The Pion Velocity in Dense Skyrmion Matter

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Abstract

We have developed a field theory formalism to calculate *in-medium* properties of hadrons within a unified approach that exploits a single Lagrangian to describe simultaneously both matter background and meson fluctuations. In this paper we discuss the consequences on physical observables of a possible phase transition of hadronic matter taking place in the chiral limit. We pay special attention to the pion velocity v_π, which controls, through a dispersion relation, the pion propagation in the hadronic medium. The v_π is defined in terms of parameters related to the matrix element in matter of the axial-vector current, namely, the in-medium pion decay constants, f_t and f_s. Both of the pion decay constants change dramatically with density and even vanish in the chiral limit when chiral symmetry is restored, but the pion velocity does not go to zero, decreasing at most 10% over the whole density range studied. A possible pseudogap structure is indicated.

1 Introduction

The initial motivation for building high energy heavy ion colliders was the quest for the quark gluon plasma. However, the data and the theoretical developments have shown that the phase diagram of the hadronic matter is far richer and more interesting than initially thought. At high temperature and/or density, hadrons are expected to possess properties that are very different from those at normal conditions. Chiral symmetry, which under normal conditions is spontaneously broken, is believed to be restored under extreme conditions. The value of the quark condensate $\langle \bar{q}q \rangle$ of QCD is an order parameter of this symmetry, and is expected to drop as the temperature and/or density of hadronic matter is increased. Since the pion is the "litmus" indicator for spontaneously broken chiral symmetry, the various patterns in which the symmetry is realized in QCD, will be directly reflected in the properties of the pions in medium.

The most fundamental quantities governing the dynamics of the pion are its mass m_π and its decay decay constant f_π. These quantities have been the subject of previous studies in the formalism adopted here [1, 2]. The property we are interested in is encoded in the pion dispersion relation in medium, which involves, besides the mass, the so-called pion velocity in medium, v_π. This allows us to gain more insight into the real time properties of the system under extreme conditions and enable us to analyze how the phase transition from normal matter to deconfined QCD phase takes place *bottom-up* from the hadronic side. This issue has been addressed recently in heat bath [3, 4, 5].

At nonzero temperature and/or density, we have to take into account that the Lorentz symmetry is broken by the medium. In the dispersion relation for the pion modes (in the chiral limit)

$$p_0^2 = v_\pi^2 |\vec{p}|^2, \tag{1}$$

the velocity v_π which is 1 in free-space must depart from 1. This may be studied reliably at least at low temperatures and at low densities via chiral perturbation theory [3]. The in-medium pion velocity can be expressed in terms of the time component of the pion decay constant, f_π^t and the space component, f_π^s, [6]

$$\begin{aligned} \langle 0|A_a^0|\pi^b(p)\rangle_{\text{in-medium}} &= if_\pi^t \delta^{ab} p^0, \\ \langle 0|A_a^i|\pi^b(p)\rangle_{\text{in-medium}} &= if_\pi^s \delta^{ab} p^i. \end{aligned} \tag{2}$$

The conservation of the axial vector current leads to the dispersion relation (1) with the pion velocity given by

$$v_\pi^2 = f_\pi^s / f_\pi^t. \tag{3}$$

Other authors [4] define two decay constants, f_t and f_s, different from those of eq. (2)), through the effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{f_t^2}{4} \text{Tr}(\partial_0 U^\dagger \partial_0 U) - \frac{f_s^2}{4} \text{Tr}(\partial_i U^\dagger \partial_i U) + \cdots, \tag{4}$$

where U is an SU(2)-valued chiral field whose phase describes the in-medium pion. In terms of these constants, the pion velocity is defined by

$$v_\pi = f_s / f_t. \tag{5}$$

What we are interested in here is the pion velocity in dense medium as one approaches chiral restoration at the critical density ρ_c. Before entering into this matter, we briefly review the

status of v_π at high temperature which is more relevant to heavy-ion collisions and the early Universe. There are two basically different predictions depending upon what degrees of freedom are taken into account, namely at $T = T_c$, $v_\pi = 0$ in one version which might be referred to as "standard" and $v_\pi \approx 1$ in another version which is non-standard. This stark difference is the motivation for our investigation of dense matter.

Consider the scenario described by the two-flavor linear sigma model where the relevant degrees of freedom in heat bath are the pions $\vec{\pi}$ and a scalar σ. In this model, as the temperature approaches the critical temperature T_c, chiral symmetry becomes restored, the pions and the scalar join in an $O(4)$ multiplet, the phase transition belonging to the $O(4)$ universality class. This is the standard scenario largely accepted by the community in the field. It has been shown [4] that this scenario predicts that at $T = T_c$, the pion velocity must vanish. The argument is simple. Elevating the isovector axial chemical potential μ_A to an $U(1)$ gauge field – a powerful trick, it follows from the low-energy effective action of QCD that the isovector axial susceptibility (ASUS) χ_A is proportional to f_t^2 that figures in eq. (4) where f_t is the time component of the fully renormalized pion decay constant in heat bath. While one cannot reliably compute f_t at low orders, one can use the information that the ASUS must equal the isovector vector susceptibility (VSUS) χ_V at the phase transition. Now one knows that $\chi_V|_{T=T_c} \neq 0$ from lattice measurements which implies that $f_t|_{T=T_c} \neq 0$ and that $f_s|_{T=T_c} = 0$ from a general consideration. Thus we have at $T = T_c$

$$v_\pi = f_s/f_t = 0. \tag{6}$$

This is a simple and unambiguous prediction of the standard linear sigma model.

The situation is markedly different when other light degrees of freedom are relevant. It has been shown [5] that when light vector mesons are in the picture as predicted by the hidden local symmetry theory with the "vector manifestation" à la Harada and Yamawaki [7] (referred to as HLS/VM), the pion velocity is close to 1 at all temperatures near T_c. The crucial element that leads to this result is that the HLS theory with the vector mesons ρ and the pions coupled gauge invariantly and matched to QCD at the matching scale Λ_M has the vector manifestation fixed point at which the parametric mass of the vector meson M_ρ and gauge coupling g vanish. To this fixed point flows the system as one approaches the criticality (in temperature, in density or in the number of flavors). To one-loop order in a generalized chiral perturbation theory in the presence of light vectors – expected to be a good approximation near the critical point, the VM assures that $\chi_A = \chi_V$ at $T = T_c$, $(f_\pi^t, f_\pi^s) \rightarrow (0, 0)$ (with f_π's defined in (2)) as $T \rightarrow T_c$. As a consequence [1]

$$v_\pi \sim 1. \tag{7}$$

We now turn to the main objective of this paper, which is to see whether there is a similar dramatic dependence of the pion velocity on the degrees of freedom also in dense matter. At present, although it has been established that hadronic matter at high density flows to the VM fixed point [9], describing dense matter in HLS/VM is found not to be easy because of coupled renormalization group equations in the renormalization scale and in density. Up to date, no conclusive result on physical variables (e.g., f_π, susceptibilities etc) has been obtained. We need therefore to resort to other formalisms.

At low density, the standard chiral perturbation theory without vector degrees of freedom should be applicable. The in-medium pion decay constants have been computed to first order

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[1]The pion velocity in this scenario is not exactly equal to 1. There is a small deviation from 1 due to the fact that the heat bath violates Lorentz invariance by a small amount at the matching point Λ_M. This has been computed [8].

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in the baryon number density ρ [10]. The results are $f_\pi^t = f_\pi(1 - 0.26\rho/\rho_0)$ and $f_\pi^s = f_\pi(1 - 1.23\rho/\rho_0)$, where ρ_0 is normal nuclear matter density. Note that the space component is highly suppressed with respect to the time one. Therefore the in-medium pion velocity scales as

$$v_\pi^2 = f_\pi^s/f_\pi^t \sim 1 - \rho/\rho_0. \qquad (8)$$

The rapid drop of f_π^s – and hence the pion velocity – suggests that leading order-order chiral perturbation theory cannot be trusted for density near that of normal nuclear matter. This observation calls for a careful re-examination of power counting.

QCD sum rules have also been applied to calculate the in-medium pion decay constants [11, 12]. Unfortunately in the pseudoscalar axial-vector correlation function used, the dimension-5 condensate in the nucleon which cannot be calculated reliably is found to play the most important role in splitting the time and space components of the pion decay constant. This renders quantitative estimates difficult. For instance, a positive value of the condensate requires $f_\pi^s/f_\pi^t > 1$ and makes the pion mass tachyonic [11], at odds with causality. In ref. [12], this problem was circumvented by taking the in-medium pion mass as an *input parameter* and obtained a negative condensate. It was found that the contribution of the intermediate Δ states is important in getting a sensible result. Using the input values for the in-medium mass in the range 139 MeV $< m_\pi^* <$ 159 MeV, the authors in [12] obtained $f_\pi^s/f_\pi = 0.37 \sim 0.78$ and $f_\pi^t/f_\pi = 0.63 \sim 0.79$ and as a consequence the in-medium pion velocity in the range of

$$1/3 < v_\pi < 1. \qquad (9)$$

In this paper, we study the in-medium pion velocity in the systematic field theory scheme developed in our recent works [1, 2]. There, the Skyrme picture is adopted to describe in a unified way both the pions and dense baryonic matter. A contact with HLS/VM theory of Harada and Yamawaki was suggested there. A static soliton solution having an FCC crystal structure [13, 14, 15, 16] [2]

is found to provide a classical description of dense baryonic matter. The pion is incorporated as fluctuating fields on top of this dense skyrmion matter. The interactions between the pions and matter modify the pion effective mass and decay constants. Since the background skyrmion matter breaks Lorentz invariance, the resulting space and time components of the pion decay constant become different.

In the following section, we describe the Skyrme model approach used for investigating the properties of the pions in dense matter. A possible pseudogap structure is described in Section 3. In Sec. 4, we study the in-medium pion velocity by taking into account the background matter effect up to the second order in the interactions. We make a brief conclusion in Sec. 5.

2   Model Lagrangian

The aim of our series of investigations has been to develop the full dynamics of dense matter from a unique Lagrangian, which we have taken for simplicity of Skyrme type. Here, we use a

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[2]A slightly different approach – but similar in spirit – to a crystal structure of dense matter was discussed by Diakonov and Mirlin [17]. These authors studied the behavior of nuclear matter with density by implementing the nucleon-nucleon interaction by means of collective coordinate quantization of the skyrmion-skyrmion interaction and arrived at the conclusion that at twice or three time nuclear matter density a crystalline structure could arise. This calculation rends an additional support that a crystalline structure may indeed figure at high density, as has been shown within our approach [1, 2].

modified Skyrme model Lagrangian [18, 19, 20, 21], which incorporates in an effective way the scale anomaly of QCD in terms of a scalar dilaton field [3]:

$$\mathcal{L} = \frac{f_\pi^2}{4}\left(\frac{\chi}{f_\chi}\right)^2 \mathrm{Tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - \frac{m_\chi^2 f_\chi^2}{4}\left((\chi/f_\chi)^4(\ln(\chi/f_\chi) - \tfrac{1}{4}) + \tfrac{1}{4}\right) \qquad (10)$$

where $U = \exp(i\vec{\tau}\cdot\vec{\pi}/f_\pi) \in SU(2)$ and χ is the scalar field. The parameters in eq. (10) are related to physical properties, f_π is the pion decay constant in free space, f_χ the corresponding quantity for the scalar field and m_χ its mass. For simplicity we will work with massless pions.

The Lagrangian (10) in the $B = 0$ sector describes only mesons and yields the following pion and scalar field solutions in the vacuum,

$$U = 1, \qquad \chi = f_\chi. \qquad (11)$$

The fluctuations about this vacuum can be described by

$$U_\pi = \exp(i\vec{\tau}\cdot\vec{\varphi}/f_\pi), \quad \text{and} \quad \chi = f_\chi + \tilde{\chi}. \qquad (12)$$

Expanded up to second order in φ and $\tilde{\chi}$, the Lagrangian (10) takes the form

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \varphi_a \partial^\mu \varphi_a + \tfrac{1}{2}\partial_\mu \tilde{\chi} \partial^\mu \tilde{\chi} - \tfrac{1}{2}m_\chi^2 \tilde{\chi}^2 + \cdots. \qquad (13)$$

This Lagrangian can also describe, à la Skyrme [23], baryons, i.e., the skyrmions, as topological solitons if a proper stabilizing term, e.g., the Skyrme term

$$\mathcal{L}_{\mathrm{sk}} = \frac{1}{32e^2}\mathrm{Tr}\left([U^\dagger \partial_\mu U, U^\dagger \partial_\nu U]^2\right) \qquad (14)$$

is added. The winding number associated with the homotopy of the static solution $U_0(\vec{r})$ is taken as the baryon number,

$$B = \frac{1}{24\pi^2}\varepsilon^{ijk}\int d^3r \mathrm{Tr}(U_0^\dagger \partial^i U_0 U_0^\dagger \partial^j U_0 U_0^\dagger \partial^k U_0). \qquad (15)$$

The numerical results are sensitively dependent on the parameter values chosen for the Lagrangian. Given that the Lagrangian used is highly approximate, our results cannot be taken at their face values. However we believe that the qualitative structure of the results can be trusted. In this work, we will take the pion decay constant to be the empirical value, say, ~ 93 MeV. As for the parameters associated with the dilaton, we have no direct empirical information: neither the mass nor its structure is experimentally identified. In the literature [24, 25], they are adjusted so that the model fits finite nuclei as well as nuclear matter. The dilaton decay constant comes out to be ~ 240 MeV, while the its mass ranges widely from 0.5 GeV to 1.5 GeV. We shall favor the lower masses, i.e. ~ 700 MeV, as proposed by several investigations [22, 26, 27] although we shall report results for the wide range.

The parameter e of the Skyrme term that figures crucially in stabilizing the skyrmion also plays an important role in setting the scale of dense matter. Unfortunately it is not obvious how to implement density dependence in this parameter. One may naively interpret the Skyrme term as arising when the ρ vector meson is considered to be much heavier than the scale involved and so is integrated out. This cannot be correct however because other mesons can contribute in the

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[3]The role of the dilaton field in our model can be better understood as an interpolating field of the "soft gluon" degree of freedom that locks to chiral symmetry as described in e.g. [22]. The "hard or epoxy gluon" degree of freedom can be considered as integrated out with its effects embedded in the coefficients of the Lagrangian.

heavy-mass limit and can destabilize the soliton. Furthermore, this term cannot be representing the effect of the ρ vector in dense medium since the ρ meson mass drops due to the vector manifestation [7]. On the other hand, the Skyrme term figures at extreme short distances as in the process of the proton decay à la Rubakov [28]. Therefore it seems reasonable to think that the single Skyrme term subsumes a large number of massive degrees of freedom having the appropriate quantum numbers lying above the scale involved in the process. This implies that the constant e – representing short-distance physics – must be varying only slowly as a function of density. This is somewhat analogous to what happens in nuclear physics where the coefficients of zero-range counter terms are "universal" in that they are more or less independent of the density. See [29] where such a nuclear physics phenomenon is encountered. For our purpose here we simply take it as a constant. This assumption can be lifted in a model in which the heavy degrees of freedom are explicitly treated for soliton structure as in hidden gauge symmetry theory. In fact the explicit account of vector mesons in the skyrmion structure is found to be crucial for the bound state description of the pentaquark Θ^+ in the soliton model [30]. Although we do not have a proof, we believe that the explicit presence of the vector mesons would not significantly modify the qualitative feature we are finding at and above nuclear matter density.

In this work, we pick the value of e to fit, with the model, either the baryon mass spectrum [31] or the axial vector coupling constant g_A [32]. The former fit leads to $e \sim 5.45$ (but in this case, f_π cannot have its empirical value, and comes out 64 MeV), while the latter fit produces $e = 4.75$ (with f_π at the empirical value).

Two skyrmions are most attractive when they are relatively rotated by an angle π about an axis perpendicular to the line joining their centers. Thus the lowest energy configuration of skyrmions for a given number density is in the crystal phase with all the neighboring skyrmions in the most attractive orientations. Moreover, the ground state of the scalar field χ is shifted from the constant value f_χ to a static field configuration $\chi_0(\vec{r})$ in order to reduce the energy of the whole skyrmion-scalar system.

3 Phase Transitions

3.1 Chiral restoration

We review and update the numerical results on skyrmion matter given in ref. [2]. The aim here is to give the background for the calculation of the pion velocity discussed in the following section with the error committed in the scale factor in [2] corrected.

In Fig.1 we show the energy per skyrmion E/B of the skyrmion crystal as a function of the baryon number density ρ. The quantities in the chiral symmetry broken phase characterized by a non-vanishing $\langle\chi\rangle$ are drawn by filled symbols, while those in the chiral symmetry restored phase with $\langle\chi\rangle = 0$ are presented by unfilled symbols. In the numerical calculation, the size and energy of the skyrmion have been scaled in units of $(ef_\pi)^{-1}$ and $6\pi^2 f_\pi/e$, respectively. When the decay constants are fixed to $f_\pi = 93$ MeV and $f_\chi = 240$ MeV, the numerical results depend only on the parameter $\mu_\chi = m_\chi/ef_\pi$. We show the results for three different values of μ_χ, namely 1.25 (by triangles), 1.63 (by circles) and 2.26 (by squares), corresponding to $m_\chi = 550$, 720 and 1000 MeV for $f_\pi = 93$MeV and $e = 4.75$. The density is given in units of $0.015(ef_\pi)^3$, where the constant 0.015 is chosen so that it equals the normal nuclear density $0.17(\text{fm})^{-3}$ for $f_\pi = 93$ MeV and $e = 4.75$. We should stress that the exact value of the density does not have a precise physical meaning, since the density scales with e, the Skyrme parameter, which as mentioned above, is not well defined. However we believe it to be a useful, albeit qualitative, guide to the magnitudes of the densities involved.
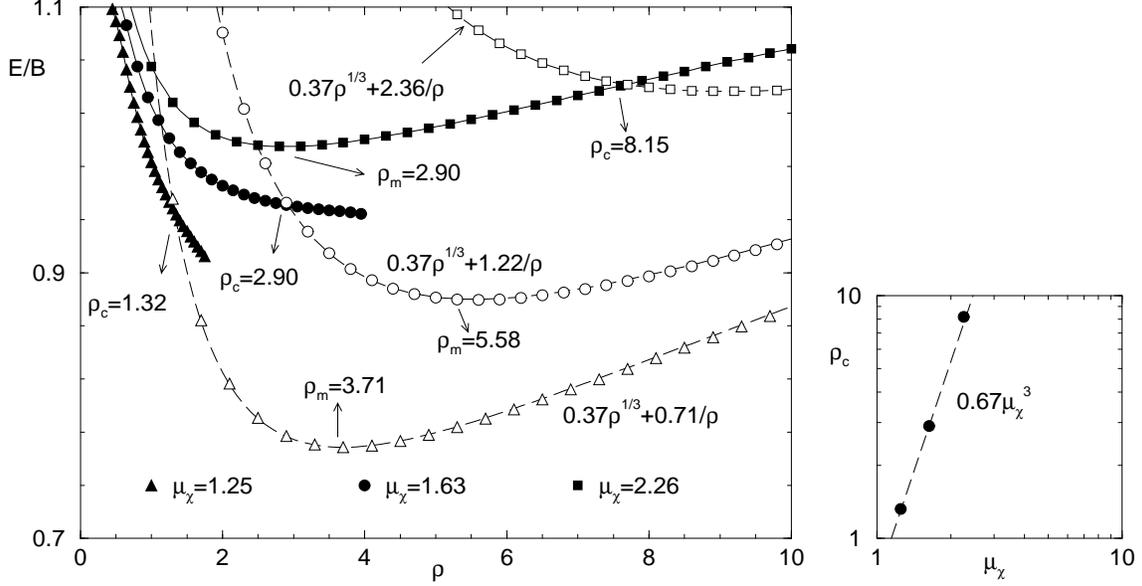
Figure 1: E/B as a function of density. Numerical data for the $\langle\chi\rangle \neq 0$ phase are presented by filled symbols and those for the $\langle\chi\rangle = 0$ phase are presented by empty symbols. The density is in unit of $0.015(ef_\pi)^3$, which equals normal nuclear matter density $\rho_0 = 0.17$ (fm)$^{-3}$ when the conventional values, $e = 4.75$ and $f_\pi = 93$ MeV, are used. E/B is given in unit of $6\pi^2 f_\pi/e$.

As the density increases we reach a value at which the chiral symmetry restored phase has a lower energy than the chiral symmetry broken phase, i.e., a chiral phase transition takes place. Note that, even beyond the phase transition density, $\langle\chi\rangle \neq 0$ is a *local minimum*. On the other hand, in the presence of the background matter, $\langle\chi\rangle = 0$ is always a local minimum whereas in free space, it was just an unstable extremum of $V(\chi)$. This phenomenon can be easily understood using the approximation introduced in ref. [2], namely, taking $\chi(\vec{r})$ as a constant χ_0. In that approximation, the energy per baryon E/B becomes

$$
\begin{aligned}
E/B &= (E_2/B)(\chi_0/f_\chi)^2 + E_4/B \\
&\quad + V(\chi_0) \times [\text{Volume occupied by a single skyrmion}],
\end{aligned}
\tag{16}
$$

where $E_{2,4}/B$ are the contributions to the energy per baryon of the quadratic and quartic in the Lagrangian (10) and $V(\chi)$ is the potential energy density of the dilaton field. Now, it is evident that

$$
\left.\frac{\partial^2(E/B)}{\partial(\chi_0/f_\chi)^2}\right|_{\chi_0=0} = 2(E_2/B),
\tag{17}
$$

and the system is quasi-stable at $\chi_0 = 0$ as long as $2(E_2/B) > 0$. These local minima become true minima depending on the density, the former phase for low densities and the latter for densities above the phase transition.

As an aside, we note that the numerical data for the $\langle\chi\rangle = 0$ phase can be fit by a simple curve

$$
E/B = a\rho^{1/3} + b/\rho,
\tag{18}
$$

where the first term comes from the Skyrme term contribution E_4/B that scales as $\rho^{1/3}$ and the second comes from the potential energy of the dilaton field. The coefficient a is almost independent of μ_χ.
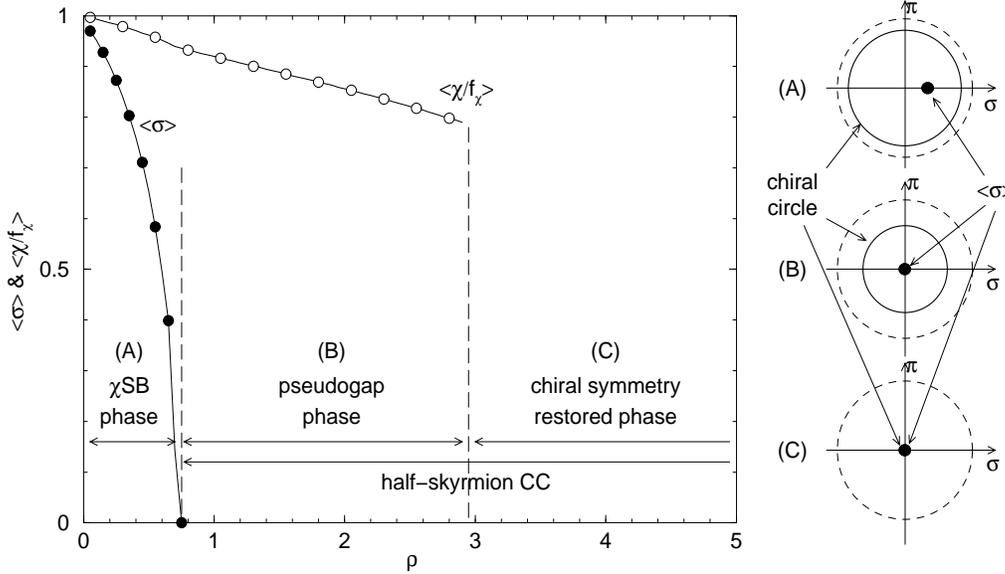
Figure 2: Average values of $\sigma = \frac{1}{2}\text{Tr}(U)$ and χ/f_χ of the lowest energy crystal configuration at a given baryon number density.

The chiral phase transition takes place at the density, ρ_c, where the phase characterized by $\langle\chi\rangle = 0$ and that by $\langle\chi\rangle \neq 0$ have the same E/B. The critical density depends sensitively on the parameter μ_χ. In Fig. 1, we show results for, $\mu_\chi = 1.25$, 1.63 and 2.26 for which ρ_c becomes 1.32, 2.90 and 8.15, respectively. The figure drawn in a small box is the critical density ρ_c as a function of μ_χ. Note that it scales as μ_χ^3. The dashed line in the figure is not a fit but $0.67\mu_\chi^3$ line just passing through the data point at $\mu_\chi = 1.63$.

3.2 Pseudogap phase

In Fig. 2 we show the average values of $\sigma = \frac{1}{2}\text{Tr}U$ and χ/f_χ over space, which provide direct information on the structure of the crystal and can be used as order parameters of the phase transition. These data show that there is a "phase transition" before the chiral restoration transition, at the density at which the expectation value of σ vanishes. Recalling that the expectation of σ is proportional to the quark condensate, this may indicate a change in the chiral symmetry structure due to destructive interference of the phase of the quark condensate with a nonvanishing modulus. In the case of the massive pions, such a phase transition is not abrupt but continuous and this is the reason why we failed to notice it in our previous work [2]. We identify this phase transition with $\langle\sigma\rangle = 0$ while $\langle\chi\rangle \neq 0$ as of a *pseudogap type*. This phase persists in an intermediate density region, where the $\langle\chi/f_\chi\rangle$ does not vanish while $\langle\sigma\rangle$ does [33]. We denote the density at which the pseudogap phase transition appears as ρ_p. Since the vanishing of σ is possible because each skyrmion maps the chiral circle onto space in order to carry nontrivial baryon number, it may be that the pseudogap phase is an artifact of the skyrmion matter. We point out however that a similar pseudogap structure was proposed in hot QCD [34].

The two step phase transition is schematically illustrated in Fig. 2.

(A) At low density ($\rho < \rho_p$), matter slightly reduces the vacuum value of the dilaton field from that of the baryon free vacuum. This implies a shrinking of the radius of the chiral
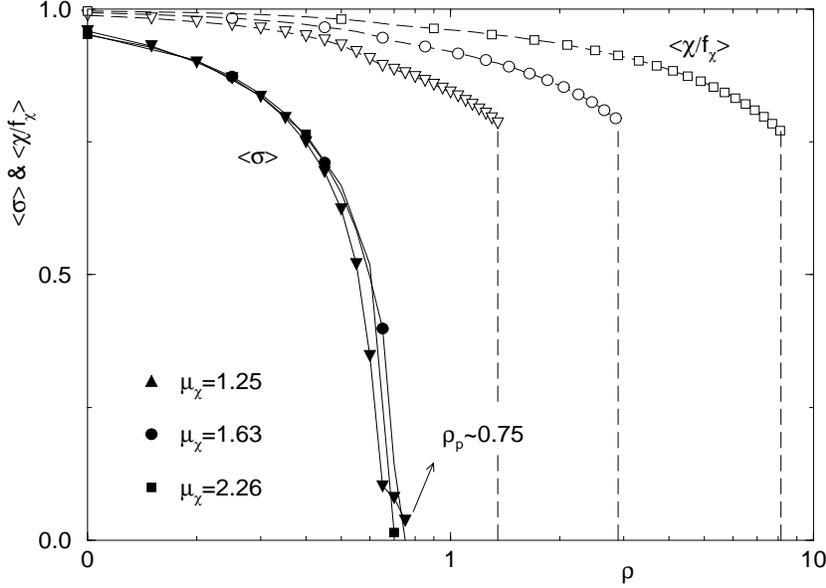
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Figure 3: μ_χ dependence of two step phase transitions.

circle by the same ratio. Since the skyrmion takes all the values on the chiral circle, the expectation value of σ is not located on the circle but inside the circle. Skyrmion matter at this density is in the chiral symmetry broken phase.

(B) At some intermediate densities ($\rho_p < \rho < \rho_c$), the expectation value of σ vanishes while that of the dilaton field is still nonzero. The skyrmion crystal is in a CC configuration made of half skyrmions localized at the points where $\sigma = \pm 1$. Since the average value of the dilaton field does not vanish, the radius of the chiral circle is still finite. Here, $\langle \sigma \rangle = 0$ does not mean that chiral symmetry is completely restored. We interpret this as a pseudogap phase.

(C) At higher density ($\rho > \rho_c$), the phase characterized by $\langle \chi / f_\chi \rangle = 0$ becomes energetically favorable. Then, the chiral circle, describing the fluctuating pion dynamics, shrinks to a point.

In the case of massive pions, the chiral circle is tilted by the explicit symmetry breaking term. Thus, the exact half-skyrmion CC, that requires a symmetry between points with values $\sigma = +1$ and $\sigma = -1$ cannot be constructed and consequently the phase characterized by $\langle \sigma \rangle = 0$ cannot exist at any density. However, $\langle \sigma \rangle$ is always inside the chiral circle and its value drops much faster than that of $\langle \chi / f_\chi \rangle$. Thus, if the pion mass is small enough, a pseudogap phase can appear in the model.

In Fig. 3 we show the two step phase transition at various values of the parameter μ_χ. Contrary to ρ_c, ρ_p depends only insensitively on μ_χ. For small $\mu_\chi \sim 1$, $\rho_p \sim \rho_c$ and its value is smaller than that for large μ_χ. As μ_χ increases, ρ_c increases proportionally to μ_χ^3 as was seen in Fig. 1, while ρ_p saturates to 0.78. Since the density unit is $(ef_\pi)^3$ and recalling that $\sqrt{2}ef_\pi \sim m_\rho$, we can summarize the phenomena in terms of the parameters e and m_χ as,

1. At density $\rho_p \sim 0.78 \times 0.015(ef_\pi)^3 \sim 0.004 m_\rho^3$, a pseudogap phase transition occurs.

2. At density $\rho_c \sim 0.67\mu_\chi^3 \times 0.015(ef_\pi)^3 \sim 0.01 m_\chi^3$, the chiral phase transition takes place,

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with $m_\chi \sim m_\rho$.

4　In-Medium Pion Velocity

The pion and scalar in dense medium can be described by fluctuations on skyrmion matter through the Ansätze

$$U = \sqrt{U_\pi} U_0(\vec{r}) \sqrt{U_\pi}, \quad \text{and} \quad \chi = \chi_0(\vec{r}) + \tilde{\chi}. \tag{19}$$

Expanding up to the second order in the fluctuating fields, we obtain

$$\begin{aligned}
\mathcal{L} &= \tfrac{1}{2} G^{ab}(\vec{r}) \partial_\mu \varphi_a \partial^\mu \varphi_b + \epsilon_{abc} \varphi_a \partial_i \varphi_b V_c^i(\vec{r}) \\
&\quad + \tfrac{1}{2} \partial_\mu \tilde{\chi} \partial^\mu \tilde{\chi} - \tfrac{1}{2} M(\vec{r}) \tilde{\chi}^2 + P_a^i(\vec{r}) \tilde{\chi} \partial_i \varphi_a.
\end{aligned} \tag{20}$$

This Lagrangian describes *single particles* moving under "local background potentials" provided by the static field configurations $U_0(\vec{r})$ and $\chi_0(\vec{r})$. We have shortened the lengthy expressions for the local potentials introducing $G^{ab}(\vec{r})$, $V_c^i(\vec{r})$, $M(\vec{r})$, $P_a^i(\vec{r})$. Their explicit forms are shown in ref. [2]. Assuming background matter to be known, eq. (20) shows, when compared with eq. (13), that a direct φ-χ (not only φ^2-χ) coupling might appear, which is absent in free space. Hereafter, we will drop the tildes for the fluctuating scalar field.

In refs. [1, 2], by replacing the local potentials with their averages over space, we obtained

$$\mathcal{L}_{\text{eff}} = \tfrac{1}{2} \langle G^{aa} \rangle \partial_\mu \varphi_a \partial^\mu \varphi_a + \tfrac{1}{2} \partial_\mu \chi \partial^\mu \chi - \tfrac{1}{2} \langle M \rangle \chi^2 + \cdots. \tag{21}$$

We interpret this result as an effective Lagrangian for the in-medium pions and scalar with "intrinsic" parameters $\langle G^{aa} \rangle$ and $\langle M \rangle$, which correspond to the in-medium pion decay constant and the effective mass for the scalar respectively. In this naive approximation, Lorentz symmetry is still preserved. Thus, the pion velocity is 1 at this level. However, static local potentials will definitely break the Lorentz symmetry. In this work, we are going to evaluate the deviation of the pion velocity from 1 due to this background.

Our strategy in performing the calculation is based on the following result. When local interactions with background matter are taken into account they lead to an effective Lagrangian for pion dynamics in the form of eq. (4) which defines the quantities necessary for the calculation. Let us be more specific. As far as the single pion is concerned, to lowest order in the fields, eq. (4) with $U = \exp(i\vec{\tau} \cdot \vec{\varphi}/f_\pi)$ becomes [4]

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} \left(\frac{f_t}{f_\pi} \right)^2 \partial_0 \varphi_a \partial_0 \varphi_a - \frac{1}{2} \left(\frac{f_s}{f_\pi} \right)^2 \partial_i \varphi_a \partial_i \varphi_a + \cdots. \tag{22}$$

Thus the procedure consists of calculating the propagators for the pion and the scalar fields from (20) by solving for the background potentials. Then, matching these propagators with those obtained from the effective Lagrangian (22), we have for the pion

$$\frac{1}{Z_t^{-1} p_0^2 - Z_s^{-1} \vec{p}^2} \tag{23}$$

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[4]If one substituted $U = \exp(i\vec{\tau} \cdot \vec{\varphi}^*/f_t)$ with *renormalized pion fields* φ_a^*, one would have

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} (\partial_0 \varphi_a^* \partial_0 \varphi_a^* - v_\pi^2 \partial_i \varphi_a^* \partial_i \varphi_a^*) + \cdots.$$

where $Z_{t,s}^{-1} = (f_{t,s}/f_\pi)^2$.

Before proceeding to the calculation, we first obtain relations between the parameters, $f_{t,s}$ and $f_\pi^{t,s}$, defined in Introduction. The effective Lagrangian (4) yields the following axial vector current components

$$A_0^a = -i\frac{f_t^2}{4}\text{Tr}\left(\partial_0 UU^\dagger\frac{\tau^a}{2} - \partial_0 U^\dagger U\frac{\tau^a}{2}\right) = (f_t^2/f_\pi)\partial_0\varphi,$$
$$A_i^a = -i\frac{f_t^2}{4}\text{Tr}\left(\partial_i UU^\dagger\frac{\tau^a}{2} - \partial_i U^\dagger U\frac{\tau^a}{2}\right) = (f_s^2/f_\pi)\partial_i\varphi, \tag{24}$$

where the last equality is obtained by substituting $U = \exp(i\vec{\tau}\cdot\vec{\varphi}/f_\pi)$. When the expectation value of these axial vector current components is taken as in eq. (2), we obtain the relations,

$$f_t = f_\pi^t, \text{ and } f_s = \sqrt{f_\pi^s f_\pi^t}. \tag{25}$$

Similar relations are derived in ref. [5] for the real part of the complex pion decay constants. In what follows, the results will be given in terms of f_t and f_s.

In order to obtain nontrivial results for the pion velocity in the medium we need to take into account the higher order effects of the background potentials. We shall proceed here *in a perturbative* scheme. To do so we decompose the Lagrangian into an unperturbed part, $\mathcal{L}_0$, and an interaction part, $\mathcal{L}_I$. The simplest way to do so is to take the free Lagrangian (13) as $\mathcal{L}_0$ and the rest as $\mathcal{L}_I$. Explicitly,

$$\mathcal{L}_I = \tfrac{1}{2}(G^{ab}(\vec{r}) - \delta^{ab})\partial_\mu\varphi_a\partial^\mu\varphi_b + \epsilon^{abc}\varphi_a\partial_i\varphi_b V_c^i(\vec{r})$$
$$-\tfrac{1}{2}(M(\vec{r}) - m_\chi^2)\tilde{\chi}^2 + P_a^i(\vec{r})\tilde{\chi}\partial_i\varphi_a. \tag{26}$$

The presence of the local interaction potential $G^{ab}(\vec{r})$ in the pion kinetic energy term makes the quantization process of the fluctuating pions nontrivial. The conjugate momenta of the pion fields φ_a are given by

$$\Pi_a \equiv \frac{\partial\mathcal{L}_0}{\partial\dot{\varphi}_a} = \dot{\varphi}_a. \tag{27}$$

We obtain

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I, \tag{28}$$

where

$$\mathcal{H}_0 = \tfrac{1}{2}(\dot{\varphi}^a\dot{\varphi}^a + \partial_i\varphi^a\partial_i\varphi^a) + \tfrac{1}{2}(\dot{\chi}\dot{\chi} + \partial_i\chi\partial_i\chi + m_\chi^2\chi^2), \tag{28a}$$

and

$$\mathcal{H}_I = -\mathcal{L}_I. \tag{28b}$$

Note that the interaction Hamiltonian $\mathcal{H}_I$ is simply $-\mathcal{L}_I$. The free propagators defined by $\mathcal{H}_0$ and the interaction potentials appearing due to $\mathcal{H}_I$ are summarized in Fig. 4. In Fig. 4, $G^{ab}(\vec{\ell})$, for example, is the Fourier transformed of the local potential $G^{ab}(\vec{r})$:

$$G^{ab}(\vec{\ell}) = \frac{1}{V_{\text{box}}}\int_{\text{box}} d^3r\, e^{i\vec{\ell}\cdot\vec{r}}G^{ab}(\vec{r}), \tag{29}$$

where the integration is over a unit box of the crystal and V_{box} is its volume. Due to the periodic structure of the crystal only discrete momentum values are possible.

We show in Fig. 5 the diagrams used to evaluate the self-energy. Only the diagrams for $\Sigma_{\varphi_a\varphi_b}$ appear. The symmetry structure of the skyrmion matter allows a nonvanishing self-energy only for $a = b$, One easily gets the corresponding diagrams for $\Sigma_{\varphi_a\chi}$ or $\Sigma_{\chi\chi}$. To first order, $\Sigma^{(1)}$ is

$$\frac{\delta_{ab}}{p_0^2 - \vec{p}^2}$$

$$\frac{1}{p_0^2 - \vec{p}^2 - m_\chi^2}$$

$$-(p_0^2 - \vec{p}\cdot\vec{q})(G^{ab}(\vec{\ell}) - \delta^{ab})$$
$$+i\epsilon_{abc}\vec{p}\cdot\vec{V}^c(\vec{\ell})$$

$$i\vec{q}\cdot\vec{P}^a(\vec{\ell})$$

$$M(\vec{\ell}) - m_\chi^2$$

Figure 4: Free propagators and interactions for the pion and the scalar fields in the presence of a background skyrmion matter. The energy-momentum conservation δ functions are not shown.

Figure 5: Diagrams used to evaluate the self-energy of the φ_a propagation up to second order in the interaction. Here, b runs over $1, 2, 3$ and the intermediate states run over all $\vec{\ell} \neq 0$.

nothing but $\mathcal{H}_I(\vec{\ell} = \vec{0})$. Since $\mathcal{H}^{\varphi\chi}(\vec{0}) = 0$, no mixing between the fluctuating pions and the fluctuating scalar occurs. Thus, the pion propagator for φ_a can be expressed as

$$\frac{1}{p_0^2 - \vec{p}^2 - \Sigma^{(1)}(p_0, \vec{p})} = \frac{1}{G^{aa}(\vec{0})(p_0^2 - \vec{p}^2)}. \tag{30}$$

where we have used that the self energy to this order is $\Sigma^{(1)}_{\varphi_a\varphi_a}(p_0, \vec{p}) = -p^2(G^{aa}(\vec{0}) - 1)$. The superscript "(1)" means that the quantities are evaluated to first order. Comparing these results with the propagator (23), we obtain $f_t = f_s = f_\pi \sqrt{G^{aa}(\vec{0})}$. Since $G^{aa}(\vec{0})$ is nothing but the average of $G^{aa}(\vec{r})$ over the space, our calculation thus far reproduces the results of ref. [2]. To the same order, the self-energy of the scalar field is $\Sigma^{(1)}_{\chi\chi} = M(\vec{0}) - m_\chi^2$. Since it is constant, this self-energy modifies just the scalar mass from the free value m_χ to $\sqrt{M(\vec{0})}$.

Now, let us evaluate the second order diagrams shown in Fig. 5. Again, the symmetry of the background skyrmion matter does not allow nonvanishing off diagonal components such as $\Sigma^{(2)}_{\varphi_a\varphi_b}(a \neq b)$ and $\Sigma^{(2)}_{\chi\varphi_a}$. We obtain

$$\Sigma^{(2)}_{\varphi_a\varphi_a}(p_0, \vec{p}) = \sum_{\vec{\ell} \neq 0} \left\{ \frac{|(p_0^2 - \vec{p} \cdot (\vec{p} + \vec{\ell}))G^{ab}(\vec{\ell}) + \epsilon_{abc}\vec{p} \cdot \vec{V}_c(\vec{\ell})|^2}{p_0^2 - (\vec{p} + \vec{\ell})^2} \right.$$
$$\left. + \frac{|\vec{p} \cdot \vec{P}_a(\vec{\ell})|^2}{p_0^2 - (\vec{p} + \vec{\ell})^2 - m_\chi^2} \right\}, \tag{31}$$

$$\Sigma^{(2)}_{\varphi_a\chi}(p_0, \vec{p}) = 0, \tag{32}$$

$$\Sigma^{(2)}_{\chi\chi}(p_0, \vec{p}) = \sum_{\vec{\ell} \neq 0} \left\{ \frac{|\vec{p} \cdot \vec{P}_a(\vec{\ell})|^2}{p_0^2 - (\vec{p} + \vec{\ell})^2} + \frac{|M(\vec{\ell})|^2}{p_0^2 - (\vec{p} + \vec{\ell})^2 - m_\chi^2} \right\}. \tag{33}$$

Here and in what follows, we economize on notation, $|O^b|^2 = \sum_b O^b(O^b)^*$ (also for lower indices), and we sum over index c. Again, in spite of the $\varphi - \chi$ coupling term in the interaction Lagrangian (26), $\Sigma^{(2)}_{\chi\varphi_a}$ vanishes so that the pion propagator and the scalar propagator can be simply written as

$$\frac{1}{p_0^2 - \vec{p}^2 - \Sigma^{(1+2)}_{\varphi_b\varphi_a}}, \qquad \frac{1}{p_0^2 - \vec{p}^2 - m_\chi^2 - \Sigma^{(1+2)}_{\chi\chi}}, \tag{34}$$

respectively.

The p_0 and $\vec{p}$ dependence of $\Sigma^{(2)}$ is not so simple as that of $\Sigma^{(1)}$. Let's consider the *small* energy-momentum region. If we expand the self-energy $\Sigma_{\varphi_a\varphi_b}$ in powers of p_0 and $\vec{p}$, then the coefficients to second order in p_0 and $\vec{p}^2$ lead to

$$\left(\frac{f_t}{f_\pi}\right)^2 = 1 - \frac{1}{2} \left.\frac{\partial^2 \Sigma^{(1+2)}_{\varphi_a\varphi_a}}{\partial p_0^2}\right|_{0,\vec{0}} = G^{aa}(\vec{0}), \tag{35}$$

$$\left(\frac{f_s}{f_\pi}\right)^2 = 1 + \frac{1}{2} \left.\frac{\partial^2 \Sigma^{(1+2)}_{\varphi_a\varphi_a}}{\partial p_i^2}\right|_{0,\vec{0}} = G^{aa}(\vec{0}) - \sum_{\vec{\ell} \neq 0} \left\{ \frac{|V_c^i|^2 + \ell_i^2|G^{ab}(\vec{\ell})|^2}{\vec{\ell}^2} + \frac{|P_a^i(\vec{\ell})|^2}{\vec{\ell}^2 + m_\chi^2} \right\}. \tag{36}$$

To obtain this result, we have used the observation that $\partial^2 \Sigma^{(1+2)}/\partial p_i \partial p_j$ is diagonal in the limit of $p_0 \to 0$ and $\vec{p} \to 0$. Note that the second order correction terms contribute only to f_s/f_π and since they are all negative the pion velocity becomes $v_\pi < 1$.

In a similar way, the scalar mass gets a second order correction given by

$$m_\chi^{*2} = m_\chi^2 + \Sigma_{\chi\chi}^{(1+2)}(0, \vec{0}) = M(\vec{0}) - \sum_{\vec{\ell} \neq 0} \frac{|M(\vec{\ell})|^2}{\ell^2 + m_\chi^2}. \tag{37}$$

The results of our calculation are shown in Figs. 6. Both of the pion decay constants change significantly as a function of density and vanish – in the chiral limit – when chiral symmetry is restored. However, the second-order contributions to the f_s and f_π, which break Lorentz symmetry, turn out to be rather small, and thus their ratio, the pion velocity, stays $v_\pi \sim 1$. The lowest value found is ~ 0.9. Note, however, the drastic change in its behavior at two different phase transition densities. At the lower density, where the skyrmion matter is in the chiral symmetry broken phase, the pion velocity decreases and has the minimum at $\rho = \rho_p$. If one worked only at low density in a perturbative scheme, one would conclude that the pion velocity decreases all the way to zero. However, the presence of the pseudogap phase transition changes this behavior. In the pseudogap phase, the pion velocity not only stops decreasing but also starts increasing with increasing density. In the chiral symmetry restored phase both f_t and f_s vanish. This result can be comparable to that of ref. [5] found in the heat bath, where the pion velocity approaches 1 while both the spatial and temporal pion decay constants vanish at $T = T_c$.

As stressed in Sec. 2, because of the strong parameter dependence, e.g., on e and m_χ, the precise values at which these transitions occur may not be meaningful. What is relevant independently of the precise value of e and m_χ is the overall qualitative behavior of the pion velocity as a function of the density. A qualitative idea can be gained by the rough estimates: The pseudogap type phase transition occurs at

$$\rho_p \sim 0.004 m_\rho^3 (\sim 1.4\rho_0 \text{ with } m_\rho = 770 \text{ MeV}), \tag{38}$$

with the minimum of the pion velocity at $v_\pi \sim 0.9$ and the chiral phase transition takes place at

$$\rho_c \sim 0.01 m_\chi^2 (\sim 2.9\rho_0 \text{ with } m_\chi = 720 \text{MeV}), \tag{39}$$

where the pion decay constants vanish. For the above estimates, we have used, for reference, the result of the analysis of the deeply bound states of pions in heavy nuclei [35]:

$$(f_\pi^*/f_\pi)^2|_{\rho=0.6\rho_0} = 0.78 \pm 0.05 \tag{40}$$

which is consistent with our naive estimates given above.

5 Conclusion

We have developed a formalism to calculate in-medium properties of hadrons within a unified approach which describes both matter and meson fluctuations with the same Lagrangian. In this paper, we have focused on the pion velocity, which controls through a dispersion relation, the pion propagation in the medium. For simplicity, we have chosen massless pions, i.e., the chiral limit and therefore the propagation is entirely controlled by the velocity.

A novel aspect that was uncovered here in the skyrmion picture in the chiral limit is the presence of a psuedogap-like phase. This phase was not noticed in the previous work [2] with massive pions. As density increases, the system first undergoes a phase transition to the pseudogap phase where the average value of σ vanishes while the chiral circle preserves a finite radius.
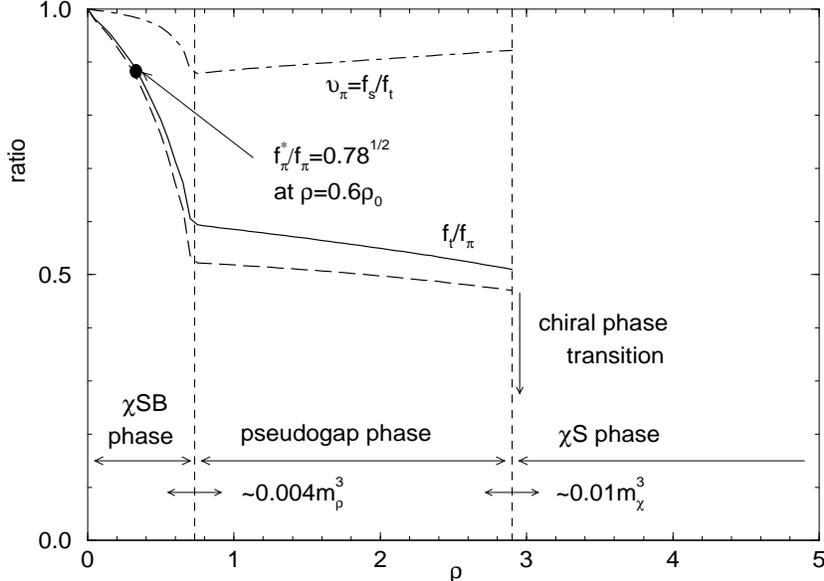
Figure 6: In-medium pion decay constants and their ratio, the pion velocity.

Roughly speaking, chiral symmetry is only partially restored in this phase. If we increase the density further, the radius of the chiral circles continuously decreases to zero and the chiral symmetry is truly restored. The density range where the skyrmion matter is in such a pseudogap phase depends strongly on the scalar mass.

Whether or not this pseudogap phase is an artifact of the Skyrme model at a classical level is not clear. The skyrmion comes out as a topological object in mapping the $SU(2)$ space defined on the chiral circle into the space. As a natural consequence, the space occupied by the skyrmions would have a value for $\langle \sigma \rangle$ which is smaller than the radius of the chiral circle. It would be very interesting to check on lattice whether such a pseudogap phase can be realized in QCD as suggested by Zarembo in heat bath [34].

The behavior of the pion decay constants, f_t and f_s, drastically changes in the various phases of skyrmion matter. The constants vanish in the chiral symmetry restored phase in the chiral limit. The pion velocity obtained as the ratio of these two pion decay constants can deviate at most by 10% from its vacuum value 1, since the second order terms of the interaction between the pion and background matter which break Lorentz symmetry have a small effect in the decay constants. After achieving its minimum value at $\rho = \rho_p$, this ratio increases asymptotically to 1 up to the density where the pion velocity is well-defined.

Our model calculation produces a complex phase transition scenario which seems to differ from the HLS/VM scenario in heat bath [5], despite the fact that the pion velocity ultimately tends to 1 as in the HLS/VM theory modulo Lorentz symmetry breaking effect, and also from the linear sigma model in the $O(4)$ universality class [4].

Finally we should point out the short-comings in the calculation and further works to be done. First of all, ours is not a realistic calculation in that the model we have used may not be describing nuclear matter properly. Specifically we do not have the Fermi liquid structure of normal nuclear matter. Even so, we do nonetheless expect the qualitative features of our calculation to be more or less unaffected even after quantum fluctuations are introduced and the liquid structure is recovered. Next we must note that our calculation was made to second order in perturbation theory which may be justified at low densities but we do not have at present a

control of this approximation for larger densities.

Other short-comings are that the kinetic energy contributions are missing from the calculation [36] and that the subtlety in the interchange of the chiral limit and infinite volume limit to address phase transitions needs to be clarified.

The immediate improvements to be made in the calculation are as follows. Apart from the quantum effects that would bring a liquid structure to the matter, it would be necessary to assure that the in-medium "vacuum" state is stable under mesonic fluctuations. For instance, we need to examine the eigen-modes and eigenvalues of eq.(20), focusing on zero-modes and negative modes if any. This is an interesting issue connected with the possible pion condensation etc. In this connection, we need to depart from the chiral limit.

All these short-comings and improvements are under investigation and will be reported in a later publication.

Acknowledgements

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