Instantons and the $\Delta I = 1/2$ Rule

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The instanton induced interaction leads to a significant enhancement of the A_0 weak amplitude determining the $\Delta I = 1/2$ rule, through the contribution of operators with dimension $d = 9$, as we show in the weak $K \rightarrow \pi\pi$ decay.

Recent experiments confirmed the large CP violation in $K \rightarrow \pi\pi$ decays [1,2]. One of the cornerstones of this problem is famous $\Delta I = 1/2$ rule [3,4]. This phenomenological rule is related to the observation of a large enhancement of the weak decays with an isospin change of $\Delta I = 1/2$ with respect to those decays with one of $\Delta I = 3/2$. Several contributions have been considered responsible for the enhancement [5–7]. One of them is the well known perturbative QCD contribution due to the exchange of hard gluons [8] . It arises from short distances, and large quark and gluon virtualities. Typically the enhancement factor of these calculations is four, far away from the data [9]. Another possible source of the rule comes from long distance hadronic final state interaction (FSI) and can lead to an enhancement of the A_0 amplitude which reaches about half of the experimental value [10,11].

We propose a new large contribution to the weak amplitudes arising from QCD through the nonperturbative multi-quark 't Hooft interaction [12], induced by strong fluctuations of the gluon fields known as instantons, which strongly favors the rule. This interaction has flavor properties, very distinct from those of the perturbative gluon exchange, which magnify the interaction in channels , like the $I = 0$ channel of the weak decays, with vacuum quantum numbers. These properties are instrumental in the resolution of the $U(1)_A$ problem [13] explaining the large mass of η' meson. We show here that the same mechanism is relevant for understanding the $\Delta I = 1/2$ rule.

A multi-quark interaction arises from the existence of quark zero modes in the instanton field. For $N_f = 3$ and for zero current quark masses, $m_u = m_d = m_s = 0$, this interaction is given by [14,15]:

$$\mathcal{H}_{tHooft} = \int d\rho n(\rho)(4\pi^2\rho^3)^3 \frac{1}{6N_c(N_c^2 - 1)} \epsilon_{f_1 f_2 f_3} \epsilon_{g_1 g_2 g_3}$$

$$\{\frac{2N_c + 1}{2N_c + 4} \bar{q}_R^{f_1} q_L^{g_1} \bar{q}_R^{f_2} q_L^{g_2} \bar{q}_R^{f_3} q_L^{g_3}$$

$$+ \frac{3}{8(N_c + 2)} \bar{q}_R^{f_1} q_L^{g_2} \bar{q}_R^{f_2} \sigma_{\mu\nu} q_L^{g_2} \bar{q}_R^{f_3} \sigma_{\mu\nu} q_L^{g_3}$$

$$+ (R \leftrightarrow L)\}, \tag{1}$$

where ρ is the instanton size and $n(\rho)$ is the density of the instantons. In calculations we use the instanton liquid model for the QCD vacuum [16,17](see reviews ref. [18]). For quarks with nonzero virtualities, k_i^2, the vertex (1) should be multiplied by the product of Fourier transformed quark zero modes in the instanton field

$$Z = \prod_i F(k_i^2), \tag{2}$$

which in the singular gauge has the form

$$F(k_i^2) = -x\frac{d}{dx} \{I_0(x)K_0(x) - I_1(x)K_1(x)\}, \tag{3}$$

where $x = \rho\sqrt{k_i^2}/2$ [19].

This interaction has a large quark helicity flip $\Delta Q = 2N_f$, which comes from the definite helicity of the quarks on zero modes. Moreover the Pauli Principle of these quarks implies that the interaction is antisymmetric under permutations of any incoming and any outgoing quark. This property leads to a single instanton contribution to the weak $\Delta I = 1/2$ amplitude (see Fig.1).

The standard $\Delta S = 1$ weak effective Hamiltonian is given by

$$\mathcal{H}_{eff}^{\Delta S=1} = \sqrt{2}G_F V_{ud}V_{us}^* \sum_{i=1}^{8} C_i(\mu)Q_i(\mu), \tag{4}$$

where the Q_i operators, with dimension $d = 6$, used are those of [8] and the coefficients $C_i(\mu)$ come from calculation of the hard perturbative gluon coupling to the weak amplitudes. The scale $\mu \approx 1GeV$ of these calculations

determines the kinematical region where one can believe pQCD, and the lower limit of the integration over the virtuality of the quarks and gluons in loops created by gluon exchanges. The matrix elements of the $Q_i(\mu)$ operators in (4) are calculated within nonperturbative approaches, for example by bosonising the quark operators or by performing lattice simulations.
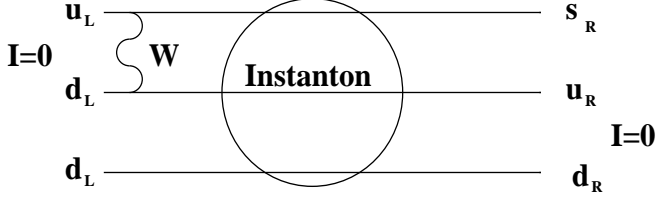

Fig.1. The contribution of the six-quark instanton induced interaction to the $\Delta I = 1/2$ weak amplitude. W denotes the W-boson exchange.

Our important observation is that there is some additional term in the weak interaction Hamiltonian with $\Delta S = 1$, coming from the six-quark interaction (Fig.1), which corresponds to the operator of dimension $d = 9$,

$$Q_{I+\bar{I}}^{d=9} = \frac{2N_c + 1}{2N_c + 4} \bar{u}_R d_L \bar{s}_R u_L \bar{d}_R d_L$$
$$+ \frac{3}{8(N_c + 2)} \bar{u}_R d_L \bar{s}_R \sigma_{\mu\nu} u_L \bar{d}_R \sigma_{\mu\nu} d_L$$
$$+ (-1)^P perm.(u_R, d_R, s_R) + (R \leftrightarrow L) \quad (5)$$

where P is number of the quark permutations and which contributes only to $\Delta I = 1/2$ transitions. Compared with the gluon induced operators (4), this operator violates helicity conservation.

With respect to the scale of the new six-quark operators a comment is needed. We would like to treat the operators in (4) as local, therefore the integration over the quark virtualities in the loop of Fig.1 should be limited by the hadronization scale $\tilde{\mu} \approx \Lambda_{QCD} \approx 1/R \approx m^* = 260 MeV$, where $m^* = -2\pi^2 \rho_c^2 < 0|\bar{q}q|0 > /3$ is the effective quark mass in the instanton vacuum.

One of the manifestations of the $\Delta I = 1/2$ rule is a huge enhancement of the $K \to \pi\pi$ amplitude in the isospin- zero state A_0 as compared with amplitude to the isospin-two $\pi\pi$, A_2, i.e., 22.2.

Our instanton induced weak interaction, which only contibutes to the A_0 amplitude, is shown in Fig.2, for the K^0 decays. We use the normalization for the $K \to \pi\pi$ amplitude of Bel'kov et al. in [4]

$$M_{K^0 \to \pi^+\pi^-} = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} + \frac{1}{\sqrt{3}} A_2 e^{i\delta_2}$$

$$M_{K^0 \to \pi^0\pi^0} = \sqrt{\frac{2}{3}} A_0 e^{i\delta_0} - \frac{2}{\sqrt{3}} A_2 e^{i\delta_2}$$

$$M_{K^+ \to \pi^+\pi^0} = \frac{\sqrt{3}}{2} A_2 e^{i\delta_2}. \quad (6)$$
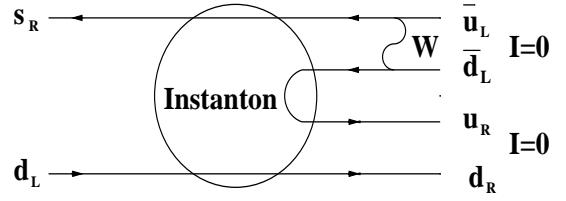

Fig.2. The contribution of the six-quark instanton induced interaction to the $K^0 \to \pi\pi$ decay.

The calculation of the diagram in Fig.2 gives for the induced effective Hamiltonian, responsible for the $K^0 \to \pi^+\pi^-$ decay and $N_c = 3$,

$$\mathcal{H}_{inst}^{K^0 \to \pi^+\pi^-} = \frac{C_1(\mu)G_F}{\sqrt{2}} V_{ud}V_{us}^* \int d\rho \frac{n(\rho)}{2\pi^2} (\frac{4\pi^2\rho^3}{3})^3$$
$$\int_{\tilde{\mu}}^{\mu} dk k F^2(k\rho/2) \bar{u}_R d_L \{\bar{s}_R u_L \bar{d}_R d_L$$
$$+ \frac{3}{32}(\bar{s}_R \lambda^a u_L \bar{d}_R \lambda^a d_L$$
$$- \frac{3}{4}\bar{s}_R \sigma_{\mu\nu} \lambda^a u_L \bar{d}_R \sigma_{\mu\nu} \lambda^a d_L)\}, \quad (7)$$

where $C_1(\mu)$ is the Wilson coefficient of Q_1 with $I = 0$ in (4), which is related to the pQCD contribution for values of the quark virtualities between μ and M_W.

By using appropriate Fierz transformations and making the PCAC substitutions

$$\bar{d}\gamma_5 u = \frac{-i\sqrt{2}F_\pi m_\pi^2}{m_u + m_d}\phi_{\pi^+},$$
$$\bar{u}\gamma_5 d = \frac{-i\sqrt{2}F_\pi m_\pi^2}{m_u + m_d}\phi_{\pi^-},$$
$$\bar{s}\gamma_5 d = \frac{-i\sqrt{2}F_K m_K^2}{m_s + m_d}\phi_{K^0}, \quad (8)$$

where $F_\pi = 93 MeV$, we arrive at the following matrix element

$$M_{K^0 \to \pi^+\pi^-} = -C_1(\mu)G_F V_{ud}V_{us}^* \frac{11}{96\pi^2}$$
$$(\frac{F_\pi m_\pi^2}{m_u + m_d})^2 (\frac{F_K m_K^2}{m_s + m_d})$$
$$\int d\rho n(\rho)(\frac{4\pi^2\rho^3}{3})^3 \int_{\tilde{\mu}}^{\mu} dk k F^2(k\rho/2). \quad (9)$$

We obtain an estimate of the instanton contribution by using Shuryak's instanton liquid model [16] with the density given by

$$n(\rho) = \frac{n_{eff}}{(m^*\rho)^3}\delta(\rho - \rho_c) \quad (10)$$

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and

$$n_{eff} = 1 fm^{-4}, \ \rho_c = 1.6 GeV^{-1} \qquad (11)$$

In chiral limit, $m_u = m_d = m_s = 0$, from (9) we obtain

$$M_{K^0 \to \pi^+ \pi^-} = -C_1(\mu) G_F V_{ud} V_{us}^* \frac{11}{12\pi^2 F_\pi^3} n_{eff}$$
$$\int_{\tilde{\mu}}^{\mu} dk k F^2(k\rho_c/2) \qquad (12)$$

where the Gell-Mann-Oakes-Renner relations

$$F_\pi^2 m_\pi^2 = -m_u < 0|\bar{u}u|0> -m_d < 0|\bar{d}d|0>,$$
$$F_K^2 m_K^2 = -m_s < 0|\bar{s}s|0> -m_d < 0|\bar{d}d|0> \qquad (13)$$

have been used.

Therefore our final result for the six-quark instanton interaction contribution to A_0 amplitude is

$$A_0^{d=9} = -\frac{\sqrt{3}C_1(\mu)G_F}{\sqrt{2}} V_{ud} V_{us}^* \frac{11}{12\pi^2 F_\pi^3} n_{eff}$$
$$\int_{\tilde{\mu}}^{\mu} dk k F^2(k\rho_c/2), \qquad (14)$$

With the chosen value of the parameters (11) and LO value $C_1(1GeV) \approx c_1(1GeV) - c_2(1GeV) \approx 1.9$, where the values of c_1 and c_2 are given in [4] for $\Lambda_{\overline{MS}}^{(4)} = 215 MeV$, we have for ratio

$$\frac{A_0^{d=9}}{A_0^{exp}} = 0.5, \qquad (15)$$

where the A_0^{exp} is the experimental amplitude [21].

The contribution of the instantons leads to a strong enhancement of the A_0 amplitude in weak K meson decays. Let us discuss the various contributions to the final number by using the, large N_C, A_0 amplitude

$$A_0^{N_c \to \infty} = -\sqrt{\frac{3}{2}} G_F V_{ud} V_{us}^* F_\pi (m_K^2 - m_\pi^2) \qquad (16)$$

as a scale. The pQCD corrections provide us with a factor of about 1.9, while the pure instanton contribution leads to a factor of about 2.0. One should not forget that additional contributions to the ratio, for example FSI, will further increase it.

Away from chiral limit there are corrections to Eq.(15) coming from additional terms proportional to the current quark masses in the PCAC relations Eq.(8). Moreover there are other contributions arising also from $SU(3)$ breaking in $K \to \pi\pi$ amplitude, e.g., those arising from $d = 6$ instanton induced operators, others from the quark non-zero mode contributions to the instanton field, etc. Their analysis is beyond the skope of this paper. Anyway we expect the chiral expansion to be relatively soft and the ratio to change at most at the level of $m_K^2/\Lambda_\chi^2 \approx 25\%$.

The most intriguing term, because of its lower dimensionality, is the one with dimension $d = 6$. Its corresponding operators arise from the reduction of the six-quark 't Hooft interaction to a four-quark interaction by closing one of the quark lines by a quark condensate. In the $SU(3)_f$ limit and for $N_c = 3$ the effective $\Delta S = 1$ lagrangian term induced by this interaction has the form Eq.(4) with

$$Q_{I+\bar{I}}^{d=6} = \bar{u}_R d_L \bar{s}_R u_L + \frac{1}{4} \bar{u}_R \sigma_{\mu\nu} d_L \bar{s}_R \sigma_{\mu\nu} u_L$$
$$-(u_R \leftrightarrow d_R) + (R \leftrightarrow L) \qquad (17)$$

and

$$C(\mu)^{d=6} = \frac{2C_1(\mu) n_{eff}}{3 < 0|\bar{q}q|0 >^2 \pi^2}$$
$$\int_{\tilde{\mu}}^{\mu} dk k F^2(k\rho_c/2), \qquad (18)$$

The use of the vacuum insertion method [20] shows that in chiral limit the contribution of this operator to $K^0 \to \pi^+\pi^-$ decay amplitude is zero.

We have shown that a novel mechanism arising from the $N_f = 3$ 't Hooft instanton induced interaction contributes considerably to the empirical $\Delta I = 1/2$ rule found in the weak $\Delta S = 1$ decays. This instanton induced multi-quark interaction, due to its specific flavor dependence, is able to contribute to the strong enhancement of A_0 amplitude in $K \to \pi\pi$ decays. Moreover it proclaims the importance of the contribution of higher dimensional operators [22], in particular $d = 9$ in our case, and the quantum numbers of the instanton induced interaction, in the weak decays.

We should mention that recently an attempt to incorporate instanton physics into the description of weak processes has been carried out by Franz et al. [23] within the Chiral Quark Model of Diakonov and Petrov [17,24]. This approach takes into account only those terms of the $N_f = 2$ instanton induced lagrangian which are leading order in the $1/N_C$ expansion and therefore does not consider the mechanism we propose here, based on the full $N_f = 3$ instanton induced lagrangian. Our mechanism is an alternative to that proposed by Franz et al. which requires, in order to produce a relevant ratio, anomalously small values of the constituent quark masses at zero virtuality, thus destroying the previous achievements of the Diakonov-Petrov scheme.

Our calculation has been performed in the chiral limit. Much work needs to be done to relax this limit. We have discussed some of the new mechanisms one might encounter. However, the fact that in the chiral limit our result is quantitatively relevant signals the importance of instanton physics in this field. One may thus conclude, that direct instanton contributions of the type discussed here cannot be omitted in any serious study of the non-leptonic decays and are important in closing the gap be-

tween the theoretical interpretation and the experimental value.

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