

# A Comment on Form Factor Mass Singularities in Flavor-Changing Neutral Currents

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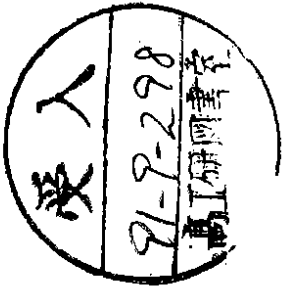
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## Abstract

Flavor-changing effective vertices  $g\bar{q}q, V^0$ , where  $V^0$  represents a neutral gauge boson ( $\gamma, Z^0, g$ ), involving a heavy external quark, are discussed within the standard model at one-loop level and second-order approximation in external momenta and masses: the logarithmic singular terms in the form factors at vanishing mass of the internal quark in the loop have to be replaced by pieces coming from next order in external momenta. Implications in the  $b \rightarrow d + X$  penguin transitions are commented.



The possibility of testing the standard model (SM) of the electroweak interactions and its possible extensions, as well as non-conventional physics, has increased in recent years the interest in rare decays of particles. The fact that such processes occur via diagrams involving particles like the gauge boson  $W$  and its possible extensions, or the yet undiscovered top quark, makes them especially suitable as complementary tests of other searches like those performed at collider physics. In particular, neutral gauge bosons ( $\gamma$ ,  $Z^0$  and  $g$ ) couple to quarks with different flavors at lowest order through one-loop level diagrams, giving rise to flavor-changing neutral current (FCNC) transitions.

Among rare processes, FCNC decays of B mesons have the intrinsic advantage over others (like kaon decays) of the relative large mass of the constituent b quark, offering the reasonable option of neglecting QCD effects at a first approximation and rendering credible a spectator model. Furthermore, in the usual low energy limit, one neglects external fermion masses and momenta whenever and as early as possible to simplify the calculation<sup>1</sup>. However, although such approximation is in general well-founded, a logarithmic singularity arises in the flavor-changing (FC) vertex amplitude when the mass of the internal fermion of the loop is set to zero. In fact as we shall see this is a manifestation of a careless low energy limiting process performed at second-order in external momenta.

In our discussion we will essentially follow the framework and notation employed in the Barroso and Soares' paper [2], where they present an exact calculation within the SM of the FC vertices  $q\bar{q}\gamma V^0$  with  $V^0 = \gamma$ ,  $Z^0$  in the 't Hooft-Feynman gauge at one-loop level using the on-shell renormalization scheme<sup>2</sup>. Throughout this paper we will not discuss QCD corrections to the effective vertex, and we will impose the following constraints: i) on-shell external quarks, ii) the mass of the light external quark ( $q$ ) is set to zero. Then, both FC renormalized vertices  $q\bar{q}\gamma$ ,  $q\bar{q}\gamma Z^0$  can be expressed as

$$\Xi^\mu_{ren|on-shell} = C [\alpha_1 q^\mu + \alpha_2 \gamma^\mu + \alpha_3 i\sigma^{\mu\nu} q_\nu] L \quad (1)$$

where

$$\begin{aligned} \alpha_1 &= -\frac{m_b}{2} A_3 + m_b A_4 + A_{11} - \frac{1}{2} A_{12} + A_{13} \\ \alpha_2 &= \frac{m_b^2}{2} A_3 + A_6 + q^2 A_7 + \frac{m_b^2 - q^2}{2} A_8 + \frac{m_b}{2} A_{12} \\ \alpha_3 &= -\frac{m_b}{2} A_3 - A_{11} - \frac{1}{2} A_{12} \end{aligned} \quad (2)$$

and

$$C = \left[ \begin{array}{c} e \\ -g/\cos\theta_W \end{array} \right] \frac{g^2}{(4\pi)^2} V_i$$

the upper(lower) line corresponding to the  $\gamma(Z^0)$  case, and  $L$  stands for the left projection operator. We denote by  $p$  and  $q$  the incoming momenta of the light  $q$

<sup>1</sup>Certainly computer evaluations of the one-loop FCNC are available at present [1], but sometimes may hide a more intuitive approach.

<sup>2</sup>In what matters in this paper the extension to the  $q\bar{q}g$  vertex is straightforward [3].

quark and the neutral gauge boson  $V^0$  respectively (Fig.1.a). For external down-type quarks in the vertex, the Kobayashi-Maskawa coefficient is  $V_i = V_{cb}^* V_{ub}$ . The  $A_n$  coefficients depend on the momenta of the external particles, and on the masses and quantum numbers of the external and internal quarks. Their full expressions will not be reproduced here but are available in ref. [2] and for the photon vertex can also be found up to minor notation differences in ref. [4] (in both, they are presented as Feynman parametric integrals). Instead, we will pick up the relevant terms giving rise to the logarithmic singularity discussed in this paper. Let us remark that the right-handed piece of the analog expression of (1) in ref. [2] has disappeared since we have set  $m_t = 0$ .

In particular, for each intermediate flavor  $i$ ,<sup>3</sup> the  $q\bar{q}g\gamma$  vertex can be written as

$$\Gamma_i^\mu = \frac{1}{(4\pi)^2} e \frac{g^2}{2M_W^2} V_i \bar{q}_h [F_1(q^2)(q^\mu \gamma^\mu - q^\mu \not{A})L - m_b F_2(q^2) i\sigma^{\mu\nu} q_\nu L] q_i \quad (3)$$

The piece containing the  $F_1$  form factor represents an electric (charge-radius) transition and does not contribute for a real photon ( $q^2 = 0$ ). The piece containing the  $F_2$  form factor corresponds to a magnetic transition and contributes in the latter process. In the limit of vanishing external masses and momenta, one recovers the well-known result of Inami-Lin [5] at lowest (second) order in external momenta and masses for the effective  $q\bar{q}g\gamma$  vertex. Introducing  $x_i = m_i^2/M_W^2$ , where  $m_i$  and  $M_W$  stand for the mass of the internal fermion and the mass of the  $W$  gauge boson respectively,  $F_1$  and  $F_2$  are given in the 't Hooft-Feynman gauge by:

$$\begin{aligned} F_1(q^2 = 0) &= Q' \left\{ \left[ -\frac{1}{12(1-x_i)} + \frac{13}{12} \frac{1}{(1-x_i)^2} + \frac{1}{2} \frac{1}{(1-x_i)^3} \right] x_i \right. \\ &\quad \left. + \left[ -\frac{2}{3} \frac{1}{1-x_i} + \left( -\frac{2}{3} \frac{1}{(1-x_i)^2} + \frac{5}{6} \frac{1}{(1-x_i)^3} + \frac{1}{2} \frac{1}{(1-x_i)^4} \right) x_i \right] \ln x_i \right\} \\ &\quad + \left[ \frac{7}{12} \frac{1}{1-x_i} - \frac{13}{12} \frac{1}{(1-x_i)^2} - \frac{1}{2} \frac{1}{(1-x_i)^3} \right] x_i \\ &\quad + \left[ \frac{1}{6} \frac{1}{1-x_i} + \frac{7}{6} \frac{1}{(1-x_i)^2} - \frac{5}{6} \frac{1}{(1-x_i)^3} - \frac{1}{2} \frac{1}{(1-x_i)^4} \right] x_i \ln x_i \end{aligned} \quad (4)$$

$$\begin{aligned} F_2(q^2 = 0) &= -Q' \left\{ \left[ \frac{1}{4} \frac{1}{1-x_i} + \frac{3}{4} \frac{1}{(1-x_i)^2} - \frac{3}{2} \frac{1}{(1-x_i)^3} \right] x_i - \frac{3}{2} \frac{x_i^2}{(1-x_i)^4} \ln x_i \right\} \\ &\quad + \left[ -\frac{1}{2} \frac{1}{1-x_i} + \frac{9}{4} \frac{1}{(1-x_i)^2} - \frac{3}{2} \frac{1}{(1-x_i)^3} - \frac{3}{2} \frac{x_i^3}{(1-x_i)^4} \right] x_i \ln x_i \end{aligned} \quad (5)$$

where  $Q'$  stands for the electric charge of the internal quark running inside the loop.

Attention should be paid to the  $1/(1-x_i) \ln x_i$  term standing in the  $F_1$  form factor, which would diverge logarithmically as  $x_i \rightarrow 0$ , that is, if we let the mass of the internal quark  $i$  tend to zero. As we shall see in the limit:  $M_W^2 \gg |q^2| \gg m_i^2 \simeq 0$ , such term has to be substituted by another one dominated by the piece:  $\ln(-q^2/M_W^2)$  [6] [7] [8].

The mathematical origin of this singularity can be traced to lie in the  $A_4$ ,  $A_6$  and  $A_7$  coefficients, as we shall show below (it is worth to mention that such divergence comes from diagram 1.b).

In the coefficients  $A_4$  and  $A_7$  one deals (among other pieces of regular behavior in the massless internal fermion limit) with parametric Feynman integrals of the type:

$$I(p^2, (p+q)^2, q^2, m_i^2) = \int_0^1 du_1 \int_0^{u_1} dv_2 \frac{f(u_1, v_2)}{\Delta'}$$

with  $f(u_1, v_2) = (v_1 - v_2)(1 + v_2 - u_1)$ , and

$$\Delta' = 1 - u_1 + \frac{m_1^2}{M_W^2} u_1 + \frac{q^2}{M_W^2} (u_2 - u_1)(1 + u_2 - u_1) + \frac{2pq}{M_W^2} (1 - u_1)(u_2 - u_1) \quad (6)$$

where we have taken  $p^2 = m_1^2 = 0$ . Moreover, let us point out henceforth that it is possible to forget about the term " $2pq$ " since it leads, by expanding  $1/\Delta'$ , to a convergent integral (when  $x_i \rightarrow 0$ ) because of the " $(1 - u_1)^n$ " factor (notice that " $q^2 \gamma^\mu - q^\mu \gamma^\nu$ " in (3) already provides the required second-order in external momenta). Therefore, we write

$$\frac{1}{\Delta'} \simeq \frac{1}{1 - (1 - x_i)u_1 + \beta(u_2 - u_1)(1 + u_2 - u_1)} + O(\geq \beta) \quad (7)$$

where we have defined  $\beta = q^2/M_W^2$ .

Let us observe that, in the limit  $x_i \rightarrow 0$ , if we drop the  $\beta$  term in the denominator, we are confronted to a divergent integral. Hence, under this circumstance, one is compelled to keep such term, perform the integral and then take the limit  $x_i \rightarrow 0$ . On the other hand, if there is an overall multiplicative  $x_i$  factor (as in those parametric integrals coming from diagram 1.c) there is no problem of convergence and it is legitimate to drop the  $q^2$ -dependent term in  $\Delta'$ . The same happens if the numerator is defined as  $f(u_1, v_2) = (1 - u_1)(1 + u_2 - u_1)$ . Furthermore, those parametric integrals in the  $A_n$  coefficients involving

$$\Delta = u_1 + x_i(1 - u_1) + \beta(u_2 - u_1)(1 + u_2 - u_1)$$

in the denominator of the integrand instead of  $\Delta'$ , have a regular behaviour even when  $x_i \rightarrow 0$ . Finally, those integrals involving  $\delta = 1 - (1 - x_i)u_1 - \beta u_1(1 - u_1)$  are not problematic because of either a " $(1 - u_1)^n$ " factor in the integrand, or an overall multiplicative  $x_i$  factor which cancels the singularity.

By means of the change of one of the integration variables:  $z = u_1 - u_2$  and reversing the order of integration one gets for such parametric integrals in the  $A_4$  and  $A_7$  coefficients

$$I = \int_0^1 dz \int_z^1 du_1 \frac{z(1-z)}{1 - (1-x_i)u_1 - \beta z(1-z)}$$

After integrating over  $u_1$ , one gets

$$I = \frac{1}{1-x_i} [(K_1 - J_1) - (K_2 - J_2)]$$

where

$$J_n = \int_0^1 dz x^n \ln [x_i - \beta z(1-z)]$$

$$K_n = \int_0^1 dz x^n \ln [1 - (1-x_i)z - \beta z(1-z)]$$

Explicit evaluation of these integrals are given in the Appendix at the end of the paper.

With respect to the divergent integral in  $A_6$  by an analog procedure as before we write

$$\int_0^1 du_1 \int_0^{u_1} dv_2 \ln \Delta'(u_1, v_2) = \int_0^1 dz \int_z^1 du_1 \ln \Delta'(u_1, z)$$

Integrating over  $u_1$  and retaining terms proportional to  $\beta$ , we get

$$\frac{1}{1-x_i} [(K_1 - J_1) - (K_2 - J_2)] \beta$$

as for the  $A_4$  and  $A_7$  coefficients, except the overall  $\beta$  factor which provides the required order in external momenta.

In the low energy regime several possible limits are:

- i)  $x_i \gg |\beta| \rightarrow 0$

In the limit  $\beta \rightarrow 0$  one gets

$$I = -\frac{5}{36} \frac{1}{1-x_i} - \frac{1}{3} \frac{1}{(1-x_i)^2} + \frac{1}{3} \frac{1}{(1-x_i)^3} + \left( -\frac{1}{2} \frac{1}{(1-x_i)^3} + \frac{1}{3} \frac{1}{(1-x_i)^4} \right) \ln x_i \quad (8)$$

The relevant term for small  $x_i$  will be then

$$I \simeq -\frac{1}{6} \ln x_i \quad (9)$$

- ii)  $1 > |\beta| \gg x_i \rightarrow 0$

In the limit  $x_i \rightarrow 0$  we have

$$I = -\frac{1}{3\beta} + \frac{1}{3\beta^2} + \left( \frac{1}{6} - \frac{1}{2\beta^2} + \frac{1}{3\beta^3} \right) \ln(1-\beta) - \frac{1}{6} \ln(-\beta) \quad (10)$$

The relevant term for small  $|\beta|$  will be then

$$I \simeq -\frac{1}{6} \ln(-\beta) \quad (\beta < 0) \quad (11)$$

$$I \simeq -\frac{1}{6} \ln(\beta) + i\frac{\pi}{6} \quad (\beta > 0)$$

Observe that if  $\beta \gg x_i \simeq 0$  the stemming imaginary part in the form factor  $F_1$  comes precisely from diagram 1.b as expected since then  $q^2 > 4m_1^2$  and the internal quarks can go on shell.

- iii)  $1 \gg x_i \simeq \beta$

$$I = \frac{2}{3} \frac{1}{-x_i} + \frac{1}{3x_i^2} + \frac{1}{2x_i} + \left( \frac{1}{4x_i^2} + \frac{1}{6x_i^3} + \frac{1}{6x_i^2} + \frac{1}{6x_i^3} \right) \ln x_i - h(x_i, x_i) + \left( \frac{5}{6x_i^2} - \frac{1}{3x_i^3} \right) H(x_i, x_i) \quad (12)$$

where the  $h$  and  $H$  functions are defined in the Appendix. In the limit of small  $x_i$ , the dominant term will be

$$I \simeq -\frac{1}{6} \ln x_i \quad (13)$$

If the external  $V^0$  is a gluon instead of a  $\gamma$  the effective vertex  $gq_k g$  has a similar structure, but now the form factors are: [3] [9]

$$F_1(q^2 = 0) = \left(-\frac{1}{12} \frac{1}{1-x_i} + \frac{13}{12} \frac{1}{(1-x_i)^2} + \frac{1}{2} \frac{1}{(1-x_i)^3}\right) x_i \\ + \left[-\frac{2}{3} \frac{1}{1-x_i} + \left(-\frac{2}{3} \frac{1}{(1-x_i)^2} + \frac{5}{6} \frac{1}{(1-x_i)^3} + \frac{1}{2} \frac{1}{(1-x_i)^4}\right) x_i\right] \ln x_i;$$

$$F_2(q^2 = 0) = \left(-\frac{1}{4} \frac{1}{1-x_i} - \frac{3}{4} \frac{1}{(1-x_i)^2} + \frac{3}{2} \frac{1}{(1-x_i)^3}\right) x_i + \frac{3}{2} \frac{x_i^2}{(1-x_i)^4} \ln x_i;$$

In the low energy limit the charge-radius form factor  $F_1$  contains a divergent term under the assumption of a massless internal quark which comes from diagram 1.b as well. Hence the "rule" also holds: for small  $|\beta|$  but greater than  $x_i \simeq 0$ , the dominant piece  $-\frac{2}{3} \ln x_i$  in  $F_1$  has to be replaced by  $-\frac{2}{3} \ln(-\beta)$ .

If the external gauge boson is a  $Z^0$ , neglecting all external momenta, only the coefficient  $A_6$  contributes to the effective vertex (since current conservation does not hold now, one is not forced to extract two powers of external momenta as in  $V^0 = \gamma, g$ ). Going to next (second) order, let us focus our attention again in the divergent terms as  $x_i$  tends to zero, which would have once more the origin in diagram 1.b. We pick up in a parallel manner as before the troublesome pieces coming from coefficients  $A_4, A_6$  and  $A_7$ . From (1) and (2) one concludes that they have a common structure  $q^2 \gamma^\mu - q^\mu \cancel{A}$ . For  $|\beta| > x_i \rightarrow 0$  again a term  $\ln(-\beta)$  would appear. In fact in all these cases discussed so far  $q^2$  plays the role of an infrared cut-off.

This discussion can be applied to rare decays of particles induced by the quark-level process:  $q_h \rightarrow q_l^+ t^-$  and particularly to the QCD-induced decays by the transition:  $q_h \rightarrow q_l g^*$ . If  $g^*$  is timelike it leads through the corresponding quark-parton level subprocesses, to:  $q_h \rightarrow q_l q \bar{q}, q_h \rightarrow q_l g g$ . As  $q^2$  ranges from its minimum up to its maximum value, one can make use of the approximations calculated before. This can be considered as midway between the full expressions of the Appendix (leading to a very complicated evaluation when integrated over the available phase space) and the crude approximation of ignoring the  $q^2$ -dependence in the form factors at all. On the other hand, it may happen that  $q^2$  could be more or less fixed as in the exclusive two-body decays. For instance, let us consider the  $B^+ \rightarrow \pi^+ \phi$  decay occurring through a QCD-induced quark-parton penguin transition:  $\bar{b} \rightarrow \bar{d} s \bar{s}$  (the tree-level diagram is absent). Notice that  $q^2$  would be greater than the masses of both up and charm quarks. Hence there should be a (approximate) cancellation between the dominant pieces of the charge-radius form factors  $F_1$  corresponding to the  $u$  and  $c$  contributions (both:  $\ln(-\beta)$ ) because  $V_{ub}^* V_{ud} \simeq -V_{cb}^* V_{cd}$ , leading to a further suppression of this decay mode.

Furthermore, the differential distribution  $(1/\Gamma) d\Gamma/dq^2$  will be similarly suppressed for large  $q^2$  besides the falling of phase space in the decays induced by the  $b \rightarrow d + X$  penguin transitions ( $X = t^+ t^-, g^*$ ).

Finally let us point out that these considerations should be also taken into account in  $e^+ e^-$  physics, for instance looking for FCNC decays  $\gamma(Z^0) \rightarrow b \bar{q} l$  ( $q_l = d, s$ ) below the  $b \bar{b}$  threshold, since  $q^2$  should be greater than the mass of the internal up and charm quarks running inside the loop of the FC vertex.

## Appendix

In the main text we are involved with the following integrals: <sup>3</sup>

$$J_1 = -1 + \frac{1}{2} \ln x_i + h(x_i, \beta)$$

$$J_2 = \frac{1}{3} \left[ -\frac{13}{6} + \frac{2x_i}{\beta} + \ln x_i + 2 \left(1 - \frac{x_i}{\beta}\right) h(x_i, \beta) \right]$$

$$K_1 = -\frac{1}{2} - \frac{y}{2\beta} + \left(\frac{1}{2} + \frac{1}{2\beta} - \frac{y^2}{4\beta^2}\right) \ln x_i + \frac{y}{2\beta^2} H(x_i, \beta)$$

$$K_2 = \frac{1}{3} \left[ -\frac{2}{3} + \frac{4-y}{2\beta} - \frac{y^2}{\beta^2} + \left(1 + \frac{3y}{2\beta^2} - \frac{y^3}{2\beta^3}\right) \ln x_i - \frac{1}{\beta^2} \left(1 - \frac{y^2}{\beta}\right) H(x_i, \beta) \right]$$

where we have defined  $y = 1 + \beta - x_i$  and

$$h(x_i, \beta) = \frac{1}{2} \sqrt{1 - 4x_i/\beta} \ln \frac{\sqrt{1 - 4x_i/\beta} + 1}{\sqrt{1 - 4x_i/\beta} - 1} = \sqrt{1 - 4x_i/\beta} \tanh^{-1} \left( \frac{1}{\sqrt{1 - 4x_i/\beta}} \right)$$

$$H(x_i, \beta) = \frac{1}{2} \sqrt{y^2 - 4\beta} \ln \frac{y-2 + \sqrt{y^2 - 4\beta}}{y-2 - \sqrt{y^2 - 4\beta}} = \sqrt{y^2 - 4\beta} \tanh^{-1} \left( \frac{\sqrt{y^2 - 4\beta}}{y-2} \right)$$

Remark that the origin of the divergence when  $x_i \rightarrow 0$  stays in the  $J_n$  (these integrals also appear in the familiar vacuum polarization in QED), not in the  $K_n$ . Analytic continuation allows to extend the validity of the last expressions even for negative values of the arguments of the root and the logarithm: it was assumed that internal masses had in the propagators a small negative imaginary part, which avoids crossing the respective branch cuts ( $x_i/\beta \rightarrow x_i/\beta - i\epsilon$ ). If  $\beta > 4x_i$  an imaginary part stems from  $J_1$  and  $J_2$ . Conversely, if  $\beta < 4x_i$ :

$$h(x_i, \beta) = \sqrt{4x_i/\beta - 1} \tan^{-1} \left( \frac{1}{\sqrt{4x_i/\beta - 1}} \right)$$

and no imaginary part stems.

Next let us distinguish some limits in the low energy regime

- i)  $x_i > |\beta| \rightarrow 0$

When  $\beta \rightarrow 0$  we get

$$J_1 = \frac{1}{2} \ln x_i ; \quad J_2 = \frac{1}{3} \ln x_i ;$$

$$K_1 = -\frac{1}{4} - \frac{1}{2} \frac{1}{(1-x_i)} + \left( \frac{1}{2} - \frac{1}{2(1-x_i)^2} \right) \ln x_i ;$$

$$K_2 = -\frac{1}{9} - \frac{1}{6(1-x_i)} - \frac{1}{3(1-x_i)^2} + \left( \frac{1}{3} - \frac{1}{3(1-x_i)^3} \right) \ln x_i ;$$

For small values of  $x_i$ ,

$$K_1 = -\frac{3}{4} ; \quad K_2 = -\frac{11}{18}$$

<sup>3</sup>In ref. [8] an equivalent evaluation with a slightly different notation can be found.

- ii)  $1 \gg |\beta| \gg x_i \rightarrow 0$

In the limit  $x_i \rightarrow 0$

$$J_1 = -1 + \frac{1}{2} \ln(-\beta) ; \quad J_2 = -\frac{13}{18} + \frac{1}{3} \ln(-\beta)$$

$$K_1 = -1 - \frac{1}{2\beta} + \left( \frac{1}{2} - \frac{1}{2\beta^2} \right) \ln(1-\beta)$$

$$K_2 = -\frac{13}{18} - \frac{1}{6\beta} - \frac{1}{3\beta^2} + \left( \frac{1}{3} - \frac{1}{3\beta^3} \right) \ln(1-\beta)$$

In the limit of small  $|\beta|$  values

$$K_1 = -\frac{3}{4} ; \quad K_2 = -\frac{11}{18}$$

- iii)  $1 \gg x_i \approx \beta$

$$J_1 = -1 + \frac{1}{2} \ln x_i + h(x_i, x_i) ; \quad J_2 = -\frac{1}{18} + \frac{1}{3} \ln x_i$$

$$K_1 = -\frac{1}{2} - \frac{1}{2x_i} + \left( \frac{1}{2} + \frac{1}{2x_i} - \frac{1}{4x_i^2} \right) \ln x_i + \frac{1}{2x_i^2} H(x_i, x_i)$$

$$K_2 = \frac{1}{3} \left[ -\frac{2}{3} + \frac{1}{2x_i} - \frac{1}{x_i^2} + \left( 1 + \frac{1}{2x_i^2} - \frac{1}{2x_i^3} \right) \ln x_i - \frac{1}{x_i^2} \left( 1 - \frac{1}{x_i} \right) H(x_i, x_i) \right]$$

where

$$h(x_i, x_i) = \frac{\pi}{2\sqrt{3}}$$

$$H(x_i, x_i) = \frac{1}{2} \sqrt{1 - 4x_i} \ln \frac{1 - \sqrt{1 - 4x_i}}{1 + \sqrt{1 - 4x_i}}$$

For small  $x_i$ ,

$$K_1 = -\frac{3}{4} ; \quad K_2 = -\frac{11}{18}$$

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## Figure Captions

Fig.1 :  $qq_A V^0$  vertex ( $q = p' - p$ ) : a) The blob represents 10 one-loop diagrams in the 't Hooft-Feynman gauge for  $V^0 = \gamma, Z^0$ , and 6 one-loop diagrams for  $V^0 = g$  ; b) and c) two of these diagrams involving the gauge boson  $W$  and the charged unphysical Higgs  $\sigma$  respectively;  $\pi_1^m$  labels the internal flavor.