Color Anomaly and Flavor-Singlet Axial Charge of the Proton in the Chiral Bag:
The Cheshire Cat Revisited

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ABSTRACT

Quantum effects inside the chiral bag induce a color anomaly which requires a compensating surface term to prevent breakdown of color gauge invariance. We show that the presence of this surface term first discovered several years ago allows one to derive in a gauge-invariant way a chiral-bag version of the Shore-Veneziano two-component formula for the flavor-singlet axial charge of the proton. This has relevance to what is referred to as the "proton spin problem" on the one hand and to the Cheshire-Cat phenomenon in hadron structure on the other. We show that when calculated to the leading order in the color gauge coupling and for a specific color electric monopole configuration in the bag, one can obtain a striking Cheshire-Cat phenomenon with a negligibly small singlet axial charge.

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1 Introduction

It was discovered some years ago [1, 2] that the vacuum fluctuation inside a chiral bag that induces the leakage of baryon charge into the hedgehog pion field outside induces color leakage if one allows for a coupling to a pseudoscalar isoscalar field η'. This would break color gauge invariance in the model unless it is canceled by a surface counter term of the form (which will be referred to as NRWZ counter term in what follows)

$$\mathcal{L}_{CT} = i\frac{g^2}{32\pi^2}\oint_{\Sigma} d\beta K^\mu n_\mu (\mathrm{Tr}\ln U^\dagger - \mathrm{Tr}\ln U) \tag{1}$$

where N_F is the number of flavors (here taken to be =2), β is a point on a surface Σ, n^μ is the outward normal to the bag surface, U is the $U(N_F)$ matrix-valued field written as $U = e^{i\pi/f}e^{i\eta/f}$ and K^μ the properly regularized Chern-Simons current $K^\mu = \epsilon^{\mu\nu\alpha\beta}(G^a_\nu G^a_{\alpha\beta} - \frac{2}{3}f^{abc}gG^a_\nu G^b_\alpha G^c_\beta)$ given in terms of the color gauge field G^a_μ. Note that (1) manifestly breaks color gauge invariance, so the action of the chiral bag model with this term is not gauge invariant but as shown in [1], when quantum fluctuations are calculated, there appears an induced anomaly term on the surface which exactly cancels this term. Thus gauge invariance is restored at the quantum level.

In this paper, we show that a proper account of this term allows us to formulate a fully consistent gauge invariant treatment of the flavor-singlet axial current (FSAC) matrix element of the proton removing a serious conceptual error committed in the previous work done by us together with Park and Brown [3, 4]. In the work of refs.[3, 4], the axial anomaly in the FSAC was introduced explicitly in terms of a Chern-Simons current inside the bag and of a heavy η' field (which we shall denote simply η in the equations) outside the bag arguing that the diagonal matrix element was gauge invariant while off-diagonal terms are not. However this argument strictly speaking is incorrect although it turns out that the conclusion reached there remains more or less correct. See Cheng [5] for a recent discussion on this point. In this paper, we propose to formulate the theory without invoking *ab initio* the problematic Chern-Simons current inside the bag. For this, the NRWZ color boundary condition plays a crucial role.

A complete description calls for a full Casimir calculation which is highly subtle and yet to be performed. In this paper, we shall limit ourselves to the lowest non-trivial order in the color gauge coupling constant and find that the Cheshire Cat principle – that physics should be more or less independent of the confinement bag radius[6] – found in the non-anomalous sector is also applicable in the anomalous sector for a particular field configuration for the color electric field. This conclusion differs from that of Dreiner, Ellis and Flores [7] who obtained the opposite result by ignoring the perturbative gluon effect inside the bag. We shall see that the Dreiner-Ellis-Flores scenario can be recovered in a particular limit of our theory.

This paper is organized as follows. In Section 2, the formulation of the theory implementing the color anomaly is presented. The relevant axial charge with the $U_A(1)$ anomly suitably incorporated is computed in terms of the degrees of freedom that figure in the chiral bag model. In Section 3, the results of the calculation are given. Further discussions and open problems are found in Section 4.

2 Formulation

2.1 Boundary conditions

The equations of motion for the gluon and quark fields inside and the η' field outside are the same as in [3, 4]. However the boundary conditions on the surface now read

$$\hat{n}\cdot\vec{E}^a = -\frac{N_F g^2}{8\pi^2 f}\hat{n}\cdot\vec{B}^a\eta \tag{2}$$

$$\hat{n}\times\vec{B}^a = \frac{N_F g^2}{8\pi^2 f}\hat{n}\times\vec{E}^a\eta \tag{3}$$

and

$$\frac{1}{2}\hat{n}\cdot(\bar{\psi}\gamma\gamma_5\psi) = f\hat{n}\cdot\partial\eta + C\hat{n}\cdot K \tag{4}$$

where $C = \frac{N_F g^2}{16\pi^2}$ and $\vec{E}^a$ and $\vec{B}^a$ are, respectively, the color electric and color magnetic fields. Here ψ is the QCD quark field.

As it stands, the boundary condition for the η' field (4) looks gauge non-invariant because of the presence of the normal component of the Chern-Simons current on the surface. However this is not so. As shown in [2], the term on the LHS of (4) is not well-defined without regularization and when properly regularized, say, by point-splitting, it can be written in terms of a well-defined term which we will write as $\frac{1}{2} : \bar{\psi}\hat{n}\cdot\gamma\gamma_5\psi :$ plus a gauge non-invariant term (see eq.(2) of [2]) which cancels exactly the second term on the RHS. The resulting boundary condition

$$\frac{1}{2}\hat{n}\cdot : (\bar{\psi}\gamma\gamma_5\psi) := f\hat{n}\cdot\partial\eta \tag{5}$$

is then perfectly well-defined and gauge-invariant. However it is useless as it stands since there is no simple way to evaluate the left-hand side without resorting to a model. Our task in the chiral bag model is to express the well-defined operator $: (\bar{\psi}\vec{\gamma}\gamma_5\psi) :$ in terms of the bagged quark field Ψ. In doing this, our key strategy is to eliminate gauge-dependent surface terms by the NRWZ surface counter term.

2.2 Flavor-singlet axial current

Let us write the flavor-singlet axial current in the model as a sum of two terms, one from the bag and the other from the outside populated by the meson field η' (we will ignore the Goldstone pion fields for the moment)

$$A^\mu = A^\mu_B \Theta_B + A^\mu_M \Theta_M. \tag{6}$$

We shall use the short-hand notations $\Theta_B = \theta(R-r)$ and $\Theta_M = \theta(r-R)$ with R being the radius of the bag which we shall take to be spherical in this paper. We demand that the $U_A(1)$ anomaly be given in this model by

$$\partial_\mu A^\mu = \frac{\alpha_s N_f}{2\pi} \sum_a \vec{E}^a \cdot \vec{B}^a \Theta_B + f m_\eta^2 \eta \Theta_M. \tag{7}$$

Our task is to construct the FSAC in the chiral bag model that is gauge-invariant and consistent with this anomaly equation. Our basic assumption is that in the nonperturbative sector outside of the bag, the only relevant $U_A(1)$ degree of freedom is the massive η' field. (The possibility that there might figure additional degrees of freedom in the exterior of the bag co-existing with the η' and/or inside the bag co-existing with the quarks and gluons will be discussed later.) This assumption allows us to write

$$A^\mu_M = f \partial^\mu \eta \tag{8}$$

with the divergence

$$\partial_\mu A^\mu_M \;=\; f m_\eta^2 \eta. \tag{9}$$

Now the question is: what is the gauge-invariant and regularized A^μ_B such that the anomaly (7) is satisfied? To address this question, we rewrite the current (6) absorbing the theta functions as

$$A^\mu = A^\mu_1 + A^\mu_2 \tag{10}$$

such that

$$\partial_\mu A^\mu_1 \;=\; f m_\eta^2 \eta \Theta_M, \tag{11}$$

$$\partial_\mu A^\mu_2 \;=\; \frac{\alpha_s N_f}{2\pi} \sum_a \vec{E}^a \cdot \vec{B}^a \Theta_B. \tag{12}$$

We shall deduce the appropriate currents in the lowest order in the gauge coupling constant α_s and in the cavity approximation for the quarks inside the bag.

2.2.1 The "quark" current A_1^μ

Let the bagged quark field be denoted Ψ. Then to the *lowest order* in the gauge coupling and ignoring possible additional degrees of freedom alluded above, the boundary condition (5) is

$$\frac{1}{2}\hat{n}\cdot(\bar{\Psi}\gamma\gamma_5\Psi) = f\hat{n}\cdot\partial\eta \tag{13}$$

and the corresponding current satisfying (11) is

$$A_1^\mu = A_{1q}^\mu + A_{1\eta}^\mu \tag{14}$$

with

$$A_{1q}^\mu = (\bar{\Psi}\gamma^\mu\gamma_5\Psi)\Theta_B, \tag{15}$$

$$A_{1\eta}^\mu = f\partial^\mu\eta\Theta_M. \tag{16}$$

We shall now proceed to obtain the explicit form of the bagged axial current operator. In momentum space, the quark contribution is

$$\begin{aligned} A_{1q}^j(q) &= \frac{1}{2}\int d^3r e^{i\vec{q}\cdot\vec{r}}\langle N_{Bag}|\Psi^\dagger\sigma^j\Psi|N_{Bag}\rangle \\ &= (a(q)\delta_{j\,k} + b(q)(3\hat{q}_j\hat{q}_k - \delta_{j\,k}))\,\langle\frac{1}{2}\sum_{quarks}\sigma^k\rangle \end{aligned} \tag{17}$$

where

$$a(q) = N^2\int drr^2(j_0^2(\omega r) - \frac{1}{3}j_1^2(\omega r))j_0(qr), \tag{18}$$

$$b(q) = \frac{2}{3}N^2\int drr^2 j_1^2(\omega r)j_2(qr) \tag{19}$$

where N is the normalization constant of the (bagged) quark wave function. In the limit that $q \to 0$ which is what we want to take for the axial charge, both terms are non-singular and only the $a(0)$ term survives, giving

$$A_{1q}^j(0) = g_{A,quark}^0\langle\frac{1}{2}\sum_{quarks}\sigma^j\rangle \tag{20}$$

where $g_{A,quark}^0$ is the singlet axial charge of the bagged quark which can be extracted from (18). In the numerical estimate made below, we shall include the Casimir effects associated with the hedgehog pion configuration to which the quarks are coupled [8, 9], so the result will differ from the naive formula (18).

To obtain the η' contribution, we take the η' field valid for a static source

$$\eta(\vec{r}) = -\frac{g}{4\pi M}\int d^3r'\chi^\dagger\vec{S}\chi\cdot\vec{\nabla}\frac{e^{-m_\eta|\vec{r}-\vec{r'}|}}{|\vec{r}-\vec{r'}|} \tag{21}$$

where g is the short-hand for the $\eta' NN$ coupling constant, M is the nucleon mass, χ the Pauli spinor for the nucleon and S the spin operator. The contribution to the FSAC is

$$\begin{aligned} A^j_{1\eta}(q) &= \int_{V_M} d^3 r e^{i\vec{q}\cdot\vec{r}} f \partial_j \eta, \\ &= (c(q)\delta_{j\,k} + d(q)(3\hat{q}_j\hat{q}_k - \delta_{j\,k}))\langle \frac{1}{2}\sum_{quarks}\sigma^k\rangle \end{aligned} \tag{22}$$

with

$$c(q) = \frac{fg}{2M}\int_R^\infty dr r^2 \frac{e^{-m_\eta r}}{r} m_\eta^2 j_0(qr), \tag{23}$$

$$d(q) = -\frac{fg}{2M}\int_R^\infty dr \frac{e^{-m_\eta r}}{r}[r^2 m_\eta^2 + 3(m_\eta r + 1)] j_2(qr). \tag{24}$$

In the zero momentum transfer limit[1], we have

$$A^j_{1\eta}(0) = \frac{gf}{2M}\left[(y_\eta^2 + 2(y_\eta + 1))\delta_{j\,k} - y_\eta^2 \hat{q}_j\hat{q}_k\right] e^{-y_\eta}\langle S^k\rangle \tag{25}$$

where $y_\eta = m_\eta R$.

The boundary condition (13) provides the relation between the quark and η' contributions. In the integrated form, (13) is

$$\int d\Sigma f x_3 \hat{r}\cdot\vec{\nabla}\eta = \int_{V_B} d^3 r \frac{1}{2}\bar{\Psi}\gamma_3\gamma_5\Psi \tag{26}$$

from which follows

$$\frac{gf}{M} = 3\,\frac{e^{y_\eta}}{y_\eta^2 + 2(y_\eta + 1)}\, g^0_{A,quark}. \tag{27}$$

This is a Goldberger-Treiman-like formula relating the asymptotic pseudoscalar coupling to the quark singlet axial charge. From (20), (25) and (27), we obtain

$$A^j_1 = g^0_{A_1}\langle S^j\rangle \tag{28}$$

with

$$g^0_{A_1} = \frac{gf}{3M}\frac{y_\eta^2 + 2(y_\eta + 1)}{e^{y_\eta}} = \frac{3}{2} g^0_{A,quark}. \tag{29}$$

This is completely analogous to the isovector axial charge g^3_A coming from the bagged quarks inside the bag plus the perturbative pion fields outside the bag. Note that the singlet charge $g^0_{A_1}$ goes to zero when the bag is shrunk to zero, implying that the coupling constant g goes to zero as $R \to 0$ as one can see from eq.(27). This

[1] With however $m_\eta \neq 0$. The limiting processes $q \to 0$ and $m_\eta \to 0$ do not commute as we will see shortly.

is in contrast to g_A^3 where the axial charge from the bag "leaks" into the hedgehog pion outside the bag and hence even when the bag shrinks to zero, the isovector axial charge remains more or less constant in agreement with the Cheshire Cat[10].

An interesting check of our calculation of $\vec{A}_1$ can be made by looking at the $m_\eta \to 0$ limit. From (17) and (25), we find that our current satisfies

$$\hat{q} \cdot \vec{A}_1(0) = \frac{gf}{M}(y_\eta + 1)e^{-y_\eta}\langle \hat{q} \cdot \vec{S} \rangle \tag{30}$$

which corresponds to eq.(11). Now eq.(11) is an operator equation so one can take the limit $m_\eta \to 0$ and expect the right-hand side to vanish, obtaining $\hat{q} \cdot \vec{A}_1 \to 0$. Equation (30) fails to satisfy this. The reason for this failure is that the $q \to 0$ and $m_\eta \to 0$ limits do not commute. To obtain the massless limit, one should take the η' mass to go to zero first.

Before taking the zero-momentum limit, the expression for $c(q)$ for the η field, (23), is

$$c(q) = \frac{fg}{3M}\left(\frac{m_\eta^2}{q^2}\frac{e^{-y_\eta}(\cos(qr) + \frac{m_\eta}{q}\sin(qr))}{1 + \frac{m_\eta^2}{q^2}}\right) \tag{31}$$

which vanishes in the $m_\eta \to 0$ limit. On the other hand, the $d(q)$, (24), which before taking the zero-momentum limit, is of the form

$$\begin{aligned} d(q) \quad = \quad & -\frac{fg}{2M}(\frac{e^{-y_\eta}(y_\eta^2 + 3y_\eta + 3)}{qR}j_1(qR) \\ & -\frac{m_\eta^2}{q^2}e^{-y_\eta}(y_\eta + 1)j_0(qR) + \frac{m_\eta^4}{q^4}\frac{e^{-y_\eta}(\cos(qr) + \frac{m_\eta}{q}\sin(qr))}{1 + \frac{m_\eta^2}{q^2}}) \end{aligned} \tag{32}$$

becomes in the $m_\eta \to 0$ limit

$$-\frac{fg}{2M}\frac{j_1(qR)}{qR}. \tag{33}$$

Adding the quark current (17) in the $q \to 0$ limit, we get

$$A_1^j(0) = \frac{fg}{M}(\delta^{jk} - \hat{q}^j\hat{q}^k)S_k \tag{34}$$

which satisfies the conservation relation. This shows that our formulas are correct.

2.2.2 The gluon current A_2^μ

The current A_2^μ involving the color gauge field is very intricate because it is not possible in general to write a gauge-invariant dimension-3 local operator corresponding to the singlet channel. We will see however that it is possible to obtain a consistent *axial charge* within the model. Here we shall calculate it to the lowest nontrivial order in the gauge coupling constant. In this limit, the right-hand sides of the boundary conditions (2) and (3) can be dropped, reducing to the original MIT boundary conditions [11]. Furthermore the gauge field decouples from the other degrees of freedom precisely because of the color anomaly condition that prevents the color leakage, namely, the condition (5). In its absence, this decoupling could not take place in a consistent way.[2]

We start with the divergence relation

$$\partial_\mu A_2^\mu = \frac{\alpha_s N_f}{2\pi} \sum_a \vec{E}^a \cdot \vec{B}^a \Theta_{V_B}. \tag{35}$$

In the lowest-mode approximation, the color electric and magnetic fields are given by

$$\vec{E}^a = g_s \frac{\lambda^a}{4\pi} \frac{\hat{r}}{r^2} \rho(r) \tag{36}$$

$$\vec{B}^a = g_s \frac{\lambda^a}{4\pi} \left(\frac{\mu(r)}{r^3} (3\hat{r}\vec{\sigma} \cdot \hat{r} - \vec{\sigma}) + (\frac{\mu(R)}{R^3} + 2M(r))\vec{\sigma} \right) \tag{37}$$

where ρ is related to the quark scalar density ρ' as

$$\rho(r) = \int_\Gamma^r ds \rho'(s) \tag{38}$$

and μ, M to the vector current density

$$\begin{aligned} \mu(r) &= \int_0^r ds \mu'(s), \\ M(r) &= \int_r^R ds \frac{\mu'(s)}{s^3}. \end{aligned}$$

The lower limit Γ usually taken to be zero in the MIT bag model will be fixed later on. It will turn out that what one takes for Γ has a qualitatively different consequence on the Cheshire-Cat property of the singlet axial current. Substituting these fields into the RHS of eq.(35) leads to

$$\vec{q} \cdot \vec{A}_2 = \frac{8\alpha_s^2 N_f}{3\pi} \vec{\sigma} \cdot \hat{q} \int_0^R dr \rho(r) \left(2\frac{\mu(r)}{r^3} + \frac{\mu(R)}{R^3} + 2M(r) \right) j_1(qr) \tag{39}$$

where $\alpha_s = \frac{g_s^2}{4\pi}$ and we have used $\sum_{i\neq j} \sum_a \lambda_i^a \lambda_j^a = -\frac{8}{3}$ for the baryons[3].

[2]To higher order in the gauge coupling, the situation would be a lot more complicated. A full Casimir calculation will be required to assure the consistency of the procedure. This problem will be addressed in a future publication.

[3]Here we are making the usual assumption as in ref.[12] that the $i = j$ terms in the color factor are to be excluded from the contribution on the ground that most of them go into renormalizing the single-quark axial charge. If one were to evaluate the color factor without excluding the diagonal terms using only the lowest mode, the anomaly term would vanish, which of course is incorrect. As emphasized in [12], there may be residual finite contribution with $i = j$ but no one knows how to compute this and so we shall ignore it here. It may have to be carefully considered in a full Casimir calculation yet to be worked out.

In order to calculate the axial charge, we take the zero momentum limit and obtain

$$\lim_{q\to 0} \vec{A}_2(\vec{q}) = \frac{8\alpha_s^2 N_f}{9\pi} \tilde{A}_2(R)\vec{S} \tag{40}$$

where

$$\tilde{A}_2(R) = \int_0^R r dr \rho(r) \left(2M(r) + \frac{\mu(R)}{R^3} + 2\frac{\mu(r)}{r^3}\right) \equiv 2\int_0^R dr r\rho(r)\alpha(r). \tag{41}$$

The quantity $\alpha(r)$ is defined for later purposes. It is easy to convince oneself that (40) is gauge-invariant, i.e., it is $\propto \int_{V_B} d^3r \vec{r} \sum_a \vec{E}^a \cdot \vec{B}^a$ which is manifestly gauge-invariant. The result (40) was previously obtained in [13].

2.2.3 The Chern-Simons current and NRWZ counter term

The Chern-Simons current K_μ whose divergence is gauge-invariant is not by itself gauge-invariant. The question that can be raised here is: How is the gauge-invariant object (40) related to the Chern-Simons current incorrectly used in refs.[3, 4] ? To answer this question, we first take the λ^a outside from the field operators

$$\begin{aligned} G_\mu^a &= \frac{g_s}{4\pi}\lambda^a \mathcal{G}_\mu, \\ G_{\mu\nu}^a &= \frac{g_s}{4\pi}\lambda^a \mathcal{G}_{\mu\nu}. \end{aligned} \tag{42}$$

This is convenient in abelianizing the theory.

From the electric and magnetic fields in the cavity (see eqs. (36) and (37)) and using

$$\mathcal{E}_i = -\partial_i \mathcal{G}_0, \tag{43}$$

$$\mathcal{B}_i = \varepsilon_{ijk}\partial_j \mathcal{G}_k \tag{44}$$

we get, up to gauge transformations,

$$\mathcal{G}_0(\vec{r}) = \int_0^r ds \frac{\rho(s)}{s^2}, \tag{45}$$

$$\mathcal{G}_i(\vec{r}) = \left(\frac{\mu(r)}{r^3} + \frac{1}{2}\frac{\mu(R)}{R^3} + M(r)\right)(\vec{r}\wedge\vec{\sigma})_i. \tag{46}$$

The curly fields behave under gauge transformations as

$$\mathcal{G}_\mu^\Lambda = \mathcal{G}_\mu + \partial_\mu \Lambda. \tag{47}$$

Consider a static $\mathcal{G}_\mu$ and restrict ourselves to time-independent field transformations. Then

$$\Lambda(\vec{r}, t) = \Lambda_1 t + \Lambda_2(\vec{r}) \tag{48}$$

where Λ_1 is a constant so that

$$\mathcal{G}_0^\Lambda = \mathcal{G}_0 + \Lambda_1 \tag{49}$$

and $\Lambda_2(\vec{r})$ is a time-independent function such that

$$\mathcal{G}_i^\Lambda = \mathcal{G}_i + \partial_i \Lambda_2. \tag{50}$$

For these fields the Chern-Simons current is given by

$$\mathcal{K}_i = -2\mathcal{G}_0 \mathcal{B}_i + 2\varepsilon^{ijk} \mathcal{G}_j \mathcal{E}_k + O(g_s^3). \tag{51}$$

At the surface of the bag

$$\hat{r} \cdot \vec{\mathcal{K}} \sim \mathcal{G}_0^\Lambda \hat{r} \cdot \vec{\mathcal{B}} \tag{52}$$

which is in general different from zero. We may choose the constant Λ_1 so that $\mathcal{G}_0$ vanishes at the surface of the bag. In general, there may be a finite contribution. However this is no cause for worry since the crucial point of our reasoning is that such a contribution, if non-vanishing, will be canceled by the NRWZ surface counter term.

The gauge dependence of the Chern-Simons current is given by

$$\mathcal{K}_i^\Lambda - \mathcal{K}_i \sim -\Lambda_1 \vec{\mathcal{B}}_i + (\vec{\partial} \Lambda_2 \wedge \vec{\mathcal{E}})_i \tag{53}$$

where we have denoted by $\mathcal{E}$ and $\mathcal{B}$ the color electric and magnetic fields with the λ factor taken out as in eq.(42). Since our fields are static ($\vec{\partial} \wedge \vec{\mathcal{E}} = 0$), we may write the RHS of eq.(53) as an exact differential, i.e.,

$$\varepsilon_{ijk} \partial_j (\Lambda_2 \mathcal{E}_k - \Lambda_1 \mathcal{G}_k). \tag{54}$$

This term when calculating the charge, i.e., integrating over the bag, will be killed by the NRWZ surface coupling. This shows that the Chern-Simons current cannot be injected into the interior of the bag without properly imposing the NRWZ counter term, an error committed in refs.[3, 4].

2.2.4 The structure of the η'

In our discussion on the boundary condition eq.(5), we emphasized the role of the NRWZ mechanism in removing gauge-non-invariant terms accumulating on the surface. An important point to note here is that this mechanism imposed no condition on the normal component of the Chern-Simons current itself. It is just that the normal flux of the Chern-Simons current was canceled by the surface counter term.

Thus far we have assumed that the only relevant degrees of freedom are the quarks and gluons inside the bag and the η' (and pions) outside the bag. This is the minimal picture. Now suppose that there are additional degrees of freedom (in addition to (15) and (16)) either outside or inside of the bag or both inside and outside. We shall assume for simplicity that there is one such degree of freedom outside. The same result will be obtained for the other cases except for possibly different physical interpretations. Now from the divergence condition (7), the additional current must be gauge-invariant and divergenceless, i.e.,

$$\delta A^{\mu}_{1\eta} = -\Delta^{\mu}\Theta_M \tag{55}$$

with

$$\partial_\mu \Delta^\mu = 0. \tag{56}$$

A possible candidate for such a degree of freedom could be a heavy quarkonium or a heavy gluonium. The condition (11) would remain unchanged provided the boundary condition (13) is modified to

$$\frac{1}{2}\hat{n}\cdot(\bar{\Psi}\gamma\gamma_5\Psi) + \hat{r}\cdot\vec{\Delta} = f\hat{n}\cdot\partial\eta. \tag{57}$$

To see what the consequences of the boundary condition (57) are, consider a $\vec{\Delta}$ that can be written in terms of the harmonic function[4]

$$\vec{\Delta} = \vec{\nabla}\Phi \tag{58}$$

with in the cavity

$$\Phi = \sum_{l,m} C_l r^l Y_{l,m}. \tag{59}$$

The boundary condition, eq.(57), gets a contribution from $l = 1$ and hence only the coefficient C_1 enters. This modifies the asymptotic normalization (27) to

$$\frac{gf}{M} = \frac{e^{y_\eta}}{y_\eta^2 + 2(y_\eta + 1)}\left(\frac{3}{2}g^0_{A,quarks} + 4\pi R^4 c_1\right) \tag{60}$$

where the normalization constant is chosen so that $\Phi(\vec{r}) = c_1\vec{S}\cdot\vec{r}$. The new term contributing to the singlet current in momentum space is given by

$$\Delta_i(q) = 4\pi c_1 R^2 S_i \frac{j_1(qr)}{q}. \tag{61}$$

[4] The argument given here is actually more general, applying as well to the case where $\vec{\nabla}\times\vec{\Delta} \neq 0$.

This new current adds a contribution to $g^0_{A_1}$. Using eqs.(60) and (61) together, we obtain

$$g^0_{A_1} = \frac{gf}{3M}\frac{y_\eta^2 + 2(y_\eta + 1)}{e^{y_\eta}} = \frac{3}{2}(g^0_{A,quarks} + \frac{4\pi}{3}R^4 c_1). \tag{62}$$

The expression (62) has an interesting physical interpretation. In terms of the $\eta\prime$ parameters, it is exactly the same as what we obtained before, i.e., eq.(29). However this is not so in terms of the quarks and the additional degree of freedom. One may interpret this as describing the quark-glueball undressing of the $\eta\prime$. It is not clear what this additional degree of freedom could be: One could perhaps relate it to (a part of) the pseudoscalar field $G = \mathrm{TrG}_{\mu\nu}\tilde{\mathrm{G}}^{\mu\nu}$ (where $\tilde{G}_{\mu\nu}$ is dual to $G_{\mu\nu}$) introduced by the authors of [14]. Without knowing its content or structure, one can however infer its role if one adopts the Cheshire-Cat principle. Equation (62) shows that $g^0_{A_1}$ will become large in magnitude as the radius grows if c_1 is non-negligible and this will violate the Cheshire Cat. Thus the Cheshire Cat will require that $c_1 \sim 0$. Since we do not know how to compute it within the model anyway, we shall simply assume it to be zero. An interesting possibility is that when the η' nucleon coupling is measured with accuracy, we will not only determine g^0_{A1} unambiguously but also learn more about this mysterious degree of freedom if it is not completely negligible.

2.2.5 The two-component formula

The main result of this paper can be summarized in terms of the two component-formula for the singlet axial charge (with $c_1 = 0$),

$$g^0_A = g^0_{A_1} + g^0_{A_2} = \frac{3}{2}g^0_{A,quarks} + \frac{8\alpha_s^2 N_f}{9\pi}A_2(R). \tag{63}$$

The first term is the "matter" contribution (29) and the second the gauge-field contribution (40). This is the chiral-bag version of Shore-Veneziano formula[15, 14] relating the singlet axial charge to a sum of an η' contribution and a glue-ball contribution.

3 Results

In this section, we shall make a numerical estimate of (29) and (40) in the approximation that is detailed above. In evaluating (29), we shall take into account the Casimir effects due to the hedgehog pions but ignore the effect of the η' field on the quark spectrum. The interaction between the internal and external degrees of freedom occurs at the surface. Our approximation consists of neglecting in the expansion of the boundary condition in powers of $\frac{1}{f}$ all η dependence, i.e.

$$i\hat{r}\cdot\gamma\Psi = e^{i\gamma_5\vec{\tau}\cdot\hat{r}\frac{\varphi(\vec{r})}{f_\pi}} e^{i\gamma_5\frac{\eta}{f}}\Psi \sim e^{i\gamma_5\vec{\tau}\cdot\hat{r}\frac{\varphi(\vec{r})}{f_\pi}}\Psi \tag{64}$$

This approximation is justified by the massiveness of the η' field in comparison to the Goldstone pion field that supports the hedgehog configuration, φ. Within this approximation, we can simply take the numerical results from [3, 4] changing only the overall constants in front.

The same is true with the gluon contribution. To the lowest order in α_s, the equation of motion for the gluon field is the same as in the MIT bag model. This is easy to see, since the modified boundary conditions eqs.(2) and (3) become

$$\hat{r}_i G^{i\mu} = -\frac{\alpha_s N_F}{2\pi}\frac{\eta}{f}\hat{r}_i \tilde{G}^{i\mu} \sim 0. \tag{65}$$

The only difference from the MIT model is that here the quark sources for the gluons are modified by the hedgehog pion field in (64). Again the results can be taken from [3, 4] modulo an overall numerical factor.

In evaluating the anomaly contribution (41), we face the same problem with the monopole component of the $\vec{E}^a$ field as in [3, 4]. If we write

$$\mathbf{E}_i^a(r) = f(r)\hat{\mathbf{r}}\lambda_i^a \tag{66}$$

where the subscript i labels the ith quark and a the color, the $f(r)$ satisfying the Maxwell equation is

$$f(r) = \frac{1}{4\pi r^2}\int_\Gamma^r ds\rho'(s) \equiv \frac{1}{4\pi r^2}\rho(r). \tag{67}$$

If one takes only the valence quark orbit – which is our approximation, then ρ' in the chiral bag takes the same form as in the MIT model. However the quark orbit is basically modified by the hedgehog boundary condition, so the result is of course not the same. The well-known difficulty here is that the bag boundary condition for the monopole component

$$\hat{\mathbf{r}} \cdot \mathbf{E}_i^a = 0, \quad \text{at} \quad r = R \tag{68}$$

is not satisfied for $\Gamma \neq R$. Thus as in [3, 4], we shall consider both $\Gamma = 0$ and $\Gamma = R$.

The existence of a solution which satisfies explicitly and locally the boundary condition suggests an approach different from the one in the original MIT calculation [11], where the boundary condition of the electric field was imposed as an expectation value with respect to the physical hadron state. In [11], the E and B field contributions to the spectrum were treated on a completely different footing. While in the former the contribution arising from the quark self-energies was included, thereby leading to the vanishing of the color electric energy, in the latter they were not. This gave the color magnetic energy for the source of the nucleon-Δ splitting. We have performed a calculation for the energy with the explicitly confined E and B field treated in a symmetric fashion [4]. Although in this calculation the contribution of the color-electric energy was non-vanishing, it was found not to affect the nucleon-Δ mass splitting, and therefore could be absorbed into a small change of the unknown parameters, i.e., zero point energy, bag radius, bag pressure etc. As we shall see shortly, the two ways of treating the confinement with $\Gamma = R$ and $\Gamma = 0$ give qualitatively different results for the role of the anomaly. One could consider therefore that the singlet axial charge offers a possibility of learning something about confinement within the scheme of the chiral bag. At present, only in heavy quarkonia [16] does one have an additional handle on these operators.

The numerical results for both cases are given in Table 1.

Table 1: The flavor-singlet axial charge of the proton as a function of radius R and the chiral angle θ. The column labeled $g^0_{A_1}$ corresponds to the total contribution from the quarks inside the bag and η' outside the bag (eq.(29)) and $g^0_{A_2}(\Gamma = R)$ and $g^0_{A_2}(\Gamma = 0)$ to the gluon contribution eq.(40) evaluated with $\Gamma = R$ and $\Gamma = 0$ in (67), respectively. The parameters are: $\alpha_s = 2.2$, $m_\eta = 958$ MeV and $f = 93$ MeV. The row with $R = \infty$ corresponds to the unrealistic (and extreme) case of an MIT bag model with the same parameters for the same degrees of freedom but containing *no pions*.

R(fm)	θ/π	$g^0_{A_1}$	$g^0_{A_2}(\Gamma = R)$	$g^0_{A_2}(\Gamma = 0)$	$g^0_A(\Gamma = R)$	$g^0_A(\Gamma = 0)$
0.0	-1.000	0.000	0.000	0.000	0.000	0.000
0.2	-0.742	0.033	-0.015	0.009	0.018	0.042
0.4	-0.531	0.164	-0.087	0.046	0.077	0.210
0.6	-0.383	0.321	-0.236	0.123	0.085	0.444
0.8	-0.277	0.494	-0.434	0.232	0.060	0.726
1.0	-0.194	0.675	-0.635	0.352	0.040	1.027
∞	0.00	0.962	-1.277	0.804	-0.297	1.784

4 Discussion

The quantity we have computed here is relevant to two physical issues: the so-called "proton spin" issue and the Cheshire-Cat phenomenon in the baryon structure. A more accurate result awaits a full Casimir calculation which appears to be non-trivial. However we believe that the qualitative feature of the given model with the specified degrees of freedom will not be significantly modified by the full Casimir effects going beyond the lowest order in α_s.

In the current understanding of the polarized structure functions of the nucleon, the FSAC matrix element or the flavor-singlet axial charge of the proton is related to the polarized flavor-singlet structure function $\Delta\Sigma = \Delta u + \Delta d + \Delta s$ [5, 17]. The presently available analyses give [17, 18]

$$\Delta\Sigma = 0.27 \pm 0.04 \pm \cdots \tag{69}$$

$$= 0.10 \pm 0.05(\mathrm{exp}) \pm^{0.17}_{0.11} (\mathrm{th}) \tag{70}$$

Our predictions for g_A^0 – which can be compared with $\Delta\Sigma$ – differ drastically depending upon whether one takes $\Gamma = 0$ for which the color electric monopole field satisfies *only globally* the boundary condition at the leading order (that is, as a matrix element between color-singlet states) as in the standard MIT bag-model phenomenology or $\Gamma = R$ which makes the boundary condition satisfied locally. The former configuration severely breaks the Cheshire Cat with the bag radius R constrained to 0.5 fm or less ("little bag scenario") to describe the empirical values (69) and (70). This is analogous to what Dreiner, Ellis and Flores [7] obtained. In this scenario, there is no way that the Cheshire Cat can be recovered in the singlet channel unless a hitherto unknown degree of freedom discussed above which contributes the surface term $\hat{r} \cdot \vec{\Delta}$ in the boundary condition (57) intervenes massively with the right sign to cancel the rest, a possibility which we find to be highly unlikely although not totally excluded.

On the other hand, the configuration with $\Gamma = R$ which we favor leads to a remarkably stable Cheshire Cat in consistency with other non-anomalous processes where the Cheshire Cat is seen to hold within, say, 30% [4, 10]. The resulting singlet axial charge $g_A^0 < 0.1$ is consistent with (70) though perhaps somewhat too low compared with (69). One cannot however take the near zero value predicted here too literally since the value taken for α_s is perhaps too large. Moreover other short-distance degrees of freedom not taken into account in the model (such as the light-quark vector mesons and other massive mesons) can make a non-negligible additional contribution[14]. What is noteworthy is that there is a large cancellation between the "matter" (quark and η') contribution and the gauge field (gluon) contribution in agreement with the interpretation anchored on $U_A(1)$ anomaly[18].

As mentioned above – and also noted in [3, 4], the electric monopole configuration with $\Gamma = R$ is non-zero at the origin and hence is ill-defined there. This feature does not affect, however, other phenomenology as shown in [4]. We do not know yet if this ambiguity can be avoided if other multipoles and higher-order and Casimir effects are included in a consistent way. This caveat notwithstanding, it seems reasonable to conclude from the result that if one accepts that the singlet axial charge is small *because of the cancellation* in the two-component formula and if in addition one demands that the Cheshire Cat hold in the $U_A(1)$ channel *as in other non-anomalous sectors*, we are led to (1) adopt the singular monopole configuration that satisfies the boundary condition *locally* and (2) to the possibility that within the range of the bag radius that we are considering, the η' is primarily quarkish with $c_1 \approx 0$. This issue will be addressed further in a forthcoming publication which will include Casimir effects.

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