The Gluon Spin in the Chiral Bag Model

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Abstract

We study the gluon polarization contribution at the quark model renormalization scale to the proton spin, Γ, in the chiral bag model. It is evaluated by taking the expectation value of the forward matrix element of a local gluon operator in the axial gauge $A^+ = 0$. It is shown that the confining boundary condition for the color electric field plays an important role. When a solution satisfying the boundary condition for the color electric field, which is not the conventionally used but which we favor, is used, the Γ has a positive value for *all* bag radii and its magnitude is comparable to the quark spin polarization. This results in a significant reduction in the relative fraction of the proton spin carried by the quark spin, which is consistent with the small flavor singlet axial current measured in the EMC experiments.

The EMC experiment[1] revealed the surprising fact that less than 30% of the proton spin may be carried by the quark spin. This is at variance with what one expects from non-relativistic or relativistic constituent quark models. This discrepancy – so called "the proton spin crisis" – can be understood as an effect associated with the axial anomaly[2]. For example it has been argued [3] that the experimentally measured quantity is not merely the quark spin polarization $\Delta\Sigma$ but rather the flavor singlet axial charge, to which the gluons contribute through the axial anomaly. Another interpretation [4] is that the anomalous gluons can induce a sea-quark polarization through the axial anomaly, which cancels the spin from the valence quarks, if the gluon spin component is positive. These explanations, and possibly others, could be reconciled if one would establish that they are gauge dependent statements, while the measured quantity is gauge-invariant [5].

Although not directly observable, an equally interesting quantity related to the proton spin is the fraction of spin in the proton that is carried by the gluons. In Ref.[6], the gluon spin Γ is introduced as

$$\tfrac{1}{2} = \tfrac{1}{2}\Sigma + L_Q + \Gamma + L_G, \tag{1}$$

where $L_{Q,G}$ is the orbital angular momentum of the corresponding constituent and Γ is defined as the integral of the polarized gluon distribution in analogy to Σ. The spin of course is gauge-invariant but the individual components in (1) may not be. Γ can be expressed as a matrix element of products of the gluon vector potentials and field strengths in the nucleon rest frame and in the $A^+ = 0$ gauge. When evaluated with the gluon fields responsible for the $N - \Delta$ mass splitting, Γ turns out to be negative, $\Gamma \sim -0.1\alpha_{\text{bag}}$, in the MIT bag model and even more so in the non-relativistic quark model.

By contrast, there are several other calculations that give results with opposite sign. For example, the QCD sum rule calculation[7] yields a positive value $2\Gamma \sim 2.1 \pm 1.0$ at 1 GeV^2. In Ref.[8], it is suggested that the negative Γ of Ref.[6] could be due to neglecting "self-angular momentum." The authors of [8] show that when self-interaction contributions are included, one obtains a positive value $\Gamma \sim +0.12$ in the Isgur-Karl quark model at the scale $\mu_0^2 \approx 0.25$ GeV^2. Once the gluons contribute a significant fraction to the proton spin, due to the normalization Eq.(1), the relative fraction of the proton spin lodged in the quark spin changes. Thus, the positive gluon spin seems to be consistent with the EMC experiment.

In this letter, we address this issue in the chiral bag model and pay special attention to the confining boundary condition for the gluon fields.

Let us start by briefly reviewing how the gluon spin operator is derived in Ref.[6, 11]. From the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\text{Tr}\left(F^{\mu\nu} F_{\mu\nu} \right) \tag{2}$$

with $F^{\mu\nu} = \frac{\lambda^a}{2} F^{a\mu\nu}$, one gets the gluon angular momentum tensor

$$M^{\mu\nu\lambda} = 2\text{Tr}\left(x^\nu F^{\mu\alpha} F_\alpha{}^\lambda - x^\lambda F_{\mu\alpha}{}^\nu \right) - (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu})\mathcal{L}. \tag{3}$$

Integrating by parts, we have

$$\begin{aligned}
M^{\mu\nu\lambda} &= 2\text{Tr}\Big(-F^{\mu\alpha}(x^\nu\partial^\lambda - x^\lambda\partial^\nu)A_\alpha + F^{\mu\lambda}A^\nu + F^{\nu\mu}A^\lambda \\
&\quad + \partial_\alpha(x^\nu F_{\mu\alpha}A^\lambda - x^\lambda F_{\mu\alpha}A^\nu) + \frac{1}{4}F^{\mu\nu}F_{\mu\nu}(x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}) \Big).
\end{aligned} \tag{4}$$

It seems reasonable to interpret the first term as the gluon orbital angular momentum contribution and the second as that of the gluon spin, while recalling that this is a gauge dependent statement. We will not consider the fourth term hereafter, since it contributes only to boosts. In Ref.[6, 11], the third term is also dropped as is done in the open space field theory. When finite space is involved, as in the bag model, dropping this term requires that the gluon fields satisfy boundary conditions on the surface of the region, as we next show. Let us express the gluon angular momentum operator in terms of the Poynting vector, i.e.,

$$\mathbf{J}_G = 2\mathrm{Tr}\int_V d^3r[\mathbf{r} \times (\mathbf{E} \times \mathbf{B})]. \tag{5}$$

Now doing the partial integration for $\mathbf{B} = \nabla \times \mathbf{A}$, we have

$$J_G^k = 2\mathrm{Tr}\left\{\int_V d^3r\left(E^l(\mathbf{r} \times \nabla)^k A_l + (\mathbf{E} \times \mathbf{A})^k\right) - \int_{\partial V} d^2r(\mathbf{r} \cdot \mathbf{E})(\mathbf{r} \times \mathbf{A})^k\right\}. \tag{6}$$

The surface term is essential to make the whole angular momentum operator gauge-invariant, but the surface term only vanishes, if the electric field satisfies the boundary condition on the surface,

$$\mathbf{r} \cdot \mathbf{E} = 0. \tag{7}$$

This is just the MIT boundary condition for gluon confinement. However, the static electric field traditionally used [9] does not satisfy this condition. Instead color singlet nature of the hadron states is imposed to assure confinement globally.

We next show that the negative Γ of Ref.[6] results if this procedure to confine color is imposed. To proceed, we choose the $A^+ = 0$ gauge and write the gluon spin in a local form as

$$\Gamma = \langle p, \uparrow | 2\mathrm{Tr}\int_V d^3x\left((\mathbf{E} \times \mathbf{A})^3 + \mathbf{B}_\perp \cdot \mathbf{A}_\perp\right)|p, \uparrow\rangle, \tag{8}$$

where $\perp$ denotes the direction perpendicular to the proton spin polarization and the superscript $+$ indicates the light cone coordinates defined as $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^3)$. We shall evaluate this expression by incorporating the exchange of the static gluon fields between i-th and j-th quarks $(i \neq j)$ which are responsible for the $N - \Delta$ mass splitting in the bag model.

In the chiral bag model, the static gluon fields are generated by the color charge and current distributions of the i-th valence quark given by[10]

$$J_i^{0a}(\mathbf{r}) = \frac{g_s}{4\pi}\rho(r)\frac{\lambda_i^a}{2}, \tag{9}$$

$$\mathbf{J}_i^a(\mathbf{r}) = \frac{g_s}{4\pi}3(\hat{\mathbf{r}} \times \mathbf{S})\frac{\mu'(r)}{r^3}\frac{\lambda_i^a}{2}, \tag{10}$$

where $\rho(r)$ and $\mu'(r)$ are, respectively, the quark number and current densities determined by the valence quark wave functions. (See Ref.[10] for their explicit formulas.) They are very similar in form to those of the MIT bag model. There is, however, an essential difference, namely, that the spin in the chiral bag model is given by the collective rotation of the whole system while in the MIT bag it is given by a individual contribution of each constituent, i.e., there is no index i in the spin operator in Eq.(10).

The charge and current densities yield the color electric and magnetic fields as

$$\mathbf{E}_i^a = \frac{g_s}{4\pi} \frac{Q(r)}{r^2} \frac{\lambda_i^a}{2} \hat{\mathbf{r}}, \tag{11}$$

$$\mathbf{B}_i^a = \frac{g_s}{4\pi} \left\{ \mathbf{S} \left(2M(r) + \frac{\mu(R)}{R^3} - \frac{\mu(r)}{r^3} \right) + 3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{S}) \frac{\mu(r)}{r^3} \right\} \frac{\lambda_i^a}{2}, \tag{12}$$

where

$$\mu(r) = \int_0^r dr' \mu'(r'), \tag{13}$$

$$M(r) = \int_r^R dr' \frac{\mu'(r')}{r'^3}. \tag{14}$$

The quantity $Q(r)$ can be determined from Maxwell's equations. The most general solution can be written as

$$Q_\lambda(r) = 4\pi \int_\lambda^r dr' r'^2 \rho(r'), \tag{15}$$

with an arbitrary λ. The standard procedure is to choose $\lambda = 0$ so that the electric field is regular at the origin. This has been the adopted convention in the early days[9]. We will refer to this solution as $Q_0(r)$. However, $Q_0(r)$ does not satisfy the *local* boundary condition (7), since it is normalized as $Q_0(R) = 1$. In Ref.[9], the fact that hadrons are color singlet states, had to be imposed in order to justify the use of this solution.

Another solution is obtained by setting $\lambda = R$ [10] and we will look for its consequences here. This choice satisfies the local boundary condition but requires the relaxation of the continuity of the electric fields inside the bag. It has been shown in [10], and will be shown here again, that these two solutions, $Q_0(r)$ and $Q_R(r)$, lead to dramatic differences for certain observables.

By using the static Green functions and the Coulomb gauge condition, one can obtain time-independent scalar and vector potentials from the charge and current densities, (9) and (10),

$$\Phi_i^a(\mathbf{r}) = \frac{g_s}{4\pi} \frac{Q_\lambda(r)}{r} \frac{\lambda_i^a}{2}, \tag{16}$$

$$\mathbf{U}_i^a(\mathbf{r}) = \frac{g_s}{4\pi} h(r)(\mathbf{S} \times \mathbf{r}) \frac{\lambda_i^a}{2}, \tag{17}$$

where

$$h(r) = \left(\frac{\mu(R)}{2R^3} + \frac{\mu(r)}{r^3} + M(r) \right). \tag{18}$$

From these, the appropriate scalar and vector potentials satisfying the $A^+ = 0$ gauge condition can be constructed:

$$A_i^{a0}(\mathbf{r}) = \Phi_i^a(\mathbf{r}),$$
$$\mathbf{A}_i^a(\mathbf{r}) = \mathbf{U}_i^a(\mathbf{r}) - \nabla \int_0^z d\zeta \; \Phi_i^a(x, y, \zeta), \tag{19}$$

where the direction of the proton polarization is taken as that of the z-axis. Finally, we obtain

$$\Gamma_\lambda = \sum_{i \neq j} \sum_{a=1}^8 \langle p, \uparrow | \left(2 \int_V d^3x (\mathbf{E}_i^a \times \mathbf{U}_j^a)^3 \right.$$

3

$$+ \int_{\partial V} d^2 s \hat{\mathbf{z}} \cdot \hat{\mathbf{r}} \left(U_i^{a1}(\mathbf{x}) \int_0^z d\zeta \, E_i^{a2}(x, y, \zeta) \right.$$

$$\left. \left. - U_i^{a2}(\mathbf{x}) \int_0^z d\zeta \, E_i^{a1}(x, y, \zeta) \right) \right) |p, \uparrow\rangle$$

$$= \frac{4}{3}\alpha \int_0^R r dr \, Q_\lambda(r)(h(R) - 2h(r)), \tag{20}$$

where $\alpha = g_s^2/4\pi$. The numerical factor in front of the final formula comes from the fact that $\sum_a \langle \lambda_i^a \lambda_j^a \rangle_{\text{baryon}} = -8/3$ for $i \neq j$ so that

$$\sum_{i \neq j} \sum_{a=1}^{8} \langle p, \uparrow | S_3 \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2} | p, \uparrow \rangle = -2, \tag{21}$$

and the integration over the angle yields $1/3$. It is different from $8/9$ of the MIT bag model[6], which comes from the expectation value

$$\sum_{i \neq j} \sum_{a=1}^{8} \langle p, \uparrow | \sigma_i^3 \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2} | p, \uparrow \rangle = -4/3. \tag{22}$$

It is interesting to note that, if we naively substitute the static gluon fields Φ_i^a and $\mathbf{U}_i^a$ of Eqs.(16) and (17) satifying the Coulomb gauge condition into the second term of Eq.(6), we get

$$\Gamma' = -\frac{4}{3}\alpha \int_0^R r dr \, Q_\lambda(r)h(r), \tag{23}$$

which is the same expresion that was used in Refs.[10, 12] to evaluate the anomalous gluon contribution to the flavor singlet axial current a_0 with the extra factor $(-N_f \alpha/2\pi)$, i.e., $a_0 = \Sigma - (N_f \alpha/2\pi)\Gamma'_\lambda$. On the other hand, in Ref.[8], the gluon spin Γ instead of Γ' is used for the anomaly correction term because the calculation is performed in the $A^+ = 0$ gauge.

If the gluons can contribute to the proton spin, then the collective coordinate quantization scheme of the chiral bag model has to be modified to incorporate their contribution. That is because there is a natural sum rule namely that the total proton spin must come out to be $\frac{1}{2}$, whatever the various contributions are. In the chiral bag model, where the mesonic degrees of freedom also play an important role, the proton spin is described by the following contributions

$$\tfrac{1}{2} = \tfrac{1}{2}\Sigma + L_Q + \Gamma + L_G + L_M, \tag{24}$$

where L_M, the orbital angular momentum of the mesons, has to be added to Eq.(1). The proton spin is generated by quantizing the collective rotation associated with the zero modes of the classical soliton solution of the model Lagrangian. To the collective rotation, each constituent responds with the corresponding moment of inertia. The moments of inertia of the quarks and mesons, $\mathcal{I}_Q$ and $\mathcal{I}_M$, have been extensively studied in the literature[14]. Substitution of the color electric and magnetic fields, given by Eqs.(11) and (12) respectively, into Eq.(5) defines a new moment of inertia of the static gluon fields with respect to the collective rotation as

$$\langle \mathbf{J}_G \rangle = -\mathcal{I}_G \omega, \tag{25}$$

where the expectation value is taken keeping only the exchange terms, and ω is the classical angular velocity of the collective rotation.

We show in Fig.1(a) and (b) the gluon moment of inertia evaluated by using the color electric fields with $Q_R(r)$ and $Q_0(r)$. In the case of $Q_R(r)$, $\mathcal{I}_G$ is positive for all bag radii and comparable in size to $\mathcal{I}_Q$, the quark moment of inertia. On the other hand, $Q_0(r)$ results in a negative $\mathcal{I}_G$. This "negative" moment of inertia may appear to be bizarre but it may not be a problem from the conceptual point of view. The $\mathcal{I}_G$ defined by Eq.(25) can be interpreted as the one-gluon exchange correction to the corresponding quantity of the quark phase, which is still positive anyway. The point is that the spin fractionizes in the same way as the moment of inertia does. This means that we have

$$
\begin{aligned}
L_Q + \tfrac{1}{2}\Sigma &= \frac{\mathcal{I}_Q}{2(\mathcal{I}_Q + \mathcal{I}_G + \mathcal{I}_M)}, \\
L_G + \Gamma &= \frac{\mathcal{I}_G}{2(\mathcal{I}_Q + \mathcal{I}_G + \mathcal{I}_M)}, \\
L_M &= \frac{\mathcal{I}_M}{2(\mathcal{I}_Q + \mathcal{I}_G + \mathcal{I}_M)}.
\end{aligned}
\tag{26}
$$

Each fraction as a function of the bag radius is presented in the small boxes inside each figure. Note in the case of adopting $Q_R(r)$ that at the large bag limit the proton spin is equally carried by quarks and gluons somewhat like the momentum of the proton. The negative $\mathcal{I}_G$ obtained with $Q_0(r)$, thus, yields a scenario where the gluons are anti-aligned with the proton spin.

The dashed and dash-dotted curves in Figs.2(a) and (b) show the values for Γ_λ and Γ'_λ. For comparison, we draw $\tfrac{1}{2}\Sigma$ by a solid curve. Note that, because of the difference in $\mathcal{I}_G$, even $\tfrac{1}{2}\Sigma$ is different according to which Q_λ is used. Again, both Γ_0 and Γ'_0 are anti-aligned with the proton spin. Note of course that the negative Γ'_0 is apparently at variance with the general belief that the anomaly is to cure the proton spin problem.

To conclude, we show in Figures 3(a) and (b) the flavor-singlet axial current including the $U_A(1)$ anomaly given by

$$
a_0 = \Sigma - (N_f \alpha/2\pi)\Gamma'_\lambda.
\tag{27}
$$

For simplicity, we neglect other contributions to a_0 studied in [13]. They show that the positive Γ is consistent with the small a_0 measured in the EMC experiments. The radius dependence of each component may be viewed as gauge dependence both in color gauge symmetry and in the "Cheshire Cat" gauge symmetry discussed by Damgaard, Nielsen and Sollacher [15].

Acknowledgments

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Figure 1: The moment of inertia associated with the collective rotation as a function of the bag radius and the proton spin fraction carried by each constituents. In the calculation, we have used (a) the "confined" color electric field with $Q_R(r)$ and (b) the conventional one with $Q_0(r)$.
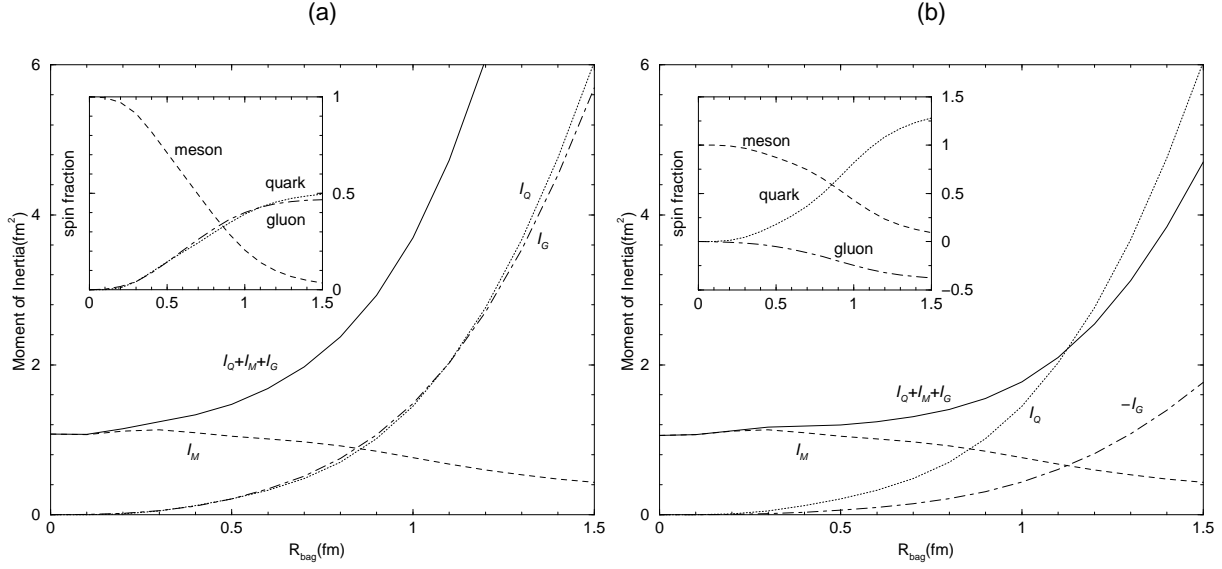
(a)

(b)

Figure 2: The gluon spin Γ as a function of the bag radius. (a) and (b) are obtained with the color electric fields explained in Fig. 1.
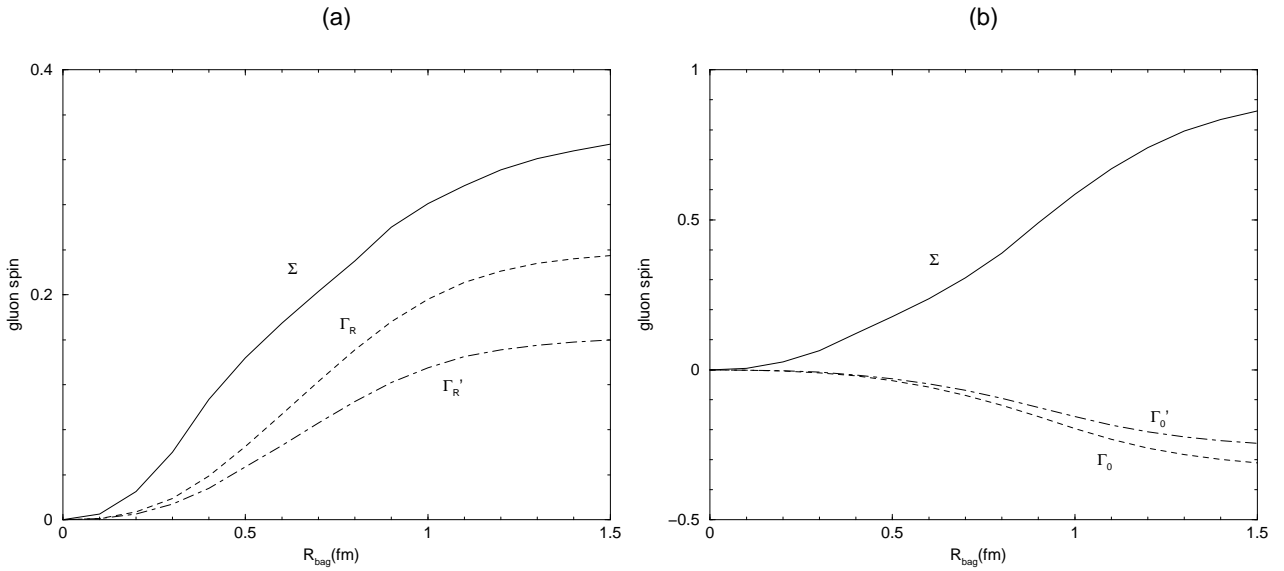
(a)

(b)

Figure 3: The flavor singlet axial current a_0 as a function of the bag radius. (a) and (b) are obtained with the color electric fields explained in Fig. 1.

(a)

(b)