A quark model analysis of the Sivers function

A. Courtoy[1], F. Fratini[2], S. Scopetta[2,3*], V. Vento[1,4]

(1) Departament de Fisica Teòrica,

Universitat de València

and Institut de Fisica Corpuscular,

Consejo Superior de Investigaciones Científicas

46100 Burjassot (València), Spain

(2) Dipartimento di Fisica,

Università degli Studi di Perugia,

via A. Pascoli 06100 Perugia, Italy.

(3) INFN, sezione di Perugia,

via A. Pascoli 06100 Perugia, Italy

(4) TH-Division, PH Department,

CERN, CH-1211 Genève 23, Switzerland

Abstract

We develop a formalism to evaluate the Sivers function. The approach is well suited for calculations which use constituent quark models to describe the structure of the nucleon. A non-relativistic reduction of the scheme is performed and applied to the Isgur-Karl model of hadron structure. The results obtained are consistent with a sizable Sivers effect and the signs for the u and d flavor contributions turn out to be opposite. This pattern is in agreement with the one found analyzing, in the same model, the impact parameter dependent generalized parton distributions. The Burkardt Sum Rule turns out to be fulfilled to a large extent. We estimate the QCD evolution of our results from the momentum scale of the model to the experimental one and obtain reasonable agreement with the available data.

* corresponding author: E-mail address: sergio.scopetta@pg.infn.it

I. INTRODUCTION

The partonic structure of transversely polarized nucleons is one of their less known features (for a review, see, e.g., Ref. [1]). Nevertheless, experiments for its determination are progressing very fast and the relevant experimental effort has motivated a strong theoretical activity (for recent developments, see Ref. [2]). The present work aims to contribute to this effort by using a successful theoretical scenario for the calculation of the Sivers function.

Semi-inclusive deep inelastic scattering (SIDIS), i.e. the process $A(e, e'h)X$, with the detection in the final state of a produced hadron h in coincidence with the scattered electron e', is one of the proposed processes to access the parton distributions (PDs) of transversely polarized hadrons. For several years it has been known that SIDIS off a transversely polarized target shows azimuthal asymmetries, the so called "single spin asymmetries" (SSAs) [3]. As a matter of fact, it is predicted that the number of produced hadrons in a given direction or in the opposite one, with respect to the reaction plane, depends on the orientation of the transverse spin of a polarized target with respect to the direction of the unpolarized beam. It can be shown that the SSA in SIDIS off transverse polarized targets is essentially due to two different physical mechanisms, whose contributions can be technically distinguished [4, 5, 6, 7]. One of them is the Collins mechanism, due to parton final state interactions in the production of a hadron by a transversely polarized quark [3], and will not be discussed here. The other is the Sivers mechanism [8], producing a term in the SSA which is given by the product of the unpolarized fragmentation function with the Sivers PD, describing the number density of unpolarized quarks in a transversely polarized target. The Sivers function is a Transverse Momentum Dependent (TMD) PD; it is a time-reversal odd object [1] and for this reason, for several years, it was believed to vanish due to time reversal invariance. However, this argument was invalidated by a calculation in a spectator model [9], following the observation of the existence of leading-twist Final State Interactions (FSI) [10]. The current wisdom is that a non-vanishing Sivers function is generated by the gauge link in the definition of TMD parton distributions [11, 12, 13], whose contribution does not vanish in the light-cone gauge, as happens for the standard PD functions. For the same reason it is difficult to relate the Sivers Function to the target helicity-flip, impact parameter dependent (IPD), generalized parton distribution (GPD) E. Although simple relations between the two quantities are found in models [14, 15], a clear model independent formal relation is still to be proven, as shown in Ref. [16].

Recently, the first data of SIDIS off transversely polarized targets have been published, for the proton [17] and the deuteron [18]. It has been found that, while the Sivers effect is sizable for the proton, it becomes negligible for the deuteron, so that apparently the neutron contribution cancels the proton one, showing a strong flavor dependence of the mechanism. Experiments on transversely polarized ^{3}He target, aimed at extracting the neutron information, addressed in [19], are being performed at JLab [20, 21]. A realistic calculation of nuclear effects for a proper extraction of the neutron information has also been performed [22]. Different parameterizations of the available SIDIS data have been published [23, 24, 25], still with large uncertainties. Further analyses are in progress (see, i.e. [26]). New data, which will reduce the uncertainties on the extracted Sivers function and will help discriminate between different theoretical predictions, will be available soon.

This experimental scenario motivates the formulation of theoretical estimates. One would like to perform a calculation from first principles in QCD, however this is not yet possible. Lacking this possibility it becomes relevant to perform model calculations of the Sivers function. Several estimates exist, in a quark-diquark model [9, 12, 27]; in the MIT bag model, in its simplest version [28] and introducing an instanton contribution [29]; in a light-cone model [30]; in a nuclear framework, relevant to establish the manifestation of the Sivers function in proton-proton collisions [31].

To our knowledge, no calculations of the Sivers function have been performed in a Constituent Quark Model (CQM), i.e. a model described in terms of constituent quarks and whose properties have been fixed from hadronic observables. The CQMs have a long history of successful predictions in studies of the hadronic spectrum and the low energy electroweak structure of hadrons. Ascribing a scale to the model calculations [32, 33] and using QCD evolution [34, 35] one can evolve the leading twist component of the observable calculated in this low energy scale to the high momentum one where DIS experiments are carried out. Such procedure has proven successful in describing the gross features of PDs (see, e.g., [36, 37, 38]) and GPDs (see, e.g. [39, 40]), by using different CQMs. Similar expectations motivate the present study of the Sivers function.

In here we propose a formalism to calculate the valence quark contribution to the Sivers function from any CQM. Thereafter, we choose the Isgur-Karl model [41] to perform a detailed calculation in order to describe the performance of the approach. A difference in the calculation of TMDs, with respect to calculations of PDs and GPDs, is that the leading twist contribution to the one-gluon-exchange (OGE) FSI has to be evaluated. This is done through a non-relativistic (NR) reduction of the relevant operator, according to the philosophy of constituent quark models [42].

The paper is structured as follows. In the second section, the main quantities of interest are introduced. In the following section, the formalism for the calculation of the Sivers function in a CQM is developed. The Isgur–Karl model is presented in the fourth section, together with the numerical results of the calculation and their discussion. The following section is devoted to the QCD evolution of the model results and to the comparison with the available data. In the last section we draw conclusions from our study.

II. THE THEORETICAL FRAMEWORK

The Sivers function, $f_{1T}^{\perp \mathcal{Q}}(x, k_T)$, the quantity of interest here, is formally defined, according to the Trento convention [43, 44], for the quark of flavor $\mathcal{Q}$, through the following expression[1]:

$$\begin{aligned} \Phi^{\mathcal{Q}}(x, \vec{k}_T, S) &= f_1^{\mathcal{Q}}(x, k_T) - \frac{\epsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^{\perp \mathcal{Q}}(x, k_T) \\ &= \frac{1}{2} \int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{-i(x\xi^- P^+ - \vec{\xi}_T \cdot \vec{k}_T)} \langle P, S | \hat{O}_{\mathcal{Q}} | P, S \rangle \ , \end{aligned} \tag{1}$$

where $\vec{S}_T$ is the transverse spin of the target hadron, the normalization of the covariant spin vector is $S^2 = -1$, M is the target mass and $f_1^{\mathcal{Q}}(x, k_T)$ is the k_T-dependent unpolarized PD. The operator $\hat{O}_{\mathcal{Q}}$ is defined as follows [12, 13]:

$$\hat{O}_{\mathcal{Q}} = \bar{\psi}_{\mathcal{Q}}(0, \xi^-, \vec{\xi}_T) \mathcal{L}^{\dagger}_{\vec{\xi}_T}(\infty, \xi^-) \gamma^+ \mathcal{L}_0(\infty, 0) \psi_{\mathcal{Q}}(0, 0, 0) \ , \tag{2}$$

where $\psi_{\mathcal{Q}}(\xi)$ is the quark field and the gauge link is:

$$\mathcal{L}_{\vec{\xi}_T}(\infty, \xi^-) = P \exp\left(-ig \int_{\xi^-}^{\infty} A^+(\eta^-, \vec{\xi}_T) d\eta^- \right) \ , \tag{3}$$

where g is the strong coupling constant. One should notice that this definition for the gauge link holds in covariant (non singular) gauges, and in SIDIS processes, since the definition of the Sivers function is process dependent. As observed in Ref. [9] for the first time, and

[1] Here and in the following, $a^{\pm} = (a_0 \pm a_3)/\sqrt{2}$ and $k_T = |\vec{k}_T|$.

later confirmed using factorization theorems in [45, 46], the gauge link, which represents the exchange of gluons, provides a scaling contribution which makes the Sivers function non vanishing in the Bjorken limit.

Taking the proton polarized along the y axis one has therefore:

$$f_{1T}^{\perp\mathcal{Q}}(x,k_T) = -\frac{M}{4k_x}\int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)}\langle \hat{O}_{\mathcal{Q}}\rangle \ , \tag{4}$$

where the following matrix element has been defined:

$$\langle \hat{O}_{\mathcal{Q}}\rangle = \{\, \langle PS_y = 1|\hat{O}_{\mathcal{Q}}|PS_y = 1\rangle - \langle PS_y = -1|\hat{O}_{\mathcal{Q}}|PS_y = -1\rangle \,\} \ . \tag{5}$$

Considering a helicity basis for the target, the Sivers function Eq. (4) can be written:

$$f_{1T}^{\perp\mathcal{Q}}(x,k_T) = \Im\left\{ \frac{M}{2k_x}\int \frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3} e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)}\langle PS_z = 1|\hat{O}_{\mathcal{Q}}|PS_z = -1\rangle \right\} \ . \tag{6}$$

This equation, finite in the limit of $k_x \to 0$, will be used to evaluate the Sivers function, using a CQM to describe the proton. We will now proceed to expand the gauge link, Eq. (3), in the coupling constant, g :

$$P\exp\left(-ig\int_{\xi^-}^{\infty} A^+(\eta^-,\vec{\xi}_T)d\eta^-\right) = 1 - ig\int_{\xi^-}^{\infty} A^+(\eta^-,\vec{\xi}_T)d\eta^- + ... \tag{7}$$

If the gauge link were not taken into account, it is clear from Eqs. (2)-(6) that the matrix element Eq. (5) would be zero and the Sivers function would vanish. For this reason, the first term on the right-hand side of Eq. (7) does not contribute to the Sivers function.

A few theoretical predictions have been formulated for the Sivers function. Let us recall two of them.

The first one, based on rather general principles, is the so called Burkardt Sum Rule [47], stating that the total average transverse momentum of the partons in a hadron, $\langle \vec{k}_T\rangle$, which can be defined in terms of the sum of the first moments of the Sivers function for all the partons in the target, has to vanish.

The second one is the conjecture according to which the Sivers function could be related to the formalism of the IPD GPDs [48], although, as it has been discussed in the Introduction, simple relations between the two quantities are found only in models [14, 15] and a clear model independent formal relation is still to be proven [16]. The IPD GPDs are the Fourier transform of the GPDs with respect to the transverse momentum transfer $\vec{\Delta}_T$, at vanishing skewness ξ. In the case of the helicity independent GPD, $H_{\mathcal{Q}}(x,\xi,\Delta^2)$, one has:

$$H_{\mathcal{Q}}(x,\xi=0,b^2) = \int \frac{d^2\vec{\Delta}_T}{(2\pi)^2} e^{-i\vec{b}\cdot\vec{\Delta}_T} H_{\mathcal{Q}}(x,\xi=0,\Delta^2) \ , \tag{8}$$

and analogous definitions hold for the helicity independent, target spin-flip GPD $E_{\mathcal{Q}}(x,\xi,\Delta^2)$ and for the other GPDs. It has been shown that these quantities have a probabilistic interpretation, describing the location of the quarks of flavor $\mathcal{Q}$ in the transverse plane and providing us with a three dimensional picture of the proton [48]. In Ref. [49, 50] (see also Ref. [51] for a recent review on this subject), it has also been shown that, in a transversely polarized proton, for example along the y direction, the quantity describing the distribution of the quarks of flavor $\mathcal{Q}$, with longitudinal momentum x, in the transverse plane, independently of their helicity, is

$$\tilde{\rho}_{\mathcal{Q}}(x,\xi=0,\vec{b}) = \frac{1}{2} H_{\mathcal{Q}}(x,0,b^2) - \frac{b_x S_y}{2M} \frac{d}{db^2} E_{\mathcal{Q}}(x,0,b^2) \ , \tag{9}$$

i.e., the transverse polarization of the proton produces a shift in the transverse location of the quarks. As explained before, this effect in the partonic structure of transversely polarized protons has been related, in peculiar models, in a qualitative way, to a nonvanishing Sivers effect [49, 50].

III. THE SIVERS FUNCTION IN CONSTITUENT QUARK MODELS

The constituent quark, one of the most fruitful concepts in 20th century physics, was proposed to explain the structure of the large number of baryons being discovered in the sixties [52]. The constituent quark concept was incorporated into a QCD scheme by taking into account gluon exchanges [42]. The chosen description was a potential model in order to establish an immediate connection with all previous work.

The constituent quark scheme has guided some of the most successful parameterizations of parton distributions [53]. Besides, the philosophy that has guided these parameterizations is precisely the one used to establish the link between constituent models and parton distributions. More specifically, model calculations are ascribed to a scale determined by their partonic content [32, 33]. In most models that scale is characterized by the existence of valence quarks only. From that low scale one uses DGLAP evolution to describe the partonic regime [38].

The models based on constituent quarks (CQMs) have produced beautiful results in the description of PDs and GPDs, leading to a phenomenological understanding of them in terms of momentum densities and wave functions [36, 37, 38, 39, 40]. This success in the description of many parton distributions makes us confident that the application of the approach to the calculation of the Sivers function will also serve to guide the experimental observations and help the physical interpretation of this observable.

Let us specify in detail the scheme in which we are going to develop our formalism for the Sivers function. We shall assume that the nucleon at a certain low energy scale is made up of valence quarks only. These valence quarks are held together by a confining interaction; in addition, there is a residual interaction, governed by the structure of perturbative QCD, e.g. the One Gluon Exchange Interaction. The strong confining interaction maintains the quarks together, while the residual one governs the splittings within the same flavor multiplet. Any scheme with these hypothesis is a constituent quark model framework.

This scheme has never to be understood in a trivial perturbative sense. The parameters absorb much of the non perturbative features of the dynamics and this relation between the parameters and some chosen observables makes the scheme predictive. If one goes to higher order in the perturbative expansion, one needs to find new parameters from the chosen observables. Thereafter, the predictions do not change much with respect to the lowest order result [54]. Certainly we are dealing with models and not with QCD and therefore one should not expect precision. Nevertheless, the scheme has been so successful that particles which do not fit approximately under it are called exotics, hybrids or other peculiar names.

Using this scheme we evaluate a formula for the Sivers function, defined according to Eq. (6), valid for any CQM. Let us proceed to the analysis having in mind Fig. 1. To the first non vanishing order giving a contribution to the asymmetry, the Sivers function for the flavor $\mathcal{Q}$ is obtained as follows:

$$f_{1T}^{\perp\mathcal{Q}}(x,k_T) = \Im\left\{\frac{M}{2k_x}\int\frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3}e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)}\langle\hat{O}^{\mathcal{Q}}\rangle\right\}\ , \tag{10}$$

where

$$\begin{aligned}\langle\hat{O}^{\mathcal{Q}}\rangle \ &= \ \langle PS_z=1|\bar{\psi}_{\mathcal{Q}i}(0,\xi^-,\vec{\xi}_T)(ig)\left\{\hat{O}_a(0,\xi^-,\vec{\xi}_T)T^a_{ij}\right\} \\ &\times \ \gamma^+\psi_{\mathcal{Q}j}(0)|PS_z=-1\rangle + \text{h.c.}\ ,\end{aligned} \tag{11}$$

where $T^a_{ij} = \lambda^a_{ij}/2$ with λ^a_{ij} being a Gell-Mann matrix, and

$$\begin{aligned}\hat{O}_a(0,\xi^-,\vec{\xi}_T) &= \int_{\xi^-}^{\infty} A^+_a(0,\eta^-,\vec{\xi}_T)d\eta^- \\ &= e^{i\hat{P}^+\xi^- - i\hat{P}_T\cdot\vec{\xi}_T}\hat{O}_a(0)e^{-i\hat{P}^+\xi^- + i\hat{P}_T\cdot\vec{\xi}_T} \ .\end{aligned} \tag{12}$$

In the above equations, use is made of light-cone states[2], defined as $|\tilde{p}\rangle = |p^+,\vec{p}_T\rangle$, with $p^- = (m^2+p_T^2)/(2p^+)$. The light-cone states are normalized as follows:

$$\langle\tilde{p}'r'|\tilde{p}r\rangle = (2\pi)^3 2p^+\delta(p'^+ - p^+)\delta(\vec{p}'_T - \vec{p}_T)\delta_{rr'} \ , \tag{13}$$

where the label r represents a set of discrete quantum numbers. The creation and annihilation operators of the quark fields are normalized accordingly:

$$\{b^\dagger_l(\tilde{p}), b_{l'}(\tilde{p}')\} = (2\pi)^3 2p^+\delta(p'^+ - p^+)\delta(\vec{p}'_T - \vec{p}_T)\delta_{ll'} \ , \tag{14}$$

where the set $l = \{m, c, \mathcal{F}\}$ includes the helicity, color and flavor quantum numbers of the quark, respectively.

Using the approximation of expanding Eq. (11) in terms of free quark fields [39], one gets

$$\begin{aligned}f_{1T}^{\perp\mathcal{Q}}(x,k_T) &= \Im\left\{\frac{M}{2k_x}\int\frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3}e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)}\langle PrS_z = 1|\right. \\ &\times \int d\tilde{k}_3\sum_{m_3} b^{\mathcal{Q}\dagger}_{m_3 i}(\tilde{k}_3)e^{ik_3^+\xi^- - i\vec{k}_{3T}\cdot\vec{\xi}_T}\bar{u}_{m_3}(\vec{k}_3) \\ &\times (ig)\left\{\hat{O}_a(0,\xi^-,\vec{\xi}_T)T^a_{ij}\right\}\gamma^+ \\ &\times \left.\sum_{m'_3}\int d\tilde{k}'_3\, b^{\mathcal{Q}}_{m'_3 j}(\tilde{k}'_3)u_{m'_3}(\vec{k}'_3)|PrS_z = -1\rangle + h.c.\right\} \ ,\end{aligned} \tag{15}$$

where $d\tilde{k}_i = dk_i^+ d\vec{k}_{Ti}/(2k_i^+(2\pi)^3)$. Inserting now proper complete sets of intermediate free one quark states, the previous equation becomes

$$\begin{aligned}f_{1T}^{\perp\mathcal{Q}}(x,k_T) &= \Im\left\{\frac{M}{2k_x}\int\frac{d\xi^- d^2\vec{\xi}_T}{(2\pi)^3}e^{-i(x\xi^- P^+ - \vec{\xi}_T\cdot\vec{k}_T)}\langle PrS_z = 1|\right. \\ &\times \int d\tilde{k}_3\sum_{m_3} b^{\mathcal{Q}\dagger}_{m_3 i}(\tilde{k}_3)e^{ik_3^+\xi^- - i\vec{k}_{3T}\cdot\vec{\xi}_T}\bar{u}_{m_3}(\vec{k}_3) \\ &\times (ig)\sum_{l_n,l_1}\int d\tilde{k}_n\int d\tilde{k}_1|\tilde{k}_1 l_1\rangle|\tilde{k}_n l_n\rangle\langle\tilde{k}_n l_n|\langle\tilde{k}_1 l_1|\end{aligned}$$

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[2] Here and in the following, $\tilde{x} = (x^+, x^-, \vec{x}_T)$ is a four vector in light-cone coordinates, while obviously $\vec{x} = (x_1, x_2, x_3)$ and $\vec{x}_T = (x_1, x_2)$.

$$
\begin{aligned}
&\times \ \left\{\hat{O}_a(0,\xi^-,\vec{\xi}_T)T^a_{ij}\right\} \\
&\times \ \sum_{l'_n,l'_1} \int d\tilde{k}'_n \int d\tilde{k}'_1 |\tilde{k}'_1 l'_1\rangle |\tilde{k}'_n l'_n\rangle \langle \tilde{k}'_n l'_n| \langle \tilde{k}'_1 l'_1| \, \gamma^+ \\
&\times \ \sum_{m'_3} \int d\tilde{k}'_3 \, b^{\mathcal{Q}}_{m'_3 j}(\tilde{k}'_3) u_{m'_3}(\vec{k}'_3) |P r S_z = -1\rangle + h.c. \Bigg\} \ . \qquad (16)
\end{aligned}
$$

If there is no further interaction within the recoiling system, one has:

$$
\langle \tilde{k}_n l_n | \tilde{k}'_n l'_n \rangle = (2\pi)^3 2k_n^+ \delta(k_n'^+ - k_n^+) \delta(\vec{k}_{n'T} - \vec{k}_{nT}) \delta_{l_n,l'_n} \ , \qquad (17)
$$

$$
\begin{aligned}
\langle P\, r\, S_z = 1 \ \ | \ \ & \{ b^{\mathcal{Q}\dagger}_{m_3 i}(\tilde{k}_3) |\tilde{k}_1 l_1\rangle |\tilde{k}_n l_n\rangle \} \\
= \ & (2\pi)^3 2k_n^+ \delta(P^+ - k_1^+ - k_3^+ - k_n^+) \delta(\vec{P}_T - \vec{k}_{1T} - \vec{k}_{3T} - \vec{k}_{nT}) \delta_{(S_z,r,l_1,l_3,l_n)} \\
\times \ & \langle P\, r\, S_z = 1 | \, \tilde{k}_3\{m_3, i, \mathcal{Q}\}; \, \tilde{k}_1\{m_1, c_1, \mathcal{F}_1\}; \, \tilde{P} - \tilde{k}_3 - \tilde{k}_1, l_n \rangle \\
= \ & (2\pi)^3 2k_n^+ \delta(P^+ - k_1^+ - k_3^+ - k_n^+) \delta(\vec{P}_T - \vec{k}_{1T} - \vec{k}_{3T} - \vec{k}_{nT}) \delta_{(S_z,r,l_1,l_3,l_n)} \\
\times \ & \Psi^\dagger_{r\, S_z=1} \left(\tilde{k}_3\{m_3, i, \mathcal{Q}\}; \, \tilde{k}_1\{m_1, c_1, \mathcal{F}_1\}; \, \tilde{P} - \tilde{k}_3 - \tilde{k}_1, l_n \right) \ . \qquad (18)
\end{aligned}
$$

In the last equation, the definition of the intrinsic proton wave function, Ψ, in momentum space[3], has been recovered. In the same equation, the terms $\delta_{(S_z,r,\ldots)}$ are showing that all the discrete quantum numbers of the quarks have to be properly combined to recover those of the parent proton. In order to obtain a workable expression for the Sivers function given by Eq. (16), other three relations have to be used. One is written using Eq. (12) and translational invariance:

$$
\langle \tilde{k}_1 l_1 | \left\{ \hat{O}_a(0,\xi^-,\vec{\xi}_T) \right\} |\tilde{k}'_1 l'_1\rangle \ = \ e^{ik_1^+ \xi^- - i\vec{k}_{1T}\cdot\vec{\xi}_T} \langle \tilde{k}_1 l_1 | \left\{ \hat{O}_a(0) \right\} |\tilde{k}'_1 l'_1\rangle e^{-ik_1'^+ \xi^- + i\vec{k}'_{1T}\cdot\vec{\xi}_T} \ . \quad (19)
$$

Another one is the identity [13]:

$$
\hat{O}_a(0) = \int_0^\infty A_a^+(\eta^-, 0_T) d\eta^- \ = \ - \int \frac{d^4 q}{(2\pi)^4} \frac{i}{q^+ - i\epsilon} A_a^+(q) \ . \qquad (20)
$$

The last one is obtained by evaluating the matrix element of the perturbative free gluon operator appearing in Eq. (19). Assuming, as an approximation, that this operator is time-independent, one gets, in the Landau gauge:

$$
\begin{aligned}
\langle \tilde{k}_1 l_1 | A_a^+(q) | \tilde{k}'_1 l'_1 \rangle \ = \ & \frac{g}{q^2} T^a_{c_1 c'_1} \bar{u}_{m_1}(\vec{k}_1) \gamma^+ u_{m'_1}(\vec{k}'_1) \delta_{\mathcal{F}\mathcal{F}'} (2\pi) \delta(q_0) \\
\times \ & (2\pi)^3 2k_1^+ \delta(k_1^+ - k_1'^+ - q^+) \delta(\vec{k}_{1T} - \vec{k}'_{1T} - \vec{q}) \ . \qquad (21)
\end{aligned}
$$

[3] In the class of models to be later used, the separation of the center of mass and intrinsic motion is always possible.

Substituting in Eq. (16) the identity:

$$\frac{1}{q^+ - i\epsilon} - \frac{1}{q^+ + i\epsilon} = i(2\pi)\delta(q^+) \ , \tag{22}$$

together with Eqs. (17) – (21), one is left with the following expression for the Sivers function:

$$\begin{aligned} f_{1T}^{\perp\mathcal{Q}}(x,k_T) \ &= \ \Im\left\{ -ig^2 \frac{M}{2k_x} \int d\tilde{k}_1 d\tilde{k}_3 \frac{d^4q}{(2\pi)^3} \delta(q^+) \right. \\ &\times \ \delta(k_3^+ + q^+ - xP^+)\delta(\vec{k}_{3T} + \vec{q}_T - \vec{k}_T)(2\pi)\delta(q_0) \\ &\times \sum_{\mathcal{F}_1, m_1, c_1, m_1', c_1', m_3, i, m_3', j} \delta_{(S_z, r, m_3', m_1', l_n, m_3, m_1, i, j, c_1, c_1')} \\ &\times \ \Psi^\dagger_{r\, S_z=1}\left(\tilde{k}_3\{m_3, i, \mathcal{Q}\};\, \tilde{k}_1\{m_1, c_1, \mathcal{F}_1\};\, \tilde{P} - \tilde{k}_3 - \tilde{k}_1, l_n\right) \\ &\times \ T^a_{ij} T^a_{c_1 c_1'} V(\vec{k}_1, \vec{k}_3, \vec{q}) \\ &\times \ \left. \Psi_{r\, S_z=-1}\left(\tilde{k}_3 + \tilde{q}, \{m_3', j, \mathcal{Q}\};\, \tilde{k}_1 - \tilde{q}, \{m_1', c_1', \mathcal{F}_1\};\, \tilde{P} - \tilde{k}_3 - \tilde{k}_1, l_n\right) \right\} \ , \end{aligned} \tag{23}$$

with the interaction determined by:

$$V(\vec{k}_1, \vec{k}_3, \vec{q}) = \frac{1}{q^2} \bar{u}_{m_3}(\vec{k}_3)\gamma^+ u_{m_3'}(\vec{k}_3 + \vec{q})\bar{u}_{m_1}(\vec{k}_1)\gamma^+ u_{m_1'}(\vec{k}_1 - \vec{q}) \ . \tag{24}$$

Since the final aim is the evaluation of the Sivers function within a NR model, a NR reduction of the interaction has to be performed. Using therefore the definitions of free four-spinors in Eq. (24), performing a NR expansion leaving out terms of second order in momentum, as it is commonly done in nuclear physics (cf. Ref. [55]), one gets the potential:

$$V_{NR}(\vec{k}_1, \vec{k}_3, \vec{q}) \ = \ \frac{1}{2q^2}\left[(V_0)_{m_1, m_1', m_3, m_3'} + (V_S)_{m_1, m_1', m_3, m_3'} \right] \ , \tag{25}$$

with:

$$V_0(\vec{k}_1, \vec{k}_3, \vec{q})_{m_1, m_1', m_3, m_3'} = \left[1 + \frac{k_3^z}{m} + \frac{\vec{q}\cdot\vec{k}_3}{4m^2} + \frac{k_1^z}{m} - \frac{\vec{q}\cdot\vec{k}_1}{4m^2} + O\left(\frac{k_1^2}{m^2}, \frac{k_3^2}{m^2} \right) \right] \delta_{m_1, m_1'}\delta_{m_3, m_3'} \tag{26}$$

$$\begin{aligned} V_S(\vec{k}_1, \vec{k}_3, \vec{q})_{m_1, m_1', m_3, m_3'} \ &= \ -i\left(1 + \frac{k_3^z}{m} + \frac{\vec{q}\cdot\vec{k}_3}{4m^2} \right) \delta_{m_3, m_3'} \frac{[\vec{q}\times(\vec{\sigma}_1)_{m_1, m_1'}]_z}{2m} \\ &+ i\left(1 + \frac{k_1^z}{m} - \frac{\vec{q}\cdot\vec{k}_1}{4m^2} \right) \delta_{m_1, m_1'} \frac{[\vec{q}\times(\vec{\sigma}_3)_{m_3, m_3'}]_z}{2m} \end{aligned}$$

$$
\begin{aligned}
&+i\delta_{m_1,m_1'}\frac{(\vec{\sigma}_3)_{m_3,m_3'}\cdot(\vec{k}_3\times\vec{q})}{4m^2}\\
&-i\frac{(\vec{\sigma}_1)_{m_1,m_1'}\cdot(\vec{k}_1\times\vec{q})}{4m^2}\delta_{m_3,m_3'}\\
&+\frac{[\vec{q}\times(\vec{\sigma}_1)_{m_1,m_1'}]_z(\vec{\sigma}_3)_{m_3,m_3'}\cdot(\vec{k}_3\times\vec{q})}{8m^3}\\
&+\frac{(\vec{\sigma}_1)_{m_1,m_1'}\cdot(\vec{k}_1\times\vec{q})[\vec{q}\times(\vec{\sigma}_3)_{m_3,m_3'}]_z}{8m^3}\\
&+\frac{[\vec{q}\times(\vec{\sigma}_1)_{m_1,m_1'}]_z[\vec{q}\times(\vec{\sigma}_3)_{m_3,m_3'}]_z}{4m^2}+O\left(\frac{k_1^2}{m^2},\frac{k_3^2}{m^2}\right)\ . \qquad (27)
\end{aligned}
$$

A few remarks are in order. First of all, the helicity conserving part, V_0, Eq. (26), of the global interaction Eq. (25), does not contribute to the Sivers function. One should notice that, in an extreme NR limit, the Sivers function would turn out to be identically zero. In our approach, it is precisely the interference of the small and large components in the four-spinors of the free quark states which leads to a non-vanishing Sivers function, even from the component with $l = 0$ of the target wave function. Effectively, these interference terms in the interaction are the ones that, in other approaches, arise due to the wave function (see, e.g., the MIT bag model calculation [28]).

The scheme is now completely set up and any CQM can be used to evaluate the Sivers function. We next use properly normalized NR wave functions to transform Eq. (23) in:

$$
f_{1T}^{\perp\mathcal{Q}}(x,k_T) \;=\; \Im\left\{-ig^2\frac{M^2}{k_x}\int d\vec{k}_1 d\vec{k}_3\frac{d^2\vec{q}_T}{(2\pi)^2}\delta(k_3^+-xP^+)\delta(\vec{k}_{3T}+\vec{q}_T-\vec{k}_T)\mathcal{M}^{\mathcal{Q}}\right\}\ , \qquad (28)
$$

where

$$
\begin{aligned}
\mathcal{M}^{\mathcal{Q}} \;=\; & \sum_{\mathcal{F}_1,m_1,c_1,m_1',c_1',m_3,i,m_3',j}\delta_{(S_z,r,m_3',m_1',l_n,m_3,m_1,i,j,c_1,c_1')}\\
& \times\ \Psi^{\dagger}_{r\,S_z=1}\left(\vec{k}_3\{m_3,i,\mathcal{Q}\};\ \vec{k}_1\{m_1,c_1,\mathcal{F}_1\};\ \vec{P}-\vec{k}_3-\vec{k}_1,l_n\right)\\
& \times\ T^a_{ij}T^a_{c_1c_1'}V(\vec{k}_1,\vec{k}_3,\vec{q})\\
& \times\ \Psi_{r\,S_z=-1}\left(\vec{k}_3+\vec{q},\{m_3',j,\mathcal{Q}\};\ \vec{k}_1-\vec{q},\{m_1',c_1',\mathcal{F}_1\};\ \vec{P}-\vec{k}_3-\vec{k}_1,l_n\right)\ . \qquad (29)
\end{aligned}
$$

Each wave function $\Psi_{r\,S_z}$ describing a possible proton state can be factorized into a completely antisymmetric color wave function, χ, and a symmetric spin-flavor-momentum state, Φ_{sf}, as follows:

$$
\Psi_{r\,S_z}=\Phi_{sf,S_z}\left(\vec{k}_3\{m_3,\mathcal{Q}\};\ \vec{k}_1\{m_1,\mathcal{F}_1\};\ \vec{P}-\vec{k}_3-\vec{k}_1,\{m_n,\mathcal{F}_n\}\right)\chi(i,c_1,c_n)\ . \qquad (30)
$$

The matrix element of the color operator in Eq. (29) can be therefore immediately evaluated:

$$\sum_{c_1,c_1',i,j} \chi^\dagger(i,c_1,c_n) T^a_{ij} T^a_{c_1 c_1'} \chi(j,c_1',c_n) = -\frac{2}{3} \ , \tag{31}$$

which is the well-known result for the exchange of one gluon between quarks in a color singlet 3-quark state [56]. Besides, as a consequence of the symmetry of the state Φ_{sf}, one can assume that the interacting quark is the one labeled "3", so that, after the evaluation of the summation on the flavors $\mathcal{F}_1$, $\mathcal{M}^\alpha$ can be written, for the u and d flavors, as follows:

$$\begin{aligned} \mathcal{M}^{u(d)} &= \left(-\frac{2}{3}\right) \cdot 3 \cdot \sum_{m_1,m_1',m_3,m_3'} \Phi^\dagger_{sf,S_z=1}\left(\vec{k}_3, m_3; \vec{k}_1, m_1; \vec{P}-\vec{k}_3-\vec{k}_1, m_n\right) \\ &\times \frac{1 \pm \tau_3(3)}{2} V_{NR}(\vec{k}_1, \vec{k}_3, \vec{q}) \\ &\times \Phi_{sf,S_z=-1}\left(\vec{k}_3+\vec{q}, m_3'; \vec{k}_1-\vec{q}, m_1'; \vec{P}-\vec{k}_3-\vec{k}_1, m_n\right) \ . \end{aligned} \tag{32}$$

Eq. (28), with $\mathcal{M}^{u(d)}$ given by Eq. (32), provides us with a suitable formula to evaluate the Sivers function, once the spin-flavor wave function of the proton in momentum space, i.e. the quantity Φ_{sf}, is available in a given constituent quark model.

IV. THE CALCULATION OF THE SIVERS FUNCTION IN THE ISGUR-KARL MODEL

As an illustration, in this section we present the results of our approach in the CQM of Isgur and Karl (IK) [41]. In this model the proton wave function is obtained in a OGE potential added to a confining harmonic oscillator (H.O.); including contributions up to the $2\hbar\omega$ shell, the proton state is given by the following admixture of states

$$|N\rangle = a|^2S_{1/2}\rangle_S + b|^2S'_{1/2}\rangle_S + c|^2S_{1/2}\rangle_M + d|^4D_{1/2}\rangle_M \ , \tag{33}$$

where the spectroscopic notation $|^{2S+1}X_J\rangle_t$, with $t = A, M, S$ being the symmetry type, has been used. The coefficients were determined by spectroscopic properties to be $a = 0.931$, $b = -0.274$, $c = -0.233$, $d = -0.067$ [56]. If $a = 1$ and $b = c = d = 0$, a simple H.O. model is recovered. The parameter $\alpha^2 = m\omega$ of the H.O potential is fixed to the value 1.23 fm^{-2}, in order to reproduce the slope of the proton charge form factor at zero momentum transfer [56].

The formal expressions of the wave functions appearing in Eq. (33) in the IK model can be found in [56, 57], given in terms of the following sets of conjugated intrinsic coordinates

$$\begin{aligned}
\vec{R} &= \frac{1}{\sqrt{3}}(\vec{r_1} + \vec{r_2} + \vec{r_3}) \;\leftrightarrow\; \vec{K} = \frac{1}{\sqrt{3}}(\vec{k_1} + \vec{k_2} + \vec{k_3}) \;, \\
\vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2}) \;\leftrightarrow\; \vec{k_\rho} = \frac{1}{\sqrt{2}}(\vec{k_1} - \vec{k_2}) \;, \\
\vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}) \;\leftrightarrow\; \vec{k_\lambda} = \frac{1}{\sqrt{6}}(\vec{k_1} + \vec{k_2} - 2\vec{k_3}) \;.
\end{aligned} \tag{34}$$

There are many good reasons to use the IK model as a test of the developed formalism. First of all, the IK is the typical CQM, succesful in reproducing the low-energy properties of the nucleon, such as the spectrum and the elastic and transition form factors at small momentum transfer [41, 56]. In particular, as was shown in Ref. [58], in the IK model, $\langle k^2 \rangle / m^2 \sim 0.3$ and therefore one expects small corrections from terms $O\left(k^2/m^2\right)$. Besides, one of the features of the IK model is that the OGE mechanism [42], which reduces the degeneracy of the spectrum, is taken into account. It is therefore natural to study our formalism, based on OGE FSI, within the IK framework. Concerning PDs, it has been shown that the IK model can describe their gross features, once QCD evolution of the proper matrix elements of the corresponding twist-2 operators is performed from the scale of the model to the experimental one [36, 37, 38]. Reasonable predictions of GPDs have also been obtained [39], and this makes particularly interesting the evaluation of the Sivers function in the IK model. In section II, the relation between the Sivers function and the impact parameter dependent GPDs has been discussed. In a model where a shift of the quark location in the transverse plane is found, a sizable Sivers function should arise. In order to investigate whether the IK model is suitable for the analysis of the Sivers function, the quantity $\rho_{\mathcal{Q}}(x, \xi = 0, \vec{b})$ has been calculated in this model [59], performing the Fourier transforms, Eq. (8), of GPDs evaluated along the lines of Ref. [39]. The quantity:

$$\rho_{\mathcal{Q}}(\vec{b}) = \int dx \tilde{\rho}_{\mathcal{Q}}(x, \xi = 0, \vec{b}) \;, \tag{35}$$

representing the distribution of the quarks of flavor $\mathcal{Q}$, with any longitudinal momentum, in the transverse plane, independently of their helicity, in a proton polarized along the positive y direction, is shown in Fig. 2. It is clear that a slight shift along the x direction is observed, with a different sign for the u and d flavor. Therefore, according to the present wisdom, a small Sivers function is expected, with different sign for the u and d flavors [49].

After this discussion, the IK model appears as a promising framework for the evaluation of the Sivers function.

The Sivers function has been calculated according to Eq. (28), using the proton states Eq. (33), neglecting the small D component, and the potential Eq. (25) in $\mathcal{M}^{u(d)}$ given by Eq. (32).

The results of the calculation can be cast in the following form:

$$
\begin{aligned}
f_{1T}^{\perp \mathcal{Q}=u,d}(x,k_T) = & -\frac{\sqrt{2}g^2M^2}{k_x}\left(\frac{3}{2}\right)^{\frac{3}{2}}\frac{1}{2\pi^{3/2}\alpha^3}\int\frac{d^2\vec{q}_T}{(2\pi)^2}\frac{k_\lambda^0}{|k_\lambda^0-k_\lambda^z|}e^{-\frac{1}{\alpha^2}\left[k_\lambda^2+\frac{7}{8}q_T^2-\sqrt{\frac{3}{2}}\vec{q}.\vec{k}_\lambda\right]} \\
& \times\frac{1}{2m}\left[a^2\frac{q_x}{q^2}p_{SS}^{(\mathcal{Q})}+ab\frac{q_x}{q^2}\left(p_{S'S}^{(\mathcal{Q})}+p_{SS'}^{(\mathcal{Q})}\right)+ac\frac{q_x}{q^2}\left(p_{MS}^{(\mathcal{Q})}+p_{SM}^{(\mathcal{Q})}\right)\right. \\
& +ac\left(p_{SM'}^{(\mathcal{Q})}+p_{M'S}^{(\mathcal{Q})}\right)+b^2\frac{q_x}{q^2}p_{S'S'}^{(\mathcal{Q})}+bc\frac{q_x}{q^2}p_{S'M}^{(\mathcal{Q})}+bc\frac{q_x}{q^2}p_{MS'}^{(\mathcal{Q})} \\
& \left.+bc\left(p_{S'M'}^{(\mathcal{Q})}+p_{M'S'}^{(\mathcal{Q})}\right)+c^2\frac{q_x}{q^2}\left(p_{MM}^{(\mathcal{Q})}+p_{M'M'}^{(\mathcal{Q})}\right)+c^2\left(p_{MM'}^{(\mathcal{Q})}+p_{M'M}^{(\mathcal{Q})}\right)\right],
\end{aligned}
\tag{36}
$$

with $k_\lambda^0=\sqrt{m^2+k_\lambda^2}$, and

$$
\vec{k}_\lambda=\sqrt{\frac{3}{2}}(\vec{q}-\vec{k})\ , \qquad k_\lambda^z=\frac{\frac{3}{2}m^2+\vec{k}_{\lambda T}^2-3x^2P^{+2}}{2\sqrt{3}P^+x}
$$
$$
k_\lambda^2 \ = \ k_\lambda^{z2}+\vec{k}_{\lambda T}^2 \quad . \tag{37}
$$

The expressions of the functions $p_{XX}^{(\mathcal{Q})}$ are given in the Appendix.

To evaluate numerically Eq. (36), the strong coupling constant g, and therefore $\alpha_s(Q^2)$, has to be fixed. Here, the prescription introduced in the past for calculations of PDFs in quark models (see, i.e., Ref. [38]) will be used. It consists in fixing the momentum scale of the model, the so-called hadronic scale μ_0^2, according to the amount of momentum carried by the valence quarks in the model. In the approach under scrutiny, only valence quarks contribute. Assuming that all the gluons and sea pairs in the proton are produced perturbatively according to NLO evolution equations, in order to have $\simeq 55\%$ of the momentum carried by the valence quarks at a scale of 0.34 GeV^2, as in typical low-energy parameterizations [53], one finds, that $\mu_0^2\simeq 0.1$ GeV^2 if $\Lambda_{QCD}^{NLO}\simeq 0.24$ GeV. This yields $\alpha_s(\mu_0^2)/(4\pi)\simeq 0.13$ [38].

For an easy presentation, the quantity which is usually shown for the results of calculations or for data of the Sivers function is its first moment, defined as follows :

$$
f_{1T}^{\perp(1)\mathcal{Q}}(x)=\int d^2\vec{k}_T\frac{k_T^2}{2M^2}f_{1T}^{\perp\mathcal{Q}}(x,k_T)\ . \tag{38}
$$

The results of the present approach for the moments Eq. (38) are given by the dashed curves in Fig. 3 (4) for the u (d) flavor. They are compared with a parameterization of the HERMES data, corresponding to an experimental scale of $Q^2 = 2.5$ GeV^2 [24][4] The patterned area represents the $1-\sigma$ range of the best fit proposed in Ref. [24].

As expected from the IPD GPDs analysis, shown in Fig. 2, a different sign for the u and d flavor is found.

Let us see now how the results of the calculation compare with the Burkardt sum rule [47], which follows from general principles and must be satisfied at any scale. If the proton is polarized in the positive y direction, in our case, where only valence quarks are present, the Burkardt sum rule reads:

$$\sum_{\mathcal{Q}=u,d} \langle k_x^{\mathcal{Q}} \rangle = 0 \ , \tag{39}$$

where

$$\langle k_x^{\mathcal{Q}} \rangle = - \int_0^1 dx \int d\vec{k}_T \frac{k_x^2}{M} f_{1T}^{\perp \mathcal{Q}}(x, k_T) \ . \tag{40}$$

Within our scheme, at the scale of the model, it is found $\langle k_x^u \rangle = 10.85\, MeV$, $\langle k_x^d \rangle = -11.25\, MeV$ and, in order to have an estimate of the quality of the agreement of our results with the sum rule, we define the ratio

$$r = \frac{\langle k_x^d \rangle + \langle k_x^u \rangle}{\langle k_x^d \rangle - \langle k_x^u \rangle} \ , \tag{41}$$

obtaining $r \simeq 0.02$, so that we can say that our calculation fulfills the Burkardt sum rule to a precision of a few percent.

Another prediction has been derived in the framework of large N_c [60] and it reads, when $xN_c \sim O(1)$ and the large N_c predictions are supposed to be applicable:

$$r_{NC} = \frac{|f_{1T}^{\perp(1)u}(x) + f_{1T}^{\perp(1)d}(x)|}{|f_{1T}^{\perp(1)u}(x) - f_{1T}^{\perp(1)d}(x)|} \simeq \frac{1}{N_c} \ . \tag{42}$$

We get the closest value to the prediction above, 0.26, in a narrow region around $x = 0.4$.

[4] It has been chosen to compare the results with the parameterization of [24] and not with that of [23] or [25] just because, in the first case, it is easier to reconstruct the parameterization of the data, and their 1-sigma range has been kindly provided by the authors of Ref. [24]. The discussion of the quality of the agreement of the present results with data would not change substantially if the comparison were made with the parameterization of Refs. [23, 25].

We note that the contribution of the states $|^2S'_{1/2}\rangle_S$ and $|^2S_{1/2}\rangle_M$, in spite of their small probability in the proton state Eq. (33), turns out to be important in the evaluation of the Sivers function.

The magnitude of the results is close to that of the data, although they have a different shape: the maximum (minimum) is predicted at larger values of x. One should anyway realize that one step of the analysis is still missing: the scale of the model, μ_0^2, is much lower than the one of the data, which is $Q^2 = 2.5$ GeV^2. For a proper comparison, the QCD evolution from the model scale to the experimental one would be necessary. This issue is discussed in the next session.

V. QCD EVOLUTION OF THE MODEL CALCULATION

The Sivers function is a TMD PDs and the evolution of this class of functions is, to a large extent, still to be understood. In any case, recent interesting developements can be found in Ref. [61].

In order to have an indication of the effect of the evolution, we perform a NLO evolution of the model results assuming, for the moments of the Sivers function, the ones defined in Eq. (38), the same anomalous dimensions of the unpolarized PDFs. As described in the previous section, the parameters of the evolution have been fixed in order to have a fraction $\simeq 0.55$ of the momentum carried by the valence quarks at 0.34 GeV^2, as in typical parameterizations of PDFs [53], starting from a scale of $\mu_0^2 \simeq 0.1$ GeV^2 with only valence quarks. The final result is given by the full curve in Fig. 3 (4) for the u (d) flavor. As it is clearly seen, the agreement with data improves dramatically and their trend is reasonably reproduced at least for $x \geq 0.2$.

Of course a word of caution is in order: the performed evolution is not really correct. In any case, an indication of two very important things is obtained:

i) The evolution of the model result is necessary to estimate the quantities at the momentum scale of experiments, as it happens for standard PDs [36, 37, 38];

ii) after evolution, the present calculation could be consistent with data, at least with the present ones, still affected by large statistical and systematic errors.

VI. CONCLUSIONS

A rather general formalism for the evaluation of the Sivers function, to be used in any CQM, has been developed. The crucial ingredient has been the NR reduction of the leading twist part of the OGE diagram in the final state. It has been shown that the IK model, based also on a OGE contribution to the Hamiltonian, is a proper framework for the estimate of the Sivers function. The obtained results show a sizable effect, with an opposite sign for the u and d flavors. This is in agreement with the pattern found from an analysis of impact parameter dependent GPDs in the IK model.

Let us compare our approach with previous calculations. The diquark model with scalar diquarks has no contribution for the d-quark [27] and therefore does not satisfy the Burkardt sum rule (BSR), Eq. (39). The diquark model with axial-vector diquarks has contributions to both u and d-quarks and with opposite sign, but with the magnitude of the d 10 times smaller than that of the u. The BSR is not satisfied. The MIT bag model calculation [28] has non-vanishing u and d-quarks contribution of opposite sign which are proportional in magnitude. The d-quark contribution is much smaller than ours and therefore does not satisfy the BSR. The MIT bag model modified by instanton effects [29] has u and d-quark contributions of the same sign and therefore does not satisfy the BSR. As a summary, we can say that our calculation, despite the naive wave function used, is in better agreement with the data with respect to the other approaches, and fulfills the BSR.

In order to compare with the data, one has to evolve the model calculation to the experimental scale. Although a consistent QCD evolution of the model results to the experimental momentum scale is not yet possible, due to the lack of the calculation of the corresponding anomalous dimensions, an estimate of the evolution has been attempted. It has been found that, once properly evolved, the model results could be in reasonable agreement with the available data.

The formalism presented here can be used with any CQM and it will be interesting in the near future to implement other calculations with different models, performing a correct evolution as soon as the corresponding ingredients become available. The connection of the Sivers function with IPD GPDs deserves a careful analysis and will be discussed elsewhere.

APPENDIX: THE SIVERS FUNCTION IN THE IK MODEL

The functions $p^{(\mathcal{Q})}_{XX}$ appearing in Eq. (36) are listed below.

$$
\begin{aligned}
p^{(u)}_{SS} &= (A - \frac{q^2}{18m^2}) \quad , \\
p^{(d)}_{SS} &= (B + \frac{q^2}{72m^2}) \quad , \qquad \text{(A.1)} \\
p^{(u)}_{S'S} &= \frac{1}{\sqrt{3}\alpha^2}\left[A\left(\frac{3}{2}\alpha^2 + \frac{q^2}{8}\right) - 5\alpha^2\frac{q^2}{36m^2} - \frac{q^4}{144m^2} + (A - \frac{q^2}{18m^2})(k_\lambda^2 - 3\alpha^2)\right] \quad , \\
p^{(d)}_{S'S} &= \frac{1}{\sqrt{3}\alpha^2}\left[B\left(\frac{3}{2}\alpha^2 + \frac{q^2}{8}\right) + 5\alpha^2\frac{q^2}{144m^2} + \frac{q^4}{576m^2} + (B + \frac{q^2}{72m^2})(k_\lambda^2 - 3\alpha^2)\right] \quad , \qquad \text{(A.2)} \\
p^{(u)}_{SS'} &= \frac{1}{\sqrt{3}\alpha^2}\Big[A\left(\frac{3}{2}\alpha^2 + \frac{q^2}{8}\right) - 5\alpha^2\frac{q^2}{36m^2} - \frac{q^4}{144m^2} + (A - \frac{q^2}{18m^2}) \\
&\quad (k_\lambda^2 - 3\alpha^2 + 2q^2 - 3\vec{q}\cdot(\vec{q} - \vec{k})) - A\frac{q^2}{2} + \frac{q^4}{36m^2} + \alpha^2\frac{q^2}{9m^2}\Big] \quad , \\
p^{(d)}_{SS'} &= \frac{1}{\sqrt{3}\alpha^2}\Bigg[B\left(\frac{3}{2}\alpha^2 + \frac{q^2}{8}\right) + 5\alpha^2\frac{q^2}{144m^2} + \frac{q^4}{576m^2} + (B + \frac{q^2}{72m^2}) \\
&\quad (k_\lambda^2 - 3\alpha^2 + 2q^2 - 3\vec{q}\cdot(\vec{q} - \vec{k})) - B\frac{q^2}{2} - \alpha^2\frac{q^2}{36m^2} - \frac{q^4}{144m^2}\Bigg] \quad , \qquad \text{(A.3)} \\
p^{(u)}_{MS} &= \frac{1}{\sqrt{6}\alpha^2}\left[-k_\lambda^2\left(D - \frac{5q^2}{72m^2}\right) + D\left(\frac{3}{2}\alpha^2 + \frac{q^2}{8}\right) - 25\alpha^2\frac{q^2}{144m^2} - 5\frac{q^4}{576m^2}\right] \quad , \\
p^{(d)}_{MS} &= \frac{2}{\sqrt{6}\alpha^2}\Big[k_\lambda^2\left(B + \frac{q^2}{72m^2}\right) - B\left(\frac{3}{2}\alpha^2 + \frac{q^2}{8}\right) - 5\alpha^2\frac{q^2}{144m^2} - \frac{q^4}{576m^2}\Big] \quad , \qquad \text{(A.4)} \\
p^{(u)}_{SM} &= p^{(u)}_{MS} + \frac{1}{\sqrt{6}\alpha^2}\left[-(q^2 - \sqrt{6}\vec{q}\cdot\vec{k}_\lambda)\left(D - \frac{5q^2}{72m}\right) + \left(-D\frac{q^2}{2} + 5\alpha^2\frac{q^2}{36m^2} + 5\frac{q^4}{144m^2}\right)\right] \quad , \\
p^{(d)}_{SM} &= p^{(d)}_{MS} + \frac{1}{\sqrt{6}\alpha^2}\left[\left(q^2 - \sqrt{6}\vec{q}\cdot\vec{k}_\lambda\right)\left(B + \frac{q^2}{72m^2}\right) + \left(B\frac{q^2}{2} + \frac{\alpha^2 q^2}{36m} + \frac{q^4}{144m^2}\right)\right] \quad , \qquad \text{(A.5)} \\
p^{(u)}_{M'S} &= -\frac{2}{q^2\sqrt{18}\alpha^2}\Bigg[-\alpha^2 q_x\frac{k^z_\lambda}{4\sqrt{2}m} - q_y\alpha^2\frac{(\vec{k}\times\vec{q})^z}{8\sqrt{2}m^2} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\,C^{M'S}_{MSMA} + \\
&\quad \frac{1}{3}\left(-\alpha^2 q_x\frac{k^z_\lambda}{4\sqrt{2}m} + q_y\alpha^2\frac{(\vec{k}\times\vec{q})^z}{8\sqrt{2}m^2} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\,C^{M'S}_{MAMS}\right)\Bigg] \quad , \\
p^{(d)}_{M'S} &= -\frac{2}{q^2\sqrt{18}\alpha^2}\frac{2}{3}\left[-\alpha^2 q_x\frac{k^z_\lambda}{4\sqrt{2}m} + q_y\alpha^2\frac{(\vec{k}\times\vec{q})^z}{8\sqrt{2}m^2} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\,C^{M'S}_{MAMS}\right] \quad , \qquad \text{(A.6)} \\
p^{(u)}_{SM'} &= p^{(u)}_{M'S} - \frac{2}{q^2\sqrt{18}\alpha^2}\left(C^{M'S}_{MSMA} + \frac{1}{3}C^{M'S}_{MAMS}\right)\left(\frac{\sqrt{3}q^2}{4} - \frac{\vec{q}\cdot\vec{k}_\lambda}{\sqrt{2}}\right) \quad , \\
p^{(d)}_{SM'} &= p^{(d)}_{M'S} - \frac{2}{q^2\sqrt{18}\alpha^2}\frac{2}{3}C^{M'S}_{MAMS}\left(\frac{\sqrt{3}q^2}{4} - \frac{\vec{q}\cdot\vec{k}_\lambda}{\sqrt{2}}\right) \quad , \qquad \text{(A.7)}
\end{aligned}
$$

$$
\begin{aligned}
p^{(u)}_{S'S'} &= \frac{1}{3\alpha^4}\Bigg[F\,p^{(u)}_{SS}+G\left(A\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)-5\alpha^2\frac{q^2}{36m^2}-\frac{q^4}{144m^2}\right)+H\Big(-A\,\frac{q^2}{2}\\
&\quad+\frac{q^4}{36m^2}+\alpha^2\frac{q^2}{9m}\Big)-\sqrt{2}\left(C^{S'S'(1)}_{MA}+\frac{1}{3}C^{S'S'(1)}_{MS}\right)+\left(C^{S'S'(2)}_{MA}+\frac{1}{3}C^{S'S'(2)}_{MS}\right)\Bigg]\quad,\\
p^{(d)}_{S'S'} &= \frac{1}{3\alpha^4}\Bigg[F\,p^{(d)}_{SS}+G\left(B\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)+5\alpha^2\frac{q^2}{144m^2}+\frac{q^4}{576m^2}\right)+H\Big(-B\frac{q^2}{2}\\
&\quad-\frac{q^4}{144m^2}-\alpha^2\frac{q^2}{36m}\Big)-\sqrt{2}\frac{2}{3}C^{S'S'(1)}_{MS}+\frac{2}{3}C^{S'S'(2)}_{MS}\Bigg]\quad, && \text{(A.8)}\\
p^{(u)}_{S'M} &= \frac{1}{3\sqrt{2}\alpha^4}\Bigg[K\,p^{(u)}_{SS}+L\left(A\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)-5\alpha^2\frac{q^2}{36m^2}-\frac{q^4}{144m^2}\right)+H\Big(-A\frac{q^2}{2}\\
&\quad+\frac{q^4}{36m^2}+\alpha^2\frac{q^2}{9m}\Big)-\sqrt{2}\left(C^{S'S'(1)}_{MA}+\frac{1}{3}C^{S'S'(1)}_{MS}\right)+\left(C^{S'S'(2)}_{MA}+\frac{1}{3}C^{S'S'(2)}_{MS}\right)\Bigg]\quad,\\
p^{(d)}_{S'M} &= \frac{1}{3\sqrt{2}\alpha^4}\Bigg[K\,p^{(d)}_{SS}+L\left(B\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)+5\alpha^2\frac{q^2}{144m^2}+\frac{q^4}{576m^2}\right)\\
&\quad+H\left(-B\frac{q^2}{2}-\frac{q^4}{144m^2}-\alpha^2\frac{q^2}{36m}\right)-\sqrt{2}\frac{2}{3}C^{S'S'(1)}_{MS}+\frac{2}{3}C^{S'S'(2)}_{MS}\Bigg]\quad, && \text{(A.9)}\\
p^{(u)}_{MS'} &= \frac{1}{3\sqrt{2}\alpha^4}\Bigg[N\,p^{(u)}_{SS}+O\left(A\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)-5\alpha^2\frac{q^2}{36m^2}-\frac{q^4}{144m^2}\right)-k_\lambda^2\Big(-A\frac{q^2}{2}\\
&\quad+\frac{q^4}{36m^2}+\alpha^2\frac{q^2}{9m}\Big)-\sqrt{2}\left(C^{S'S'(1)}_{MA}+\frac{1}{3}C^{S'S'(1)}_{MS}\right)+\left(C^{S'S'(2)}_{MA}+\frac{1}{3}C^{S'S'(2)}_{MS}\right)\Bigg]\quad,\\
p^{(d)}_{MS'} &= \frac{1}{3\sqrt{2}\alpha^4}\Bigg[N\,p^{(d)}_{SS}+O\left(B\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)+5\alpha^2\frac{q^2}{144m^2}+\frac{q^4}{576m^2}\right)-k_\lambda^2\Big(-B\frac{q^2}{2}\\
&\quad-\frac{q^4}{144m^2}-\alpha^2\frac{q^2}{36m}\Big)-\sqrt{2}\frac{2}{3}C^{S'S'(1)}_{MS}+\frac{2}{3}C^{S'S'(2)}_{MS}\Bigg]\quad, && \text{(A.10)}\\
p^{(u)}_{S'M'} &= -\frac{2}{3\sqrt{18}\,q^2\alpha^4}\left(C^{S'M'}_{MSMA}+\frac{1}{3}C^{S'M'}_{MAMS}\right)\quad,\\
p^{(d)}_{S'M'} &= -\frac{2}{3\sqrt{18}\,q^2\alpha^4}\frac{2}{3}\,C^{S'M'}_{MAMS}\quad, && \text{(A.11)}\\
p^{(u)}_{M'S'} &= -\frac{2}{3\sqrt{18}\,q^2\alpha^4}\left(C^{M'S'}_{MSMA}+\frac{1}{3}C^{M'S'}_{MAMS}\right)\quad,\\
p^{(d)}_{M'S'} &= -\frac{2}{3\sqrt{18}\,q^2\alpha^4}\frac{2}{3}\,C^{M'S'}_{MAMS}\quad, && \text{(A.12)}\\
p^{(u)}_{MM} &= \frac{1}{6\alpha^4}\Bigg[S\,p^{(u)}_{SS}+U\left(A\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)-5\alpha^2\frac{q^2}{36m^2}-\frac{q^4}{144m^2}\right)-k_\lambda^2\Big(-A\frac{q^2}{2}\\
&\quad+\frac{q^4}{36m^2}+\alpha^2\frac{q^2}{9m}\Big)-\sqrt{2}\left(C^{S'S'(1)}_{MA}+\frac{1}{3}C^{S'S'(1)}_{MS}\right)+\left(C^{S'S'(2)}_{MA}+\frac{1}{3}C^{S'S'(2)}_{MS}\right)\Bigg]\quad,\\
p^{(d)}_{MM} &= \frac{1}{6\alpha^4}\Bigg[S\,p^{(d)}_{SS}+U\left(B\left(\frac{3}{2}\alpha^2+\frac{q^2}{8}\right)+5\alpha^2\frac{q^2}{144m^2}+\frac{q^4}{576m^2}\right)-k_\lambda^2\Big(-B\frac{q^2}{2}\\
&\quad-\frac{q^4}{144m^2}-\alpha^2\frac{q^2}{36m}\Big)-\sqrt{2}\frac{2}{3}C^{S'S'(1)}_{MS}+\frac{2}{3}C^{S'S'(2)}_{MS}\Bigg]\quad, && \text{(A.13)}
\end{aligned}
$$

$$
\begin{aligned}
p^{(u)}_{M'M'} &= \frac{2}{3\alpha^4}\Bigg[\left(C^{M'M'(1)}_{MS}+C^{M'M'(2)}_{MS}+C^{M'M'(3)}_{MS}\right) \\
&\quad +\frac{1}{3}\left(C^{M'M'(1)}_{MA}+C^{M'M'(2)}_{MA}+C^{M'M'(3)}_{MA}\right)\Bigg] \quad , \\
p^{(d)}_{M'M'} &= \frac{2}{3\alpha^4}\frac{2}{3}\left(C^{M'M'(1)}_{MA}+C^{M'M'(2)}_{MA}+C^{M'M'(3)}_{MA}\right) \quad , && \text{(A.14)} \\
p^{(u)}_{MM'} &= -\frac{1}{3\,q^2\alpha^4}\left(C^{MM'}_{MAMS}-\frac{1}{3}C^{MM'}_{MSMA}\right) \quad , \\
p^{(d)}_{MM'} &= \frac{1}{3\,q^2\alpha^4}\,\frac{2}{3}\,C^{MM'}_{MSMA} \quad , && \text{(A.15)} \\
p^{(u)}_{M'M} &= -\frac{1}{3\,q^2\alpha^4}\left(C^{M'M}_{MAMS}-\frac{1}{3}C^{M'M}_{MSMA}\right) \quad , \\
p^{(d)}_{M'M} &= \frac{1}{3\,q^2\alpha^4}\frac{2}{3}\,C^{M'M}_{MSMA} \quad ; && \text{(A.16)}
\end{aligned}
$$

with

$$
\begin{aligned}
A &= \frac{2}{3}+\frac{k^z_\lambda}{3\sqrt{6}m}-\frac{(q^2-\vec{k}\cdot\vec{q})}{18m^2} \quad , \\
B &= -\frac{2}{3}+\frac{k^z_\lambda}{3\sqrt{6}m}+\frac{5}{36m^2}(q^2-\vec{k}\cdot\vec{q}) \quad , \\
C &= \frac{4}{3}-\frac{k^z_\lambda}{3\sqrt{6}m}-7\frac{(q^2-\vec{k}\cdot\vec{q})}{36m^2} \quad , \\
F &= k^4_\lambda+k^2_\lambda q^2-\sqrt{6}k_\lambda\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda 6\alpha^2-6q^2\alpha^2+3\alpha^2\sqrt{6}\vec{q}\cdot\vec{k}_\lambda+9\alpha^4 \quad , \\
G &= 2k^2_\lambda+2q^2-\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-6\alpha^2 \quad , \\
H &= k^2_\lambda-3\alpha^2 \quad , \\
K &= -k^4_\lambda-k^2_\lambda q^2+\sqrt{6}k^2_\lambda\vec{q}\cdot\vec{k}_\lambda+k^2_\lambda 3\alpha^2+3q^2\alpha^2-3\alpha\sqrt{6}\vec{q}\cdot\vec{k}_\lambda \quad , \\
L &= -q^2+\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-3\alpha^2 \quad , \\
T &= 1-\frac{\vec{q}\cdot\vec{k}_\lambda}{4\sqrt{6}m^2} \quad , \\
T' &= 1-5\frac{\vec{q}\cdot\vec{k}_\lambda}{12\sqrt{6}m^2}-\frac{k^z_\lambda}{\sqrt{6}m} \quad , \\
N &= -k^4_\lambda-2k^2_\lambda q^2+\sqrt{6}k^2_\lambda\vec{q}\cdot\vec{k}_\lambda+k^2_\lambda 3\alpha^2 \quad , \\
O &= 2q^2-\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-3\alpha^2 \quad , \\
S &= k^4_\lambda+k^2_\lambda q^2-\sqrt{6}k^2_\lambda\vec{q}\cdot\vec{k}_\lambda \quad , \\
U &= -q^2+\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-2k^2_\lambda \quad ; && \text{(A.17)}
\end{aligned}
$$

and with

$$
C^{M'S}_{MSMA} = -q_x+3q_x\frac{k^z_\lambda}{2\sqrt{6}m}+\frac{k^x_\lambda}{2\sqrt{6}m^2}\left(q^2_x-\frac{3}{2}q^2_y\right)+5\frac{k^y_\lambda q_x q_y}{4\sqrt{6}m^2} \quad ,
$$

$$
\begin{aligned}
C^{M'S}_{MAMS} &= -q_x + 3q_x\frac{k^z_\lambda}{2\sqrt{6}m} + \frac{k^x_\lambda}{2\sqrt{6}m^2}\left(q_x^2 + \frac{3}{2}q_y^2\right) - \frac{k^y_\lambda q_x q_y}{4\sqrt{6}m^2} \quad ,\\
C^{S'M'}_{MSMA} &= \left(\sqrt{\frac{3}{2}}q^2 - \vec{q}\cdot\vec{k}_\lambda\right)\left(k_\lambda^2 - 5\frac{\alpha^2}{2} + \frac{q^2}{8}\right)\frac{1}{8\sqrt{3}}((-2\sqrt{6} + 3\frac{k^z_\lambda}{m} + \frac{\vec{q}\cdot\vec{k}_\lambda}{m^2})q_x\\
&\quad -3q_y\frac{(\vec{k}_\lambda\times\vec{q})_z}{2m^2}) - q_x\alpha^2\frac{k^z_\lambda}{4\sqrt{2}m}(k_\lambda^2 - 3\alpha^2) - q_y(\vec{k}_\lambda\times\vec{q})_z\frac{\alpha^2}{16\sqrt{2}m^2}\left(5\alpha^2 + \frac{q^2}{4}\right) \quad ,\\
C^{S'M'}_{MAMS} &= \left(\sqrt{\frac{3}{2}}q^2 - \vec{q}\cdot\vec{k}_\lambda\right)\left(k_\lambda^2 - 5\frac{\alpha^2}{2} + \frac{q^2}{8}\right)\frac{1}{8\sqrt{3}}\big((-2\sqrt{6} + 3\frac{k^z_\lambda}{m} + \frac{\vec{q}\cdot\vec{k}_\lambda}{m^2})q_x\\
&\quad +3q_y\frac{(\vec{k}_\lambda\times\vec{q})_z}{2m^2}) - q_x\alpha^2\frac{k^z_\lambda}{4\sqrt{2}m}(k_\lambda^2 - 3\alpha^2) + q_y(\vec{k}_\lambda\times\vec{q})_z\frac{\alpha^2}{16\sqrt{2}m^2}\left(5\alpha^2 + \frac{q^2}{4}\right) \quad ,\\
C^{M'S'}_{MSMA} &= q_x\Bigg[\frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\left(-5\frac{\alpha^2}{2} + 13\frac{q^2}{8} + k_\lambda^2 - \sqrt{6}\vec{q}\cdot\vec{k}_\lambda\right)\left(-1 + 3\frac{k^z_\lambda}{2\sqrt{6}m} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{6}m^2}\right)\\
&\quad +\alpha^2\frac{k^z_\lambda}{4\sqrt{2}m^2}\left(5\frac{q^2}{2} + k_\lambda^2 - \sqrt{6}\vec{q}\cdot\vec{k}_\lambda - 3\alpha^2\right)\Bigg]\\
&\quad -q_y\frac{(\vec{k}_\lambda\times\vec{q})_z}{4\sqrt{2}}\Bigg[\left(3\frac{\vec{q}\cdot\vec{k}_\lambda}{\sqrt{6}} + \alpha^2\right)\left(13\frac{q^2}{16} + \frac{k_\lambda^2}{2} - \sqrt{6}\frac{\vec{q}\cdot\vec{k}_\lambda}{2}\right) - 3\frac{5}{4}\alpha^2\frac{\vec{q}\cdot\vec{k}_\lambda}{\sqrt{6}} - \frac{\alpha^4}{4}\Bigg] \quad ,\\
C^{M'S'}_{MAMS} &= q_x\Bigg[\frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\left(-5\frac{\alpha^2}{2} + 13\frac{q^2}{8} + k_\lambda^2 - \sqrt{6}\vec{q}\cdot\vec{k}_\lambda\right)\left(-1 + 3\frac{k^z_\lambda}{2\sqrt{6}m} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{6}m^2}\right)\\
&\quad +\alpha^2\frac{k^z_\lambda}{4\sqrt{2}m^2}\left(5\frac{q^2}{2} + k_\lambda^2 - \sqrt{6}\vec{q}\cdot\vec{k}_\lambda - 3\alpha^2\right)\Bigg]\\
&\quad +q_y\frac{(\vec{k}_\lambda\times\vec{q})_z}{4\sqrt{2}}\Bigg[\left(3\frac{\vec{q}\cdot\vec{k}_\lambda}{\sqrt{6}} + \alpha^2\right)\left(13\frac{q^2}{16} + \frac{k_\lambda^2}{2} - \sqrt{6}\frac{\vec{q}\cdot\vec{k}_\lambda}{2}\right) - 3\frac{5}{4}\alpha^2\frac{\vec{q}\cdot\vec{k}_\lambda}{\sqrt{6}} - \frac{\alpha^4}{4}\Bigg] \quad ,\\
C^{MM'}_{MSMA} &= q_x\Bigg[\frac{1}{2\sqrt{2}}\left(-1 + 3\frac{k^z_\lambda}{2\sqrt{6}m} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{6}m^2}\right)\left(\sqrt{\frac{3}{2}}q^2 - \vec{q}\cdot\vec{k}_\lambda\right)\left(-k_\lambda^2 + \frac{\alpha^2}{2} + \frac{q^2}{8}\right)\\
&\quad +\alpha^2 k_\lambda^2\frac{k^z_\lambda}{4\sqrt{2}m^2}\Bigg] - q_y(\vec{k}_\lambda\times\vec{q})_z\Bigg[\frac{3}{4\sqrt{6}m^2}\frac{1}{2\sqrt{2}}\left(\sqrt{\frac{3}{2}}q^2 - \vec{q}\cdot\vec{k}_\lambda\right)\left(-k_\lambda^2 + \frac{\alpha^2}{2} + \frac{q^2}{8}\right)\\
&\quad +\frac{\alpha^2}{8\sqrt{2}m^2}\left(5\frac{\alpha^2}{2} + \frac{q^2}{8} - k_\lambda^2\right)\Bigg] \quad ,\\
C^{MM'}_{MAMS} &= q_x\Bigg[\frac{1}{2\sqrt{2}}\left(-1 + 3\frac{k^z_\lambda}{2\sqrt{6}m} + \frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{6}m^2}\right)\left(\sqrt{\frac{3}{2}}q^2 - \vec{q}\cdot\vec{k}_\lambda\right)\left(-k_\lambda^2 + \frac{\alpha^2}{2} + \frac{q^2}{8}\right)\\
&\quad +\alpha^2 k_\lambda^2\frac{k^z_\lambda}{4\sqrt{2}m^2}\Bigg] - q_y(\vec{k}_\lambda\times\vec{q})_z\Bigg[\frac{3}{4\sqrt{6}m^2}\frac{1}{2\sqrt{2}}\left(\sqrt{\frac{3}{2}}q^2 - \vec{q}\cdot\vec{k}_\lambda\right)\left(-k_\lambda^2 + \frac{\alpha^2}{2} + \frac{q^2}{8}\right)\\
&\quad +\frac{\alpha^2}{8\sqrt{2}m^2}\left(5\frac{\alpha^2}{2} + \frac{q^2}{8} - k_\lambda^2\right)\Bigg] \quad ,
\end{aligned}
$$

$$
\begin{aligned}
C^{M'M}_{MSMA} &= q_x\Bigg[\frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\left(-1+3\frac{k^z_\lambda}{2\sqrt{6}m}+\frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{6}m^2}\right)\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda+2\alpha^2+11\frac{q^2}{8}\right)\\
&\quad -\alpha^2\frac{k^z_\lambda}{4\sqrt{2}m^2}\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda-3\frac{q^2}{2}\right)\Bigg]\\
&\quad -q_y(\vec{k}_\lambda\times\vec{q})_z\Bigg[3\frac{\vec{q}\cdot\vec{k}_\lambda}{8\sqrt{12}m^2}\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda+2\alpha^2+11\frac{q^2}{8}\right)\\
&\quad +\frac{\alpha^2}{8\sqrt{2}m^2}\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda+5\frac{q^2}{2}-11\frac{q^2}{8}\right)\Bigg] \quad ,\\
C^{M'M}_{MAMS} &= q_x\Bigg[\frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{2}}\left(-1+3\frac{k^z_\lambda}{2\sqrt{6}m}+\frac{\vec{q}\cdot\vec{k}_\lambda}{2\sqrt{6}m^2}\right)\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda+2\alpha^2+11\frac{q^2}{8}\right)\\
&\quad -\alpha^2\frac{k^z_\lambda}{4\sqrt{2}m^2}\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda-3\frac{q^2}{2}\right)\Bigg]\\
&\quad +q_y(\vec{k}_\lambda\times\vec{q})_z\Bigg[3\frac{\vec{q}\cdot\vec{k}_\lambda}{8\sqrt{12}m^2}\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda+2\alpha^2+11\frac{q^2}{8}\right)\\
&\quad +\frac{\alpha^2}{8\sqrt{2}m^2}\left(\sqrt{6}\vec{q}\cdot\vec{k}_\lambda-k^2_\lambda+5\frac{q^2}{2}-11\frac{q^2}{8}\right)\Bigg] \quad ,
\end{aligned}
\tag{A.18}
$$

$$
\begin{aligned}
C^{S'S'(1)}_{MA} &= q^2\Big(-5\frac{\alpha^4}{16\sqrt{2}m^2}+\alpha^2\left(5\frac{T}{4\sqrt{2}}-\frac{q^2}{8\sqrt{2}m^2}\right)+T\frac{q^2}{16\sqrt{2}}-\frac{q^4}{256\sqrt{2}m^2}\Big) \quad ,\\
C^{S'S'(2)}_{MA} &= q^2\Big(-70\frac{\alpha^4}{128m^2}+\alpha^2\left(5\frac{T}{8}-\frac{7q^2}{128m^2}\right)+T\frac{q^2}{64}-\frac{q^4}{1288m^2}\Big) \quad ,\\
C^{S'S'(1)}_{MS} &= -q^2\Big(-5\frac{\alpha^4}{48\sqrt{2}m^2}+\alpha^2\left(5\frac{T'}{4\sqrt{2}}-\frac{q^2}{24\sqrt{2}m^2}\right)+T'\frac{q^2}{16\sqrt{2}}-\frac{q^4}{768\sqrt{2}m^2}\Big) \quad ,\\
C^{S'S'(2)}_{MS} &= -q^2\Big(-70\frac{\alpha^4}{384m^2}+\alpha^2\left(5\frac{T'}{8}-\frac{7q^2}{384m^2}\right)+T'\frac{q^2}{64}-\frac{q^4}{3848m^2}\Big) \quad ,\\
C^{M'M'(1)}_{MA} &= \frac{1}{4}\left(\sqrt{\frac{3}{2}}q^2-\vec{q}\cdot\vec{k}_\lambda\right)\left(\alpha^2\frac{k^z_\lambda}{m}+\vec{q}\cdot\vec{k}_\lambda T-\alpha^2\frac{\vec{q}\cdot\vec{k}_\lambda}{4m^2}-q^2\frac{\vec{q}\cdot\vec{k}_\lambda}{16m^2}\right) \quad ,\\
C^{M'M'(1)}_{MS} &= -\frac{1}{4}\left(\sqrt{\frac{3}{2}}q^2-\vec{q}\cdot\vec{k}_\lambda\right)\left(\frac{2}{3}\alpha^2\frac{k^z_\lambda}{m}+\vec{q}\cdot\vec{k}_\lambda T'-\alpha^2\frac{\vec{q}\cdot\vec{k}_\lambda}{12m^2}-q^2\frac{\vec{q}\cdot\vec{k}_\lambda}{48m^2}\right) \quad ,\\
C^{M'M'(2)}_{MA} &= -\sqrt{\frac{3}{2}}\left(T\vec{q}\cdot\vec{k}_\lambda\left(\frac{\alpha^2}{2}+\frac{q^2}{8}\right)+\alpha^2q^2\frac{k^z_\lambda}{8m}-3\vec{q}\cdot\vec{k}_\lambda\frac{\alpha^2q^2}{32m^2}-q^4\frac{\vec{q}\cdot\vec{k}_\lambda}{128m^2}\right) \quad ,\\
C^{M'M'(2)}_{MS} &= \sqrt{\frac{3}{2}}\left(T'\vec{q}\cdot\vec{k}_\lambda\left(\frac{\alpha^2}{2}+\frac{q^2}{8}\right)+\alpha^2q^2\frac{2}{3}\frac{k^z_\lambda}{8m}-\vec{q}\cdot\vec{k}_\lambda\frac{\alpha^2q^2}{32m^2}-q^4\frac{\vec{q}\cdot\vec{k}_\lambda}{384m^2}\right) \quad ,\\
C^{M'M'(3)}_{MA} &= T\frac{\alpha^2k^2_\lambda}{2}+\frac{T}{8}(\vec{q}\cdot\vec{k}_\lambda)^2-\alpha^2\left(\vec{q}\cdot\vec{k}_\lambda\frac{k^z_\lambda}{4m}+q^2\frac{k^2_\lambda}{32m^2}+\frac{(\vec{q}\cdot\vec{k}_\lambda)^2}{16m^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{q^2(\vec{q}\cdot\vec{k}_\lambda)^2}{128m^2} \quad , \\
C_{MS}^{M'M'(3)} &= -\Bigg[T'\frac{\alpha^2 k_\lambda^2}{2} + \frac{T'}{8}(\vec{q}\cdot\vec{k}_\lambda)^2 - \alpha^2\left(\frac{2}{3}\vec{q}\cdot\vec{k}_\lambda\frac{k_\lambda^z}{4m} + q^2\frac{k_\lambda^2}{96m^2} + \frac{(\vec{q}\cdot\vec{k}_\lambda)^2}{48m^2}\right) \\
& -\frac{q^2(\vec{q}\cdot\vec{k}_\lambda)^2}{384m^2}\Bigg] \quad . \qquad \text{(A.19)}
\end{aligned}
$$

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FIGURE CAPTIONS

Fig. 1: The contributions to the Sivers function in the present approach. The graph has been drawn using JaxoDraw [62].

Fig. 2: In the upper (lower) panel, the quantity $\rho_Q(\vec{b})$, Eq. (35), is shown for the u (d) flavor.

Fig. 3: The quantity $f_{1T}^{\perp(1)q}(x)$, Eq. (38), for the u flavor. The dashed curve is the result of the present approach at the hadronic scale μ_0^2, Eq. (36). The full curve represents the evolved distribution after standard NLO evolution (see text). The patterned area represents the $1-\sigma$ range of the best fit of the HERMES data proposed in Ref. [24].

Fig. 4: The same as in Fig. 3, but for the d flavor.

FIG. 1:

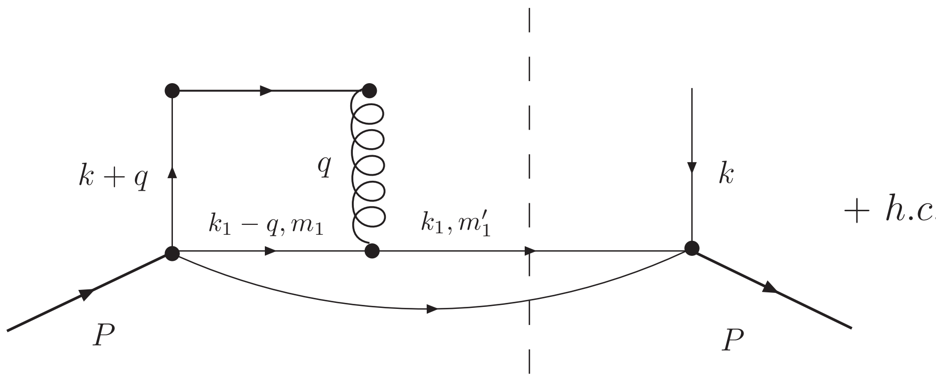

FIG. 2:

$$\rho_q(\vec{b}) = \int dx\, \rho_q(x, \xi = 0, \vec{b})$$

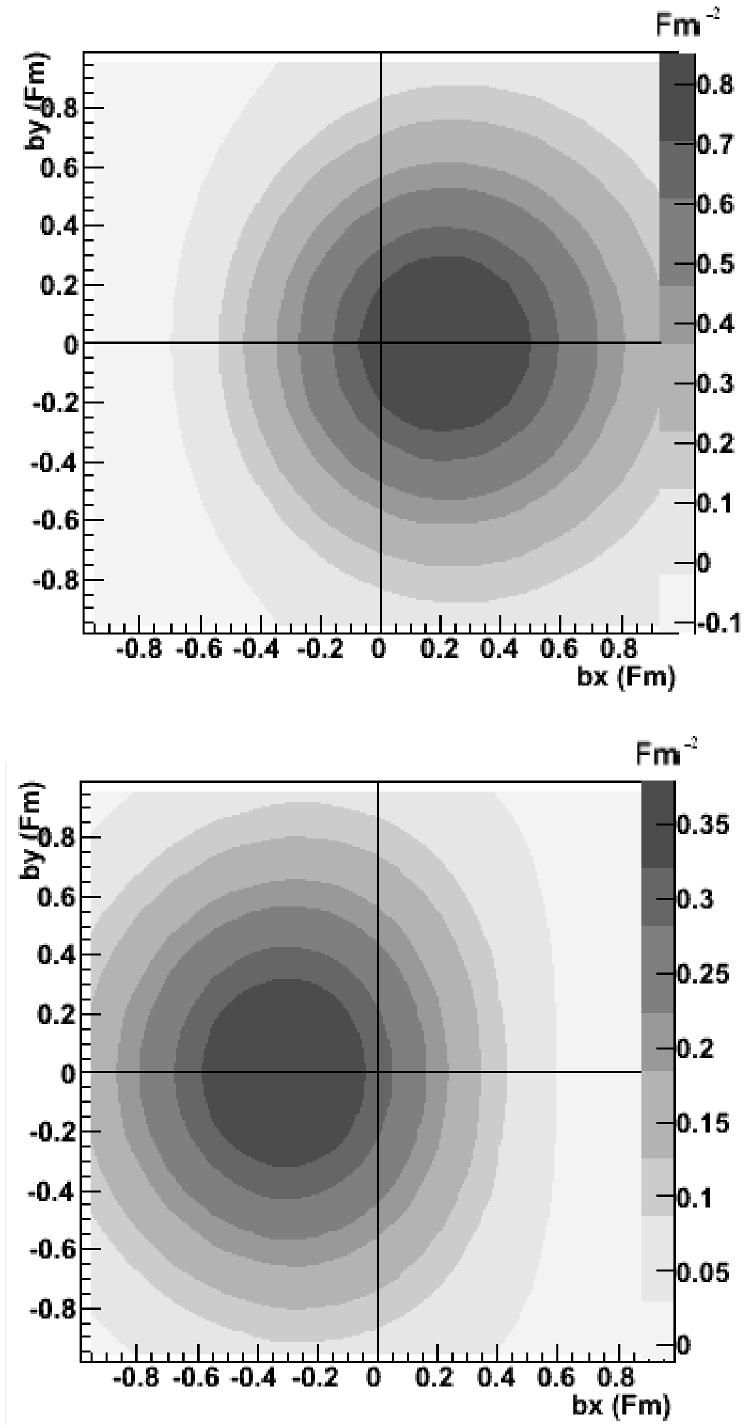

FIG. 3:

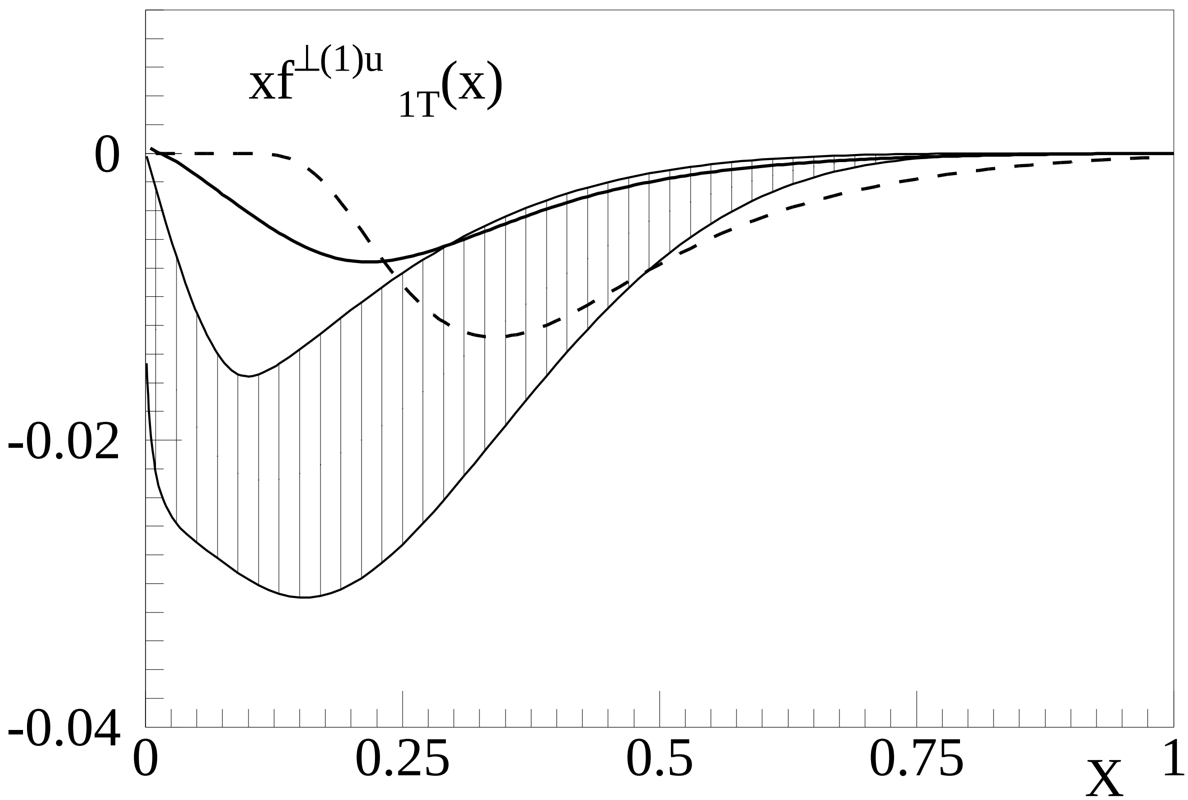

FIG. 4: