Generalized Parton Distributions and Constituent Quarks

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An approach is described to calculate Generalized Parton Distributions (GPDs) in Constituent Quark Models (CQM). The GPDs are obtained from wave functions to be evaluated in a given CQM. The general relations linking the twist-two GPDs to the form factors and to the leading twist quark densities are recovered. Results for the leading twist, unpolarized GPD in the Isgur and Karl model are presented.

1. INTRODUCTION

Generalized Parton Distributions (GPDs) are becoming one of the main topics of interest in hadronic physics [1]. GPDs are a natural bridge between exclusive processes, such as elastic scattering, described in terms of form factors, and inclusive ones, described in terms of structure functions. As it happens for the usual Parton Distributions (PDs), the measurement of GPDs provides us with a unique way to access several features of the structure of the nucleon, such as the quark orbital angular momentum contribution to the proton spin [2,3]. Therefore, relevant experimental efforts to measure GPDs will take place in the next few years and it becomes urgent to produce predictions for these quantities. Several calculations have been already performed by using different approaches [1,4] and an impressive effort has been devoted to study their QCD evolution properties [5].

A step towards calculations of GPDs in Constituent Quark Models (CQM) can be found in [6], and a consistent approach has been proposed in [7]. The CQM has a long story of successful predictions in low energy studies of the structure of the nucleon. In the high energy sector, in order to compare model predictions with data, one has to evolve, according to QCD, the leading twist component of the physical structure functions obtained at the low momentum scale associated with the model. Such a procedure, already addressed in [8], has proven successful in describing the gross features of standard PDs by using different CQM (see, e.g., [9]). Similar expectations motivated the study of GPDs described in [7], where a simple formalism is described to calculate GPDs from any model. In this talk, the approach of [7] is reviewed and applied to the Isgur and Karl (IK) [10] model.

2. GENERAL FORMALISM

Let us think to diffractive DIS off a nucleon target, with initial and final momenta P and P', respectively. GPDs describe the amplitude for finding a quark with momentum fraction $x+\xi/2$ (in the IMF) in the nucleon and replacing it back into the nucleon with a momentum transfer Δ^μ. The GPD $H(x,\xi,\Delta^2)$ is introduced by defining the twist-two part of the light-cone correlation function [2]

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\bar{\psi}(-\frac{\lambda n}{2})\gamma^\mu \psi(\frac{\lambda n}{2})|P\rangle = \\ = H(x,\xi,\Delta^2)\bar{U}(P')\gamma^\mu U(P) + E(x,\xi,\Delta^2)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + ... \quad (1)$$

where ellipses denote higher-twist contributions, ψ is a quark field and M is the nucleon mass. The ξ variable, the so called "skewedness", is defined by the relation $\xi = -n \cdot \Delta$, where $n = (1,0,0,-1)/(2\Lambda)$ and Λ depends on the reference frame. The ξ variable is bounded by 0 and $\sqrt{-\Delta^2}/\sqrt{M^2 - \Delta^2/4}$. Besides, one has $t = \Delta^2 = \Delta_0^2 - \vec{\Delta}^2$. When the longitudinal momentum fraction of the quark is less than $-\xi/2$, GPDs describe antiquarks; when it is larger than $\xi/2$, they describe quarks; when it is between $-\xi/2$ and $\xi/2$, they describe $q\bar{q}$ pairs. There are two natural limits for $H(x,\xi,\Delta^2)$: i) when $P' = P$, i.e., $\Delta^2 = \xi = 0$, the so called "forward" limit, one recovers the usual PDs

$$H(x,0,0) = q(x) \quad (2)$$

ii) the integration over x yields the Dirac Form Factor (FF)

$$\int dx H(x,\xi,\Delta^2) = F_1(\Delta^2) \ . \quad (3)$$

Any model estimate of the GPDs has to respect the above two crucial constraints.

In Ref. [7], the Impulse Approximation (IA) expression for the GPD $H(x,\xi,\Delta^2)$, suitable to perform CQM calculations, has been obtained. In particular it has been found that, substituting the quark fields in the left-hand-side of Eq.(1), taking into account the quarks degrees of freedom only, using IA, considering a process with $\vec{\Delta}^2 \ll M^2$ in the Nucleon rest frame, using a symmetric wave function (as the one given in a NR quark model once color has been taken into account), one obtains

$$H(x,\xi,\Delta^2) = \int d\vec{k} \ \delta\left(x + \frac{\xi}{2} - \frac{k^+}{M}\right) \tilde{n}(\vec{k},\vec{k}+\vec{\Delta}) \ , \quad (4)$$

where $\tilde{n}(\vec{k},\vec{k}+\vec{\Delta})$ is the one-body non-diagonal momentum distribution:

$$\tilde{n}(\vec{k},\vec{k}+\vec{\Delta}) = 3\int \psi^*(\vec{k}_1,\vec{k}_2,\vec{k}+\Delta)\psi(\vec{k}_1,\vec{k}_2,\vec{k})d\vec{k}_1 d\vec{k}_2 = \\ = \int e^{i((\vec{k}+\vec{\Delta})\vec{r}-\vec{k}\vec{r}')}\rho(\vec{r},\vec{r}')d\vec{r}d\vec{r}' \ , \quad (5)$$

defined through the one-body non diagonal charge density

$$\rho(\vec{r},\vec{r}') = \int \psi^*(\vec{r}_1,\vec{r}_2,\vec{r}')\psi(\vec{r}_1,\vec{r}_2,\vec{r})d\vec{r}_1 d\vec{r}_2 \ . \quad (6)$$

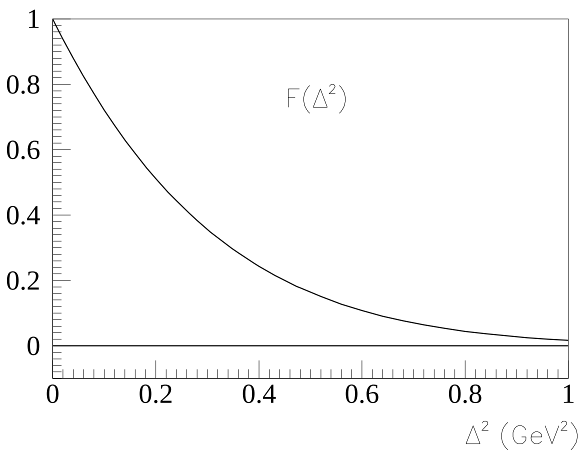

Figure 1. The charge FF in the IK model.

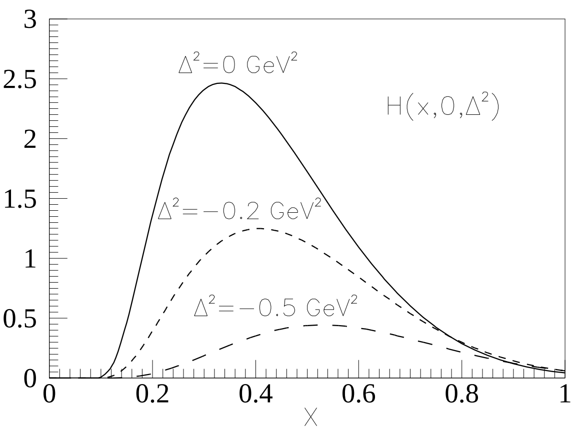

Figure 2. The GPD $H(x, \xi, \Delta^2)$ at $\xi = 0$ and three values of Δ^2.

Eq. (4) allows the calculation of the GPD $H(x, \xi, \Delta^2)$ in any CQM, and it naturally verifies the two crucial constraints, Eqs. (2) and (3) [7]. With respect to Eq. (4), a few caveats are necessary. Due to the use of CQM wave functions, only quarks GPDs can be evaluated, i.e., only the region $x \geq \xi/2$ can be explored. The approach has to be improved in order to study the sea region ($-\xi/2 \leq x \leq \xi/2$). Besides, in the argument of the δ function in Eq. (4), due to the used approximations, the x variable is not defined in its natural support, i.e. it can be larger than 1. Although the support violation is small in most models, this problem has to be considered a serious drawback of all CQM calculations of parton distributions, in particular if pQCD evolution of the model prediction is performed. We stress that our definition of the GPD $H(x, \xi, \Delta^2)$ can be easily generalized to other GPDs, which can be obtained in any CQM.

3. RESULTS IN THE ISGUR AND KARL MODEL

We consider the Isgur-Karl model [10], with a proton wave function given by a harmonic oscillator potential including contributions up to the $2\hbar\omega$ shell. In this case the proton state is given by the following admixture of states

$$|N\rangle = a_{\mathcal{S}}|^2S_{1/2}\rangle_S + a_{\mathcal{S}'}|^2S'_{1/2}\rangle_S + a_{\mathcal{M}}|^2S_{1/2}\rangle_M + a_{\mathcal{D}}|^4D_{1/2}\rangle_M \ , \qquad (7)$$

where we have used the spectroscopic notation $|^{2S+1}X_J\rangle_t$, with $t = A, M, S$ being the symmetry type. The coefficients were determined by spectroscopic properties to be: $a_{\mathcal{S}} = 0.931$, $a_{\mathcal{S}'} = -0.274$, $a_{\mathcal{M}} = -0.233$, $a_{\mathcal{D}} = -0.067$.

The obtained behavior of the FF is shown in Fig. 1. As it is well known, such a FF underestimates the data at high t. Results evaluated using the IK model, Eq. (7), in the general formula, Eq. (4), are shown in Figs. 2 to 4. In Fig. 2, we show the t dependence of our results. The full line corresponds to the usual PD. One immediately realizes that a strong t dependence is found, in comparison with other estimates, for example, with the one in [4]. This has to do with the too a strong t dependence of the FF in the IK model. In Figs. 3 and 4 we have the full t and ξ dependences. These are similar to the ones obtained in [4], although the ξ dependence is stronger.

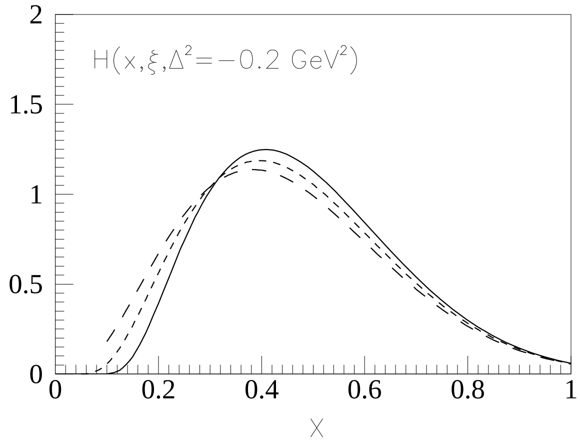

Figure 3. The GPD $H(x, \xi, \Delta^2)$ at $\Delta^2 = -0.2$ GeV^2 and $\xi = 0$ (full line), $\xi = 0.1$ (dotted line), $\xi = 0.2$ (dashed line). Notice that $H(x, \xi, \Delta^2)$ is shown for $x \geq \xi/2$.

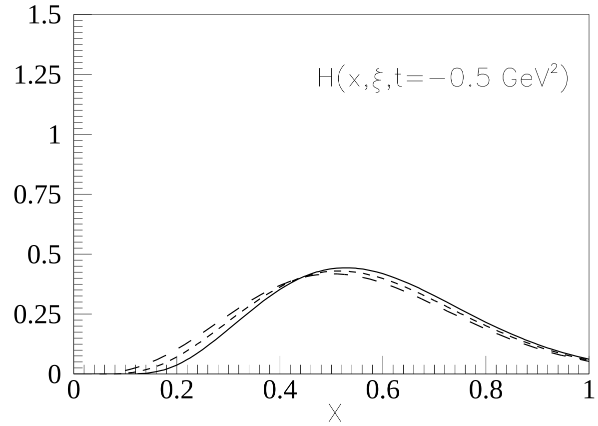

Figure 4. The GPD $H(x, \xi, \Delta^2)$ at $\Delta^2 = -0.5$ GeV^2 and $\xi = 0$ (full line), $\xi = 0.1$ (dotted line), $\xi = 0.2$ (dashed line). Notice that $H(x, \xi, \Delta^2)$ is shown for $x \geq \xi/2$.

Our results for $H(x, \xi, \Delta^2)$ correspond to the low momentum scale associated with the model. In order to compare them with the data which are going to be taken in future experiments, one has to evolve them to the experimental high-momentum scale.

The proposed approach can have many interesting developments, such as the use of more realistic models, the inclusion of QCD evolution from the scale of the model to the experimental one, the addition of corrections due to relativistic dynamics, or the ones due to a possible finite size and complex structure of the constituent quarks, as proposed by several authors [11].

REFERENCES

1. X. Ji, J. Phys. G24 (1998) 1181, A.V. Radyushkin, hep-ph/0101225. K. Goeke *et al*, Prog. Part. Nucl. Phys.47 (2001) 401; M. Diehl *et al*, Nucl. Phys. B596 (2001) 33.
2. X. Ji, Phys. Rev. Lett. 78 (1997) 610; Phys. Rev. D 55 (1997) 7114.
3. R.L. Jaffe, A.V. Manohar, Nucl. Phys. B 337 (1990) 509.
4. X. Ji, W. Melnitchouk, and X. Song, Phys. Rev. D 56 (1997) 5511.
5. A.V. Belitsky *et al* Phys. Lett. B474 (2000) 163 and references there in.
6. M. Burkardt, Phys. Rev. D 62 (2000) 071503; hep-ph/0105324.
7. S. Scopetta and V. Vento, hep-ph/0201265.
8. G. Parisi, R. Petronzio, Phys. Lett. B 62 (1976) 331; R.L. Jaffe, G.G. Ross, Phys. Lett. B 93 (1980) 313.
9. M. Traini *et al*, Nucl. Phys. A 614 (1997) 472; S. Scopetta and V. Vento, Phys. Lett. B 460 (1999) 8.
10. N. Isgur and G. Karl, Phys. Rev. D 18 (1978) 4187, D 19 (1979) 2653.
11. G. Altarelli *et al.* Nucl. Phys. B 69 (1974) 531; F. Cardarelli *et al* Phys. Lett. B 357 (1995) 267; S. Scopetta *et al* Phys. Lett. B 421 (1998) 64.