

# The scalar glueball spectrum

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## Abstract

We discuss scenarios for scalar glueballs using arguments based on sum rules, spectral decomposition, the  $\frac{1}{N_c}$  approximation, the scales of the strong interaction and the topology of the flux tubes. We analyze the phenomenological support of those scenarios and their observational implications. Our investigations hint a rich low lying glueball spectrum.

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# 1 Introduction

The glueball spectrum has attracted much attention since the formulation of the theory of the strong interactions Quantum Chromodynamics (QCD)[1, 2]. QCD sum rules [3] and models [4, 5] have been used to determine their spectra and properties. Lattice QCD computations, both in the pure glue theory and in the quenched approximation of QCD, have been used to determine their spectra [6, 7]. It has become clear by now that it is difficult to single out which states of the hadronic spectrum are glueballs because we lack the necessary knowledge to determine their decay properties [8]. Moreover the strong expected mixing between glueballs and quark states leads to a broadening of the possible glueball states which does not simplify their isolation [10]. The wishful sharp resonances which would confer the glueball spectra the beauty and richness of the baryonic and mesonic spectra are lacking. This confusing picture has led to a loss of theoretical and experimental interest in these hadronic states. However, it is important to stress, that if they were to exist they would be a beautiful and unique consequence of QCD .

Glueballs have not been an easy subject to study and much debate has been associated with their properties[10]. Even the quantum numbers of the lowest lying glueball have not been agreed upon until recently. There is now a general consensus that the lightest glueball is a  $0^{++}$  [9]. However, its properties, i.e., mass and widths still differ among the various calculations. Dominguez and Paver [11], Bordes, Peñarrocha and Giménez [12], and Kisslinger and Johnson [13] obtain by means of low energy theorems and/or sum rule calculations with (or without) instanton contributions a low lying (mass  $< 700$  MeV), narrow ( $\Gamma_{\pi\pi} < 100$  MeV) scalar glueball. Narison and collaborators[14] using a two (subtracted and unsubtracted) sum rules prefer a broader (200-800 MeV), heavier (700-1000 MeV) gluonium whose properties imply a strong violation of the Okubo-Zweig-Ishimura's (OZI) rule <sup>1</sup>. In a recent state of the art sum rule calculation, Forkel [15], obtains the gluonium at  $1250 \pm 200$  MeV with a large width ( $\sim 300$  MeV). However he has some strength at lower masses which he is not able to ascribe to a resonance in the fits <sup>2</sup>. Lattice QCD [6, 7] produce heavy glueballs. Present day interpretation of experiments[16, 17] claim a heavy glueball ( $\sim 1500$  MeV). We found illuminating the discussion of Kisslinger and Johnson [13] since using their calculation they can explain the existence of two scalar glueballs, a light one ( $\sim 500$  MeV) and heavy one ( $\sim 1700$ ) MeV, by studying the influence of the higher condensates in their sum rule approach.

To investigate the scalar glueball sector we develop our description initially in a world where the OZI rule is exactly obeyed, i.e., decays into quarks which require gluons are strictly forbidden. OZI dynamics (OZID) generates a glueballs spectrum which is formed of towers of states disconnected from mesons, baryons and leptons. The lowest lying scalar glueball (hereafter called  $g$ ) is, in this world, a bound state of two strongly interacting gluons with a torus type flux tube topology [18]. OZID confers this topology a Super Selection rule inhibiting any decays from this state into other particles. It is therefore stable and (almost) invisible since it only interacts with other glueballs and gravitationally.  $g$ , arises as a pseudo-Goldstone boson of broken scale invariance and therefore its mass is provided by the gluon condensate.

However, OZID is an idealized scenario, which breaks down, and through this breaking the interactions of the glueballs with quarks, and through them with all other standard model probes, arise. The implementation of this breaking leads to scenarios, which we analyze.

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<sup>1</sup>However, a lighter glueball would be narrow since the coupling to  $\pi\pi$  is proportional to the square of the mass.

<sup>2</sup>Private communication.

## 2 QCD scalars

To transform the OZID scenario into a Gedanken picture of reality we need the support of theory. Our basic assumption is that the trace anomaly gives rise in QCD to a dilaton which is a pseudo-Goldstone boson of scale invariance in line with the arguments of anomaly cancellation of 't Hooft [19], which have been so successfully applied to the axial anomaly [19, 20, 21]. The would be dilaton will describe the  $0^{++}$  glueball ground state. In the extreme OZID picture, gluodynamics is the theory describing  $g$  and becomes effectively

$$L = \frac{1}{2} (\partial g)^2 + V(g) \quad (1)$$

where the potential  $V(g)$  has been constructed to satisfy the anomaly constraint and some low energy theorems [22, 23, 24]. A consequence of this analysis is the following relation between the mass of the dilaton and the condensate,

$$m_g^2 f_g^2 = -4 \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | 0 \rangle, \quad (2)$$

where  $f_g = \langle 0 | w | 0 \rangle$  is the dilaton's vacuum expectation value,  $m_g$  the dilaton mass, and the right hand side arises from the anomaly. This relation was also obtained by Novikov et al. [25] isolating the leading power correction in their calculation<sup>3</sup>.

The theoretical support for OZID we find in the  $\frac{1}{N_c}$  expansion of QCD. Eq.(2) is consistent with the expected behavior

$$m_g \sim 1 \quad \text{and} \quad f_g \sim N_c. \quad (3)$$

Let us introduce the following correlator

$$\Pi(q^2) = i \int dx e^{iqx} \langle 0 | T \left( \frac{\beta(\alpha_s)}{4\alpha_s} G^2(x) \frac{\beta(\alpha_s)}{4\alpha_s} G^2(0) \right) | 0 \rangle. \quad (4)$$

It is known that [25]

$$\Pi(0) = -4 \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2(0) | 0 \rangle, \quad (5)$$

which is related to the energy of the vacuum. To leading order in  $\frac{1}{N_c}$ ,

$$\Pi(q^2) = \sum_{glueballs} \frac{N_c^2 a_n^2}{M_n^2 - q^2} + \sum_{mesons} \frac{N_c c_n^2}{m_n^2 - q^2}, \quad (6)$$

where  $M_n$  and  $m_n$  represent respectively the masses of the glueballs and mesons contributing to the correlator, and the numerators are related to the following transition matrix elements

$$N_c a_n = \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | nth \ glueball \rangle \quad (7)$$

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<sup>3</sup>The validity of this approximation in their scheme sets the limit on the mass of  $g$ , i.e. full consistency in their analysis would imply a very small mass for  $g$  and the effective theory eq.(1) would be a good realization of QCD in this sector.

and

$$\sqrt{N_c}c_n = \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | nth\ meson \rangle . \quad (8)$$

In the extreme  $\frac{1}{N_c}$  limit at low  $q^2$ ,  $\Pi(q^2)$  is dominated by the lowest mass glueball  $m_g$ , and then

$$| \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | g \rangle |^2 = -4m_g^2 \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | 0 \rangle \quad (9)$$

in agreement with the effective theory, i.e.

$$m_g^2 f_g = \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | g \rangle . \quad (10)$$

The above statements contradict the results of Voloshin and Zakharov [26], which require that the matrix elements of their scalar gluonic operator with light mesons are not negligible. To avoid this contradiction we have to include the lowest lying scalar meson, which is one order down in  $\frac{1}{N_c}$ .

If we extend the analysis to include the lowest lying meson, which we call  $\sigma$ , we get

$$m_\sigma^2 f_\sigma = \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | \sigma \rangle , \quad (11)$$

and from the general properties of the  $\frac{1}{N_c}$  expansion we obtain,

$$m_\sigma \sim 1 \quad f_\sigma \sim \sqrt{N_c} . \quad (12)$$

Let us estimate the masses of these two states following the analysis of Shifman [27] although adapting his philosophy to the above scheme. To calculate the  $0^{++}$  gluonia mass he assumes that the gluonic resonances couple strongly to the quark degrees of freedom in line with the arguments of Voloshin and Zakharov [26]. We proceed in the OZID (large  $N_C$ ) limit, i.e.  $g$  does not couple to them but  $\sigma$  does, and it is therefore the latter which plays the role of saturating the matrix elements.

The glueball contributes to the spectral function as

$$Im\Pi(q^2) = \pi m_g^2 f_g^2 \delta(q^2 - m_g^2) \quad (13)$$

The  $\sigma$  follows the discussion in ref. [27] since it may decay into other mesons, i.e.  $2\pi$ ,  $2\eta$ ,  $2K$ ,  $\dots$ , thus we obtain

$$m_g^2 f_g^2 + \frac{s_r^2}{8\pi^2} = -4 \langle 0 | \frac{\beta(\alpha_s)}{4\alpha_s} G^2 | 0 \rangle , \quad (14)$$

where we have used  $m_\sigma^2 f_\sigma^2 = \frac{s_r^2}{8\pi^2}$ . The additional term appearing from the existence of the  $g$  implies a reduction of the masses with respect to the cited analysis.

Let me use  $\frac{1}{N_c}$  in here,

$$\frac{m_g^2 f_g^2}{m_\sigma^2 f_\sigma^2} = N_c .$$

Repeating the numerical estimate of ref. [27] for massless quarks and for  $N_c = 3$  we get

$$m_{res} = 600\text{MeV}$$

and therefore

$$m_g \sim m_\sigma \sim 600\text{MeV}$$

and

$$f_g \sim \sqrt{3}f_\sigma.$$

For massive quarks the Shifman's estimates lead to

$$m_g \sim m_\sigma \sim 750\text{MeV}$$

Thus  $g$  and  $\sigma$  have similar masses in this naive scenario.

In gluodynamics the absence of quarks increases the coupling constant. The right hand side of Eq.(14) is related to the energy of the vacuum which increases and the left hand side has no contribution from the quarks, i.e.  $\sigma$  meson, thus

$$(m_g f_g)^{gluodynamics} \sim \sqrt{\frac{44}{27}}(m_g f_g)^{QCD}. \quad (15)$$

Moreover we expect  $f_g$  to decrease in gluodynamics since decay channels into photons will be closed. This statement together with our previous estimate Eq.(15) leads to

$$m_g^{gluodynamics} > 1\text{GeV}.$$

This qualitative argument explains the mechanisms by which the pure gauge calculations sees higher masses.

However, in line with the arguments of Kisslinger and Johnson [13], we should expect in gluodynamics two  $0^{++}$  glueballs, separated by approximately 1 GeV and which would move lower in energy once the effects of quarks are introduced.

### 3 Topology and dynamics

Nature does not realize OZID, namely the number of colors is not very large. We have to establish a scheme for breaking OZID. How should we incorporate corrections to the leading order in the  $\frac{1}{N_c}$  expansion? In order to understand how nature departs from OZID we resort to symmetry breaking and topological arguments.

Dynamical transmutation in  $QCD$  gives rise to the confinement scale,  $\Lambda$ , which introduces dimensions into a dimensionless (apart from quark masses) theory. Conventional low energy physics is governed by the chiral symmetry breaking scale  $f_\pi$  [28], which ultimately should be a function of  $\Lambda$ . In our case low energy dynamics will be governed by  $f_g$  and  $f_\sigma$  respectively. Recalling the results of previous section we notice that  $f_\pi \sim f_\sigma \sim O(\sqrt{N_c})$ , Eq. (12), while  $f_g \sim O(N_c)$ . Eq.(3). The breaking of OZID is governed by powers of their inverses. Thus we expect the corrections to the mesons to be

$O(\frac{1}{N_c})$  while that for the glueball  $O(\frac{1}{N_c^2})$ . OZID is better realized in the glueball sector than in the meson sector.

A second idea which guides our intuition about the breaking of OZID is the topology of the flux tubes and their relation with perturbative emission. The mesonic  $q\bar{q}$  states have an elongated, almost linear structure in their flux tubes[29, 30, 31]. The glueballs in most treatments arise from twisted flux tube configurations [32, 33, 34, 35]. In particular we conjecture based on a the simplest possible non linear topology, namely a torus like configuration [18], the behavior of particle emission.

Gluon and quark emission occur inside the flux tubes and therefore the scale of the perturbative emission is limited by the confinement size, i.e. the running coupling constant takes its maximum possible value when the particles are emitted with the lowest possible momentum, which is bounded from below by Heisenberg's principle,

$$\text{i) for the meson: } L < L_{conf} \sim \frac{1}{\Lambda_{QCD}} \sim 1\text{fm.}$$

$$\text{ii) for the glueball}^4: L < \frac{L_{conf}}{\sqrt{2\pi}} \sim \frac{1}{4}\text{fm.}$$

Therefore,

$$\alpha_{meson} \sim \alpha(L_{conf}) \gg \alpha_{glueball} = \alpha\left(\frac{L_{conf}}{\sqrt{2\pi}}\right) \quad (16)$$

where  $\alpha$  is the running coupling constant.

This argument also suggests that OZID dynamics is a better approximation in the case of glueballs than in the case of mesons since for the former the perturbative emission is weak. We expect therefore that the pure perturbative emission approximation of QCD to gluon and quark emission for  $g$  is very appropriate at any scale, while for the  $\sigma$  non perturbative chiral effects will be important [36].

## 4 $g$ - $\sigma$ Mixing

Since  $g$  and  $\sigma$  have the same quantum numbers they can easily mix in broken OZID and the observed particles are coherent superpositions of them. We use the discussion of previous section and  $\frac{1}{N_c}$  arguments presented in the Appendix to construct the breaking pattern.

We consider that  $g$  and  $\sigma$  mixed due to additional terms in the hamiltonian which are of higher order in  $\frac{1}{N_c}$ . Since  $f_\sigma \sim \sqrt{N_c}$  and  $f_g \sim N_c$  the following is the most general hamiltonian in this reduced Fock space,

$$\begin{pmatrix} m & \delta \\ \delta & m + \Delta m \end{pmatrix} \quad (17)$$

where  $\Delta m \sim \frac{1}{N_c}$ ,  $\delta \sim (\frac{1}{N_c})^{\frac{3}{2}}$  and we exclude terms  $O((\frac{1}{N_c})^2)$  and higher powers. The diagonal basis of this hamiltonian can be presented as,

$$\tilde{g} = g \cos(\theta/2) - \sigma \sin(\theta/2), \quad (18)$$

$$\tilde{\sigma} = g \sin(\theta/2) + \sigma \cos(\theta/2), \quad (19)$$

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<sup>4</sup>The torus flux tube is basically a planar figure since the cross section radius of the tube is small with respect to the other radius. Thus  $\Delta p_x \sim \frac{1}{2R}$  where  $R$  is the radius of the large circle of the torus. Thus  $\Delta p = \sqrt{(\Delta p_x)^2 + (\Delta p_y)^2} = \frac{1}{\sqrt{2}R}$ . But  $L_{conf} = 2\pi R$  thus  $L < \frac{L_{conf}}{\sqrt{2\pi}}$ .

where the tilde labels the physical particles and  $\theta$  is the mixing angle.

The masses of the physical particles become

$$m_{\tilde{g}} = m + \frac{\Delta m}{2} - r$$

$$m_{\tilde{\sigma}} = m + \frac{\Delta m}{2} + r \tag{20}$$

$$\tag{21}$$

where

$$\tan \theta = \frac{2\delta}{\Delta m} \tag{22}$$

and

$$r = \frac{\Delta m}{2} \sqrt{1 + \left(\frac{2\delta}{\Delta m}\right)^2} = \frac{\Delta m}{2 \cos \theta} \tag{23}$$

In Figs.1 we represent the masses of the physical states as a function of the mixing angle  $\theta$ . The curves separate possible mass regions. The horizontal solid line represents the mass of both states for  $\delta m \rightarrow 0^5$ . The dashed lines represent the value of  $m_{\tilde{g}}$  for  $\Delta m = \pm 250 MeV$  and the short-dotted lines those for  $m_{\tilde{\sigma}}$ . To the left of the vertical line the values are consistent with the  $\frac{1}{N_c}$  expansion, the condition that defines that line is

$$\tan \theta = \frac{2\delta}{\Delta m} \sim \frac{2}{N_c} \sim \frac{2}{3}$$

In Figs.1 the curves on the left show that the two state mixing scenario for positive  $\Delta m$  leads to a "light" glueball with a mass in the range  $650 MeV < m_{\tilde{g}} < 750 MeV$  and a scalar meson with a mass in the range  $750 MeV < m_{\tilde{\sigma}} < 1050 MeV$ . The  $\frac{1}{N_c}$  expansion favors small mixings in the physical states. The curves on the right show that for negative  $\Delta m$  the meson becomes lighter  $450 MeV < m_{\tilde{\sigma}} < 750 MeV$ , while the glueball becomes heavier  $750 MeV < m_{\tilde{g}} < 850 MeV$ .

Let us speculate about strong OZID breaking. If we abandon the  $\frac{1}{N_c}$  expansion, i.e. allow the mixing matrix elements to be larger than required by this approximation, the masses separate notoriously and in particular the glueball (meson) becomes very light in the  $\Delta m > 0$  ( $\Delta m < 0$ ) scenario. Correspondingly, the associated meson (glueball) becomes heavy. In this case however the mixing is large thus it is difficult to talk about glueball or meson since both states are an almost perfect mixture, i.e. the  $\tilde{g}$  state has a large  $\sigma$  component and the  $\tilde{\sigma}$  state a large glueball component.

We have performed a mathematical analysis of our theoretical scheme, in the next section we put the present analysis under the scrutiny of data.

## 5 Discussion

The OZID glueball does not interact with quarks, neither with leptons nor electroweak gauge bosons, therefore in our approach it is sterile. However, the physical glueball does because of its admixture

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<sup>5</sup>We take a value for  $\Delta m$  small enough so that the deviation from this line which occurs for  $\theta \rightarrow \frac{\pi}{2}$ , which leads ultimately to a  $\pm\delta$  splitting for  $\delta$  finite, is beyond the shown values.

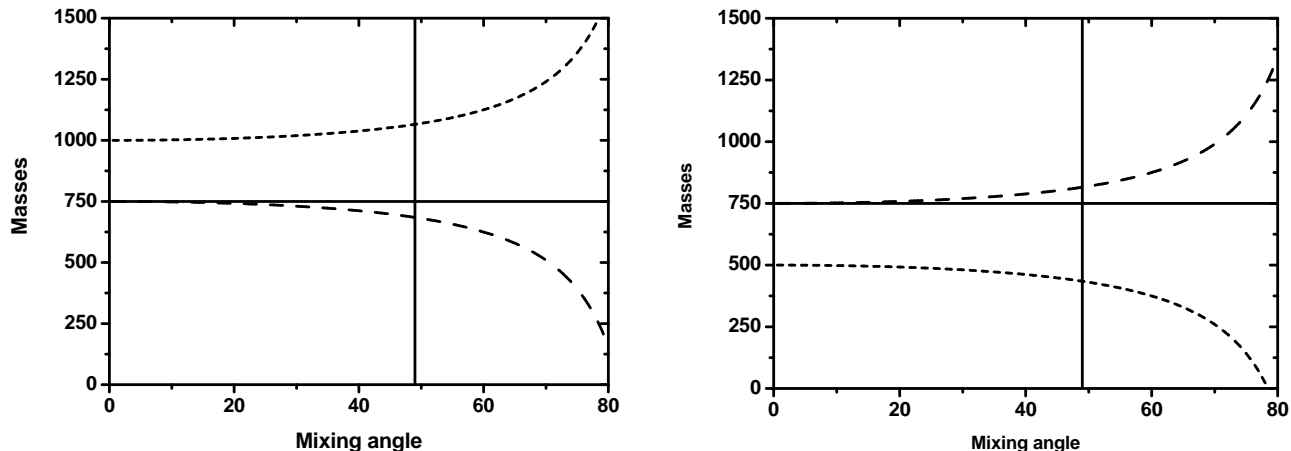


Figure 1: The limiting values for the masses of the physical  $\tilde{g}$  (solid–dashed lines) and  $\tilde{\sigma}$  (solid–short-dashed lines) are shown as a function of the mixing angle for the range  $0 < |\Delta m| < 250$  MeV . The left (right) figure corresponds to positive (negative)  $\Delta m$ . The degenerate initial mass has been taken, as discussed in the text, at  $m = 750$  MeV. The vertical line defines the approximate limit of the validity of the  $\frac{1}{N_c}$  expansion.

with the  $\sigma$ . From now on we will only talk about the physical particles and we omit their tilde in the notation. Using a  $\sigma$ -model interaction we get

$$\begin{aligned} \Gamma_{\sigma \rightarrow 2\pi} &= \frac{3}{64\pi f_\pi^2} \left( \frac{m_\sigma^2 - m_\pi^2}{m_\sigma} \right)^2 \sqrt{m_\sigma^2 - 4m_\pi^2} \\ &\sim \frac{3}{64\pi} \frac{m_\sigma^3}{f_\pi^2} \sim 1.5 \left( \frac{m_\sigma(\text{GeV})}{1\text{GeV}} \right)^3 \text{ GeV}, \end{aligned} \quad (24)$$

where we have taken  $f_\pi \sim 100$  MeV and neglected terms  $O(m_\pi^2)$  in the last line.

Let us look at the lower spectrum of scalars shown in table 1. Below 750 MeV the only existing resonance is the broad  $f_0(600)$ , whose mass and width are still quite undetermined. Using the data on the width and using Eq.(24) we obtain

$$737\text{MeV} < m_\sigma < 874\text{MeV}. \quad (25)$$

Thus the  $\Delta m < 0$  scenario is discarded by the data. Therefore the glueball is lighter than the meson, i.e. within the limits of the  $\frac{1}{N_c}$  expansion

$$650\text{MeV} < m_\sigma < 750\text{MeV}. \quad (26)$$

Note that in this approximation the mixing angle is small and therefore

$$\Gamma_{g \rightarrow 2\pi} \sim 1.5 \sin^2(\theta/2) \left( \frac{m_g(\text{GeV})}{1\text{GeV}} \right)^3 \text{ GeV} < 100\text{MeV}, \quad (27)$$



	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0(1710)$
mass (MeV)	400 – 1200	$980 \pm 10$	120 – 1500	$1507 \pm 5$	$1714 \pm 5$
width (MeV)	600 – 1000	40 – 100	200 – 500	$109 \pm 7$	$140 \pm 10$
Decay modes	$\pi\pi$ dominant $\gamma\gamma$ seen	$\pi\pi$ dominant $K\bar{K}$ seen $\gamma\gamma$ seen	$\pi\pi$ seen $4\pi$ ... $\rho\rho$ dominant ... other $4\pi$ seen $\eta\eta$ seen $K\bar{K}$ seen $\gamma\gamma$ seen	$\pi\pi$ 35% $4\pi$ 50% $\eta\eta$ 5% $\eta\eta'$ 2% $K\bar{K}$ 9% $\gamma\gamma$ not seen	$\pi\pi$ seen $K\bar{K}$ seen $\eta\eta$ seen

Table 1: The scalar spectrum according to the Particle Data Group [38]

$$\Gamma_{\sigma \rightarrow 2\pi} \sim 1.5 \cos^2(\theta/2) \left( \frac{m_\sigma(\text{GeV})}{1\text{GeV}} \right)^3 \text{ GeV} > 500\text{MeV}. \quad (28)$$

Our analysis supports that the broad  $f_0(600)$  hides, within its experimental indetermination our two states, the conventional  $\sigma$  meson and the lightest searched for glueball.

If we relax the OZID hypothesis we could arrive to an exotic scenario in which for large mixings one of the states could have a small mass close to the  $2\pi$  threshold and an extremely small width due to the kinematical threshold factor appearing in Eq.(24). This exotic scenario would be characterized by a quasi stable state close to the observed lower mass limit ( $\sim 400$  MeV) and a broad width state in the upper mass limit ( $\sim 1200$  MeV).

The  $f_0(980)$ , which belongs to the meson nonet, is too narrow to correspond to our sigma-model state, since it survives the large  $N_c$  limit [36], we ascribe it to the first mesonic excitation. Since its width is relatively low it does not seem to arise from a mixing with the lower lying states and therefore it sets the upper bound for the mass of the lowest lying  $\sigma$ -meson. Thus the existence of the  $f_0(980)$  excludes, in our view, the extreme exotic scenario and validates an approximate OZID scheme.

The  $f_0(1370)$  region is again ill-determined experimentally. In this case new channels, like  $2\eta$ , open up. The same mass analysis for the excitations could be carried out, which would lead us to conclude that two excited states, a glueball and a meson, exist. Here we should not apply our naive sigma model width and therefore our discussion for the widths is absent. However, the recent analysis of Forkel[15] concludes with the existence of a broad glueball at 1250 which corresponds to the region around the upper mass limit. We conclude from this analysis that in this region of the spectrum the  $\Delta m < 0$  scenario takes place and that the companion meson should have lower mass than the glueball. The proximity of the  $f_0(980)$ , and a minimal population hypothesis, leads us to propose that the  $f_0(980)$  is the required companion. The fact that the lower mass particle, a meson in this case, is narrower also confirms the breaking scheme.

Finally if the  $f_0(1500)$  is a glueball [16, 17], by assuming the same analysis, a new  $\Delta m > 0$  scenario may take places, which would ascribe the  $f_0(1700)$  as its companion meson. The  $f_0(1500)$  could be also the higher lying glueball of ref. [13] after mixing.

Thus, our analysis leads to the existence of three glueball states in the low lying scalar spectrum with three companion mesons. The precise dynamical mechanisms by which they arise are as of yet unknown, however more precise studies within the large  $N_c$  approximation might shed light to our

proposal. The duality of the  $\Delta m$  mechanism which leads to an ordering of the spectrum in the form

$$m_g < m_\sigma < m_{\sigma_1} < m_{g_1} < m_{g_2} < m_{\sigma_2} \dots$$

has been guided by observation and physical intuition as explained above.

The analysis could be completed by studying other decay modes. In particular  $2\gamma$  decays also hint about the mass orderings. Using the trace anomaly [24]

$$\Gamma_{\sigma \rightarrow 2\gamma} = \frac{\alpha^2}{16\pi^3} \frac{m_\sigma^3}{f_\sigma^2} \sim 10.5 \left( \frac{m_\sigma}{1\text{GeV}} \right)^3 \text{ eV} \quad (29)$$

where we have used  $N_c = 3$  and  $f_\sigma \sim f_\pi \sim 100\text{MeV}$ . We obtain therefore

$$\Gamma_{g \rightarrow 2\gamma} \sim 10.5 \sin^2(\theta/2) \left( \frac{m_g(\text{GeV})}{1\text{GeV}} \right)^3 \text{ eV} < 1\text{eV}, \quad (30)$$

$$\Gamma_{\sigma \rightarrow 2\gamma} \sim 10.5 \cos^2(\theta/2) \left( \frac{m_\sigma(\text{GeV})}{1\text{GeV}} \right)^3 \text{ eV} > 3\text{eV}. \quad (31)$$

Thus, in this weak mixing scenario, the glueball state is narrower than the meson state.

## 6 Conclusions

We have analyzed the possible existence of a  $0^{++}$  glueball low lying state from different perspectives. The analysis has been modelled by  $\frac{1}{N_c}$  physics on which we have also based our estimates. We are led to a scenario of weak OZID breaking and a low mass glueball. This glueball is narrow since only its  $\sigma$  state component is allowed to decay and the small mixing angle inhibits decays. It represents a beautiful example of OZID dynamics.

The discussion and mechanisms can be repeated for the higher lying scalars and a spectrum arises in which glueballs and scalar mesons appear in pairs, with masses ordered according to the sign of the  $1/N_c$  breaking parameter  $\Delta m$ .

The lowest lying states,  $g$  and  $\sigma$ , appear within the  $f_0(600)$  peak in agreement with previous estimates [12, 14, 13], and therefore they might be difficult to isolate [37], although their widths are vastly different for strong and electromagnetic decays. Maybe more precise experiments could manage to see the two peaks.

The existence of low lying glueballs might strongly influence the transition towards the Quark Gluon Plasma [39, 40] and it might be in this physical regime where it might appear unquestioned.

The present investigation lies at the foundations for the understanding of the scalar spectrum. Our reasonings can be made more quantitative by lattice studies and more sophisticated model studies. It opens up the possibility of understanding glueballs and their dynamics.

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