IMPLICATIONS OF THE RESTORATION OF
CHIRAL SYMMETRY IN BAG MODEL BUILDING

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ABSTRACT

We analyze via a bag model description of hadron structure the consequences of having two fundamental scales in QCD, one associated with confinement and the other with chiral symmetry restoration. The main effect is that the exterior pion field becomes weaker and therefore we are able to reproduce the axial vector coupling constant.

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1. <u>INTRODUCTION</u>

The Chiral Bag Model[1] is a quite successful phenomenological description of hadrons, whose guiding principle is Quantum Chromodynamics (QCD)[2]. This theory, supposedly a good description of the strong interactions, behaves rather differently for low energies, than for high energies. The simpler behaviour (known as asymptotic freedom) occurs at high energies when the coupling constant becomes very weak and therefore one may proceed to use perturbative methods. On the other hand for low energies, the coupling becomes strong, leading hopefully to the unobservability of the hadronic constituents (confinement) and the spontaneous breaking of chiral symmetry. All these properties constitute the foundations of the Bag Model. There is though a further assumption implicit in these models, namely, that the energy scale for confinement and spontaneous chiral symmetry breaking is the same. In this way there is only one scale parameter for both phenomena, ultimately the bag radius. The perturbative (asymptotically free) phase contains only quarks and gluons, while in the non-perturbative phase, one has Goldstone bosons, consequence of the spontaneous chiral symmetry breaking.

Studies by two different methods, semi-classical techniques[3], and numerical lattice gauge theory calculations[4] have brought to our attention a different scenario, based on two scales, one for the confinement-deconfinement transition and another for the chiral symmetry breaking-restoration transition. For example in the Monte Carlo calculation of QCD on a lattice[4], it is found that for SU(2) there exist clearly two phase transition temperatures T_{ch} and T_c, such that $T_{ch} > T_c$, while for SU(3) these temperatures seem to be almost identical. Here T_{ch} is the temperature above which the chiral symmetry is restored and T_c the one for deconfinement.

If one reinterprets these results in terms of the bag model, the alternative of having two scales, leads by simple scaling arguments to a scenario with two length scales, the confinement one R_c and the size of the Wigner phase R_{ch}. By dimensional reasons

$$\frac{R_c}{R_{ch}} \sim \frac{T_{ch}}{T_c} , \qquad (1)$$

which turns out to be slightly greater than one. The following scenario arises naturally: a simplified view of a hadron consists of two concentric spheres determined by R_{ch} and R_c. Chiral symmetry is realised in the Wigner mode inside ($r < R_{ch}$) and in the Goldstone mode outside the smaller sphere ($r > R_{ch}$). For

$r < R_{ch}$ there are only quarks and gluons, while only pions exist for $r > R_c$. All the constituents coexist in the region $R_{ch} < r < R_c$.

Two extreme limits of our model are known in the literature, the already mentioned Chiral Bag Model ($R_{ch} = R_c$) and the so-called Cloudy Bag Model[5] ($R_{ch} = 0$). One feature that attracts immediate attention, to the possibility of having the three region model is that one is able to calculate g_A correctly in a perturbative manner, since our result extrapolates between the Cloudy Bag Model value of 1.09 to the Chiral Bag Model value of 1.63.

One comment is needed at this point which should clarify our attitude in the rest of the paper. We take in this work a perturbative approach, meaning that we neglect effects arising from the solitonic nature of the pionic field[6]. The quantitative statements made throughout our work should be understood as up to normalizations due to vacuum polarizations, which we expect to be small in the perturbative regime. As has been shown[7], the expectation values of observables are flat as a function of the confinement radius and therefore perturbative results become of interest.

In the next section we proceed to formulate our model, writing down the equations of motion for the constituent fields. In section 3 we obtain perturbative solutions to the equations of motion. In section 4 we calculate the axial vector coupling constant g_A and the energy of the system then analyze the possibility of eliminating collapse by the introduction of the skin. Finally in the last section we discuss the results obtained in this investigation.

2. THE MODEL

A hadron consists of a hypertube in space-time, such that for fixed time space is divided in three regions. The core region is called Wigner phase and is characterized by the absence of collective modes (pions) and a perturbative coupling between quarks and gluons. The exterior region is characterized by the exclusive presence of collective modes. Finally the region between the two (skin), in which perturbative and collective modes coexist and interact with each other.

For simplicity in the formulation we do not consider gluonic effects and we take the surfaces to be static concentric spheres of radii R_{ch} and R_c, with $R_{ch} < R_c$. The above description is realized mathematically by the following Lagrangian density[8],

$$\mathcal{L} = \mathcal{L}_1 \, \Theta(-r + R_c) + \mathcal{L}_2 \, \Theta(R_c - r) \, \Theta(r - R_{ch})$$
$$+ \mathcal{L}_3 \, \Theta(r - R_c) + \mathcal{L}_S \, \delta(r - R_c) \qquad (2)$$

where

$$\mathcal{L}_1 = - \overline{\Psi} \, \gamma_\mu \, \frac{\overleftrightarrow{\partial_\mu}}{2} \, \Psi - B, \qquad (2a)$$

$$\mathcal{L}_2 = -m \left(1 + \frac{\phi^2}{f_\pi^2}\right)^{-1/2} \overline{\Psi} \left(1 + \frac{i \vec{\tau} \cdot \vec{\phi}}{f_\pi} \gamma_5\right) \Psi -$$
$$- \frac{1}{2} \left[\left(1 + \frac{\phi^2}{f_\pi^2}\right) (\mathcal{D}_\mu \vec{\phi})^2 - \frac{1}{4 f_\pi^2} (\mathcal{D}_\mu \phi^2)^2 \right], \qquad (2b)$$

$$\mathcal{L}_3 = - \frac{1}{2} \left[\left(1 + \frac{\phi^2}{f_\pi^2}\right) (\mathcal{D}_\mu \vec{\phi})^2 - \frac{1}{4 f_\pi^2} (\mathcal{D}_\mu \phi^2)^2 \right], \qquad (2c)$$

and finally the surface term that establishes the continuity of the axial current across the outermost surface and which is given by

$$\mathcal{L}_S = - \frac{1}{2} \left(1 + \frac{\phi^2}{f_\pi^2}\right)^{-1/2} \overline{\Psi} \left(1 + \frac{\vec{\phi} \cdot \vec{\tau}}{f_\pi} \gamma_5\right) \Psi. \qquad (2d)$$

In the above equation, ψ represents the quark field, $\vec{\phi}$ the pion field, $\mathcal{D}_\mu$ is the non-linear derivative

$$\mathcal{D}_\mu = \left(1 + \frac{\phi^2}{f_\pi^2}\right)^{-1} \partial_\mu$$

and m is an effective coupling constant with the dimensions of a mass.

The origin of the quark-pion coupling is not related to any fundamental property of the theory. Since the skin region is non-perturbative and therefore not asymptotically free, we feel one should analyze the effect of such a coupling.

We now proceed in the static approximation for the pion field and perform a perturbative expansion in powers of $1/f_\pi$, the pion-quark coupling constant, keeping only lowest chiral invariant order. This procedure leads to the following equations of motion

$$\gamma_\mu \partial_\mu \psi + m \psi \Theta (r - R_{ch}) = 0 \quad , \quad r < R_c, \qquad (3a)$$

$$\nabla^2 \phi^i = \frac{im}{f_\pi} \overline{\psi} \gamma_5 \tau^i \psi \qquad , \quad r > R_{ch} \qquad (4a)$$

and the boundary conditions at $r = R_{ch}$

$$\psi \qquad\qquad \text{continuous}, \qquad\qquad (3b)$$

$$\hat{r} \cdot \vec{\nabla} \phi^i = 0 \qquad , \qquad\qquad (4b)$$

and at $r = R_c$,

$$\hat{r} \cdot \vec{\gamma} \, \psi = \psi \quad , \qquad\qquad (3c)$$

$$\lim_{\epsilon \to 0} \left\{ \hat{r} \cdot \vec{\nabla} \phi^i \Big|_{R_c + \epsilon} - \hat{r} \cdot \vec{\nabla} \phi^i \Big|_{R_c - \epsilon} \right. = \frac{i}{2f_\pi} \overline{\psi} \tau^i \gamma_5 \psi. \qquad (4c)$$

It is worth mentioning, that the regularity condition of the Cloudy Bag Model is substituted by Eq. (4b), which is of Neuman character. Also, the spontaneous breaking of the flavour symmetry has generated an effective mass term for the quarks in the intermediate region. Finally, it is obvious from Eq. (4c) that the exterior pion field is weaker than that of the Chiral Bag Model and thus g_A will be smaller than in that case. This sharing of strength between the interior and exterior pions is the mechanism which gives rise to the small value of g_A in the Cloudy Bag Model.

3. SOLUTIONS OF THE EQUATIONS OF MOTION

The perturbative scheme defines a unique way to look for solutions, namely, one solves for the quark modes and those determine uniquely the pion modes. The first step is therefore to look for solutions of Eqs. (3), in particular the lowest quark mode is given by

$$\psi = \begin{cases} q_1 & r \le R_1 \quad , \\ q_2 & R_1 < r < R_2 \end{cases}, \qquad (5)$$

where

$$q_1 = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} i j_0(wr) \\ -j_1(wr)\, \vec{\sigma}\cdot\hat{r} \end{pmatrix} e^{-iwt} , \tag{5a}$$

and

$$q_2 = \frac{N}{\sqrt{4\pi}} \left[\beta \begin{pmatrix} i j_0(pr) \\ -K j_1(pr)\, \vec{\sigma}\cdot\hat{r} \end{pmatrix} + \gamma \begin{pmatrix} i n_0(pr) \\ -K n_1(pr)\, \vec{\sigma}\cdot\hat{r} \end{pmatrix} \right] e^{-iwt} \tag{5b}$$

we will simplify the notation and call Rch = R_1 and Rc = R_2. In Eqs. (5a) and (5b) N represents the normalization constant which is given in Appendix A, $p^2 = \sqrt{w^2 - m^2}$ and $K = \left[(w-m)/(w+m) \right]^{\frac{1}{2}} = p/(w+m)$. From the boundary condition Eq. (3b) we obtain

$$\beta = -(pR_1)^2 \left\{ j_0(wR_1) n_1(pR_1) - \frac{1}{K} j_1(wR_1) n_0(pR_1) \right\} \tag{6a}$$

$$\gamma = (pR_1)^2 \left\{ j_0(wR_1) j_1(pR_1) - \frac{1}{K} j_1(wR_1) j_0(pR_1) \right\} . \tag{6b}$$

The mode energy w is obtained from the confinement condition Eq. (3c)

$$j_0(pR_2) - K j_1(pR_2) = -\frac{\gamma}{\beta} \left(n_0(pR_2) - K n_1(pR_2) \right) . \tag{7}$$

Once the quark wave function is known, Eqs. (4) determine uniquely the pionic modes. The solution is found to be,

$$\phi^i(\vec{r}) = \phi_0^i(\vec{r}) + \phi_1^i(\vec{r}) \tag{8}$$

with

$$\phi_0^i(\vec{r}) = A_{01} \left(r + \frac{R_1^3}{2} \frac{1}{r^2} \right) \vec{\sigma}\cdot\hat{r}\, \tau^i , \tag{8a}$$

and

$$\phi_1^i(\vec{r}) = A_{11} \left\{ \int_{R_1}^{R_2} r'^2 J(r') \left\{ \frac{r_<}{r_>^2} + \frac{1}{2} \frac{R_1^3}{(r r')^2} \right\} dr' \right\} \vec{\sigma}\cdot\hat{r}\, \tau^i \tag{8b}$$

if $R_1 < r < R_2$. For $r > R_2$

$$\phi_o^i(\vec{r}) = A_{o1} \left(R_2^3 + \frac{R_1^3}{2} \right) \frac{\vec{\sigma} \cdot \hat{r}}{r^2} \tau^i , \tag{8c}$$

and

$$\phi_1^i(\vec{r}) = A_{11} \left\{ \int_{R_1}^{R_2} r'^2 J(r') \left(r' + \frac{R_1^3}{2r'^2} \right) dr' \right\} \frac{\vec{\sigma} \cdot \hat{r}}{r^2} \tau^i . \tag{8d}$$

The coefficients and the source terms are given by

$$A_{o1} = -\frac{K N^2}{12 \pi f_\pi} J(R_2) \tag{9a}$$

$$A_{11} = -\frac{m \, K N^2}{6 \pi f_\pi} \tag{9b}$$

and

$$J(r) = \beta^2 j_0(pr) j_1(pr) + \beta\gamma \left(j_0(pr) n_1(pr) + j_1(pr) n_0(pr) \right) + \\ + \gamma^2 n_0(pr) n_1(pr) \tag{10}$$

Herewith the solutions corresponding to ordinary baryons have been found.

In order to reproduce the Chiral Bag Model results and to avoid singularities in the limit $R_1 \to R_2$ we let m be a function of the skin parameter $1 - R_1/R_2$, such that it vanishes in that limit. We have no rule from general principles as to the form of this mass dependence, and we have adopted for our investigation a smooth function of the skin parameter, namely

$$m = m_o \left(1 - \frac{R_1}{R_2} \right)^\alpha \tag{11}$$

This formula leads to the correct limit in the Chiral Bag Model picture, while leaves some massive quarks in the Cloudy Bag Model limit.

4. RESULTS

We proceed in this section to calculate observables which we consider crucial in the understanding of the physics involved. Our scheme for obtaining them is the so-called cavity field theory, where quantization is performed by elevating to operators the expansion coefficients of the fields in terms of modes, and imposing upon them the appropriate anticommutation relations. Since

we shall not allow for free pions in this paper, there will be no pionic modes, just pionic c-number propagators.

Our model is invariant under chiral symmetry at the current level, although the symmetry is spontaneously broken due to the degeneracy of the non-perturbative vacuum. The vanishing of the axial current divergence leads to the so-called Goldberger-Treiman relation, which we use to calculate the axial vector coupling constant. In the model the axial current is given by

$$A_\mu^i = i \overline{\Psi} \gamma_5 \gamma_\mu \frac{\tau^i}{2} \Psi \, \theta(R_2 - r) + f_\pi \, \partial_\mu \phi^i \, \theta(r - R_1) \, . \tag{12}$$

In terms of this current the axial coupling constant for the nucleon is defined by

$$\lim_{k \to 0} \langle N | \vec{A}^i(k) | N \rangle \equiv g_A \, u_N^+ (\vec{\sigma} - \vec{\sigma} \cdot \hat{k} \, \hat{k}) \frac{\tau^i}{2} u_N , \tag{13}$$

where the right hand side is just a consequence of the conservation of axial current. The second term in the right-hand side of Eq. (13) appears due to the existence of the pion pole ($k^2 = 0$) and therefore g_A can be obtained just by computing the contribution to the term which is proportional to $\vec{\sigma} \cdot \hat{k} \, \hat{k}$. Note that only the pionic part of Eq. (12) contributes to this term, but that both quarks and pions contribute to g_A. This remark is just another way of stating the Goldberger-Treiman relation, namely that g_A is proportional to the strong coupling constant, which is given by the asymptotic pion field, i.e, by the residue of the pole at $k^2 = 0$. We thus obtain

$$g_A = \frac{10}{9} K N^2 \left[J(R_2) \left(R_2^3 + \frac{R_1^3}{2} \right) + 2m \int_{R_1}^{R_2} r^2 J(r) \left(r + \frac{R_1^3}{2r^3} \right) dr \right] \tag{14}$$

In Appendix B we show the exact expressions for the massless limit. It is trivial to prove from these expressions the two known limits, namely,

$$g_A \rightarrow \frac{5}{6} \frac{w R_2}{w R_2 - 1} \approx \frac{5}{3} \, , \quad \text{as} \quad R_1/R_2 \rightarrow 1 \text{ and } m \rightarrow 0. \tag{15a}$$

and

$$g_A \rightarrow \frac{5}{9} \frac{w R_2}{w R_2 - 1} \approx \frac{10}{9} \, , \quad \text{as} \quad R_1/R_2 \rightarrow 0 \text{ and } m \rightarrow 0. \tag{15b}$$

In Fig. 1 we show results of the numerical calculation for the massive case as function of the mass parameter and of the skin parameter. It is apparent that for all cases plotted we are able to fit the chiral symmetric limit $g_A \approx 1.36$.

Another observable which is of relevance for our study is the mass of the nucleon. This is obtained by integrating the Hamiltonian density defined by

$$\mathcal{H} = T^{oo}_1 \, \Theta\,(R_2-r) + T^{oo}_2 \, \Theta\,(R_2-r)\, \Theta(r-R_1) + T^{oo}_3 \, \Theta(r-R_2) \tag{16}$$

where the T^{00}'s are the appropriate components of the energy momemtum tensor defined in the usual way

$$T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \varphi_\mu}\, \partial \varphi_\nu - \mathcal{L}\, \delta_{\mu\nu}\,, \tag{17}$$

where φ represents any field component. Note that the surface couplings do not intervene in the definition of the energy, they just appear in the boundary conditions. Keeping to lowest order in the expansion in powers of $1/f_\pi$, we obtain for a system with three quarks occupying the lowest cavity modes

$$E = 3w + \frac{4\pi}{3} B R_2^3 - \frac{F_B}{f_\pi^2} \left\{ J(R_2)^2 \left(R_2 + \frac{R_1^3}{2}\right) + \right.$$
$$+ 2m \left(J(R_2) + K^2\right) \cdot \int_{R_1}^{R_2} r'^2 \, J(r')\left(r' + \frac{R_1^3}{2r'^2}\right) dr' +$$
$$+ 4m^2 K^2 \int_{R_1}^{R_2} r^2 J(r) \int_{R_1}^{R_2} r'^2 \, J(r') \left[\frac{r_<}{r_>^3} + \frac{1}{2}\, \frac{R_1^3}{r^2 r'^2}\right] dr dr' \right\} \tag{17}$$

where

$$F_B = \frac{1}{72\pi} \, N^4 K^2 \left\langle B\Big| \sum_{i \neq j} \vec{\sigma}(i)\cdot\vec{\sigma}(j)\, \vec{\tau}(i)\cdot\vec{\tau}(j)\Big| B\right\rangle . \tag{17a}$$

The energy coming from the quark-pion coupling is attractive. Contributions coming from the quark-gluon coupling are missing but since they are perturbative, they can just be added to the above equation. We do not worry in here about corrections for centre-of-mass spurious motion. Again we refer to Appendix B for analytical expressions in the massless case.

The calculation of other observables follows the same pattern developed above. It is certainly not the purpose of this paper to reproduce the static properties of baryons, in which certainly the model will do at least as good as the two previous models, but to explore in a qualitative manner the picture that arises once a skin is added to the bag. The results of the numerical calculation are shown in Figs. 2 and 3.

5. DISCUSSION AND CONCLUSIONS

We have defined a bag model that includes in a phenomenlogical way two scales, one associated with chiral symmetry restoration and one with deconfinement. Due to the coupling of quarks to pions, the former acquire an effective mass in the intermediate region. In order to obtain a non-singular limit for vanishing skin size, this mass parameter has been restricted to be a soft function of skin depth. A nice feature of the model is that it inter-polates between two well-known and successful hybrid models and thus we are certain of reproducing a great deal of phenomenology. This though is not our motivation in here and therefore we have avoided the technicalities involved with data fitting and have just paid attention to two crucial observables, namely g_A and the nucleon energy. In Fig. 1 we show the former and the model is able to reproduce the chiral invariant "experimental" value of 1.36. As the mass parameter increases the skin depth has to increase if g_A is to be reproduced. Moreover, in our naive approach, at least a small skin is needed if we want to fit the data.

Coming back to the energy, we show in Fig.2, the absolute value of the pionic energy as a function of skin depth and mass. There is no unique behaviour. For low masses the energy decreases as a function of skin depth, i.e, the pionic energy becomes less attractive as we increase the size of the system. For larger masses, on the contrary, the energy might become even highly attractive compared to the zero mass case.

In Fig. 3 we show the energy functional as a function of mass for fixed skin. It collapses at short distances. If one varies the skin for fixed mass the changes are small and the collapse feature remains. It is important to note that between 0.6 and 1.5 fm., i.e, the stability dip, the energy functional is very shallow, changing at most by 200 MeV. After this analysis, the only mechanisms we know that avoid collapse are Skyrme's fourth order term[6] and the coupling of an w-vector meson[9].

Our investigations have been motivated by recent analysis of QCD by semi-classical methods and fundamentally by numerical lattice calculations. Certainly the very late results for SU(3) with dynamical light fermion point to an almost simultaneous phase transition ($T_c \simeq T_{ch}$). If we accept this quantitative prediction of lattice QCD we are led to a small skin size. In our scheme this would imply a small mass parameter (quark-pion coupling) as a result of the g_A calculation. Therefore the Chiral Bag Model can be considered as the

only valid smooth limit of our model, which is consistent with lattice QCD results as interpreted here.

In our opinion there are two fundamental aspects in building hadron models. Firstly, one has to introduce in those models all the information available from the fundamental theory. Today that information comes mostly from lattice calculations and non-perturbative methods. Secondly, these models have to be able to reproduce a fair amount of phenomenology. The second condition by itself seems to us of no relevance. Models must serve to test and to obtain predictions which arise from the way we understand the theory through the model. Therefore the main difficulty results in the interpretation of the more fundamental results in terms of simple models. We have interpreted in this paper the possibility of several phase transition temperatures, as the existence of various well-defined regions in the structure of a hadron. If one looks at very recent QCD calculations, one might be tempted to conclude that $T_c = T_{ch}$, but still there might be an intermediate region, where quarks acquire mass in a slow fashion until they decouple and only Goldstone bosons remain. We have to wait and see. In the meantime our analysis of the first alternative leads us to affirm that the existence of a skin no matter how thin changes the physics of hadron models considerably and for example, the deformation required to fit g_A will not be as dramatic as could be expected without it[10].

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APPENDIX A

NORMALIZATION OF THE SINGLE PARTICLE WAVE FUNCTIONS

Performing the appropriate integrals the normalization integral is given by

$$
N^{-2} = \frac{1}{2} R_1^3 F(R_1) + \beta^2 \left(\frac{1}{2} R_2^3 G(R_2) - \frac{1}{2} R_1^3 G(R_1) \right) +
$$
$$
+ \gamma^2 \left(\frac{1}{2} R_2^3 H(R_2) - \frac{1}{2} R_1^3 H(R_1) \right) +
$$
$$
+ 2\beta\gamma \left(\frac{1}{2} \frac{R_2^2}{P} I(R_2) - \frac{1}{2} \frac{R_1^2}{P} I(R_1) \right)
$$

where

$$
F(R) = j_0^2(wR) + n_0(wR) j_1(wR) + j_1^2(wR) - j_0(wR) j_2(wR),
$$

$$
G(R) = j_0^2(pR) + n_0(pR) j_1(pR) + K^2 \left(j_1^2(pR) - j_0(pR) j_2(pR) \right),
$$

$$
H(R) = n_0^2(pR) - j_0(pR) n_1(pR) + K^2 \left(n_1^2(pR) - n_0(pR) n_2(pR) \right),
$$

$$
I(R) = n_0^2(pR) - j_0^2(pR) + K^2 pR \left[j_1(pR) n_1(pR) - \right.
$$
$$
\left. - \frac{1}{2} j_2(pR) n_0(pR) - \frac{1}{2} j_0(pR) n_2(pR) \right]
$$

and j_1 and n_1, $1 = 0,1,2$ are the spherical Bessel and Neumann functions and β, γ, k and p are defined in the main text in section 3.

APPENDIX B

THE MASSLESS LIMIT

Taking the mass to be zero is equivalent to making in the equations p=w, β=1, γ=0 and k=1. We obtain in this limit

$$N^{-2} = 2 R_2^3 j_0^2 (wR_2) \left[1 - \frac{1}{wR_2} \right]$$

which leads for g_A [Eq. 14] to,

$$g_A = \frac{5}{9} \frac{wR_2}{wR_2 - 1} \left[1 + \frac{1}{2} \left(\frac{R_1}{R_2} \right)^3 \right]$$

since $wR_2 = 2.0428...$ then

$$g_A \approx \frac{10}{9} \left[1 + \frac{1}{2} \left(\frac{R_1}{R_2} \right)^3 \right]$$

For the mass of the system our starting point is Eq. (18) and we obtain

$$E = 3w + \frac{4\pi}{3} B R_2^3 -$$

$$- \sum_{i \neq j} \frac{\langle B | \vec{\sigma}(i) \cdot \vec{\sigma}(j) \ \vec{\tau}(i) \cdot \vec{\tau}(j) | B \rangle}{288 \pi f_\pi^2 R_2^3 \left(1 - \frac{1}{wR_2} \right)^2} \left[1 + \frac{1}{2} \frac{R_1^3}{R_2^3} \right]$$
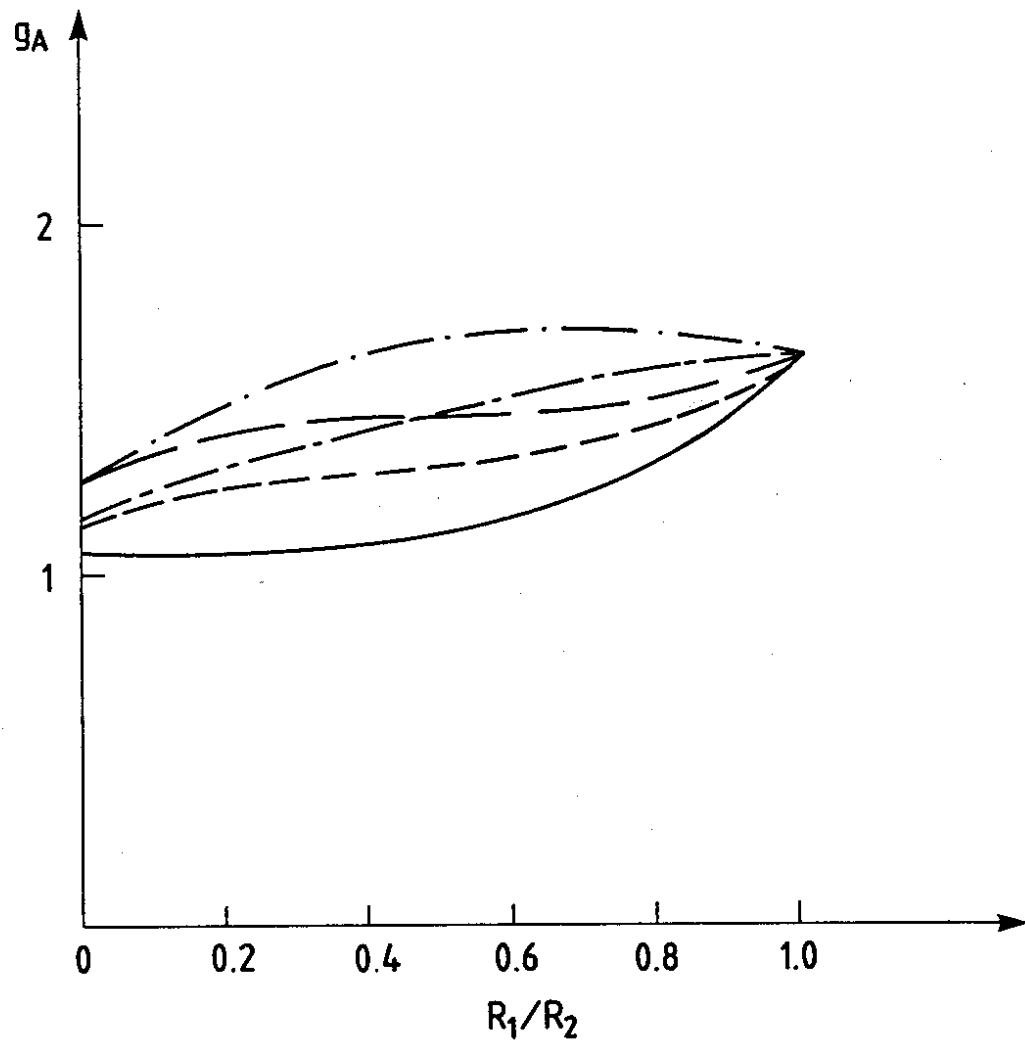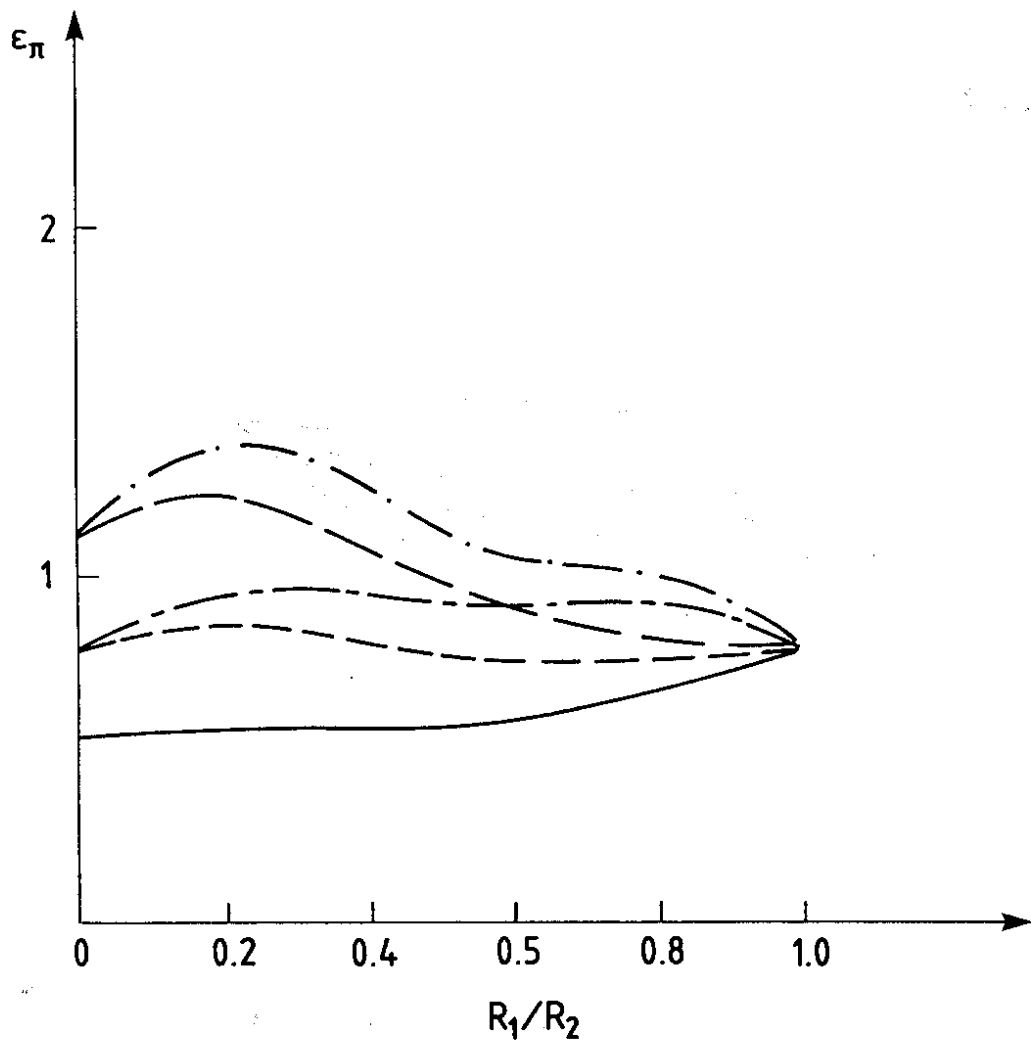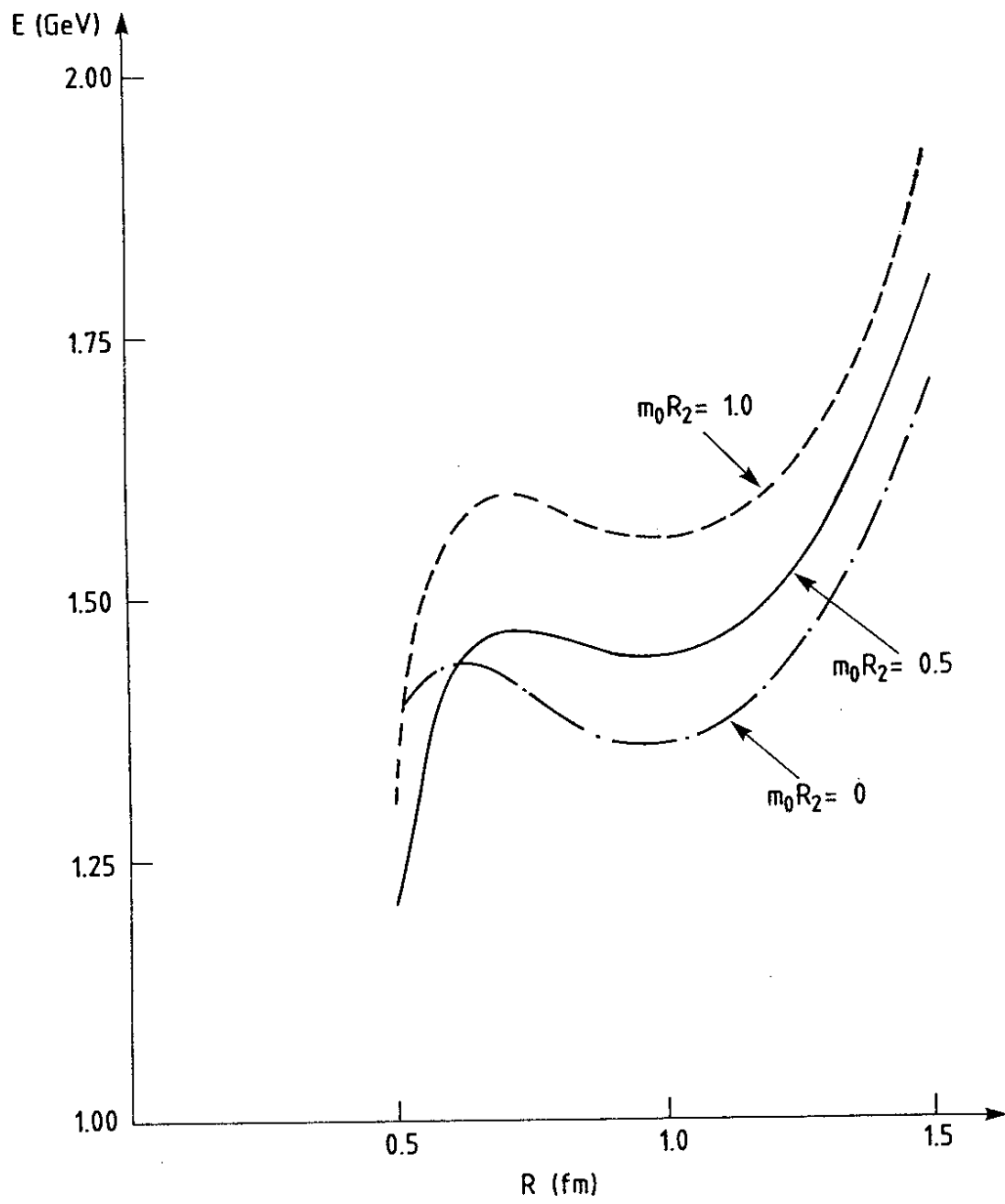
FIGURE CAPTIONS

Fig. 1:

Nuclear axial vector coupling constant g_A as a function of skin depth for different mass parameters. The following sets of parameters have been used:

$m_0 R_2 = 0$ ——— ; $m_0 R_2 = .5$, $\alpha = 1.0$ — — — —;

$m_0 R_2 = .5$, $\alpha = .1$ —— · —— ·—— ; $m_0 R_2 = 1.0$, $\alpha = 1.0$ —— ——

; $m_0 R_2 = 1.0$; $\alpha = .1$ ——·——·—— ·

Fig. 2:

Absolute value of the pionic energy related parameter

$$\epsilon_\pi = \frac{40\pi \; |E_\pi| \; f_\pi^2 \, R_2^3}{\left\langle \sum_{i \neq j} \vec{\sigma}(i) \cdot \vec{\sigma}(j) \; \vec{\tau}(i) \cdot \vec{\tau}(j) \right\rangle}$$

as a function of skin depth using the same set of parameters as in the previous figure.

Fig. 3:

Energy functional versus the bag radius keeping the skin parameter fixed to $R_1/R_2 = 0.8$ and using $B^{1/4} = 150$ MeV for different values of the mass parameter. We use $\alpha = 0.1$ and the masses as shown in the figure.

Fig. 1

Fig. 2

Fig. 3