

## Sliding Vacua in Dense Skyrmion Matter

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### Abstract

In continuation of our systematic effort to understand hadronic matter at high density, we study dense skyrmion matter and its chiral phase structure in an effective field theory implemented with the trace anomaly of QCD applicable in the large  $N_c$  limit. By incorporating a dilaton field  $\chi$  associated with broken conformal symmetry of QCD into the simplest form of skyrmion Lagrangian, we simulate the effect of “sliding vacua” influenced by the presence of matter and obtain what could correspond to the “intrinsic dependence” on the background of the system, i.e., matter density or temperature, that results when a generic chiral effective field theory of strong interactions is matched to QCD at a matching scale near the chiral scale  $\Lambda_\chi \sim 4\pi f_\pi \sim 1$  GeV. The properties of the Goldstone pions and the dilaton scalar near the chiral phase transition are studied by looking at the pertinent excitations of given quantum numbers on top of a skyrmion matter and their behavior in the vicinity of the phase transition from Goldstone mode to Wigner mode characterized by the changeover from the FCC crystal to the half-skyrmion CC crystal. We recover from the model certain features that are connected to Brown-Rho scaling and that suggest how to give a precise meaning to the latter in the framework of an effective field theory that is matched to QCD .

Pacs: 12.39-x, 13.60.Hb, 14.65-q, 14.70Dj

Keywords: chiral symmetry, dilaton, pion, dense matter, mended symmetry, BR scaling

# 1 Introduction

In trying to understand what happens to hadrons under extreme conditions, e.g., at high density as in compact stars or at high temperature as in relativistic heavy-ion collisions, it is necessary that the theory or model adopted for the description be consistent with the basic tenet of QCD. What this means in terms of effective theories using macroscopic degrees of freedom, i.e., hadrons, the effective field theory should be matched to QCD at a scale close to the chiral scale  $\Lambda_\chi \sim 4\pi f_\pi \sim 1$  GeV. In a recent important development reviewed in [1], Harada and Yamawaki show how this matching can be effectuated in the framework of hidden local symmetry (HLS) theory. In this theory, the light-quark vector meson fields  $\rho_\mu$  figure as gauge fields coupled to the Goldstone pion fields  $\pi$ . When this theory is matched to QCD via current correlators at a suitable scale  $\Lambda_M \sim \Lambda_\chi$ , among the variety of flow paths by which the theory can follow to its multiple fixed points as the scale is changed, it picks the particular flow that leads to one unique fixed point as one reaches the point at which the phase change to Wigner mode from Goldstone mode takes place. This fixed point corresponds to the “vector manifestation (VM)” in which the *parameters* of the Lagrangian have the limiting behavior:

$$g \rightarrow 0, \quad a \rightarrow 1 \tag{1}$$

where the parametric  $g$  is the (hidden) gauge coupling constant and  $a$  is the ratio  $F_\pi/F_\sigma$  where  $F_\pi$  and  $F_\sigma$  are respectively the *parametric* pion <sup>1</sup> and would-be Goldstone scalar decay constants. In [2], Harada and Yamawaki discuss how the fixed point (1) is reached when one dials the number of flavors to a critical value  $N_f^c > 3$  at which the physical pion decay constant  $f_\pi$  vanishes. Subsequently it has been shown that chiral phase transition at critical temperature  $T_c$  [3] and at critical density  $n_c$  [4] involves the same VM. The consequence is that at the critical point, independently of whether it is driven by temperature, density or large  $N_f$ , the gauge boson mass (both parametric and pole) approaches zero and so does the constituent quark mass. Thus all light-quark hadrons (except for the Goldstone pions) are expected to become massless in the chiral limit in some power of the gauge coupling constant  $g$ . This observation led Brown and Rho to conjecture that the property of VM and Brown-Rho scaling are intimately connected [6]. An important lesson one can learn from this development is that without the Wilsonian matching to QCD, there is no way to pick the right one from the multitude of different ways to describe hadron properties near chiral restoration. This means that the theories that possess all relevant symmetries but are unmatched to QCD have no predictive power for hadron properties. This caveat applies not only to HLS theory but also to all chiral field theories with or without vector-meson degrees of freedom.

The matching at  $\Lambda_M$  of the correlators between the HLS sector and the QCD sector gives the parameters of the “bare” HLS Lagrangian in terms of the quantities figuring in the QCD sector, namely, the color gauge coupling  $\alpha_s$ ,  $\Lambda_{QCD}$  and quark and gluon condensates. In medium, the condensates depend on the background, that is, on density <sup>2</sup>. Imposing that the quark condensate  $\langle \bar{q}q \rangle$  – as an order parameter – vanish at the critical density  $n_c$ , one obtains, at the matching scale, the conditions on the HLS parameters  $g(\Lambda_M, n_c) = 0$  and  $a(\Lambda_M, n_c) = 1$ . However it gives no constraint on the parametric  $F_\pi(\Lambda_M, n_c)$ . Now bringing the scale from  $\Lambda_M$  to the scale appropriate to the physics involved is done by renormalization group equations. Thus the properties both at the matching scale in matter-free vacuum and at the VM in medium are completely determined by the theory. But the HLS theory formalism does not offer any

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<sup>1</sup>We will denote the physical pion decay constant by the lower case  $f_\pi$  to be distinguished from the parametric one.

<sup>2</sup>Similar arguments hold in heat bath although we do not specifically mention it in what follows.

method to describe how the parameters behave at an arbitrary scale  $\Lambda$  and density  $n$  away from the two special points. This means that we know practically nothing on the *intrinsic* density dependence of the parameters of the Lagrangian and hence cannot compute the density dependence of physical quantities except at  $n \sim 0$  and  $n \sim n_c$ . Thus it would be futile to attempt to employ HLS/VM theory in its present form for analyzing experimental data which of course sample all ranges of density.

In this paper, we tackle the *intrinsic dependence* and related issues with the help of the unified approach to high density developed in [7]. In [7], it was shown using a skyrmion Lagrangian in its simplest form that an effective Lagrangian valid in the large  $N_c$  limit (where  $N_c$  is the number of colors) can describe both infinite nucleonic matter and pionic fluctuations thereon as density is increased and that by describing the chiral restoration as a changeover from an FCC crystal to the half-skyrmion matter, one can describe the properties of various excitations commensurate with the background given by the half-skyrmion crystal configuration. In this approach, the density dependence of the background is taken into account to all orders. No low-density approximation whose validity is in doubt except at very low density is ever made in the calculation. The power of the approach is that the dynamics of the background and excitations thereon can be treated in a unified way on the same footing with a single Lagrangian.

In addressing the problem at hand, the Lagrangian used in [7] is probably incomplete. In fact, it is not clear that the intrinsic density dependence required by the matching to QCD discussed above is fully implemented in the model. One puzzling feature we found in [7] was that the Wigner phase represented by the half-skyrmion matter with  $\langle \text{tr}U_0 \rangle = 0$  supported a non-vanishing pion decay constant. This was interpreted there as a possible signal for a pseudo-gap phase. However it can also be interpreted as an analog to Georgi's "vector limit" in which chiral symmetry is restored with pions present with non-vanishing pion decay constant. Harada and Yamawaki argue [1] that this phase is not consistent with QCD Ward identity, implying that it cannot be realized in nature.

Our proposal in this paper is that we can circumvent the above difficulty and achieve our objective within a large  $N_c$  framework by incorporating into the skyrmion description the trace anomaly of QCD which is lacking in the standard skyrmion model. In fact it is this feature that led to BR scaling in [5] when the problem was treated albeit in a schematic way. In this paper we formulate BR scaling of [5] in a more rigorous way and offer a scheme that could shed light on the *intrinsic* density dependence needed in the HLS/VM theory.

The basic idea in our approach is as follows. We consider a skyrmion-type Lagrangian with spontaneously broken chiral symmetry and scale symmetry associated, respectively, with nearly massless quarks and trace anomaly of QCD. Such a theory may be considered as an  $N_c \rightarrow \infty$  approximation to QCD. Suppose that the Lagrangian can describe not only the lowest-excitation, i.e., pionic, sector but also the baryonic sector and massive vector meson sector all lying below the chiral scale  $\Lambda_\chi$ . The skyrmion description gives not only the single baryon spectra but also multi-baryon systems including infinite nuclear matter. We are interested in how low-energy degrees of freedom in many-body systems behave in dense matter, in particular as the density reaches a density at which QCD predicts a phase transition from the broken chiral symmetry to the unbroken chiral symmetry or chiral restoration in short. For this purpose, one first looks for the ground state of the many-baryon system in question as a soliton solution of the Lagrangian and then looks at the fluctuation of effective fields in various channels of low-energy excitations with the effect of the background taken into account self-consistently. As one varies the density of the system, the parameters of the theory involved in the process must adapt to the density of the skyrmion background. The problem we are interested in is whether we can extract from the unified scheme an information on the intrinsic dependence inherent in effective

field theories matched to QCD , i.e., HLS/VM theory of Harada and Yamawaki. The hope is that by looking at the structure at the phase transition and tuning the parameters such that the features predicted by HLS/VM are reproduced, we can ultimately learn about the intrinsic dependence of the parameters and hence “derive” BR scaling. We will see that some progress in this direction can be made.

The content of this paper is as follows. In Section 2, we give our model Lagrangian which is the simplest form of skyrmion Lagrangian, namely, the original Skyrme Lagrangian implemented with trace anomaly of QCD. In Section 3, a single skyrmion is analyzed to define the parameters of the theory at zero density. The chiral phase transition from an FCC crystal to a half-skyrmion CC crystal is described in terms of the model Lagrangian in Section 4. Section 5 describes how the pseudo-Goldstone pion and the scalar of the trace anomaly (“dilaton”) behave in the skyrmion matter as a function of density. In Section 6, the chiral restoration transition is made from the inhomogeneous phase – to which the FCC crystal collapses – to the half-skyrmion crystal phase. The contact with BR scaling, and indirectly with the parametric property of HLS/VM, is made in this section. Some concluding remarks are given in Section 7.

## 2 The Model Lagrangian

The starting point of our work is the skyrmion Lagrangian introduced by Ellis and Lanik [8] and employed by Brown and Rho [5] for nuclear physics that incorporates the trace anomaly of QCD. Here we will study the same Lagrangian from a more modern perspective.

The classical QCD action of scale dimension 4 in the chiral limit is scale-invariant under the scale transformation

$$x \rightarrow \lambda x = \lambda^{-1} x, \quad \lambda \geq 0, \quad (2)$$

under which the quark field and the gluon fields transform with the scale dimension 3/2 and 1, respectively. The quark mass term of scale dimension 3 breaks this scale invariance. At the quantum level, scale invariance is also broken by dimensional transmutation even for massless quarks, as signaled by a non-vanishing trace of the energy-momentum tensor. Equivalently, this phenomenon can be formulated by the non-vanishing divergence of the dilatation current  $D_\mu$ , the so called trace anomaly,

$$\partial^\mu D_\mu = \theta_\mu^\mu = \sum_q m_q \bar{q}q - \frac{\beta(g)}{g} \text{Tr} G_{\mu\nu} G^{\mu\nu}, \quad (3)$$

where  $\beta(g)$  is the beta function of QCD.

We will implement broken scale invariance into large  $N_c$  physics by modifying the skyrmion Lagrangian,

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}[U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2 + \frac{f_\pi^2 m_\pi^2}{4} \text{Tr}(U + U^\dagger - 2). \quad (4)$$

The chiral field  $U = \exp(i\vec{\tau} \cdot \vec{\pi}/f_\pi)$  has scale dimension 0 and therefore the Lagrangian respects neither the scale invariance of QCD nor its breaking eq. (3). The current algebra term with two derivatives is of dimension 2 and the meson mass term is of dimension 0, while the Skyrme term with four derivatives has scale dimension 4.

In order to make the Skyrme model well behaved under the scaling properties of QCD , we introduce an additional degree of freedom in the form of a scalar field  $\chi$  with a scale dimension

1, whose coupling to the  $U$  fields is defined by,

$$\begin{aligned} \mathcal{L} = & \frac{f_\pi^2}{4} \left( \frac{\chi}{f_\chi} \right)^2 \text{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{32e^2} \text{Tr}([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U])^2 + \frac{f_\pi^2 m_\pi^2}{4} \left( \frac{\chi}{f_\chi} \right)^3 \text{Tr}(U + U^\dagger - 2) \\ & + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - \frac{1}{4} \frac{m_\chi^2}{f_\chi^2} \left[ \chi^4 \left( \ln(\chi/f_\chi) - \frac{1}{4} \right) + \frac{1}{4} \right]. \end{aligned} \quad (5)$$

We have denoted the non vanishing vacuum expectation value of  $\chi$  as  $f_\chi$ , a constant which describes, as we shall see, the decay of the scalar into pions. The second term of the trace anomaly (3) can be reproduced by the potential energy  $V(\chi)$  for the scalar field, which is adjusted in the Lagrangian (5) so that  $V = dV/d\chi = 0$  and  $d^2V/d\chi^2 = m_\chi^2$  at  $\chi = f_\chi$ .

The vacuum state of the Lagrangian at zero baryon number density is defined by  $U = 1$  and  $\chi = f_\chi$ . We will take the latter to be positive and therefore the field values around the vacuum will be positive. The fluctuations of the pion and the scalar fields about this vacuum, defined through

$$U = \exp(i\vec{\tau} \cdot \vec{\phi}/f_\pi), \quad \text{and} \quad \chi = f_\chi + \tilde{\chi} \quad (6)$$

give physical meaning to the model parameters:  $f_\pi$  as the pion decay constant,  $m_\pi$  as the pion mass,  $f_\chi$  as the scalar decay constant, and  $m_\chi$  as the scalar mass. For the pions, we use their empirical values as  $f_\pi = 93\text{MeV}$  and  $m_\pi = 140\text{MeV}$ . We fix the Skyrme parameter  $e$  to 4.75 from the axial-vector coupling constant  $g_A$  as in ref. [10]. However, for the scalar field  $\chi$ , no experimental values for the corresponding parameters are available yet.

The scalar field may be interpreted as a bound state of gluons, the so-called glueball [8]. If one assumes that it is a pure glueball, then one can use the gluon condensate  $G_0 = \langle 0 | (\alpha_s/\pi) G_{\mu\nu} G^{\mu\nu} | 0 \rangle$  to restrict the product of its mass  $m_\chi$  by the vacuum expectation value  $f_\chi$  as

$$\frac{1}{2} f_\chi m_\chi = \left( \frac{9}{8} G_0 \right)^{\frac{1}{2}}. \quad (7)$$

Fluctuating around the vacuum at zero density, the Lagrangian yields the relevant interactions for the process  $\chi \rightarrow \pi\pi$ . In the chiral limit, the coupling from the current algebra term is given by

$$\mathcal{L}_{\chi\pi\pi} = \frac{\tilde{\chi}}{f_\chi} \sum_{a=1}^3 (\partial_\mu \phi_a)^2. \quad (8)$$

This yields the decay width  $\Gamma(\chi \rightarrow \pi\pi)$  as

$$\Gamma(\chi \rightarrow \pi\pi) = \frac{3m_\chi^3}{32\pi f_\chi^2} = \frac{m_\chi^5}{48\pi G_0} \quad (9)$$

in the  $\chi$  rest frame, where we used eq. (7) to eliminate  $f_\chi$  in favor of  $G_0$ . Introducing the pion mass into the calculation modifies the expression slightly [8]. Using the ITEP value  $G_0 = 0.012\text{GeV}^4$  [11]<sup>3</sup>, one obtains

$$\Gamma(\chi \rightarrow \pi\pi) \approx 0.6\text{GeV} \times (m_\chi/1\text{GeV})^5. \quad (10)$$

This equation provides a restriction to the range of the possible gluonium masses

$$\begin{aligned} \Gamma(\chi \rightarrow \pi\pi) < 1 \text{ MeV} & \quad \text{for} \quad m_\chi < 0.3 \text{ GeV} \\ \Gamma(\chi \rightarrow \pi\pi) > 0.5 m_\chi & \quad \text{for} \quad m_\chi > 1 \text{ GeV}. \end{aligned} \quad (11)$$

<sup>3</sup>The more recent ITEP value [14] is  $G_0 = 0.011 \pm 0.009 \text{ GeV}^4$  which is consistent with the older value.

Table 1: Parameter sets of the model Lagrangian

	$m_\chi$	$f_\chi$	$G_0$
set A	550MeV	240MeV	0.004 GeV <sup>4</sup>
set B	720MeV	240MeV	0.007 GeV <sup>4</sup>
set C	1000MeV	240MeV	0.012 GeV <sup>4</sup>

We see that a light scalar gluonium with the mass below 400 MeV cannot be detected because of its too narrow decay width, while a heavy one with the mass greater than 1 GeV cannot be recognized as a well-defined resonance.

In the following development, we will find that the scalar  $\chi$  in this picture cannot describe pure gluonium; it must contain a light scalar quarkonium component. This “soft” component cannot vanish in the chiral broken phase in order to satisfy the symmetry properties of the fundamental theory [15]. The low-energy sum rules[9], related to the gluon condensates, are controlled by the gluonium component and can be used to fit the parameters of the theory in the free case, i.e.  $m_\chi$  and  $f_\chi$  from  $G_0$ . There can however be a subtle “mended symmetry” that puts the mass of  $\chi$  nearly degenerate with that of the  $\rho$  meson, i.e.,  $\sim 700$  MeV [12]. It is not clear what happens in dense medium. Our conjecture [13] is that as the density is increased, the two components “decouple” with only the quarkonium component interacting with matter. We can think of this as the “hard” gluonium component being integrated out in an effective field theoretic sense with their properties absorbed into the definition of the fields and the parameters [16]. This makes sense as the characteristic mass scale drops compared with the mass scale of the gluonium. Chiral symmetry restoration implies the vanishing of the “soft” component.

There are some indications that the above conjecture makes sense. In ref. [15], the scalar field is incorporated into a relativistic hadronic model for nuclear matter not only to account for the anomalous scaling behavior but also to provide the mid-range nucleon-nucleon attraction. Then, the parameters  $f_\chi$  and  $m_\chi$  are adjusted so that the model fits finite nuclei. One of the parameter sets is  $m_\chi = 550$  MeV and  $f_\chi = 240$  MeV (Set A). On the other hand, Song *et al.* [19] obtain the “best” values for the parameters of the effective chiral Lagrangian with the “soft” scalar fields so that the results are consistent with the “Brown-Rho” scaling [5]; explicitly,  $m_\chi = 720$  MeV and  $f_\chi = 240$  MeV (Set B). These observations are consistent with the notion that “mended symmetry” is operative for the scalar field to be identified with the sigma field of linear sigma model in certain “dilaton” limit [16].

For completeness, we consider also a parameter set of  $m_\chi = 1$  GeV and  $f_\chi = 240$  MeV (Set C) corresponding to a mass scale comparable to that of chiral symmetry  $\Lambda_\chi \sim 4\pi f_\pi$ .

The parameter sets defined above for the model Lagrangian are summarized in Table 1.

### 3 Single Skyrmion with the Scalar Field

The topological baryon number current associated with the homotopy of the mapping  $U_0(\vec{r}) : S^3(R^3 - \{\infty\}) \rightarrow S^3$  of  $SU(2)$  is defined as

$$B^\mu = \frac{1}{24\pi^2} \varepsilon^{\mu\nu\lambda\rho} \text{Tr}(U_0^\dagger \partial_\nu U_0 U_0^\dagger \partial_\lambda U_0 U_0^\dagger \partial_\rho U_0). \quad (12)$$

The soliton solution with the baryon number  $B = 1$  can be found by generalizing the spherical hedgehog Ansatz of the original Skyrme model as

$$U_0(\vec{r}) = \exp(i\vec{r} \cdot \hat{r}F(r)), \text{ and } \chi_0(\vec{r}) = f_\chi C(r), \quad (13)$$

with two radial functions  $F(r)$  and  $C(r)$ .

Then, the mass of the single soliton can be expressed as

$$M_{B=1} = 4\pi \int_0^\infty r^2 dr \left[ \frac{f_\pi^2}{2} C^2 \left( F'^2 + 2 \frac{\sin^2 F}{x^2} \right) + \frac{1}{2e^2} \frac{\sin^2 F}{x^2} \left( \frac{\sin^2 F}{x^2} + 2F'^2 \right) + f_\pi^2 m_\pi^2 C^3 (1 - \cos F) + \frac{f_\chi^2}{2} \left( C'^2 + \frac{m_\chi^2}{2} \{ (C^4 \ln C - \frac{1}{4}) + \frac{1}{4} \} \right) \right]. \quad (14)$$

where the prime means the derivation with respect to  $r$ . Variations of  $M_{B=1}$  with respect to  $F(r)$  and  $C(r)$  lead to the following equations of motion

$$\begin{aligned} \left( f_\pi^2 r^2 C^2 + \frac{2 \sin^2 F}{e^2} \right) F''(r) + 2 f_\pi^2 (r C^2 + r^2 C C') F'(r) - \frac{2 \sin F \cdot \cos F}{e^2} F'^2(r) \\ - 2 f_\pi^2 C^2 \sin F \cdot \cos F - \frac{2 \sin^3 F \cdot \cos F}{e^2 r^2} - f_\pi^2 m_\pi^2 C^3 r^2 \sin F = 0, \end{aligned} \quad (15)$$

for  $F(r)$  and

$$\begin{aligned} f_\chi^2 C''(r) + \frac{2 f_\chi^2}{r} C'(r) - f_\pi^2 C(r) \left( F'^2(r) + \frac{2 \sin^2 F}{r^2} \right) - 3 f_\pi^2 m_\pi^2 (1 - \cos F) C^2(r) \\ + f_\chi^2 m_\chi^2 C^3(r) \ln C(r) = 0, \end{aligned} \quad (16)$$

for  $C(r)$ .

At infinity, the fields  $U_0(\vec{r})$  and  $\chi_0(\vec{r})$  should reach their vacuum values. The asymptotic behavior of the equations of motion reveals that the radial functions reach the corresponding vacuum values as

$$F(x) \sim \frac{e^{-m_\pi r}}{r}, \text{ and } C(x) \sim 1 - \frac{e^{-m_\chi r}}{r}. \quad (17)$$

In order for the solution to carry a baryon number,  $U_0$  has the value  $-1$  at the origin, that is,  $F(x=0) = \pi$ , while there is no such topological constraint for  $C(x=0)$ . All that is required is that it be a positive number below 1. The equations of motion tell us that for small  $x$ ,

$$\begin{aligned} F(x) &\sim \pi - \alpha x + \gamma x^3 + O(x^5), \\ C(x) &\sim C_0 + \beta x^2 + O(x^4). \end{aligned} \quad (18)$$

The coupled equations of motion for  $F(x)$  and  $C(x)$  together with the boundary conditions (17) and (18) can be solved in various different ways. For example, we can use an iteration method. We first start with  $C_{i=0}(x) = 1$  for all  $x$  and solve the equation of motion (15) with  $C(x)$  fixed as  $C_0(x)$  to obtain  $F_{i=0}(r)$ , which is nothing but the profile function of the single skyrmion of the original Lagrangian. Then, we take this  $F_0(x)$  as  $F(x)$  in the equation of motion (16) to obtain  $C_{i=1}(x)$ . We repeat this iteration until  $F_{i+1}(x)$  and  $C_{i+1}(x)$  converge to  $F_i(x)$  and  $C_i(x)$ . This iteration converges quite fast and after ten iterations the solution can be found with a sufficient accuracy.

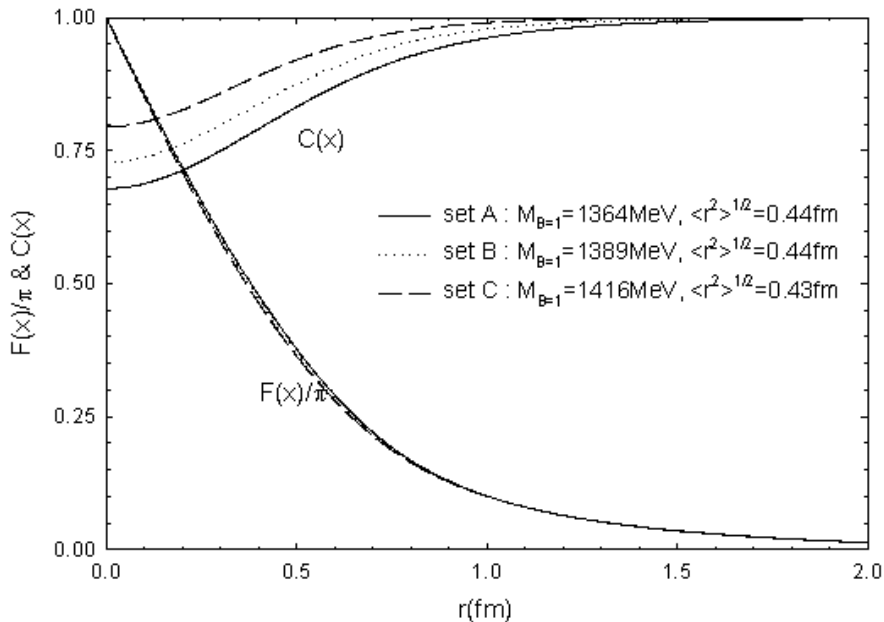


Figure 1: Profile functions  $F(x)$  and  $C(x)$  as a function of  $x$ .

Shown in Fig. 1 are the profile functions  $F(x)$  and  $C(x)$  as a function of  $x(=ef_\pi r)$ .  $F(r)$  and consequently the root mean square radius of the baryon charge

$$\langle r^2 \rangle^{1/2} = \left( \int d^3r r^2 B^0(\vec{r}) \right)^{1/2} \quad (19)$$

show little dependence on  $m_\chi$ . On the other hand, the changes in  $C(r)$  and the soliton mass are recognizable. The larger the scalar mass is, the smaller its coupling to the pionic field and the less its effect on the single skyrmion. In the limit of  $m_\chi \rightarrow \infty$ , the scalar field is completely decoupled from the pions and the model returns back to the original one, where  $C(r) = 1$ ,  $M_{sol} = 1479$  MeV and  $\langle r^2 \rangle^{1/2} = 0.43$  fm. The strong dependence of  $C(r)$  on  $m_\chi$  may come from the asymptotic behavior (17) at large  $x$ . As for the soliton mass, note that  $M_{sol}$  scales approximately as  $(f_\pi/e)$  and  $C^2(r)$  is multiplied to  $f_\pi^2$  in the current algebra term of the Lagrangian. Thus,  $C(r) \leq 1$  reduces the *effective*  $f_\pi$  inside the single skyrmion so that the soliton mass decreases accordingly. For example, for the parameter set A, the soliton mass gets 7% reduction and we can imagine that the effective  $f_\pi$  is reduced by the same amount in average in the region where the single skyrmion is located.

## 4 FCC Skyrmion Crystal with the Scalar Field

Consider a crystal configuration made up of skyrmions, where each FCC lattice site is occupied by a single skyrmion center with  $U_0 = -1$  and each nearest neighboring pair is relatively rotated in isospin space by  $\pi$  with respect to the line joining them. At low density, the system maintains the original configuration, i.e., it appears as an FCC crystal with the dense centers of single skyrmions on each lattice sites. At high density, however, the system undergoes a phase transition to a CC crystal made up of half-skyrmions. Here, only half of the baryon number



Table 2: Symmetries of the FCC skyrmion crystal

symmetry	space	$U_0$	$\chi$
reflection	$\vec{x} \rightarrow (-x, y, z)$	$U_0 \rightarrow (\sigma, -\pi_1, \pi_2, \pi_3)$	$\chi_0 \rightarrow \chi_0$
3-fold axis rotation	$\vec{x} \rightarrow (y, z, x)$	$U_0 \rightarrow (\sigma, \pi_2, \pi_3, \pi_1)$	$\chi_0 \rightarrow \chi_0$
4-fold axis rotation	$\vec{x} \rightarrow (x, z, -y)$	$U_0 \rightarrow (\sigma, \pi_1, \pi_3, -\pi_2)$	$\chi_0 \rightarrow \chi_0$
translation	$\vec{x} \rightarrow (x + L, y + L, z)$	$U_0 \rightarrow (\sigma, -\pi_1, -\pi_2, \pi_3)$	$\chi_0 \rightarrow \chi_0$

carried by the single skyrmion is concentrated at the original FCC sites, while the other half is concentrated at the center of the links connecting these points.

Let us denote the field configuration for the skyrmion field by  $U_0(\vec{x}) = \sigma + i\vec{\tau} \cdot \vec{\pi}$  (with  $\sigma^2 + \pi^2 = 1$ ) and  $\chi_0(\vec{x})$  for the scalar field. The FCC configuration we are considering has the symmetries listed in Tab. 2. There,  $2L$  is the size of the single FCC unit cell that contains 4 skyrmions. Thus, the baryon number density is  $\rho = 1/2L^3$ . Normal nuclear matter density occurs at  $\rho_0 = 0.17/\text{fm}^3$  which corresponds to  $L \sim 1.43$  fm. Note that the chiral  $\sigma$  field has exactly the same symmetries as those of the  $\chi$ .

As for the constrained fields  $(\sigma, \pi_a)$ , it is convenient to work with “*unnormalized*” fields  $(\bar{\sigma}, \bar{\pi}_a)$ , which can then be “*normalized*” as

$$\sigma = \frac{\bar{\sigma}}{\sqrt{\bar{\sigma}^2 + \bar{\pi}_1^2 + \bar{\pi}_2^2 + \bar{\pi}_3^2}}, \quad (20)$$

and similarly for  $\pi_a$  ( $a = 1, 2, 3$ ). The field configurations obeying the above symmetries can be easily found by expanding the unnormalized fields in terms of Fourier series as [21]

$$\bar{\sigma}(\vec{x}) = \sum_{a,b,c} \beta_{abc} \cos(a\pi x/L) \cos(b\pi y/L) \cos(c\pi z/L), \quad (21)$$

and

$$\bar{\pi}_1(\vec{x}) = \sum_{h,k,l} \alpha_{hkl} \sin(h\pi x/L) \cos(k\pi y/L) \cos(l\pi z/L), \quad (22)$$

$$\bar{\pi}_2(\vec{x}) = \sum_{h,k,l} \alpha_{hkl} \cos(l\pi x/L) \sin(h\pi y/L) \cos(k\pi z/L), \quad (23)$$

$$\bar{\pi}_3(\vec{x}) = \sum_{h,k,l} \alpha_{hkl} \cos(k\pi x/L) \cos(l\pi y/L) \sin(h\pi z/L), \quad (24)$$

and finally

$$\chi_0(\vec{x}) = \sum_{a,b,c} \gamma_{abc} \cos(a\pi x/L) \cos(b\pi y/L) \cos(c\pi z/L). \quad (25)$$

The symmetries of Tab. 2 restrict the modes appearing in eqs. (21-25) as follows;

(M1) if  $h$  is even, then  $k, l$  are restricted to odd numbers and  $a, b, c$  are to even numbers,

(M2) if  $h$  is odd, then  $k, l$  are restricted to even numbers and  $a, b, c$  are to odd numbers.

Furthermore,  $\alpha_{hkl} = \alpha_{hlk}$  and  $\beta_{abc} = \beta_{bca} = \beta_{cab} = \beta_{acb} = \beta_{cba} = \beta_{bac}$ .

We can locate, without loss of generality, the centers of the skyrmions at the corners of the cube and at the centers of the faces by letting  $\sigma = -1$  and  $\pi_i (i = 1, 2, 3) = 0$  at those points. For

the skyrmion field to have a definite integer baryon numbers per site, we should have  $\sigma = +1$  and  $\pi_i (i = 1, 2, 3) = 0$  at points such as  $(L, 0, 0)$ . This produces the constraint,

$$\sum_{a,b,c=\text{even}} \bar{\beta}_{abc} = 0. \quad (26)$$

As for  $\gamma_{abc}$  associated with the scalar field, there is no such constraint, but the coefficients should be arranged to satisfy  $\chi_0 \geq 0$ , a consequence of our choice of vacuum.

If we had only the modes (M2) in the expansion, the configuration would then have an additional symmetry, namely, under the translation  $\vec{x} \rightarrow (x + L, y, z)$  the field undergoes an  $O(4)$  rotation by  $\pi$  in the  $\sigma, \pi_1$  plane. Furthermore, in order to satisfy the constraint  $\chi \geq 0$ ,  $\chi$  must vanish identically. This configuration corresponds to the *half-skyrmion* CC as explained above. Because of this additional symmetry, physical quantities such as the local baryon number density and the local energy density become completely identical around the points with  $\sigma = -1$  and the points with  $\sigma = +1$ . Thus, one half of the baryon number carried by a single skyrmion is concentrated at the sites where the centers of the single skyrmion are expected to be in the FCC crystal. The other half of the baryon number is now concentrated on the links connecting those points, where the  $\sigma$  takes the value  $+1$  and, in the original FCC configuration, the local baryon number density is rather low. As a consequence, the expectation value  $\langle \sigma \rangle$  goes to zero, signaling the restoration of the spontaneously broken chiral symmetry. Both modes, (M1) and (M2), are included in the actual numerical procedure which we define next. The half-skyrmion crystal configuration arises at high density where the expansion coefficients associated with the modes (M1) become small.

In order to obtain the coefficients we minimize the energy per baryon  $E/B$  given by

$$\begin{aligned} E/B &= -\frac{1}{4} \int_{\text{Box}} d^3r \mathcal{L}_M(U_0, \chi_0) \\ &= \frac{1}{4} \int_{\text{Box}} d^3x \left\{ \frac{f_\pi^2}{4} \left( \frac{\chi_0}{f_\chi} \right)^2 \text{Tr}(\partial_i U_0^\dagger \partial_i U_0) + \frac{1}{32e^2} \text{Tr} [U_0^\dagger \partial_i U_0, U_0^\dagger \partial_j U_0]^2 \right. \\ &\quad \left. + \frac{f_\pi^2 m_\pi^2}{4} \left( \frac{\chi_0}{f_\chi} \right)^3 \text{Tr}(2 - U_0^\dagger - U_0) + \frac{1}{2} \partial_i \chi_0 \partial_i \chi_0 + V(\chi_0) \right\} \quad (27) \end{aligned}$$

by taking the coefficients of the expansions as variational parameters. In eq. (27), the subscript ‘box’ denotes that the integration is over a single FCC box and the factor  $1/4$  in front appears because the box contains baryon number four. We employ “the down-hill simplex method” [22] for the minimization process.

Before going into further details, let us do a rough study of the role the chiral field plays in the phase structure of the system. For this, we take  $\chi_0$  to be a constant

$$\chi_0/f_\chi = X. \quad (28)$$

Then eq.(27) can be approximated to

$$E/B(X, L) = X^2(E_2/B) + (E_4/B) + X^3(E_m/B) + (2L^3) \left( X^4(\ln X - \frac{1}{4}) + \frac{1}{4} \right), \quad (29)$$

where  $E_2$ ,  $E_4$  and  $E_m$  are, respectively, the contributions from the current algebra term, the Skyrme term and the pion mass term of the Lagrangian to the energy of the skyrmion system and  $(2L^3)$  is the volume occupied by a single skyrmion. It can be understood as an effective

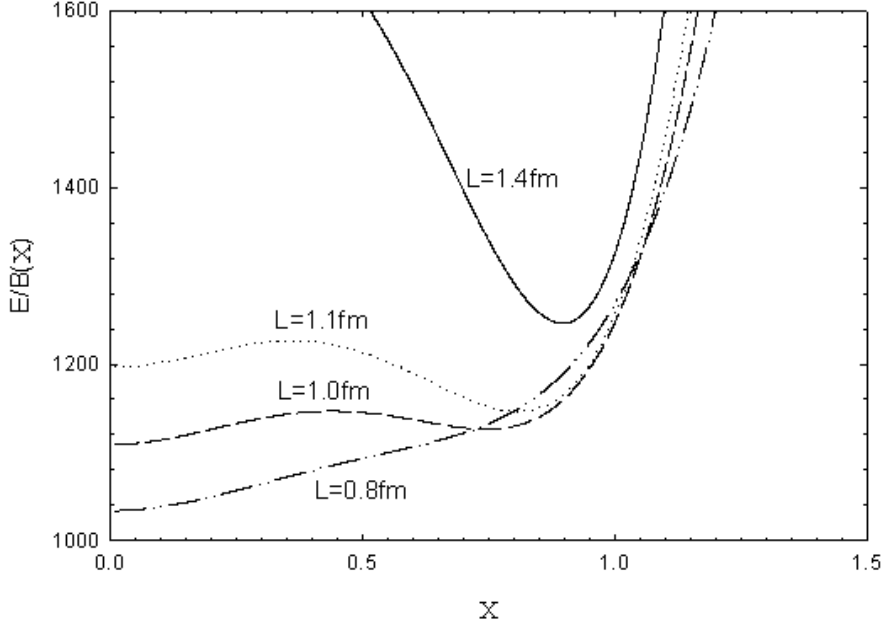


Figure 2: Energy per single skyrmion as a function of the scalar field  $X$  for a given  $L$ . The results are obtained with the  $(E_2/B)$ ,  $(E_4/B)$ , and  $(E_m/B)$  of ref.[7] and with the parameter sets B.

potential for the chiral field *in medium*, modified by the coupling of the scalar to the background matter. With the parameter values obtained in ref.[7] for the Skyrme model without the scalar fields, the effective potentials  $E/B(X)$  behave as shown in Fig. 2. At low density (larger  $L$ ), the minimum of the effective potential is located slightly from  $X = 1$ . As the density increases, the effective potential  $V(\chi)$  develops another minimum at  $X = 0$  which was an unstable extremum of the potential in free space. At  $L \sim 1$  fm, the newly developed minimum can compete with the one near  $X \sim 1$ . At higher density, the minimum gets shifted to  $X = 0$  where the system gets stabilized.

In Fig. 3, we plot  $E/B(X_{min}, L)$  as a function of  $L$ . The figure in a small box is the corresponding value of  $X_{min}$  as function of  $L$ . There we see an explicit manifestation of a first-order phase transition. Although the discussion is perhaps a bit too naive, it essentially encodes the same physics as in the more rigorous treatment of  $\chi_0$  given below.

Given in Fig. 4 is the energy per baryon,  $E/B$ , as a function of the FCC box size parameter  $L$ . Each point corresponds to the lowest energy crystal configuration for the given value of  $L$ . The solid circles, solid squares and solid triangles are obtained with the parameter sets A, B, and C, respectively. Furthermore, the black solid lines correspond to the single skyrmion FCC phase, while the gray lines to the half-skyrmion CC phase. As we squeeze the system from  $L = 6$  on, the skyrmion system undergoes at  $L = L_{pt}$  a phase transition from the FCC single skyrmion configuration to the CC half-skyrmion configuration. The transition appears to be first order. In the half-skyrmion phase,  $\chi_0(\vec{r})$  vanishes. The energy of the system comes only from the Skyrme term and the scalar field potential. The former roughly scales as  $\sim 1/L$  and

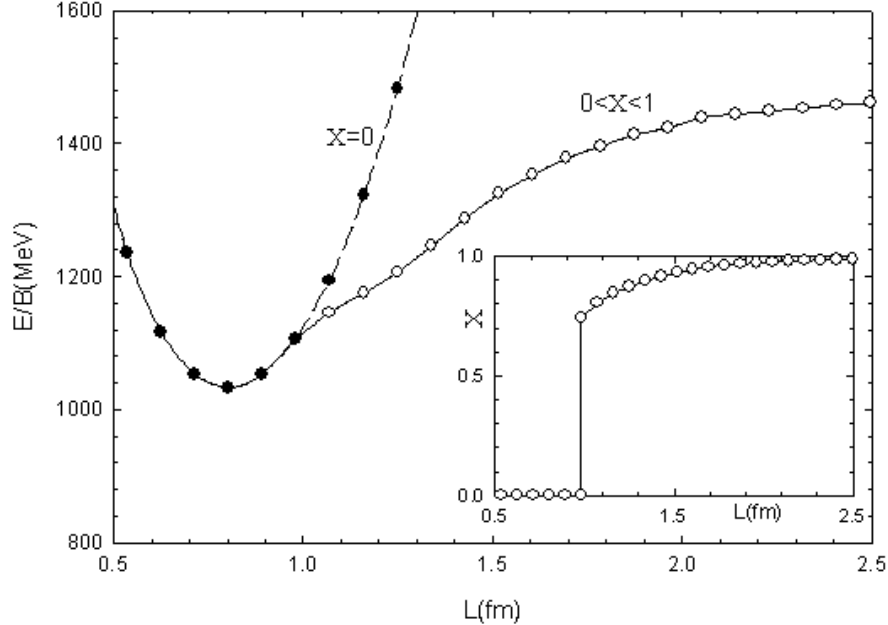


Figure 3: Energy per single skyrmion as a function of  $L$ . The results are obtained by minimizing  $E/B(X, L)$  with respect to  $X$  for a given  $L$ .

the latter exactly as  $L^3$  (the volume of the box); explicitly,

$$E/B \sim a/L + 2V(\chi_0 = 0)L^3, \quad (30)$$

$a$  being constant. After the phase transition, the energy per baryon  $E/B$  continues to decrease (even faster) and reaches its minimum point at  $L = L_{min}$ . The precise values for the phase transition point  $L_{pt}$  and the minimum energy point  $L_{min}$  depend on the parameters and are therefore different for the various sets.

The incorporation of the scalar field into the Skyrme model Lagrangian produces quite dramatic effects on the properties of skyrmion matter. For example, the energy per baryon drops down to  $\sim 700$  MeV. This can be understood since the potential energy between the skyrmions amounts to nearly 50% of the skyrmion mass and comes from the medium-range attraction generated by the scalar field.

From here on we denote by  $\langle Q \rangle$  the average value of a quantity  $Q(\vec{r})$  over the FCC box defined by

$$\langle Q \rangle = \frac{1}{8L^3} \int_{Box} d^3r Q(\vec{r}). \quad (31)$$

In Fig. 5, we represent  $\langle \sigma \rangle$  and  $\langle \chi_0/f_\chi \rangle$  – the averaged values of  $\sigma(\vec{r})$  and  $\chi_0(\vec{r})$  over space – as a function of  $L$ . Both quantities go to zero at the *same* phase transition point  $L = L_{pt}$ . However, for larger values of  $m_\chi$ , the transition properties of the two quantities look different. Note that while  $\sigma(\vec{r}) \neq 0$  locally and  $\langle \sigma \rangle = 0$  is obtained by the averaging process,  $\langle \chi_0/f_\chi \rangle = 0$  results because  $\chi_0(\vec{r}) = 0$  throughout the whole space. We will take  $\langle \chi_0 \rangle$  as the *effective* value of  $f_\chi$ , i.e.,  $f_\chi^*$ . We interpret this result as the vanishing of the “soft” part of the gluon condensate tied to the quarkonium component.

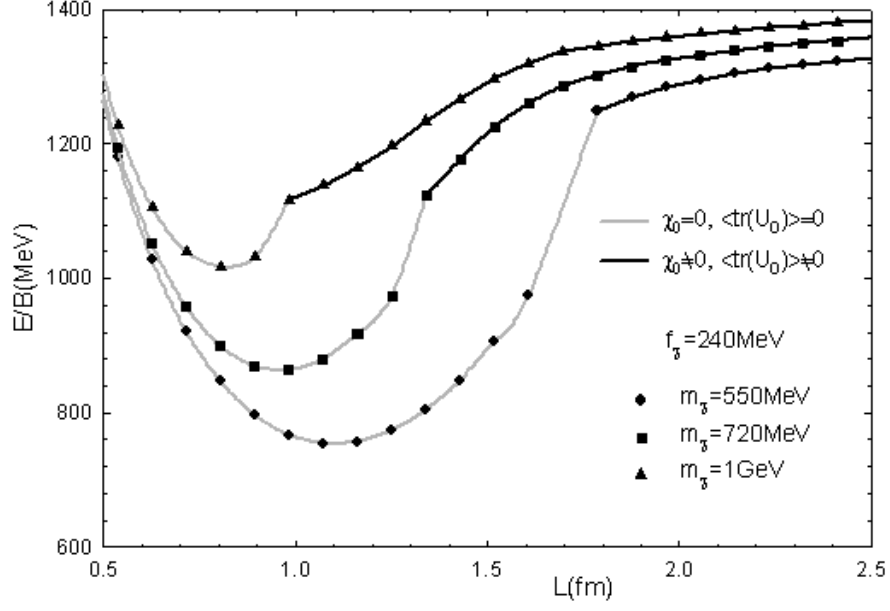


Figure 4: Energy per single skyrmion as a function of the size parameter  $L$ . The solid circles, solid squares, solid triangles are the results obtained with the parameter sets A, B, and C, respectively.

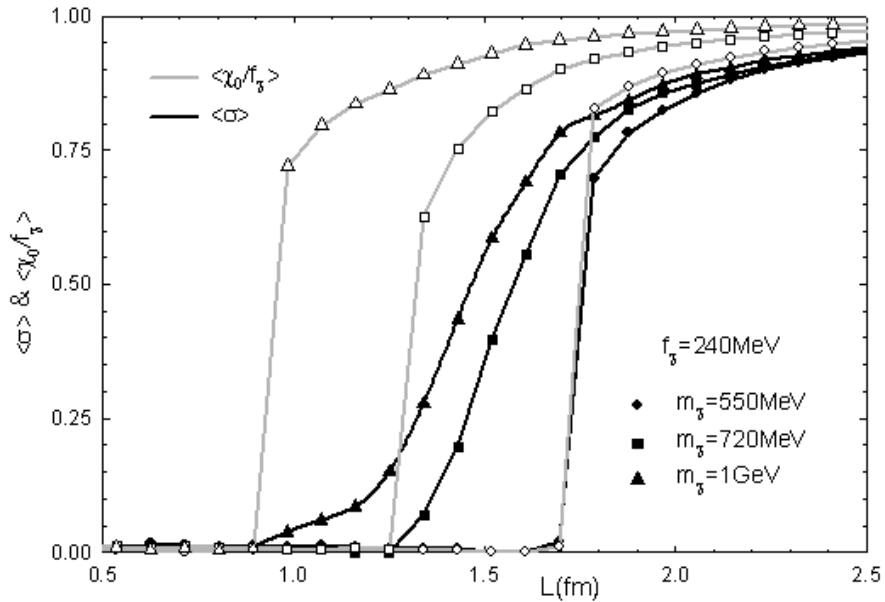


Figure 5:  $\langle \sigma \rangle$  and  $\frac{\chi_0}{f_\pi}$  as a function of the size parameter  $L$ .

## 5 Properties of Pions and Scalar in Skyrmion Matter

In this section, we study the properties of the pionic and scalar excitations in dense matter. For this purpose, we incorporate the fluctuations on top of the static skyrmion crystal as in [7]. This can be achieved using the Ansatz

$$U(x) = \sqrt{U_\pi(x)U_0(\vec{x})}\sqrt{U_\pi(x)}, \quad \chi(x) = \chi_0(\vec{x}) + \tilde{\chi}(x), \quad (32)$$

where  $U_\pi = \exp(i\vec{\tau} \cdot \vec{\phi}/f_\pi)$  as in ref. [7]. Hereafter, we will drop the tilde to denote the  $\chi$  fluctuation.

Expanding the Lagrangian (5) up to the second order in the fluctuating fields, we obtain

$$\mathcal{L}(U, \chi) = \mathcal{L}_M(U_0, \chi_0) + \mathcal{L}_{M,\chi} + \mathcal{L}_{M,\phi} + \mathcal{L}_{M,\phi\chi}, \quad (33)$$

where the subscript ‘M’ denotes the matter field,  $\mathcal{L}_M$  the Lagrangian density for the static configuration for the background matter and the various terms are given by

$$\begin{aligned} \mathcal{L}_{M,\phi} = & \frac{1}{2}G_{ab}\dot{\phi}_a\dot{\phi}_b - \frac{1}{2}H_{ab}^{ij}(\vec{x})\partial_i\phi_a\partial_j\phi_b - \frac{1}{2}\left(\frac{\chi_0}{f_\chi}\right)^3 m_\pi^2\sigma(\vec{x})\phi_a^2 \\ & + \frac{1}{2f_\pi^2}\epsilon_{abc}\partial_\mu\phi_a\phi_b V_c^\mu(\vec{x}) \end{aligned} \quad (34)$$

$$\begin{aligned} \mathcal{L}_{M,\chi} = & \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\left[m_\chi^2\left(\frac{\chi_0}{f_\chi}\right)^2\left(1 + 3\ln(\chi_0/f_\chi)\right) + \frac{2}{f_\chi^2}\frac{f_\pi^2}{4}\text{Tr}(\partial_i U_0^\dagger\partial_i U_0)\right. \\ & \left. + \frac{6\chi_0}{f_\chi^3}f_\pi^2 m_\pi^2(1 - \sigma(\vec{x}))\right]\chi^2 \end{aligned} \quad (35)$$

$$\mathcal{L}_{M,\phi\chi} = \left(\frac{2\chi_0}{f_\chi^2}\frac{f_\pi}{4}\text{Tr}(i(L_i - R_i)\tau^a)\right)\chi\partial^i\phi_a + \left(\frac{3\chi_0^2}{f_\chi^3}\frac{f_\pi m_\pi^2}{2}\text{Tr}(i\tau_a U_0)\right)\chi\phi^a. \quad (36)$$

Here,

$$G_{ab}(\vec{x}) = \left(\frac{\chi_0}{f_\chi}\right)^2 (\delta_{ab} + g_{ab}(\vec{x})) + \frac{1}{32e^2 f_\pi^2}\text{Tr}([R_i, \tilde{\tau}^a][R_i, \tilde{\tau}^b]), \quad (37)$$

$$H_{ij}^{ab}(\vec{x}) = G^{ab}\delta_{ij} + \frac{1}{32e^2 f_\pi^2}\text{Tr}\left([R_i, R_j][\tilde{\tau}^a, \tilde{\tau}^b] - [R_i, \tilde{\tau}^b][R_j, \tilde{\tau}^a]\right), \quad (38)$$

$$V_i^a(\vec{x}) = \left(\frac{\chi_0}{f_\chi}\right)^2 \frac{i}{4}f_\pi^2\text{Tr}[(L_i + R_i)\tau^a] + \frac{i}{16e^2}\text{Tr}\left([L_j, \tau^a][L_i, L_j] + [R_j, \tau^a][R_i, R_j]\right), \quad (39)$$

where for later convenience, we have defined a part of  $G^{ab}$  separately as

$$g_{ab}(\vec{x}) = \frac{1}{4}\text{Tr}(\tau_a U_0 \tau_b U_0^\dagger - \tau_a \tau_b) = -(\pi^2\delta_{ab} - \pi_a \pi_b) \quad (40)$$

which comes from the current algebra term in the Lagrangian. Here  $L_i$  and  $R_i$  are defined by

$$L_i = (\partial_i U_0^\dagger)U_0, \quad R_i = (\partial_i U_0)U_0^\dagger, \quad (41)$$

in terms of the background matter fields.

The previous equations show how the medium, represented by the background skyrmion field, influences the elementary excitations. In order to get an idea of the expected results we show the

tree level approximation where we have substituted the background field by its space average over the cell. In this way we obtain closed form expressions for the *in-medium parameters* showing their relation to their vacuum values.

Due to the symmetry of the background field,  $\langle \text{Tr}(i\tau_a U_0) \rangle$  vanishes in the averaging procedure and therefore there are no terms with  $\chi\phi_a$  in  $\mathcal{L}_{M,\chi\phi}$  to this order. The relevant terms arise from  $\mathcal{L}_{M,\phi}$  and  $\mathcal{L}_{M,\chi}$  and can be written as

$$\mathcal{L}_{M,\phi} = \frac{Z_\pi^2}{2} \dot{\phi}_a \dot{\phi}_a - \frac{m_\pi^{*2} Z_\pi^2}{2} \phi_a^2 \dots, \quad (42)$$

$$\mathcal{L}_{M,\chi} = \frac{1}{2} \dot{\chi} \dot{\chi} - \frac{m_\chi^{*2}}{2} \chi^2 \dots. \quad (43)$$

where the pion wave function renormalization constant  $Z_\pi$ , the in-medium pion mass  $m_\pi^*$  and scalar mass  $m_\chi^*$  are defined as

$$Z_\pi^2 = \left\langle \left( \frac{\chi_0(\vec{x})}{f_\chi} \right)^2 \left( 1 - \frac{2}{3} \pi^2(\vec{x}) \right) \right\rangle \equiv \left( \frac{f_\pi^*}{f_\pi} \right)^2, \quad (44)$$

$$m_\pi^{*2} Z_\pi^2 = \left\langle \left( \frac{\chi_0(\vec{x})}{f_\chi} \right)^3 \sigma(\vec{x}) m_\pi^2 \right\rangle, \quad (45)$$

$$m_\chi^{*2} = \left\langle m_\chi^2 \left( \frac{\chi_0(\vec{x})}{f_\chi} \right)^2 \left( 1 + 3 \ln(\chi_0(\vec{x})/f_\chi) \right) + \frac{2}{f_\chi^2} \frac{f_\pi^2}{4} \text{Tr}(\partial_i U_0^\dagger \partial_i U_0) + \frac{6\chi_0}{f_\chi^3} f_\pi^2 m_\pi^2 (1 - \sigma) \right\rangle. \quad (46)$$

The wave function renormalization constant  $Z_\pi$  gives the ratio of the in-medium pion decay constant  $f_\pi^*$  to the free one, and the above expression arises from the current algebra term with  $f_\pi$  in the Lagrangian. The other two equations reflect how the medium affects the effective masses of the mesons.

In Fig. 6 we show the ratios of the in-medium parameters relative to their free-space values. Only the results obtained with the parameter set B are presented. The other parameter sets yield similar results except that  $L_{pt}$  takes different values<sup>4</sup>. It is important to note that at  $L = L_{pt}$ , *all* the ratios approach zero except for  $m_\chi^*/m_\chi$ .

Let us analyze these ratios at the light of the above equations and related work. As stated before  $\chi_0$  and  $\langle \sigma \rangle$  vanish in the dense matter phase. This is the reason for the vanishing of two of the above ratios. The non-vanishing of  $m_\chi^*/m_\chi$  is due to the existence of a pure background term which appears in the contribution to the in-medium scalar mass. This term substitutes in the dense matter phase the trace anomaly relation eq. (7) by

$$\frac{1}{2} m_\chi^* f_\chi = \sqrt{\left\langle \frac{f_\pi^2}{8} \text{Tr}(\partial_i U_0^\dagger \partial_i U_0) \right\rangle}, \quad (47)$$

which remains finite. In line with our previous arguments, we see that quantities related to the condensate in free space become tied to chiral fields in dense matter.

The extra terms in eq. (46), which are not proportional to  $m_\chi^2$ , come from the properly scaled current algebra and the pion mass terms. In the (matter-free) vacuum these terms describe the

<sup>4</sup>Since the parameters are not uniquely given and will strongly depend on the details of the Lagrangian, one should not take the precise value of  $L_{pt}$  seriously.

couplings of  $\chi$  to the  $\pi$ 's. In dense medium  $\chi$  couples to the static background matter fields which contribute to the  $\chi$  mass.

The vanishing of the pion mass, which already occurred in the pure Skyrme model calculation due to the vanishing of the  $\langle\sigma\rangle$ , shows the chiral behavior of  $\chi_0$ , since the transition density for both phenomena is the same.

The vanishing of  $f_\pi^*/f_\pi$  represents the main qualitative difference from our previous calculation [7] and requires a more careful analysis. In our previous calculation the ratio was found to saturate to  $\sim 2/3$  at high density. The scalar field shifts the effective potential of the pion in matter from one where the symmetry of the ground state is spontaneously broken, to one where it is not, i.e., in the language of the sigma model, the scalar field “lifts” in the medium the matter-free vacuum constraint. One may interpret the scalar field  $\chi_0$  as the “radius field” of Chanfray *et al* [20]<sup>5</sup>.

The phenomenon discussed above is closely related to “Brown-Rho” scaling [5]. In the description of [5], the density dependence comes solely from the change in the mean field  $\chi^*$  with the corresponding change to the skyrmion structure ignored. Our present result corrects and gives a precise meaning to the scaling relation of [5]. Similarly eq. (44) compares to the corresponding  $f_\pi^*/f_\pi$  of ref. [5]. Again an additional factor,  $(1 - \frac{2}{3}\vec{\pi}^2(\vec{x}))$ , associated with the background property appears as a correction.

At low matter density, the ratio  $f_\pi^*/f_\pi$  can be fit to a linear function

$$\frac{f_\pi^*}{f_\pi} \sim 1 - 0.24(\rho/\rho_0) + \dots \quad (49)$$

At  $\rho = \rho_0$ , this yields  $f_\pi^*/f_\pi = 0.76$  which is to be compared with  $f_\pi^*/f_\pi \approx 0.78$  of ref. [5]. The ratio  $m_\chi^*/m_\chi$  scales similarly to  $f_\pi^*/f_\pi$  up to  $\rho \sim \rho_0$ . In Tab. 3, we list the slopes of the ratios. The inset figure in Fig. 6 shows the behavior of the masses  $m_\pi^*$  and  $m_\chi^*$ . They become *nearly* degenerate close to the “critical” density. This near degeneracy may be indicative of the “mended symmetry” discussed by Beane and van Kolck [16]. Note however that in contrast to the dilaton limit of [16], the scalar mass remains finite at the phase transition as pointed out before while the pion mass vanishes<sup>6</sup>.

It is interesting to look at the decay of  $\chi$  into two pions in the medium. Gathering the terms with a scalar field and two pion fields, we get the Lagrangian density for the process  $\chi \rightarrow \pi\pi$

$$\mathcal{L}_{M,\chi\pi^2} = \frac{\chi_0}{f_\chi^2}(\delta_{ab} + g_{ab})\chi\partial_\mu\phi_a\partial^\mu\phi_b + \frac{3\chi_0^2}{2f_\chi^3}m_\pi^2\sigma(\vec{x})\chi\phi_a^2, \quad (50)$$

To compare this result with eqs. (8,9) we take only the first term, i.e., the current algebra term, in the Lagrangian. Averaging the space dependence of the background field configuration modifies the coupling constant by a factor  $\langle(\chi_0/f_\chi)(1 + g_{11})\rangle = \langle(\chi_0/f_\chi)(1 - \frac{2}{3}\pi^2)\rangle$ . Taking into

<sup>5</sup>It is easy to see this in the work of Beane and van Kolck [16] who relate  $\Sigma \propto U_\chi$  to the sigma model fields  $\sigma + i\vec{\tau} \cdot \vec{\pi}$  with of course no sigma model constraint on the radius of the chiral circle. According to this interpretation,  $\chi_0$  is, trivially, what lifts the vacuum constraint by definition. Furthermore, if we take  $\langle\chi_0/f_\chi\rangle$  as  $1 + \theta/f_\pi$  of ref.[20], we can expand  $f_\pi^*/f_\pi$  of Eq.(44) as

$$f_\pi^*/f_\pi \sim \left(1 + \theta/f_\pi - \frac{1}{3}\vec{\pi}^2\right). \quad (48)$$

This is related to eq.(43) of ref.[20]. (In order for the comparison, we have to take into account that  $\vec{\pi}$  corresponds to their  $\vec{\phi}/f_\pi$ .) Note also that, in ref.[20], the last term proportional to the “*nuclear virtual pion condensation*” cannot be derived by their “shifted”  $\theta$ ; it is put in by hand.

<sup>6</sup>Our skyrmion Lagrangian is not applicable after the phase transition, so the spectra in the Wigner phase cannot be considered physically meaningful.



Table 3: The slope of the ratios near the origin

	$f_\pi^*/f_\pi$	$m_\pi^*/m_\pi$	$f_\chi^*/f_\chi$	$m_\chi^*/m_\chi$
set A	- 0.21	- 0.01	- 0.14	- 0.25
set B	- 0.25	- 0.03	- 0.14	- 0.28
set C	- 0.12	- 0.005	- 0.07	- 0.14

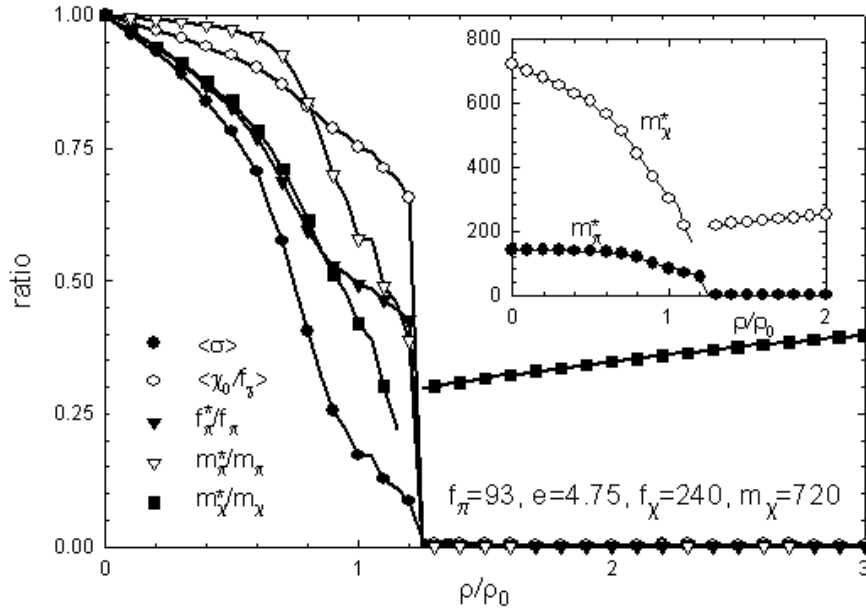


Figure 6: The ratios of the in-medium parameters to the free space parameters. The graph in a small box shows the masses of the pion and the scalar.

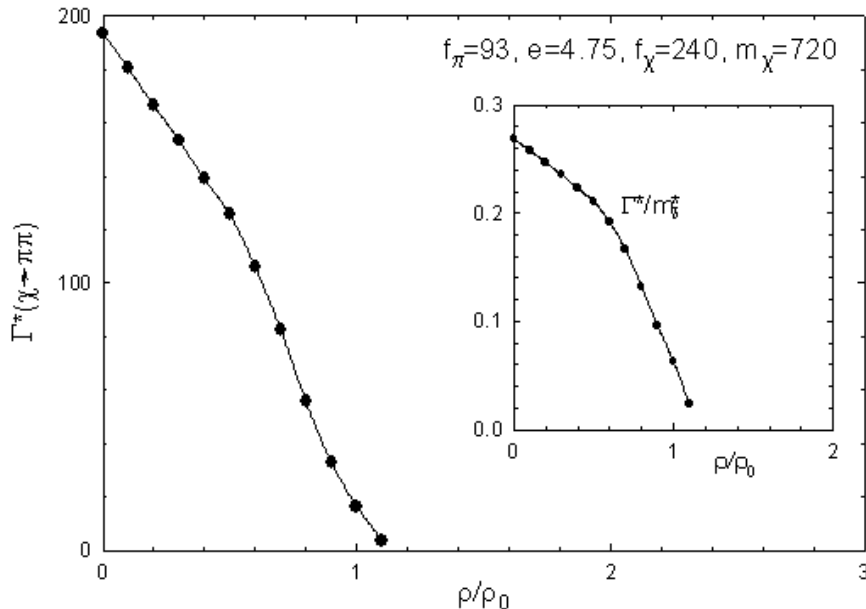


Figure 7: The in-medium decay width  $\Gamma^*(\chi \rightarrow \pi\pi)$  as a function of  $\rho$ . We use eq.(51) (not eq.(52)) to evaluate the quantities.

account the appropriate wave function renormalization factors,  $Z_\pi$ , and the change in the scalar mass, we obtain, in the chiral limit, an in-medium decay width of the form

$$\Gamma^*(\chi \rightarrow \pi\pi) = \frac{3m_\chi^{*3}}{32\pi f_\chi^2} \left| \frac{\langle (\chi_0/f_\chi)(1 - \frac{2}{3}\pi^2) \rangle}{\langle (\chi_0/f_\chi)^2(1 - \frac{2}{3}\pi^2) \rangle} \right|^2. \quad (51)$$

Applying the naive mean field approximation for averaging, we arrive at

$$\Gamma^*(\chi \rightarrow \pi\pi) \approx \frac{3m_\chi^{*3}}{32\pi f_\chi^{*2}}. \quad (52)$$

This expression corresponds to the free one where  $m_\chi$  and  $f_\chi$  are replaced by the in-medium quantities  $m_\chi^*$  and  $f_\chi^*$ . We show in Fig. 7 the in-medium decay width predicted with the parameter set B. In the region  $\rho \geq \rho_{pt}$  where  $\chi_0 = 0$ ,  $\Gamma^*$  cannot be defined to this order. Near the critical point, the scalar becomes an extremely narrow-width excitation, a feature which has been discussed in the literature as a signal for chiral restoration [17, 18].

## 6 From an Inhomogeneous Phase to the Half-Skyrmion Crystal

Up to this point, we have described the background field as an FCC skyrmion crystal. However, as can be seen in Fig. 4, in the range of  $L > L_{min}$ , the pressure of the skyrmion system  $-\partial E/\partial V$  is negative, which implies that the system is unstable. As discussed in ref. [7], the system chooses to avoid instability by going into an inhomogeneous phase instead of remaining in the homogeneous crystal structure. In this stable phase some part of the volume is occupied by the matter with a density  $\rho_{min} \equiv 1/2L_{min}^3$  and the rest of the volume is empty. We call this “inhomogeneous phase.” In this inhomogeneous phase, the spatial average is calculated by

$$\langle Q \rangle^{(i)} = 1 - (\langle Q \rangle_{vac}^{(h)} - \langle Q \rangle_{min}^{(h)})(\rho/\rho_{min}), \quad (53)$$

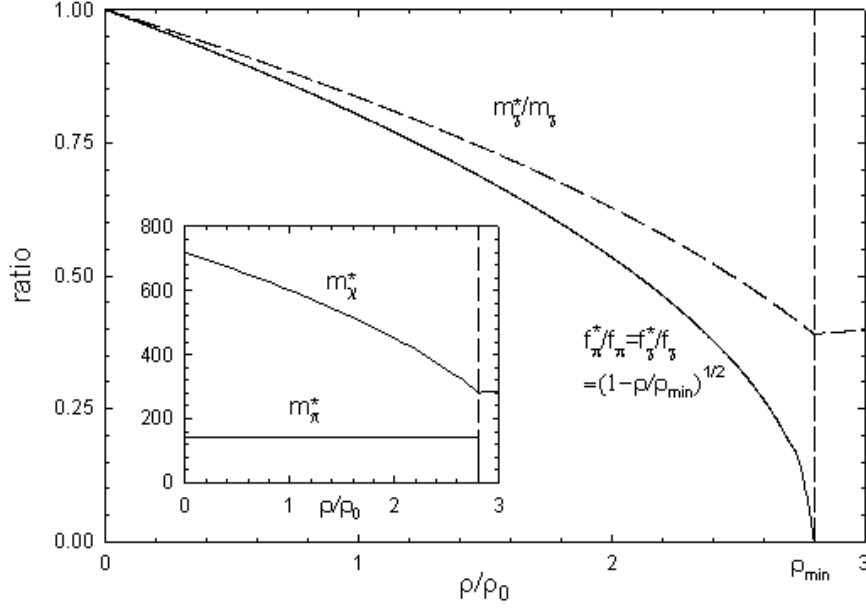


Figure 8: The scaling of the in-medium parameters in the inhomogeneous skyrmion matter. The graph in a small box shows the masses of the pion and the scalar.

where the superscripts ‘(i)’ and ‘(h)’ denote that the average values are evaluated for the inhomogeneous and homogeneous phases, respectively, and the subscripts ‘vac’ and ‘min’ denote, respectively, the vacuum and the lowest energy configuration at  $\rho = \rho_{min}$ .

What comes out from this naive averaging are the the relevant quantities in the free space and those in the minimum energy half-skyrmion phase. Making the averages in eqs. (44-46), we obtain the simple scaling

$$(f_\chi^*/f_\chi)^{2(i)} = (f_\pi^*/f_\pi)^{2(i)} = (m_\pi^* f_\pi^*/f_\pi m_\pi)^{2(i)} = 1 - \rho/\rho_{min}, \quad (54)$$

and

$$(m_\chi^*/m_\chi)^{2(i)} = 1 - (1 - (m_\chi^*/m_\chi)_{min}^{2(h)})\rho/\rho_{min}, \quad (55)$$

where we have exploited that only the ratio  $(m_\chi^*/m_\chi)_{min}^{2(h)}$  is non-vanishing.

We show in Fig. 8 the schematic scaling behavior of physical quantities in the inhomogeneous background matter. Here, the phase transition occurs at  $\rho = \rho_{min}$  from the inhomogeneous phase to the half-skyrmion phase. The scaling is given simply by eq.(54) (which depends only on  $\rho_{min}$ ) and eq.(55). Note that in this naive approximation, the pion mass stays unchanged up to  $\rho = \rho_{min}$  and then it drops to zero. At low density, we have

$$\frac{f_\chi^*}{f_\chi} \approx \frac{f_\pi^*}{f_\pi} \approx 1 - \frac{1}{2} \frac{1}{2.8} (\rho/\rho_0) + \dots = 1 - 0.18(\rho/\rho_0) + \dots, \quad (56)$$

$$\frac{m_\pi^*}{m_\pi} \approx 1, \quad (57)$$

$$\frac{m_\chi^*}{m_\chi} \approx 1 - 0.15(\rho/\rho_0) + \dots, \quad (58)$$

where we have used the parameter set B for which  $\rho_{min}/\rho_0 \approx 2.8$ . These results are consistent with the scaling found by Song *et al* [19]. See eqs. (16-18) of ref. [19].

It would be desirable to see how the same quantities scale near the phase transition at  $\rho = \rho_{min}$ . Unfortunately, our naive expression does not allow us to do so.

## 7 Further Remarks and Conclusion

This paper represents a significant step toward a systematic understanding of in-medium properties of hadrons from the perspective of a unified theory of the hadronic interactions. In our previous work [7], our starting point was the Skyrme Lagrangian whose parameters are defined in order to reproduce mesonic properties in the  $B = 0$  sector and single nucleon properties in the  $B = 1$  sector and then we extended the description to a skyrmion matter as a way to understand the properties of nuclear matter and the in-medium properties of mesons and nucleons. There the phase transition from Goldstone phase to Wigner phase took place from an FCC crystal state to a half-skyrmion CC crystal structure. The chiral-symmetry restored phase supported a non-vanishing pion decay constant. We interpreted this phase as an analog to the pseudo-gap phase of high  $T_c$  superconductivity. However there may be a different interpretation. The fact that chiral symmetry is restored with a non-vanishing pion decay constant is reminiscent of Georgi’s vector limit where chiral restoration occurs with the excitation of scalar partners of the Goldstone pions with an equal and non-vanishing decay constant [23]. It is known however that the Georgi vector limit is not consistent with the chiral Ward identity [1], so we believe that if the model is viable, a more likely possibility is the pseudo-gap type realization.

In this paper, we extended the model studied in [7] to one in which the trace anomaly of QCD is implemented in terms of a dilaton scalar field. The idea is very similar to what was adopted in [5] and is closely based on the Lagrangian introduced by Ellis and Lanik in 1985 [8].

The first question we had to address was how to interpret, in the context of the issue at hand, the scalar interpolating field introduced to represent the conformal symmetry breaking in QCD in terms of physical particles. It is generally understood that the  $B = 0$  sector is dominated by pions and gluonium with the trace anomaly more or less saturated by the glueball contribution [24, 9]. However chiral symmetry breaking and scale symmetry breaking must be intricately tied, requiring the presence of a quarkonium component. How the two components are mixed is presently a hotly debated issue and is not our concern here. We simply assume that the trace anomaly has two components, the dominant “hard” gluonium component and the subdominant “soft” quarkonium component. In a spirit close to that taken by Beane and van Kolck [16], we assume that in matter, the hard component gradually decouples with increasing density and in the so-called dilaton limit reached at the critical point, only the quarkonium remains active. Thus the “soft” component is instrumental in describing the chiral properties in dense systems<sup>7</sup>. In order to represent the degree of our ignorance, we consider three possible scenarios with the mass of the scalar  $m_\chi = 550, 720$  and  $1000$  MeV. The results obtained from the three do not differ qualitatively, but the densities at which the interesting phenomena take place do change substantially.

The physics we analyze in this new model is closely related with the one presented in our earlier work [7]. However the details are modified dramatically from the previous work. As

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<sup>7</sup>In temperature driven chiral restoration, roughly half of the gluon condensate  $\langle G^2 \rangle$  decondenses across the phase transition [25]. Since the gluonium contribution dominates at zero temperature, this means that part of the gluonium contribution must also melt at the transition, not just the quarkonium part which is small at  $T = 0$ . We have no clear idea as to how to unravel this subtlety but we believe this not to be crucial for the qualitative argument we are developing here.

before, the skyrmion matter we consider has two phases: a low-density phase, which we ultimately describe by an inhomogeneous phase, and a high-density phase, which is described by a CC half-skyrmion crystal. In the former phase which we simulate up to at most a few times nuclear matter density by an inhomogeneous structure – the closest we can get at present to a Fermi liquid phase – the pions in the medium are massive and their dynamics determined by an effective theory where chiral symmetry is spontaneously broken, while the scalar has a large mass and couples strongly to the pions. The parameters governing the phase transition,  $\langle\sigma\rangle$  and  $\chi_0$ , are non-vanishing. In the dense phase, on the contrary, skyrmion matter becomes a stable crystal where the two parameters vanish simultaneously at the same density. The pions become massless and their dynamics is governed by a chirally restored theory, while the scalar remains with a small but finite mass decoupled from the pions. The fact that both parameters vanish at the same density confirms the link between chiral symmetry restoration and the “soft” component of the  $\chi$  field. The scale anomaly is maintained by the background field and the “hard” component, which decouples from the pions and describes scalar gluonium by itself. Our model realizes specifically the “lifting” of the radius-field scenario of Chanfray *et al* [20] and provides a more precise meaning to the scaling behavior proposed in [5]. The field  $\chi_0$  defines the radius of the chiral circle for the in-medium pions and therefore when it is non-vanishing their ground state is degenerate and chiral symmetry is spontaneously broken. At the phase transition the chiral circle abruptly shrinks to zero and chiral symmetry is restored. In our model the background fields,  $\chi_0$  and  $(\sigma, \vec{\pi})$ , representing skyrmion matter play a dominant role in this “lifting” process.

The model enables one to calculate leading corrections to “Brown-Rho” scaling [5]. The in-medium  $f_\pi$  decreases with density at approximately the same rate as in [5] for low densities. However in our model, two mechanisms participate in this decrease, the changing of  $\chi_0$  and the deformation of the skyrmion background fields. As mentioned, the main difference with respect to our previous work of [7] is that here  $f_\pi$  vanishes in the dense phase whereas it does not in [7]. This vanishing is solely associated with the scalar field.

The masses of the pions and the scalar meson decrease with density in a very characteristic way. The pion mass does not change much over the range of density relevant to our consideration. Indeed for low density its mass basically remains constant. Not so for the scalar, whose mass decreases rapidly, so much so that close to the phase transition it becomes nearly degenerate with the pion. This phenomenon may be related to the “mended symmetry” in the dilaton limit of the Beane-van Kolck scenario [16] and provides a support to our understanding of the behavior of the  $\chi$  field. However the scalar mass remains finite at the phase transition and in the dense phase, while the pion mass vanishes identically. The reason for the non-vanishing of the scalar mass is associated to the scale anomaly contribution from the background fields. The corresponding term is small. However when all the remaining terms vanish according to the Beane-van Kolck scenario, the small term becomes relevant. In the true dilaton limit the scalar should also become massless and would become the  $\sigma$ -meson of the sigma model Lagrangian. However the background obstructs this limit through its contribution to the scale anomaly and gives the  $\sigma$  a non-vanishing mass.

We have also calculated the decay width of the scalar in the medium, and we realize that at the phase transition the decay width vanishes, signalling the consolidation of the scalar as a hadronic stable particle and a possible manifestation of chiral restoration as discussed in the literature [17, 18].

We now make a brief remark on the implications of this work on the parametric dependence of an effective field theory matched to QCD discussed in the introduction. What we have computed is the density dependence of the ratio  $f_\chi^*/f_\chi$  and the response to density of the background

skyrmion. They should provide density dependence to *all* variables of interest in the model. Since we have no vector degrees of freedom explicitly present in the model, we cannot address directly how the gauge coupling  $g$  scales. However we can think of implementing the vectors in a manner consistent with HLS in which case, their scaling is likely governed by the scaling of the  $\chi_0$  field as first proposed in [5] with corrections coming from the change in the background skyrmion. Thus it is highly likely that we will recover at the chiral transition the limit  $g \rightarrow 0$  together with  $m_\rho^*/m_\rho \rightarrow 0$  as in BR scaling of [5].

An important feature of the description of the medium properties, discussed in the literature [26] - [30], is the behavior of the so called pion velocity. This quantity denoted  $v_\pi$  is given by the ratio of the space part over the time part of the pion decay constant  $f_\pi^s/f_\pi^t$  and characterizes how different effective schemes approach *QCD* around the chiral phase transition as proposed in [30]. In hot/dense matter the dynamics breaks Lorentz invariance and hence the different components could differ in general. For instance, when chiral restoration is induced by temperature, there can take place two drastically different phenomena. If one assumes that in the vicinity of chiral restoration the only relevant degrees of freedom are pions, then the pion velocity is found [26] to vanish  $v_\pi \rightarrow 0$  as  $T \rightarrow T_c$ . However if there are other light degrees of freedom that can enter in the chiral transition as, e.g., in the hidden local symmetry theory of Harada and Yamawaki [1] where the light  $\rho$ -mesons with the vector manifestation play a crucial role, the prediction [30] is that the velocity approaches 1 as the critical temperature is approached. There can be a small deviation from 1 if one takes into account Lorenz-breaking terms in medium at the scale at which the effective field theory (HLS/VM) and QCD correlators are matched. In our present scheme where we are concerned with dense matter, the situation is quite analogous to the case of HLS/VM. At the leading order of pion fluctuation in the background of the skyrmion matter, the pion velocity is 1. This is because the influence of the background at this order is principally from the scalar field which is Lorentz-scalar. The deviation from 1 can come at the next order where the pion interacts with the static soliton background. It turns out that the deviation from 1 in  $v_\pi$  remains small at least in perturbation theory, resembling the case of HLS/VM at  $T \approx T_c$ . The details will be given in a subsequent publication [31].

There are several important issues with the model that remain to be addressed. First of all, we treated the normal matter to be an inhomogeneous matter to which the FCC crystal state collapses. This is not quite the Fermi-liquid state we know nuclear matter should be in. To obtain a Fermi-liquid state from skyrmion matter, we need, at the very least, to quantize the collective degrees of freedom of the skyrmion matter. This is being worked out at present. The solution to this problem will provide us a more realistic information on the intrinsic dependence on the background than what we have done here.

The phase that possesses Wigner symmetry at high density is a half-skyrmion CC crystal and it is most likely a solid. It is not clear whether this state – when the color gauge fields are suitably implemented – is color-superconducting as predicted by QCD at asymptotic density [32] or a precursor to color superconductivity. Ignoring this, suppose that such a solid state is stabilized in compact stars. An interesting question is what the solid state implies for the properties of dense stars and their possible observable signatures [33]. A related question is whether this state is different from quark stars or other exotic stars that have been discussed in the literature. This is an issue that may be confronted with the phenomenology of compact stellar systems.

## Acknowledgments

Hee-Jung Lee, Byung-Yoon Park and Vicente Vento are grateful for the hospitality extended to them by KIAS, where this work was initiated. This work was partially supported by grants

MCyT-BFM2001-0262 and GV01-26 (VV), KOSEF Grant R01-1999-000-00017-0 (BYP), M02-2002-000-10008-0 (HJL)

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