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A quark model analysis of the transversity distribution†

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Abstract

The feasibility of measuring chiral-odd parton distribution functions in polarized Drell-Yan and semi-inclusive experiments has renewed theoretical interest in their study. Models of hadron structure have proven succesful in describing the gross features of the chiral-even structure functions. Similar expectations support our study of the transversity parton distributions in the Isgur-Karl and MIT bag models. We confirm the diverse low x behavior of the transversity and spin structure functions at the experimental scale and show that it is fundamentally a consequence of the different behavior under evolution of these functions. The inequalities of Soffer establish constraints between data and model calculations of the chiral-odd transversity function. The approximate compatibility of our model calculations with these constraints confers credibility to our estimates.

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1 Introduction

The parton distributions on the lightcone are physical quantities which describe the low-energy properties of the nucleon in high-energy processes. One may define for a hadron an infinite number of parton distributions, however in high-energy processes we are interested in those with low twist. At the twist two level the quark parton model defines three parton distributions labelled conventionally as $f_1(x)$, $g_1(x)$ and $h_1(x)$. The first two have been the main focus of studies so far. The last has received little attention because it plays a minor role in deep inelastic processes. Not until the work of Ralston and Soper [1] its importance in characterizing the nucleons high-energy properties was recognized. Jaffe and Ji [2] named it transversity distribution because it measures the density of quarks (minus antiquarks) in eigenstates of the transverse Pauli-Lubanski operator.

h_1 is a twist two parton distribution which, unlike the others, is chiral-odd. This makes it unaccessible to conventional inclusive DIS experiments except if the current quark masses are considered, in which case arises as a twist three contribution [3]. However it can be measured at the leading twist in the Drell-Yan process with both beam and target polarized [1, 4]. Recently, semi-inclusive lepton nucleon DIS processes have been proposed as a way of determining h_1 [5, 6, 7, 8]. The time has come to investigate the chiral-odd and higher twist parton distribution functions in detail since measurements will appear in the next years from RHIC [9], HERA [10], CERN [11] and from new facilities such as ELFE [12]. The possibility of measuring h_1 has renewed theoretical interest and estimates to guide the experimental analysis have been presented using various methods, i.e., leading log approximation of QCD [13], QCD sum rule approach [14, 15] and model calculations [2, 16, 17].

The aim of this paper is to study the transversity distribution within different models of hadron structure. As pointed out by Jaffe and Ross [18], these calculations are associated with a low Q^2 , the so called hadronic scale. To go from the hadronic scale to the experimental conditions our scheme proceeds via perturbative QCD evolution. It has been claimed in the past that $h_1 \approx g_1$, a result which arises naturally in model calculations. We will show that the very diverse evolution properties of these two structure functions lead, even if they are similar at the hadronic scale, to large differences at the usual experimental conditions, a result known to other authors [16, 17]. This confirmation within different schemes leads to an optimal experimental scenario.

Soffer [19] has produced a series of inequalities in the parton model, relating the transversity distributions with the chiral-even distributions. When combined with data, these inequalities provide rigorous constraints for model calculations of h_1 . We analyze their applicability and obtain the limitations of the models under scrutiny.

2 The theoretical framework

The transversity structure function measures the polarization asymmetry of quarks (or antiquarks) in a transversely polarized hadron, i.e.,

$$h_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 (h_1^q(x, Q^2) + h_1^{\bar{q}}(x, Q^2)), \quad (1)$$

where the transversity parton distribution functions are given by

$$h_1^q(x, Q^2) = q_{\uparrow}(x, Q^2) - q_{\downarrow}(x, Q^2). \quad (2)$$

Here \uparrow (\downarrow) indicates that the spin of the quark of flavor q is parallel (antiparallel) to the transverse polarization of the nucleon and \bar{q} refers to the equivalent antiquark distributions.

The following inequalities arise from current-hadron amplitudes,

$$|g_1(x, Q^2)| \leq f_1(x, Q^2) \quad (3)$$

and

$$|h_1(x, Q^2)| \leq f_1(x, Q^2), \quad (4)$$

which also appear as a trivial consequence of the definition of the distribution functions in the lightcone helicity and transversity bases, respectively [2].

A third inequality was proven by Soffer for the parton model [19, 20],

$$2|h_1^q(x, Q^2)| \leq g_1^q(x, Q^2) + f_1^q(x, Q^2) \quad (5)$$

and its behavior under evolution has been clarified by Barone [21].

The structure function $h_1(x, Q^2)$ is associated with the tensor operator $\bar{\Psi}\sigma_{\mu\nu}i\gamma_5\Psi$. The $n = 1$ sum rule leads to

$$\delta q(Q^2) = \int_0^1 dx ((h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2))) \quad (6)$$

where δq is named the *tensor charge* and is given in the nucleons's rest frame by

$$\langle PS | \bar{\Psi}_q \Sigma_i \Psi_q | PS \rangle = 2\delta q S_i \quad (7)$$

where Ψ_q here labels the spinor of flavor q , Σ_i is the conventional spin operator and S_i the nucleon's spin vector defined as usual [2]. From eq.(6) we see that the tensor charge counts the number of valence quarks of opposite transversity. Since δq is charge conjugation-odd, it gets no contribution from quark-antiquark pairs of the sea.

In contrast the quark spin operator associated with g_1 is even under charge conjugation and therefore the corresponding equation is

$$\Delta q(Q^2) = \int_0^1 dx ((g_1^q(x, Q^2)) + g_1^{\bar{q}}(x, Q^2)), \quad (8)$$

where Δq , the axial charge, is given in the nucleon rest frame by

$$\langle PS|\Psi_q^+\Sigma_i\Psi_q|PS\rangle = 2\Delta q S_i, \quad (9)$$

It is evident that Δq includes the helicity of the sea. From eqs.(7) and (9) it is apparent that in non-relativistic model calculations $\delta q = \Delta q$. This argument can be generalized to all the other moments so that one may prove that $h_1 = g_1$ in these models.

The evolution properties of h_1 and g_1 are however very different. All of the local operators associated with h_1 have non-vanishing leading order anomalous dimensions. Moreover no gluon operators contribute to h_1 in any order because it is chiral-odd and the gluon operators are all chiral even. Therefore h_1 is a non-singlet structure function, which evolves homogeneously with Q^2 and none of its moments is Q^2 independent. On the contrary the non-singlet components of the first moments of g_1 have vanishing anomalous dimensions, thus they are Q^2 independent and their singlet components mix with gluons in a complicated way governed by the axial anomaly [5, 6, 22].

The formalism of our calculation was described in detail in ref.[23], although we will in some cases diverge from some of its details to respect the author's philosophy in the models. We assume that quark model calculations give the value of the matrix elements at a definite hadronic scale μ_0^2 . The leading twist contribution of the matrix elements is evolved to the experimental conditions at high Q^2 by means of renormalization group methods of perturbative *QCD*. The analysis here will be carried out to leading order (LO) since the evolution parameters of h_1 are only known to this order [3, 5].

At the hadronic scale the physical meaning of the structure functions is very intuitive in a naive non-relativistic formulation [23]:

$$g_1^q(x, \mu_0^2) = \frac{m_q}{M} \int d^3p (n_q^{\rightarrow} - n_q^{\leftarrow}) \delta\left(x - \frac{p^+}{M}\right) \quad (10)$$

$$h_1^q(x, \mu_0^2) = \frac{m_q}{M} \int d^3p (n_q^{\uparrow} - n_q^{\downarrow}) \delta\left(x - \frac{p^+}{M}\right) \quad (11)$$

$$(12)$$

where the spin dependent momentum distributions of the quark q are

$$n_q^{\rightarrow(\leftarrow)}(\vec{p}) = \langle PS_z | \sum_{i=1}^3 P_i^q \frac{1 + (-)\sigma_i^z}{2} | PS_z \rangle \quad (13)$$

$$n_q^{\uparrow(\downarrow)}(\vec{p}) = \langle PS_x | \sum_{i=1}^3 P_i^q \frac{1 + (-)\sigma_i^x}{2} | PS_x \rangle \quad (14)$$

$$(15)$$

Here P_i^q is the flavor projection operator. It is clear that due to rotational invariance the two matrix elements are identical, i.e., $h_1^q(x, \mu_0^2) = g_1^q(x, \mu_0^2)$. This is not the case in relativistic models where the contribution from the lower components makes them

different. In the latter case we will use [2, 16, 26]

$$g_1(x, \mu_0^2) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle PS_z | \bar{\Psi}(0) \not{n} \gamma_5 \Psi(\lambda n) |_{\mu_0^2} | PS_z \rangle \quad (16)$$

$$h_1(x, \mu_0^2) = \int \frac{d\lambda}{8\pi} e^{i\lambda x} \langle PS_\perp | \bar{\Psi}(0) [\not{S}_\perp, \not{n}] \gamma_5 \Psi(\lambda n) |_{\mu_0^2} | PS_\perp \rangle \quad (17)$$

where $n^\mu = \frac{1}{\sqrt{2p}}(1, 0, 0, -1)$. One should realize that these equations also incorporate the relativistic corrections to the non-relativistic calculations.

These set of equations represent the basis for the calculation of matrix elements at the hadronic scale. The sheme is completed by evolving using the conventional procedure developed for this problem in ref.[23].

3 Results

We will discuss the results in two models

- i) The non relativistic model of Isgur-Karl [24];
- ii) The relativistic MIT bag model [25].

In both cases we will use the corresponding support correction as defined in [23] and [18], respectively. In Figs. 1 we show the results corresponding to g_1 and h_1 for the first case. Fig. 1a corresponds to the pure valence quark hadronic scenario of ref.[23], characterized by a very low hadronic scale ($\mu_0^2 = 0.079 GeV^2$). Fig. 1b corresponds to the second scenario of ref.[23], where 40% of the momentum is carried by valence gluons and therefore the hadronic scale is larger ($\mu_0^2 = 0.3 GeV^2$). The initial data are evolved to $10 GeV^2$.

The figures show that the diverse evolution properties of these two structure functions lead to a large difference between the two initially identical functions. The difference occurs at small x and has been noted also by other authors [17, 21]. In Fig. 1a we show the Next to Leading Order evolution of g_1 as measure of our theoretical uncertainties. We do not expect in the case of h_1 an erratic NLO behavior and are confident that our discussion prevails beyond the leading order approximation.

In Figs. 2 we analyze the result of the same calculation in the support corrected MIT bag model. It has been argued [26] that gluons in the bag are not carrying momentum. This would imply, according to conventional QCD wisdom, that the hadronic scale should be very low ($\mu_0^2 \approx 0.079 GeV^2$)[16]. On the other hand the comparison of the bag f_1 moments with the experimental results [18] suggests a much larger hadronic scale ($\mu_0^2 \approx 0.75 GeV^2$). We show therefore in Figs 2. both scenarios.

This lack of precise definition of the hadronic scale has led to Fig. 3 where we analyze the dependence of the difference between h_1 and g_1 at small x as a function of the hadronic scale in the MIT bag model. It is clear from the figure that a small hadronic scale implies large differences between the two structure functions while the opposite is true in the

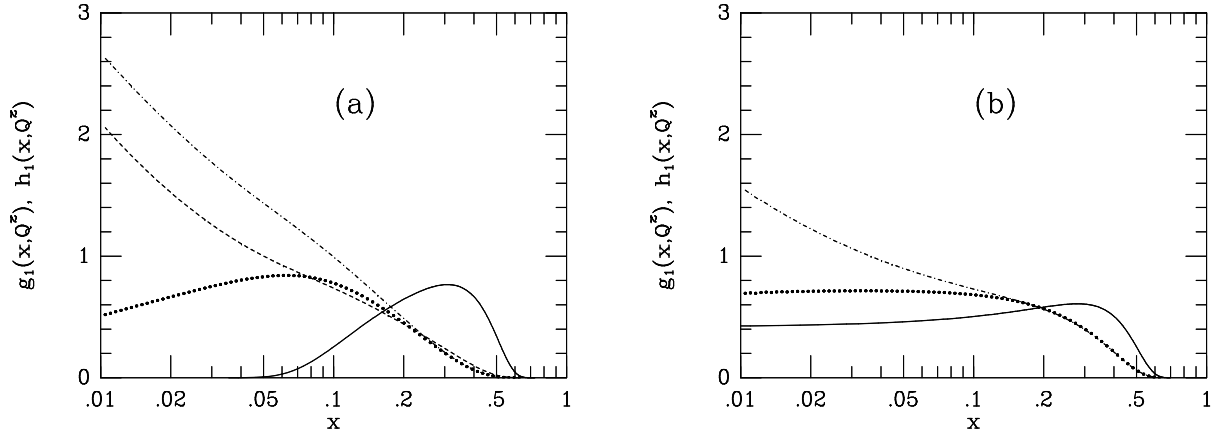


Figure 1: We show the transversity function $h_1(x, \mu_0^2)$ (continuous line) which coincides with the spin distribution function $g_1(x, \mu_0^2)$ for the a) Isgur-Karl model [24] at the hadronic scale $\mu_0^2 = 0.079 GeV^2$; b) Isgur-Karl based model with 40% valence gluons at the hadronic scale $\mu_0^2 = 0.3 GeV^2$ [23]. Their corresponding evolved (LO) distributions $h_1(x, Q^2)$ (dotted) and $g_1(x, Q^2)$ (dot-dashed) at $Q^2 = 10 GeV^2$ are also shown. In Fig a) we draw the (NLO) evolution of the spin structure function for comparison (dashed line).

case of large hadronic scales. The hadronic scale controls the magnitude of the evolution and since the difference arises mostly from evolution, the behavior at small x of these functions will control this parameter.

For completeness we show in the table the magnitude of the tensor and axial charges, noting that the latter are renormalization group invariant to leading order (see [15] where this calculation is discussed in detail for the MIT bag model.)

		μ_0^2	Q^2
δh	IK	0.270	0.183
	MIT	0.215	0.146
Δg	IK	0.270	0.270
	MIT	0.176	0.176

Table: The values of the tensor charge, $\delta h = \frac{1}{2} \sum_q e_q^2 \delta q$, and the spin charge, $\Delta g = \frac{1}{2} \sum_q e_q^2 \Delta q$, are shown, both at the hadronic scale and at the experimental scale, and for the two models studied. L.O. evolution from a hadronic scale of $\mu_0^2 = 0.079 GeV^2$ to $Q^2 = 10 GeV^2$ has been performed.

To finish this section we turn to Soffer's inequalities. Both models verify the primitive Soffer inequality eq.(5), not only at the hadronic scale, but also as we evolve the distribution function towards the physical regime. However Soffer argued [19] that his positivity bound could be used combined with data to limit the validity of models. In particular by imposing the simple relation

$$\Delta u(x) = u(x) - d(x) \quad (18)$$

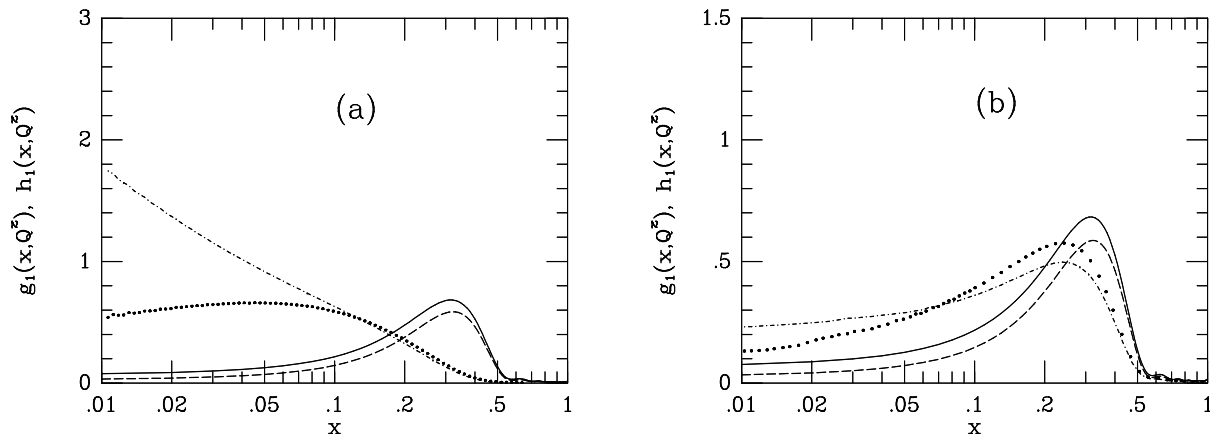


Figure 2: The transversity function $h_1(x, \mu_0^2)$ (continuous line) and the spin distribution function $g_1(x, \mu_0^2)$ (dashed line) for the support corrected MIT bag model [25] at the hadronic scale a) $\mu_0^2 = 0.079 \text{ GeV}^2$ and b) $\mu_0^2 = 0.75 \text{ GeV}^2$ are shown. Their corresponding evolved distributions $h_1(x, Q^2)$ (dotted) and $g_1(x, Q^2)$ (dot-dashed) at $Q^2 = 10 \text{ GeV}^2$ are also shown.

proposed in [27] and which is well supported by the data, it is possible to use the positivity bound to obtain the allowed range of values for h_1^u , namely

$$u(x) - d(x) \geq |h_1^u(x)| \quad (19)$$

The MIT bag model fails the bound for large values of x [19].

We show in Fig. 4 the comparison of the experimental constrain at 4 GeV^2 with the Isgur-Karl and MIT bag calculations. In the figure the allowed region is described by taking the lefthand side of eq.(19) from the data. The remaining curves represent the righthand side of the equation which we have calculated from the models. It is clear from the figure that at the hadronic scale, neither fulfils the constraint. Since these inequalities should be valid in the partonic regime [20], we show the values resulting from evolving the model calculations from the hadronic scale to the scale of the data (4 GeV^2). As the figure shows, greater consistency is achieved after this procedure. Moreover the analysis of Bourrely and Soffer [27] pays no attention to the possible errors associated with the experimental fit. If these are taken into account the consistency is even better. This result does not imply that the conventional models of hadron structure taken as a description of the physics at the hadronic scale are quantitatively succesful in explaining the deep inelastic data. As stated in previous analysis [23], these models give a qualitative description, which we have confirmed for the Soffer inequalities. However in order to obtain a quantitative description additional ingredients have to be added. By looking at Fig. 4 we rediscover the need for high momentum components in the Isgur-Karl model. Moreover the inclusion of gluons allows compatibility with a much larger hadronic scale. The same figure teaches us that the Stratmann scenario [16] with small hadronic scale for the MIT bag model is better realized then the large hadronic scale scenario of Jaffe and Ross [18].

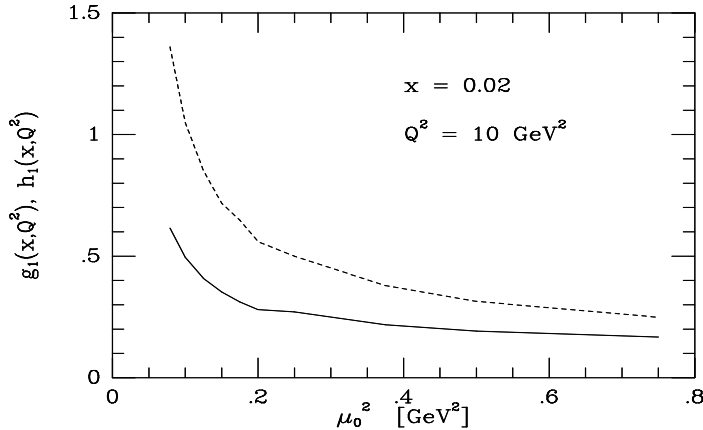


Figure 3: We show the value of h_1 (full) and g_1 (dashed) at $x = 0.02$ as a function of the hadronic scale for the support corrected MIT bag model calculation.

4 Conclusions

The use of models of hadron structure to describe the deep inelastic properties of the proton and neutron has proven successful for the chiral-even twist two structure functions [23] (and references therein). Several authors have generalized the analysis to the transversity functions [7, 16, 17]. Since these have not been measured, this analysis has the added value of prediction. We have completed the spectrum of possible calculations by including that of a well established not relativistic model, with a fine tuned technique for constructing the structure functions and performing the RGE evolution. Moreover we have returned to the highly successful field theoretic approach of the MIT bag model and reanalyzed some of the features questioning its validity, i.e., the highly discussed Soffer inequalities.

The features of the analysis which we next discuss are herewith well established. The evolution properties of the h_1 and g_1 structure functions is the main ingredient which distinguishes between them. We have analyzed the evolution from the hadronic scale to the experimental scale. In this scenario, even if the two structure functions are similar at the hadronic scale, the evolution makes them very different at the experimental scale. Moreover their difference occurs at small x . We can see that the quantitative difference between the various models is not large, therefore we are confident in the size of the estimates.

We have studied the dependence on the hadronic scale. This parameter turns out to be a main ingredient of the description since it controls the magnitude of the evolution process. In the case of the MIT bag model a small hadronic scale, as used for example by Stratmann [16] leads to considerable differences between the two structure functions at small x , while the large one proposed by Jaffe and Ross [18] implies small, but still detectable differences.

The above analysis leads to an experimental scenario characterized by precise mea-

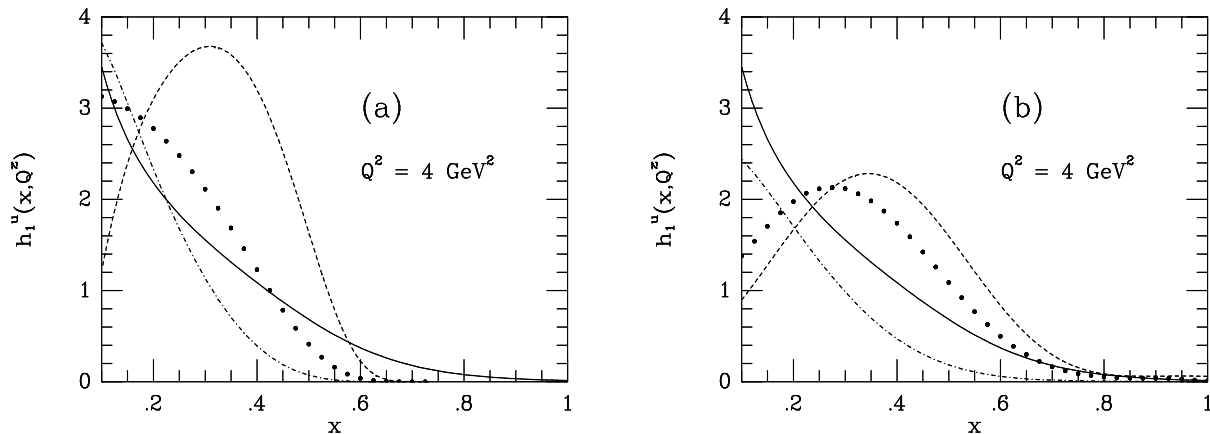


Figure 4: The allowed region of Soffer determined from the experimental data at 4GeV^2 , corresponds to the region inside the continuous line. Fig.a): the dashed line corresponds to the pure Isgur-Karl model calculation [24]; the dot-dashed line represents the evolved IK solution from an hadronic scale of 0.079GeV^2 , while the dotted line corresponds to the evolved solution of the IK model supplemented by gluons from an hadronic scale of 0.3GeV^2 . Fig b): the dashed line corresponds to the pure MIT calculation; the dotted line assumes a hadronic scale of 0.75GeV^2 , while the dot-dashed line one of 0.079GeV^2 , consistent with the Stratmann analysis [16]. The evolution has been carried out to leading order.

measurements in the small x region. The magnitude of the structure functions in that region is a measurement of the hadronic scale, while that of their difference gives more indication about the internal structure of the hadron.

Once it is realized that Soffer's inequalities are statements valid at the partonic scale and not at the hadronic scale, even if, as shown by Barone [21], they are respected by evolution, the models under scrutiny in this paper do satisfy them approximately as long as the hadronic scale is taken in the strict sense [16, 23].

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