

Dynamical confinement in bosonized QCD_2 *

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Abstract

In the bosonized version of two dimensional theories non trivial boundary conditions (topology) play a crucial role. They are inevitable if one wants to describe non singlet states. In abelian bosonization, color is the charge of a topological current in terms of a non-linear meson field. We show that confinement appears as the dynamical collapse of the topology associated with its non trivial boundary conditions.

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1 Introduction

Quantum Chromodynamics in two dimensions is a confining theory. The realization is quite simple, as can be seen from the propagator, a solution to the non-dynamical gluon equation,

$$\partial_-^{-2}(x, y) = \frac{1}{2}|x_+ - y_+|\delta(x_- - y_-) \quad (1)$$

and which to first order in $\frac{1}{N}$ is proportional to the $q\bar{q}$ interaction. Thus confinement here is a peculiarity of the dimensionality of space-time [1]. The resolution of 't Hooft's equation confirms this mechanism leading to a discrete, stable and infinite spectrum [2]. However the formalism is much more powerful eliminating those amplitudes which would violate confinement explicitly [3, 4]. Let us study for example the process $meson \rightarrow q\bar{q}$. It can be shown that the amplitude for it is given by

$$F_n^{a\bar{b}}(t, r_-) = \frac{e^2}{r_- \sqrt{\pi N}} P \int_0^1 dt' \frac{\varphi_n^{a\bar{b}}(t')}{(t-t')^2} \quad (2)$$

if $t \in [0, 1]$. The notation follows that of ref.([2]). In order to obtain the physical amplitude one has to impose the on mass shell restrictions, i.e.,

$$p^2 = M_a^2, \quad (p-r)^2 = M_b^2, \quad r^2 = r_n^2 \quad (3)$$

where M_i are the renormalized quark masses and r_n the corresponding meson mass. The on mass shell condition leads to the following relation for the adimensional momentum t

$$\mu^2 = \frac{\alpha_a}{t} + \frac{\alpha_{\bar{b}}}{1-t} \quad (4)$$

and therefore 't Hooft's equation becomes

$$P \int_0^1 \frac{\varphi_n^{a\bar{b}_n}}{(t-t')^2} = 0 \quad (5)$$

which implies the vanishing of the amplitude for the process. Therefore an expected consequence of the confinement mechanism is that no quark can be liberated from a bound state.

Let us characterize confinement from the point of view of the quarks. 't Hooft [2] used a cut-off to regulate the infrared divergences. In this way the quark propagator acquired a singularity leading to an infinite renormalized mass. This behavior was interpreted as the confinement mechanism. The pole of the quark propagator was displaced from the bare quark mass to infinity leading to an impossibility of satisfying the on mass shell condition. This argument was heavily criticized once it was discovered that the choosing of the infrared regulator is just a consequence of gauge freedom [4]. One can choose a gauge where the self-energy is perfectly finite. However we next show that one can find a relation between the self-energy and the interaction potential and in this way the latter might be used to describe confinement.

Dyson's equation for the self-energy is

$$\Sigma(t) = -\frac{e^2 r_-}{2\pi} P \int_{-\infty}^{\infty} \frac{\text{sign}(t')}{(t-t')^2} \quad (6)$$

which in position space leads to

$$\Sigma(x_+) \sim \frac{V(x_+)}{x_+} \quad (7)$$

Thus the long distance behavior of the self-energy contains information about the confining features of the potential.

It is important to stress that higher order terms in $\frac{1}{N}$ do not change the confining character of the interaction. In particular, in the so-called singular gauge [3], it can be shown that the higher order corrections are proportional to the gauge parameter α

$$V(x_+) \sim |x_+| - \alpha(1 + f(\frac{1}{N})) \quad (8)$$

and therefore do not affect the result of this calculation.

The confinement mechanism just described is not a *capriccio* of 't Hooft's model but a fundamental property of QCD_2 [5, 6].

Two dimensional theories are exactly bosonizable and we have seen that in the chiral limit many crucial properties of the spectrum and the realization of chiral symmetry appear in a clear and appealing fashion [7]. The purpose of this paper is to analyze confinement in this description of the theory. The crucial symmetry associated with confinement is color. For reasons of simplicity we shall work only with one flavor and two colors, a description which allows visualization of the results but does not imply any restriction on the physics involved.

2 Realization of color symmetry in the bosonized free massless theory

The bosonized action for a massless quark using abelian bosonization is [8]

$$S = \int d^2x \left\{ \frac{1}{2} (\partial_\mu \varphi)^2 + \frac{1}{2} (\partial_\mu \eta)^2 \right\} \quad (9)$$

It can seem surprising that such a simple action can realize the full global $SU(2)$ color symmetry. A mere counting of color degrees of freedom seems to imply that some color field might be missing¹. The answer to this apparent contradiction lies in the *topology*

¹Non-abelian bosonization would lead to an action in terms of the $SU(2)$ fields $g = \exp(i\frac{\tau^a \pi^a}{2})$, $a = 1, 2, 3$, with three color degrees of freedom, where the π^a fields transform according to the adjoint representation.

of the η field. This field does not transform under any linear transformation of $SU(2)$, but it can be related through the Mandelstam equations to the s_{\pm}^{α} operators, which transforms under the fundamental representation of the color group [8]. But in order for this relation to occur the η field must possess non-trivial boundary conditions, i.e.,

$$T^3 = \frac{1}{\sqrt{2\pi}}[\eta(+\infty) - \eta(-\infty)] \quad (10)$$

Thus the action in Eq.(9) can only realize the full $SU(2)$ group if one incorporates besides the conventional solutions $\eta_0(\pm\infty) = 0$, those associated with non-trivial boundary conditions ($\eta(+\infty) = \pm\sqrt{\frac{\pi}{2}}$, $\eta(-\infty) = 0$). That they exist for the free theory has been proven by construction [8]. Moreover they are dynamically allowed since the finiteness of the energy for physical states,

$$E_{\eta} = \int dx H(\eta) = \int dx \frac{1}{2}(\partial_x \eta)^2 < \infty \quad (11)$$

only requires the asymptotic vanishing of the derivative. Thus the η field can tend to different constants at $\pm\infty$, and generate solitonic solutions. These states are stable because they are protected from desintegrating into conventional $T^3 = 0$ states by topological conservation laws.

Despite its naive appearance, the bosonized action Eq.(9) has an internal symmetry group which is bigger than the obvious $U(1)_F \otimes U(1)_C$. The reason behind this statement is that the soliton operator appearing in Eq.(10) generates besides the conserved current

$$J_{\mu}^3 = \frac{1}{2}\varepsilon_{\mu\nu}\partial^{\nu}\eta \quad (12)$$

two others, J_{μ}^1 and J_{μ}^2 , whose explicit expressions are highly non-local [8]. The main difference between them is that for the former the conservation law is a trivial mathematical identity (topological conservation law) while for the latter the dynamics of the η field is required for conservation (Noether currents). The conserved charges associated to these currents close an $SU(2)$ algebra. Moreover the following commutator can be calculated

$$[T^a, s_{\pm}^{\alpha}] = \left(\frac{\tau^a}{2}\right)_{\alpha\beta} s_{\pm}^{\beta} \quad (13)$$

which indicates that the soliton operators transform under the color group in the fundamental representation.

It is important to stress that it is the existence of non-trivial boundary conditions which generate the full $SU(2)_C$ algebra. The raising and lowering operators connect different topological sectors, i.e.,

$$T_{-}|\eta(\infty) = \sqrt{2\pi}t \rangle_{\infty} |\eta(\infty) = \sqrt{2\pi}(t-1) \rangle \quad (14)$$

Therefore if one would restricts by *fiat* the Fock space to that determined by ordinary boundary conditions (obc's), one is eliminating without justification all states with $T_3 \neq 0$.

3 Vacuum structure of the bosonized *free* massless theory

As we have just seen well-defined non-trivial boundary conditions in the bosonized theory are essential to reach the full $SU(2)_C$ symmetry of the fermionic action. These boundary conditions are not explicit in the bosonized action. However it is possible to introduced them by adding a small mass term into the fermionic action. The bosonized form of this mass term becomes

$$S_M = \int d^2x \{ \mu M (\cos \sqrt{2\pi}\varphi)_\mu (\cos \sqrt{2\pi}\eta)_\mu \} \quad (15)$$

μ is a renormalization mass [9]. The classical potential has a minimum at

$$V_M(\tilde{\varphi}, \tilde{\eta}) = -\mu M \quad (16)$$

which implies a degenerate vacuum formed by the infinite set of points [8]

$$\emptyset = \emptyset_I \cup \emptyset_{II} \quad (17)$$

where

$$\emptyset \equiv \begin{cases} (\sqrt{2\pi}n, \sqrt{2\pi}m) & \in I \\ (\sqrt{2\pi}(n + \frac{1}{2}), \sqrt{2\pi}(m + \frac{1}{2})) & \in II \end{cases} \quad (n, m) \in Z \quad (18)$$

The structure of the vacuum gives us precise information about the solutions of the bosonized theory. We have already shown how to read particle properties from field boundary conditions . When a potential is present, the possible boundary conditions are related to the minima of the potential [1, 10]

$$\lim_{x \rightarrow \pm\infty} (\varphi(x), \eta(x)) = (\varphi_{n_\pm}, \eta_{m_\pm}) \in \emptyset \quad (19)$$

Baryon number and T^3 charge are related by the bosonization formulae to the asymptotic conditions of the fields

$$\begin{aligned} B &= \frac{1}{\sqrt{2\pi}} [\varphi(+\infty) - \varphi(-\infty)] \\ T^3 &= \frac{1}{\sqrt{2\pi}} [\eta(+\infty) - \eta(-\infty)] \end{aligned} \quad (20)$$

Eqs.(19) and (20) allow us to transform the (φ, η) vacuum structure plane, Eq.(18), into a diagram for physical states.

Figure 1: Examples of allowed states: a) Diagonal arrows: up-color quark state ($B = \frac{1}{2}, T^3 = \frac{1}{2}$); b) Horizontal arrows: up-color vector state ($B = 0, T^3 = 1$); c) Vertical arrows: color singlet baryon state ($B = 1, T^3 = 0$).

Fig.(1) shows that all states described by arrows of the same length and direction are equivalent, i.e., they represent the same physical state. Thus we can define an equivalence relation and choose just one representative per class. For example, we proceed by attaching the arrows to the same point ($\varphi(-\infty) = \eta(-\infty) = 0$), since we have the freedom to select one of the boundary conditions. With this restriction all the physical states will be represented by the points of the vacuum structure lattice, which becomes in this way a (B, T^3) plane (see Fig.(2))

Global $SU(2)$ transformations leave the potential invariant. This is easily seen in the non-abelian bosonization scheme where the potential is proportional to $tr(g) = \cos \sqrt{2\pi}\eta$, an $SU(2)$ invariant, and a function of φ , itself invariant by construction [7]. Therefore the set of physical states, the lattice, is invariant under these transformations. Moreover it is also invariant under the discrete translations [8]

$$\begin{aligned}
 \varphi &\rightarrow \varphi + \sqrt{2\pi}n \\
 \eta &\rightarrow \eta + \sqrt{2\pi}m \\
 &(n, m) \in Z
 \end{aligned}
 \tag{21}$$

Before closing this section it is important to emphasize that the vacuum structure just studied coincides with that of the chiral limit of the theory. The topological charges of the physical states are mass independent. They survive in the chiral limit together with the asymptotic conditions that generate them. The minima of the

Figure 2: Representation of the physical states in the (B, T^3) plane.

potential become the non-trivial boundary conditions, which are allowed by energy considerations.

4 The vacuum structure of QCD_2

Our starting point will be Baluni's bosonized action [11]². In this case the potential arising from the quark-gluon interaction is

$$V_i(\eta) = \frac{e^2}{32\pi}\eta^2 + \sqrt{\pi}\Lambda^2\left\{1 - \frac{\sin\sqrt{2\pi}\eta}{\sqrt{2\pi}\eta}\right\} \quad (22)$$

Here Λ is a scale parameter coming from the renormalization through normal-ordering.

Let us generalize our energy finiteness argument to the interacting case

$$E_\eta = \int_{-\infty}^{+\infty} dx \left[\frac{1}{2}(\partial_x \eta)^2 + V_i(\eta) \right] \quad (23)$$

A soliton field η_s must satisfy the equations of motion. For static solutions we have

$$E_{\eta_s} = \int_{-\infty}^{\infty} dx \ 2V_i(\eta_s) \quad (24)$$

But this integral is necessarily divergent unless $\eta_s(+\infty)$ is an absolute minimum³.

²There are indications that even in the non-abelian case it is possible to find a similar form for the bosonized action[4].

³ In a recent paper by Ellis et al. [12] they find color solitons in bosonized QCD_2 with infinite energy. We claim that their *constituent quarks* correspond to solitons which do not connect absolute minima.

Figure 3: The color singlet sector of QCD_2 .

This clearly implies that states which are not constructed at an absolute minimum have infinite energy.

The potential Eq.(22) is positive definite and it has only one absolute minimum $V_i(\eta) = 0$ which occurs for $\eta = 0$

$$V_i(\eta) = 0 \leftrightarrow \eta = 0 \quad (25)$$

This property is crucial to understand the vacuum structure of bosonized QCD_2 . Even if a mass term was added, the same result would persist. What are the implications of this result on the (B, T^3) plane?

The full potential has a lower bound

$$V(\varphi, \eta) = V_M(\varphi, \eta) + V_i(\eta) \geq -\mu M, \quad \forall(\varphi, \eta) \quad (26)$$

The equality is only saturated if the following conditions are met

$$\cos \sqrt{2\pi}\varphi \cos \sqrt{2\pi}\eta = 1 \quad \text{and} \quad V_i(\eta) = 0 \quad (27)$$

Thus the minima are the set of points

$$\emptyset_{QDC_2} = \{(\varphi_n, \eta = 0) ; n \in Z\} \quad (28)$$

which correspond to the color singlet states shown in Fig.(3).

Let us expand on this subtle point. Dynamically the topologically non-trivial solutions are pushed to infinite energy and thus the Fock space in QCD_2 has been reduced to that of only $T_3 = 0$ states. We claim however that in this case this corresponds to only the color singlet states, i.e., $T = 0$ states. Let us prove it.

In order to do so we shall calculate the matrix element of $\vec{T} \cdot \vec{T}$ between any such states,

$$\langle \Phi | \vec{T} \cdot \vec{T} | \Phi' \rangle = \langle \Phi | T_3 T_3 + \frac{1}{2} T_- T_+ + \frac{1}{2} T_+ T_- | \Phi' \rangle \quad (29)$$

where evidently these charge operators are obtainable from the space integrals of the zeroth component of the color currents J_a^μ and the states belong to the Fock space determined by obc's. In order to calculate the matrix elements of the ladder operators, we use completeness in this space

$$\langle \Phi | T_\pm T_\mp | \Phi' \rangle = \sum_n \langle \Phi | T_\pm | \Phi_n \rangle \langle \Phi_n | T_\mp | \Phi' \rangle \quad (30)$$

The result limited to this subspace is trivially zero and thus the states are necessarily color singlet. Note that the restriction in the present case has not been imposed by *fiat* but has been dynamically generated and therefore is a consequence of the interactions in QCD_2 . States with non vanishing color quantum numbers acquire infinite mass as a result of eq.(24).

The presence of the color interaction has transformed completely the structure of the vacuum with respect to that of the *free* theory. Once we turn on the interaction ($e \neq 0$), the *free* vacuum collapses immediately to the bosonized QCD_2 vacuum and therefore the set of all possible physical states reduces to the subset of color singlet states. Confinement appears in this picture as a collapse of topology, resulting from the dynamical prohibition of the non-trivial boundary conditions for the η field.

To conclude, the singlet spectrum is formed by particles of baryon number $B = n$, i.e., by mesons ($n = 0$) and by baryons ($n = 1, 2, \dots$) or antibaryons ($n = -1, -2, \dots$). Particles with half-integer baryon number are *necessarily* colored states. This result agrees with what is expected from the fermionic theory. Color singlet states are operators which in color space are of the form

$$q_\alpha^+ q_\alpha \ , \ \varepsilon^{\alpha\beta} q_\alpha q_\beta \ , \ q_\alpha^+ q_\alpha \varepsilon^{\beta\gamma} q_\beta q_\gamma \ , \ \dots \quad (31)$$

5 Confinement and topology

Confinement is recognized in the effective action by the fact that the η field is a color singlet field. No color charged states can be built from the bosonized action. As we have proven in previous work [7], resonant states belong to the color sector of the bosonized theory. Consequently the η dynamics is responsible for generating the spectrum and therefore only color singlet states can appear.

Since η is a color singlet it must have simple relations with $SU(2)_C$ invariant quantities. This is easily established when we compare simple operators in their abelian and non-abelian forms. For example, we can express the non-abelian bosonized field g in its abelian form

$$trg = g_1^1 + g_2^2 = 2 \cos \sqrt{2\pi} \eta \quad (32)$$

But because $g \in SU(2)_C$ it also has a standard representation in terms of the adjoint fields π^a , $g_\beta^\alpha = e^{i\pi^a(\frac{\tau^a}{2})_{\alpha\beta}}$ and therefore $\eta = \frac{\sqrt{\pi^a \pi_a}}{2\sqrt{2\pi}}$. Thus for ordinary boundary conditions the η field is a color singlet, moreover it is a singlet scalar particle.

We have learned that the vacuum structure of the *free* field theory is the consequence of the existence of non-trivial boundary conditions. The stability of the soliton numbers (B, T^3) is guaranteed by the existence of *topological* conservation laws for the $U(1)_F$ and $U(1)_C$ currents

$$j^\mu = \frac{1}{\sqrt{2\pi}} \varepsilon^{\mu\nu} \partial_\nu \varphi \quad , \quad J_3^\mu = \frac{1}{\sqrt{2\pi}} \varepsilon^{\mu\nu} \partial_\nu \eta \quad (33)$$

Because the (B, T^3) charges depend on the values of the (φ, η) fields on the border of the one-dimensional space, they can be related to topological properties of the groups associated with them. The (B, T^3) charges generate the $U(1)_F \otimes U(1)_C$ explicit symmetry of the *free* lagrangian. Its action can be written in terms of the $U(1)_F \otimes U(1)_C$ fields $(e^{i\sqrt{2\pi}\varphi}, e^{i\sqrt{2\pi}\eta})$ using

$$\partial_\mu \phi \partial^\mu \phi = \frac{1}{2\pi} \partial_\mu (e^{i\sqrt{2\pi}\phi}) \partial^\mu e^{-i\sqrt{2\pi}\phi} \quad (34)$$

where $\phi = \varphi, \eta$.

The compactified one-dimensional space is the S^1 sphere. We may define the *free* action in terms of the $U(1)_F \otimes U(1)_C$ fields with support on the sphere. These fields map the sphere into the group, and since $U(1)$ has a parameter space which is the unit circle, it turns out to be a mapping from S^1 into $S^1 \otimes S^1$. However this mapping is not unambiguously defined, and each of the homotopic solutions can be characterized by two integers (ν_F, ν_C) . These topological charges can be obtained by means of the integral formulas in terms of group elements [1]

$$\nu_i = \frac{i}{2\pi} \int_0^{2\pi} d\theta g_i \frac{d}{d\theta} g_i^{-1} \quad (35)$$

where $g_i \in U_i(1), i = F, C$ and therefore

$$\begin{aligned} \nu_F &= \sqrt{\frac{2}{\pi}} [\varphi(2\pi) - \varphi(0)] = 2B \\ \nu_C &= \sqrt{\frac{2}{\pi}} [\eta(2\pi) - \eta(0)] = 2T^3 \end{aligned} \quad (36)$$

The non-trivial boundary conditions produce the *winding numbers* associated with the homotopy classes of these mappings. The lattice of physical states gives just the first homotopy class of the group

$$\emptyset \approx \Pi_1[U(1)_F \otimes U(1)_C] \approx \Pi_1[U(1)_F] \times \Pi_1[U(1)_C] \approx Z_F \otimes Z_C \quad (37)$$

Let us add to the *free* action a non-confining interaction, then the quarks must be the only asymptotic in/out states. Thus asymptotically the action is that of a free (φ, η) field with non-trivial boundary conditions and therefore we obtain a free vacuum structure. Due to the conservation of the charges (B, T^3) this structure is also that of the interaction theory, because if not, asymptotic states different from free quark states could be generated, contrary to our starting assumption. Thus the addition of a non-confining interaction leaves the vacuum structure unaltered and therefore we can characterize this type of interactions topologically by Eq.(37).

Let us now add a confining interaction. The vacuum structure collapses into a colorless subset (see Fig.(3)). In topological language the non-trivial topology induced by the mapping

$$\Pi_1[U(1)_C] \approx Z_C \quad (38)$$

disappears. The confining interaction forces the physical solutions to be states of $T^3 = \nu_C = 0$. The color topology becomes trivial and therefore the topology induced by the interaction in the center of the $SU(2)_C$ gauge group ($U(1)_C$) determines that all physical states have to be color singlets.

6 Conclusion

The fermionic description indicates in a qualitative manner that QCD_2 is a confining theory beyond leading order in the $\frac{1}{N}$ expansion. The spectrum, as well as, the vanishing of the quark creation amplitudes, corroborate the non existence of *free* color non singlet states. The quark self-energy shows a confining behavior.

In the bosonized version of fermionic two dimensional theories topology plays an important role. States with baryon number and color charge are described by solitons. The properties of the vacuum, which give rise to non-trivial boundary conditions, determine the quantum number structure of the Fock space. We have analyzed initially the rich spectrum of non confining theories by discussing the role of the boundary conditions. It is appealing that in the bosonized version of the theory, this discussion can be carried out purely at the classical level.

Once the dynamics of QCD_2 is incorporated the Fock space collapses to the subset of color singlet states and we recover the fermionic result independent of any large N assumption. The mechanism can be cast in a more mathematical language by invoking homotopy groups, but one should not avoid the very naive dynamical statement, namely that color solitons are given infinite energy [12].

The abstraction associated with the mathematical language might guide one into the four dimensional case. Is it not possible to write an approximate bosonized theory in four dimensions? Skyrme type models have been extremely succesful in describing the low energy flavor properties of the theory, but they avoid confinement simply by assuming its existence. Could one find an approximate bosonized lagrangian in terms

of the color degrees of freedom ?, and if so, what is the mechanism that produces the collapse of the spectrum ? The work of 't Hooft [13] has been pioneering in this respect, but again the dimensionality of space-time makes difficult the connection. No relation between our topological scheme and his has been as of yet found, but the endeavor seems sufficiently appealing to embark.

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