominantly oscillation-length-uncertainty driven, and take into account that variations in the invariant length may be caused by metric fluctuations, since $L^2 = g_{\mu\nu}L^\mu L^\nu$, implying $(\Delta g_{\mu\nu})^2 \sim (\Delta L)^2/L^2$, in order of magnitude. To obtain the best fit results of I, then, for $L \sim 2E/\Delta m^2$, one needs quantum-gravity induced metric fluctuations in the antineutrino sector of order $(\overline{\Delta} g_{\mu\nu})^2 \sim 10^{-18} L \cdot E$. The increase with energy is not unreasonable, given that the higher the energy of the antineutrino the stronger the back reaction onto space time, and hence the stronger the quantum-gravity induced metric fluctuations. The factor 10^{-18} may be thought of as being of order E/M_P, with $M_P \sim 10^{19}$ GeV the Planck mass, although alternative interpretations may be valid (see discussion on possible cosmological interpretations at the end of the article). The increase with L is not uncommon in stochastic models of quantum foam, where the decoherence 'medium' effects build up with the distance the (anti)particle travels [13]. We also mention at this stage that, apart from these effects, in stochastic models of foam there are additional contributions to decoherence, arising from the fluctuations of the density of the medium. These too can mimic the effects of the best-fit model of I in the antineutrino sector, as discussed in some detail in [12], but their L-dependence is different from that of the above effects. Comparison between short and long baseline experiments, therefore, may differentiate between the various decoherent contributions.

Unfortunately at present, we lack a detailed microscopic model of space-time foam, and hence the above considerations should be treated with caution. Nevertheless, we think that the above plausibility arguments, as well as those in [12], attempting to explain the order and the energy dependence of the decoherence parameters of I are not unreasonable.

In view of our conjecture on the quantum-gravity origin of the mass differences between neutrino flavours, supported by the above analysis, we should stress that we are clearly dealing here with an *interacting* theory on (highly) curved space times, and the ordinary procedure of quantisation needs to be completely rethought. If the mass difference is time independent, then, as argued above, one cannot follow standard methods of free-field quantisation [15, 22], due to the non-well defined nature of the scattering matrix, leading to CPT Violation. Based on this, one might even conjecture a situation in which the foam flavour vacuum is relaxing to equilibrium as the cosmic time elapses, in such a way that the asymptotic value of the neutrino mass-squared difference vanishes, and a proper set of asymptotic states can be defined. These are very interesting, and highly non-trivial issues, that we would like to bring to the attention of the reader at this stage, merely to argue that, if our conjecture on the quantum-gravity origin of the neutrino mass difference is valid, then the flavour mixing issue is far from being resolved, and certainly it cannot be treated in the way addressed in [22]. We hope to be able to address such questions in a more detailed manner in a future publication.

As a final remark we would like to draw a connection between our decoherence scenario and the matter-antimatter asymmetry of the Universe. As is well known, sphaleron transitions occurring at and after the electroweak (EW) phase transition induce violations of $B + L$[24], which efficiently wipe out any pre-existing $B + L$ asymmetry. Leptogenesis models evade this problem by generating an early asymmetry in L, which is then converted to a baryon asymmetry by the $B - L$ conserving sphaleron processes [25]. To avoid sphaleron dilution of $B + L$, and to satisfy the Sakharov conditions [26] for baryogenesis, standard leptogenesis models require strongly out-of-equilibrium processes and new sources of CP violation beyond the Standard Model. Our model of decoherence on the contrary provides a novel and extremely economical mechanism to generate the observed baryon asymmetry, through a process of equilibrium electroweak leptogenesis (the fact that it violates CPT obviates the need for two of the three Sakharov conditions, namely the requirements of out-of-equilibrium and CP violating processes). Put it more formally, by breaking CPT and thus the axioms of quantum field theory, we have violations of the index theorem that relates the Chern-Simons winding number of the sphaleron configuration to a change in $B + L$. It is difficult to do a precise calculation of this effect, but it is easy to derive an order of magnitude estimate. In [27] the asymmetries between semileptonic decays of K_0 and those of $\overline{K}_0$ turned out to depend linearly on dimensionless decoherence parameters such as $\widehat{\gamma} = \gamma/\Delta\Gamma$; in the parametrization of Ellis et al. in [3], where $\Delta\Gamma = \Gamma_L - \Gamma_S$ was a characteristic energy scale associated with energy eigenstates of the kaon system. In fact, the dependence was such that the decoherence corrections to the asymmetry were of order $\widehat{\gamma}$ in complete positivity scenaria, where only one decoherence parameter, $\gamma > 0$ was non zero. In similar spirit, in our case of lepton-antilepton number asymmetries, one expects the corresponding asymmetry to depend, to leading order, linearly on the quantity $\widehat{\gamma} = \gamma/\sqrt{\Delta m^2}$, since the quantity $\sqrt{\Delta m^2}$ is the characteristic energy scale in the neutrino case, playing a role analogous to $\Delta\Gamma$ in the kaon case. The only difference from the kaon case, is that here, in contrast to the kaon asymmetry results, there are no zeroth order terms, and thus the result of the matter-antimatter asymmetry is proportional to the dimensionless decoherence parameter $\widehat{\gamma}$, which we are going to take as the larger of the two dechorence parameters of our model in [1], *i.e.* $\widehat{\gamma} \to \widehat{\gamma_1} = 10^{-18} \cdot E/\sqrt{\Delta m^2}$.

Cutting this long story short, the matter-antimatter asymmetry in the Universe is estimated to be $\mathcal{A} = \frac{\langle\nu\rangle - \langle\overline{\nu}\rangle}{\langle\nu\rangle + \langle\overline{\nu}\rangle} \simeq \widehat{\gamma_1} \simeq 10^{-6}$ [28]. Thus, $B = \frac{n_\nu - \bar{n_\nu}}{s} \sim \frac{\mathcal{A} n_\nu}{g_* n_\gamma}$ with n_ν ($\bar{n_\nu}$) the number density of (anti) neutrinos, n_γ the number density of photons and g_* the effective number of degrees of freedom (at the temperature where the asymmetry is developed) which depends on the exact matter content of the model but it ranges from 10^2 to 10^3 in our case. This implies a residual baryon asymmetry of order 10^{-10}, roughly the desired magnitude.

In this work we have used a minimal model of CPT violating decoherence, able to explain all observations in neutrino experiments, in an attempt to account for the vacuum energy of the Universe as well as for its matter-antimatter asymmetry. This extremely simplified model which incorporates just two (decoherence) parameters to the standard three generation scenario is able through some educated guesses to get numbers in the right ballpark for these two apparently unrelated quantities. Obviously we are in need of a detailed theoretical model of foam before definite conclusions are reached in these important issues, but we think that our discussion in this work places neutrino physics in a quite novel perspective that is worth of further study.

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[12] If the foam medium is assumed completely random (Gaussian), that is $\langle n^c_{\rm bh}(r) n^c_{\rm bh}(r')\rangle \sim \Omega^2 (\langle n^c_{\rm bh}\rangle)^2 \delta(r-r')$, then the effective (two-flavor) neutrino Hamiltonian will assume the generic form $H_{\rm eff} = H + n^c_{\rm bh}(r) H_I$, where $H_I = G_N J_{2x2}$, is an appropriate constant 2×2 matrix, whose entries depend on the details of the foam/neutrino interactions. In our case it will mimic the MSW interaction Hamiltonian, since we assume that the foam interactions discriminate between neutrino flavours. The mean value (macroscopic) part, n_0, of $n^c_{\rm bh}(r)$, assumed time independent, will contribute to the Hamiltonian part of the evolution of the neutrino density matrix, ρ, while the fluctuating part will yield a double commutator structure[11]: $\partial_t \langle\rho\rangle = i[H + n_0 H_I, \langle\rho\rangle] - \Omega^2 n_0^2 [H_I, [H_I, \langle\rho\rangle]]$, where $\langle\ldots\rangle$ indicates average with respect to the stochastic effects, and H is the massless neutrino Hamiltonian (if the neutrino mass is assumed to have conventional contributions, then H is the massive neutrino Hamiltonian). The Hamiltonian part yields space-time foam-induced mass-squared splittings (1) for neutrinos. The double commutator part, on the other hand, which is due to the non-trivial quantum fluctuations of the foam density, is *time irreversible*, unrelated in principle to CP properties, and thus CPT violating, and can be rewritten in such a way so as to contribute width-like imaginary (non hermitean) parts to the effective Hamiltonian, associated with the induced time irreversibility: $\partial_t\langle\rho\rangle = iH^-\langle\rho\rangle - i\langle\rho\rangle H^+ + 2\Omega^2 n_0^2 H_I \langle\rho\rangle H_I$ with $H^\pm = H_{\rm eff} \pm i\Omega^2 n_0^2 H_I^2$. Such double commutator evolution is similar to some energy-driven decoherence models[14], and admits pure states, that is the conditions $\rho^2(t) = \rho(t)$, ${\rm Tr}\rho(t) = 1$, are maintained during the decoherent evolution. Nevertheless, as mentioned above, CPT is violated due to the form of the decoherence terms. As a result, the decoherence parameters in the antiparticle sector may not be the same as those in the particle sector. For instance, one may reproduce the results of [1], with equality of masses and widths between neutrinos and antineutrinos, but with dominant decoherence parameters

$\overline{\gamma}_j \sim 10^{-18} \cdot E$ in the antineutrino sector only, by selecting appropriately the corresponding density fluctuation parameters $\overline{\Omega} \neq \Omega$, while maintaining $\langle n^c_{\rm bh}\rangle \equiv n_0$ the same in both sectors (c.f. (1)). This is physically meaningful, since it implies that for the same momenta for neutrinos and antineutrinos, and hence the same average number of foam particles they interact with, their back reaction (interaction) with the foam, which causes the foam-particle density fluctuations, is different, as a result of CPT violation. In fact in such a case, the order of the decoherence parameters in the antiparticle sector is $\overline{\Omega}^2 G_N^2 n_0^2$ (in Planck units), and to reproduce the dominant decoherence $10^{-18}E$ one would need an $\overline{\Omega}^2 \propto 10^{-18}k^3/((\Delta m^2)_{\rm foam})^2 \sim 10^{28}(k/GeV)^3(GeV)^{-1}$. The increase of the fluctuations with the (antineutrino) momentum scale is reasonable, since the higher the momentum, the stronger the back reaction onto the space time foam (although the lesser the number of foamy particles the antineutrino interacts with). Notice, however, that, as we discuss in the text, once a Δm^2 is generated, there will be automatically contributions to the vacuum energy Λ, proportional to $(\Delta m^2)^2$, which in turn will spoil the purity of states during the evolution, and hence they will enforce a stronger form of CPT violation, à la [17]. In such a case the additional decoherent terms in the antiparticle sector are of order [16] $\Lambda(\overline{\Delta} g_{\mu\nu})^2$, where $(\overline{\Delta} g_{\mu\nu})^2$ denote space-time metric fluctuations. As we have seen above, the density fluctuations in the antiparticle sector will also be proportional to $\overline{\Omega}^2 \propto 1/\Lambda$, and since the density fluctuations (as a result of back reaction effects) may be related to metric fluctuations in the neighborhood of the foamy defects, the latter may also be expected to be proportional to $1/\Lambda$ in the antineutrino sector, leading to unsuppressed (with respect to Λ) contributions to decoherence. In our case, however, it may be checked that the Λ-induced γ_1 decoherence coefficient will be much more suppressed, of order $10^{-18}k^3/M_p^2$. For three generation models one may have more decoherence parameters, and one may encounter a situation like that in I for the various Ω^2_{ij}, as a result of different back reaction effects. However, even in that case, the parametrisation in I would yield a suppressed Λ-induced γ_3 decoherence parameter of order $\Delta m^2 k/M_p^2$.

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