Spectroscopy, leptonic decays and the nature of heavy quarkonia *

Juan-Luis Domenech-Garret[a] and Miguel-Angel Sanchis-Lozano[b] †

[a]Departamento MACS, Física Aplicada. Universitat de Lleida
Alcalde Rovira Roure 191, 25198 Lleida (Spain)
[b]Instituto de Física Corpuscular (IFIC) and Departamento de Física Teórica,
Centro Mixto Universitat de València-CSIC
Dr. Moliner 50, E-46100 Burjassot, Valencia (Spain)

We examine the electronic width ratios of Υ resonances below the $B\bar{B}$ threshold by means of an effective (Cornell-type) QCD potential incorporating $1/m_b$ corrections obtained from a prior fit to the bottomonium spectrum. From our analysis we conclude that the $\Upsilon(2S)$ and $\Upsilon(3S)$ states should belong to the strong-coupling (nonperturbative) regime while the $\Upsilon(1S)$ state should belong to the weak-coupling (perturbative) regime, in agreement with a previous study based on radiative decays.

1. Introduction

Heavy quarkonium has historically played a role of utmost importance in the rise of the Standard Model (SM), notably regarding the quark model of hadrons and the development of QCD as the presently accepted theory to describe the strong interaction among them.

Furthermore, a large amount of data have been collected during the last decade at BEPC, B-factories, CESR, HERA, and Tevatron experiments, greatly improving the accuracy of the measured production cross sections, decay widths and branching fractions involving heavy quarkonia (see [1] for a review). In the future, a Super Flavour Factory could provide further experimental results on heavy quarkonia to an unprecedented accuracy [2,3].

On the other hand, such precise measurements are matched by the ever-growing soundness of the theoretical background, firmly based on an effective field theory, namely the Non-Relativistic QCD (NRQCD). The maturity already reached in the field even makes feasible the search for new physics, e.g. in quarkonium decays, looking for experimental deviations from the SM expectations. Let us mention the seek of light dark matter in invisible quarkonium decays [4] (followed up by experimental searches [5,6]) and Υ radiative decays into dileptons as a way of searching for a light non-standard Higgs boson [7,8,9,10,11].

With the advent of the quark model and QCD, hadronic properties have been traditionally understood with the help of (more or less QCD-motivated) potential models, some of them having reached a fairly acceptable level in predicting or postdicting level spacings, transitions rates, etc. Nevertheless, potential models have several setbacks and limitations, mainly due to the fact that they do not come directly from first principles. At this stage, effective theories enter the game in order to describe rigorously the hadron dynamics.

A prototype is the Heavy Quark Effective Theory (HQET) which naturally describes hadrons with a single heavy quark [12]. These systems are characterized by two energy scales: the heavy quark mass, m_Q, and the characteristic scale of the strong interaction Λ_{QCD}. HQET is obtained by integrating out the scale m_Q and expanding the QCD Lagrangian in powers of Λ_{QCD}.

On the other hand, bound states made of two heavy quarks are characterized by more scales whose relevance and hierarchy are usually estimated by invoking the so-called velocity counting rules [13]. Since the heavy quark relative velocity v is typically small ($v^2 \sim 0.3$ for charmonium and $v^2 \sim 0.1$ for bottomonium) the different scales obey the useful relation $E_n \sim m_Q v^2 << p \sim m_Q v << m_Q$, where E_n is the heavy-quark bound-state energy with n the principal quantum number. NRQCD is obtained by integrating out the heavy quark mass m_Q [14]. High-energy modes are not lost but encoded into short-distance coefficients and new local interaction terms in the effective Lagrangian.

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†Email:Miguel.Angel.Sanchis@uv.es

The resulting framework allows the separation between the short-distance scale of the process under study from the longer distance scales associated with quarkonium structure. Therefore operators can be expanded as a double series (perturbative and nonperturbative), with α_s and v being the expansion parameters controlling the accuracy of the truncated series, respectively. However, these expansion parameters are not completely independent in heavy quarkonium physics as the typical velocity of the heavy quark is determined by a balance between the kinetic energy $m_Q v^2$ and the potential energy which should be dominated by a Coulombic-term $\sim \alpha_s/r$. Setting $r \sim 1/m_Q v$, by invoking the virial theorem we are led to the well known relation

$$\alpha_s(m_Q v) \sim v$$

Moreover, since $\alpha_s(\mu)$ runs (decreasing) with the energy scale (for higher μ), one can approximately write

$$\alpha_s(m_b) \sim v^2$$

where m_b denotes the bottom quark mass. The above relation should play an important role in assessing the velocity counting rules to be applied in quarkonium physics. In fact, relativistic corrections of order v^2 would be of the same order as perturbative corrections of order $\alpha_s(m_b)$. Let us also remark that the α_s perturbative expansions in NRQCD may be not fast convergent series and truncation even at NNL will likely imply sizeable effects [15]. In this work, we will implicitly take the α_s corrections into account through the velocity counting rules.

2. Potential model connection to pNRQCD

A naive connection of NRQCD with potential models can be made by realizing that certain (long-distance, colour-singlet) matrix elements of NRQCD can be actually related in first approximation to wave functions at the origin (WFO) or their derivatives. Nevertheless, this simple picture does not hold too far as there are NRQCD matrix elements without an equivalence in potential models, namely, colour-octet contributions since the heavy quark antiquark pair needs not to be in a colour-singlet state. Actually, the potential picture that emerges from NRQCD [16] is quite different from the traditional one [18,19] and superior.

The observation that NRQCD still contains energy scales irrelevant for the lower-lying states of quarkonium, led to further simplifications and the resulting theory was called potential NRQCD (pNRQCD) [20,21], when only ultrasoft degrees of freedom remain dynamical. Such an effective field theory turns out to be, in fact, close to a Schrödinger-like description of the bound state [22,23]. Moreover, matrix elements in pNRQCD can be expressed as the product of WFOs and nonperturbative glue-dependent factors, yielding a formal similarity with potential models in many observable quantities (like decay widths into leptons or light hadrons [25], or magnetic dipole transition [24]), which do not happen in NRQCD.

The relation between Λ_{QCD} and the scales $m_Q v$ and $m_Q v^2$ dictates the degrees of freedom of pNRQCD:

- The *weak coupling regime*, when $m_Q v^2 \gtrsim \Lambda_{QCD} > m_Q v$ and the binding energy is mainly due to a Coulombic-like potential. Dynamics can be described using perturbative theory.

- The *strong coupling regime*, when $m_Q v \sim \Lambda_{QCD}$ and the binding energy is mainly due to a confining (nonperturbative) potential.

The assignment of each quarkonium state to any of these regimes is not such an easy task as the different scales are not directly measurable. The fact that the spectrum of excitations of the bottomonium family is not Coulombic suggests that the higher states are not in the weak coupling regime. However, there are claims in the literature that the $\Upsilon(2S)$ and $\Upsilon(3S)$ can also be understood within the weak coupling regime [26,27]. In Ref.[28] $\Upsilon(1S, 2S, 3S)$ radiative decays were used to investigate their nature. Experimental results from CLEO were confronted with the theoretical expectations, in particular as a ratio of decay widths for different energies of the photon.

In this Letter we follow a similar strategy but focusing on electronic decays of the $\Upsilon(1S, 2S, 3S)$ resonances mainly basing our analysis on a phenomenological approach. We have employed the QQ-onia package presented in Ref.[29] to determine the required WFOs, making use of a Cornell-type potential incorporating a $-c'/r^2$ term as suggested by recent lattice studies. Table 1 shows the experimental values of the electronic partial widths for all three resonances.

Table 1
Measured electronic widths (in keV) for the $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ resonances (from [30]).

$\Gamma_{ee}[\Upsilon(1S)]$	$\Gamma_{ee}[\Upsilon(2S)]$	$\Gamma_{ee}[\Upsilon(3S)]$
1.340 ± 0.018	0.612 ± 0.011	0.443 ± 0.008

3. Velocity counting rules

In this section we briefly review different velocity scaling rules appearing in both perturbative and nonperturbative regimes of heavy quarkonia. All our arguments are order-of-magnitude estimates which, moreover, are subject to uncertainties especially in the nonperturbative regime due to our ignorance of the scaling rules expected to be valid then.

Notice that, no matter the regime, it always holds the following velocity counting rules

$$\langle nS|\frac{p^2}{m_b}|nS\rangle \ \sim \ m_b v^2 \ ; \ \langle nS|V^{(0)}|nS\rangle \ \sim \ m_b v^2$$

by the definition of v and from the virial theorem, respectively. The latter is an example where the naive guess of $m_b v$ is modified by the dynamics of the bound state to $m_b v^2$.

3.1. Perturbative regime

The soft scale in heavy quarkonium is basically set by its size r which can be provided by the Bohr radius of the bound state. If r is small enough, i.e. $r \lesssim 1/\Lambda_{QCD}$, the soft scale should be perturbative and the potentials can be entirely determined in perturbation theory.

Then the following scaling rules *should* hold:

$$\langle nS|V^{(1)}|nS\rangle \ \sim \alpha_s^2/r^2 \sim m_b^2 v^4$$

since $\alpha_s \sim v$ and $r \sim 1/(m_b v)$, and

$$\langle nS|V^{(2)}|nS\rangle \ \sim \ \alpha_s/r^3 \ \sim m_b^3 v^4$$

and so on.

Let us stress that perturbation theory cannot incorporate quark confinement, so it becomes crucial to determine the potential nonperturbatively in this regime. This can be the case for high-lying quarkonium resonances, as we are checking in this work. Indeed, states below the $B\bar{B}$ threshold and not too deep (namely $\Upsilon(2S)$ and $\Upsilon(3S)$) are expected to be in the strong-coupling regime whereas the deeper $\Upsilon(1S)$ state is expected to be in the weak coupling regime. States above (or very close to) open bottom threshold are not expected to be in either regime [31].

3.2. Nonperturbative regime

Admittedly, the power counting of NRQCD is not well known in the nonperturbative regime and, in fact, has been addressed by different authors in distinct ways [25,32,33]. We might assume a very conservative counting: $m_b v^d$ with d standing for the operator dimension[3]. Thus the following counting rules *could* hold:

$$\langle nS|V^{(1)}|nS\rangle \ \sim \ m_b^2 v^2 \ ; \ \langle nS|V^{(2)}|nS\rangle \ \sim \ m_b^3 v^3$$

On the other hand, making use of the $1/m_b$ expansion of the potential, one may write

$$V(r) = V^{(0)}(r) + \frac{V^{(1)}(r)}{m_b} + \frac{V^{(2)}(r)}{m_b^2} \ + \ \cdots \qquad (1)$$

where $V^{(k)}$, $k = 0, 1, 2, \ldots$ are the leading and subleading terms respectively. The potential $V^{(2)}/m_b^2$ contains the leading-order spin-dependent potentials and the velocity-dependent potential. In this work we truncate the expansion up to $V^{(1)}/m$, neglecting $1/m_b^2$ terms and higher. In a nonperturbative regime this approximation should amount to a $\mathcal{O}(v^2)$ accuracy at least when solving the Schrödinger equation, thereby justifying the use of a non-relativistic approach.

Let us also point out that lattice calculations will be of help in determining the functional form of $V^{(1)}/m_b$. We will come back to this important point in section 5.1.

Finally, notice that the static potential $V^{(0)}(r)$ is well parametrized by a Coulomb plus linear term (i.e. a Cornell-type functional form [35,36]),

$$V^{(0)}(r) = -\frac{c}{r} + \sigma r + \mu$$

where σ stands for the string tension governing the confining potential and μ is a constant. This funnel shape will be used throughout this work as a reference, later on to be somewhat modified when defining the actual leading-order potential for heavy quarkonium.

[3] This is somewhat similar to HQET [34] where any operator counts like Λ_{QCD}^d

4. Υ leptonic decays

As is well known, leptonic partial widths are a probe of the compactness of the quarkonium system, and provide useful information complementary to spectroscopy [17]. In particular, the electronic width for $^3S_1$ states ($\Gamma[\Upsilon(nS) \to e^+e^-]$) probes the WFO according to potential models.

Likewise, if the pNRQCD framework is applied, the leptonic width can still be written in terms of the radial WFO as mentioned in the Introduction. The following expression obtained in Ref.[25] should hold up to order $v^3 \times (E_n/m_b, \Lambda^2_{QCD}/m_b^2)$:

$$\Gamma[\Upsilon(nS) \to e^+e^-] \;=\; \frac{N_C}{\pi}\;\frac{|R_n(0)|^2}{m_b^2}\;\times$$

$$\left[\;\mathrm{Im}f_{ee}(^3S_1)\;\left(1-\frac{E_n^{(0)}}{m_b}\frac{2\epsilon_3}{9}+\frac{2\epsilon_3^{(2,EM)}}{3m_b^2}+\frac{c_F^2B_1}{3m_b^2}\right)\right.$$

$$\left.+\;\mathrm{Im}g_{ee}(^3S_1)\;\left(\frac{E_n^{(0)}}{m_b}-\frac{\epsilon_1}{m_b^2}\right)\;\right] \qquad (2)$$

where $N_C = 3$ is a colour factor and $E_n^{(0)}$is the (leading-order) bound-state energy; ϵ and B stand for universal (i.e. flavour and state independent) nonperturbative parameters, which can be expressed in terms of gluonic field-strength correlators [25]. They can be determined either by experimental data or by lattice simulation, but still their numerical values are quite uncertain.

The matching coefficients $\mathrm{Im}f_{ee}(^3S_1)$ and $\mathrm{Im}g_{ee}(^3S_1)$, corresponing to the $O_1(^3S_1)$ and $P_1(^3S_1)$ operators of the NRQCD Lagrangian, are given at order α_s by the expressions [14,37]

$$\mathrm{Im}f_{ee}(^3S_1) = \frac{1}{3}\pi Q^2\alpha^2\left[1-\frac{16\alpha_s}{3\pi}+\mathcal{O}(\alpha_s^2)\right] \qquad (3)$$

$$\mathrm{Im}g_{ee}(^3S_1) = -\frac{4}{9}\pi Q^2\alpha^2\left[1-\frac{8\alpha_s}{3\pi}+\mathcal{O}(\alpha_s^2)\right] \qquad (4)$$

where $|Q| = 1/3$ for the bottom quark.

At lowest order, one recovers from (2) the well known formula [38] expressed in our notation as

$$\Gamma[\Upsilon(nS) \to e^+e^-] \;=\; \frac{N_C\;\mathrm{Im}f_{ee}(^3S_1)}{\pi}\;\frac{|R_n^{(0)}(0)|^2}{m_b^2}$$

showing that the leptonic width of a quarkonium vector state is primarily sensitive to the square of its radial WFO, though perturbative and nonperturbative corrections are large indeed. Yet the unknown parameters in Eq.(2) do not allow its direct comparison with experiment.

5. Testing the nature of heavy quarkonium

In the leptonic width ratio of two S-wave states, however, several terms cancel out, leading to

$$\frac{\Gamma[\Upsilon(nS) \to e^+e^-]}{\Gamma[\Upsilon(rS) \to e^+e^-]} = \frac{|R_n(0)|^2}{|R_r(0)|^2} \times [1+\delta_{nr}] \qquad (5)$$

up to corrections of order $\mathcal{O}(v^q)$, where q will be later determined in our analysis, providing an insight on the nature of heavy quarkonium and a hint at the velocity scaling rules to be applied.

The correcting factor δ_{nr} is given by

$$\delta_{nr} = \left(\frac{\mathrm{Im}g_{ee}(^3S_1)}{\mathrm{Im}f_{ee}(^3S_1)} - \frac{2\;\epsilon_3(2m_b)}{9}\right) \times \left[\frac{E_n^{(0)}-E_r^{(0)}}{m_b}\right]$$

and using Eqs.(3-4) we get

$$\delta_{nr} = \left(r_{ee} + \frac{2\;\epsilon_3(2m_b)}{9}\right) \times \left[\frac{E_r^{(0)}-E_n^{(0)}}{m_b}\right] \qquad (6)$$

where we have defined r_{ee} up to $\mathcal{O}(\alpha_s^2)$ corrections,

$$r_{ee} = \frac{4}{3} \times \left[1+\frac{8\alpha_s(2m_b)}{3\pi}\right]$$

Now, taking into account that $E_n = M_n - 2m_b$ with M_n being the meson mass [39], we can safely use the following relation

$$E_r^{(0)} - E_n^{(0)} \simeq E_r - E_n = M_r - M_n$$

Hence Eq.(6) can be rewritten as

$$\delta_{nr} = \left(r_{ee} + \frac{2\;\epsilon_3(2m_b)}{9}\right) \times \left[\frac{M_r-M_n}{m_b}\right] \qquad (7)$$

On the other hand, $\epsilon_3(\mu)$ stands as the only nonperturbative gluonic parameter in (6), all others cancelling out in the ratio (5) at the desired accuracy. There is an experimental determination of this long-distance parameter [4] in [40]:

$$\epsilon_3(1\text{ GeV}) = 1.8^{+1.2}_{-0.7}$$

where the error bars are experimental only. Additional theoretical uncertainties associated to subleading operators in the power counting and perturbative expansion are not taken into account in the above uncertainty.

Finally, we need to know ϵ_3 at the bottomonium scale. To this aim we can use the scale evolution

$$\epsilon_3(\mu') = \epsilon_3(\mu) + \frac{24C_F}{\beta_0}\ln\frac{\alpha_s(\mu')}{\alpha_s(\mu)} \qquad (8)$$

where $C_F = 4/3$, $\beta_0 = 11C_A/3 - 4n_fT_F/3$ with $C_A = 3$, $T_F = 1/2$ and $n_f = 5$ in our case.

Setting $\epsilon_3(1\text{ GeV}) = 1.8$, yields $\epsilon_3(2m_b) \simeq 4.2$ which can be then used as an input in Eq.(7).

[4] Let us note a factor $N_C = 3$ of difference between the definitions of the gluonic parameter ϵ in [40] and ϵ_3 in [25].

Table 2
Values of the predicted and experimental mass (in GeV), WFO squared (or derivative) in GeV$^{3+2\ell}$, mean square radius (in fm) and typical quark velocity for the $\Upsilon(1S, 2S, 3S, 4S)$, $\chi_b(1P, 2P)$ and $\Upsilon(1D)$ states when the improved Cornell-type (Corn-mod) potential of Eq.(10) is employed. For comparison, we present in the first column the masses obtained using a Cornell (Corn) potential [36].

Resonance	Mass (Corn)	Mass (Corn-mod)	Exp.	$\lvert R^{\ell'}_{n\ell}(0)\rvert^2$	$\langle r^2\rangle^{1/2}$	$\langle v^2\rangle$
$\Upsilon(1S)$	9.4603	9.4603	9.4603	12.65	0.23	0.090
$\chi_b(1P)$	9.96	9.8929	9.9001	1.409	0.40	0.071
$\Upsilon(2S)$	10.05	10.0236	10.0233	6.444	0.51	0.087
$\Upsilon(1D)$	10.20	10.1476	10.1622	0.562	0.53	0.078
$\chi_b(2P)$	10.31	10.2729	10.2600	1.854	0.63	0.089
$\Upsilon(3S)$	10.40	10.3750	10.3552	5.404	0.71	0.103
$\Upsilon(4S)$	10.67	10.6477	10.5794	5.194	0.88	0.120

5.1. Lattice estimates of $V^{(1)}/m_b$ shape and size

As commented previously, the $V^{(1)}/m_b$ potential has the form $1/r^2$ relying on perturbation theory in the short-distance region. However, since the binding energy of the $b-\bar{b}$ system, typically of order $m_b v^2$, can be similar or even smaller than Λ_{QCD} due to the non-relativistic nature of the system, it is essential to determine the potential nonperturbatively. Monte Carlo simulations of lattice QCD provide a powerful tool for a nonperturbative determination of the potential [41,42].

In a former analysis presented in Ref.[41], $V^{(1)}(r)/m_b$ was found to be comparable with the Coulombic term of the static potential (i.e. $V^{(1)}/m_b \sim 1/r$) when applied to bottomonium states up to $r = 0.6$ fm. Consequently, if $V^{(1)}(r)$ is nonperturbative, the piece $V^{(1)}(r)/m$ in the potential should not be considered as subleading with respect to $V^{(0)}(r)$.

However, in a later study involving further long distance data up to $r = 0.9$ fm [42], the same authors found that the $1/r$ function was not supported by the fit, while the functional form $1/r^2$ with the linear term could fit the data well. It is interesting to note that $V^{(1)}/m_b$ turns out to have the same functional form as expected from perturbative theory.

The $V^{(1)}/m$ term cannot be neglected as compared to the static potential $V^{(0)}$ and has to be incorporated into the quarkonium potential for the sake of coherence. Thus the leading-order potential $V^{\rm LO}$ should read

$$V^{\rm LO} = V^{(0)} + \frac{V^{(1)}}{m_b} \tag{9}$$

yielding a potential of the form

$$V_{Corn-mod}(r) = -\frac{c}{r} - \frac{c'}{r^2} + \sigma r + \mu \tag{10}$$

We will refer to (10) as a *Cornell-modified* potential, since the functional form has been improved by the additional $-c'/r^2$ piece; besides, the contribution from the $V^{(1)}/m$ term also alters the value of σ [42].

In our approach the values of the parameters m_b, c, c', σ and μ are obtained through a fitting procedure to the bottomonium spectrum ($\Upsilon(1S)$ and $\Upsilon(2S)$ states), not from lattice estimates. We obtain from the fit the following values for the parameters of the potential (10):

$$\sigma = 0.217 \text{ GeV}^2,\ c = 0.400,\ c' = 0.010 \text{ GeV}^{-1}$$

and $m_b = 4.7$ GeV [5].

The values of the predicted masses, WFOs and other properties of interest for different bottomonium states using this potential are shown in Table 2. An excellent agreement with the experimental mass values of different resonances in the spectrum can be observed.

5.2. Discussion

The WFOs corresponding to the Cornell-modified potential were obtained using our code based on a Numerov technique (See Ref.[29] for a thorough description of the QQ-onia package.). A comparison of the WFOs of the $\Upsilon(1S, 2S, 3S)$ states obtained using different potentials can be found in Table 3.

[5] A significant coincidence is found between the m_b value used in the lattice calculation [42] and required in our fit.

Table 3
$|R_n^{\rm LO}(0)|^2$ (in GeV^3) values of $\Upsilon(ns)$ states ($n=1,2,3$) for several potentials: Buchmuller-Tye ($m_b=4.88$ GeV) [18]; Cornell [18] ($m_b=5.18$ GeV); Cornell-modified (including a $-c'/r^2$ piece, $m_b=4.7$ GeV).

Resonance	$\|R_n^{\rm LO}(0)\|^2_{B-T}$	$\|R_n^{\rm LO}(0)\|^2_{Corn}$	$\|R_n^{\rm LO}(0)\|^2_{Corn-mod}$
$\Upsilon(1S)$	6.477	14.07	12.65
$\Upsilon(2S)$	3.234	5.669	6.444
$\Upsilon(3S)$	2.474	4.271	5.404

Let us remark that the inclusion of the $-c'/r^2$ term in the Cornell potential has a non-straightforward effect on the new resulting WFOs, for it implies a modification of both the Coulombic term and the b-quark mass obtained from the fit. Let us also note that the c' value is determined to a large extent by the $\Upsilon(2S)$ resonance in our fitting method (see [29] for more details).

On the other hand, as already pointed out in section 5.1, the perturbative calculation of $V^{(1)}/m_b$ yields the same functional r-dependence as suggested by lattice studies. Therefore, one may look upon $V^{(1)}/m_b$ as an interpolating term between the perturbative and non-perturbative regimes, rendering the fit meaningful even using both $\Upsilon(1S,2S)$ states. As a check of the fitting procedure (see Table 2), the predicted meson masses for the bottomonium family obtained from the Cornell-modified potential improve with respect to the Cornell potential [36], when compared with the experimental spectrum [30].

Moreover, the values of the WFOs shown in Table 3 look self-consistent: the (absolute) relative variations for the Cornell-modified versus the Cornell potential are $\sim$ 10%, 14% and 26% for the $\Upsilon(1S,2S,3S)$ resonances, respectively. Indeed, one could naively expect a $\mathcal{O}(v^2)$ effect in the perturbative regime as a consequence of incorporating the new term into the potential, but increasingly larger variations for higher (thus dominantly non-perturbative) states.

Finally, note that the $V^{(2)}/m_b^2$ (and higher) terms neglected in the expansion (1) of the QCD potential should likely provide corrections of relative order $\mathcal{O}(v^2)$ to the WFOs obtained by solving the Schrödinger equation with the leading-order potential (10). This counting is supported by the spin-dependent splitting experimentally found in the bottomonium spectrum. In fact, if the static potential were exact, then the potential model would reproduce QCD up to corrections of relative order v^2 [22].

6. Numerical results

Now we proceed to check the validity of formula (5) by recasting it onto the following double (experimental to theoretical) ratio:

$$\frac{\Gamma[\Upsilon(nS)\to ee]/\Gamma[\Upsilon(rS)\to ee]}{(|R_n^{\rm LO}(0)|^2/|R_r^{\rm LO}(0)|^2)\times[1+\delta_{nr}]}=1+\Delta_{nr} \quad (11)$$

where we have introduced the dimensionless quantity Δ_{nr} ($n,r=1,2,3,\ n\neq r$) parametrizing the deviation from unity for different combinations of all three $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ states.

The following experimental and theoretical inputs have been employed in our analysis:

1) The experimental input for the electronic widths can be readily obtained from Table 1, allowing a determination of the ratios with relative error $\sim 1\%$

$$\frac{\Gamma[\Upsilon(nS)\to e^+e^-]}{\Gamma[\Upsilon(rS)\to e^+e^-]},\quad (n,r=1,2,3\ \ n\neq r)$$

2) The $|R_n^{\rm LO}(0)|^2$ values for different potentials can be found in Table 3. We have assumed in (11) that

$$\frac{|R_n(0)|^2}{|R_r(0)|^2}\ =\ \frac{|R_n^{\rm LO}(0)|^2}{|R_r^{\rm LO}(0)|^2}\ \times\ [1+\mathcal{O}(v^q)]$$

where q is expected to be 2 on account of the arguments given in the previous section.

3) δ_{nr} were computed according to Eq.(7) (valid up to order (v^2,α_s^2) corrections). Experimental meson masses and m_b values for each potential were used (see Table 3); we set $\epsilon_3(2m_b)=4.2$ derived from Eq.(8) using $\epsilon_3(1\ {\rm GeV})=1.8$ [40].

Thus, if the two quarkonium states n and r were in the same (strong) regime, one should expect

$$\Delta_{nr}\ \lesssim\ \mathcal{O}(v^q)\ \sim\ 100\cdot v^q(\%) \quad (12)$$

Table 4
Δ_{nr} (in %) for different potentials from Table 3 using $\epsilon_3(2m_b) = 4.2$. Let us observe that Δ_{23} always remains smaller than Δ_{12} and Δ_{13}, as expected if both states $\Upsilon(2S, 3S)$ were in the same (strong-coupling) regime.

Potential	B-T	Cornell	Cornell-modified
Δ_{13}	20%	36%	12%
Δ_{12}	15%	30%	14%
Δ_{23}	9%	10%	1%

Conversely, one should expect $\Delta_{nr} > 100 \cdot v^q(\%)$, if anyone of the states is in the strong-coupling regime and the other in the weak-coupling regime.

The values for Δ_{nr} obtained in our analysis for the Buchmuller-Tye, Cornell and Cornell-modified potentials can be found in Table 4, representing our main result.

In particular, it turns out that when the Cornell potential is employed we find that Δ_{13} is greater than Δ_{12}, which in turn is larger than Δ_{23}, i.e.

$$\Delta_{13} \simeq 35\% \gtrsim \Delta_{12} \simeq 30\% > \Delta_{23} \simeq 10\%$$

in agreement with a counting rule providing $q = 2$ for the latter case.

In sum, our results for the Cornell potential are thus consistent with the expected level of accuracy (up to order v^2) of Eq.(11), provided that both $\Upsilon(2S)$ and $\Upsilon(3S)$ states belong to the strong-coupling regime, while the $\Upsilon(1S)$ state does not.

Furthermore, once the $-c'/r^2$ piece is included in the Cornell-modified potential we get

$$\Delta_{13} \simeq \ \Delta_{12} \simeq 10\% > \Delta_{23} \simeq 1\%$$

where Δ_{23} is now found to be remarkably small.

Of course, there is an uncertainty coming from the ϵ_3 value in the computation of δ_{nr} according to Eq.(7). Setting $\epsilon_3(2m_b)$ equal to zero, we find that Δ_{23} becomes appreciably worse. Thereby a non-null value of $\epsilon_3(2m_b)$ is clearly favoured in our analysis. Demanding $\Delta_{23} = 0$ in Eq.(11) we get $\epsilon_3(2m_b) = 3.2$ from Eq.(7) and, consequently, $\epsilon_3(1\ \text{GeV}) = 0.8$ using the running equation (8).

7. Conclusions

In this Letter we have presented a phenomenological study of the electronic width ratios of Υ resonances (below open bottom production), finding evidence favouring both $\Upsilon(2S)$ and $\Upsilon(3S)$ states in the strong coupling regime, at the same time disfavouring the $\Upsilon(1S)$ in it, in accordance with the conclusions from the analysis of Ref.[28] based on radiative decays of Υ resonances.

Moreover, the agreement between the experimental and predicted $\Upsilon(2S)/\Upsilon(3S)$ ratios is even better than naively expected from a conservative velocity counting once the Cornell potential becomes improved by a $-c'/r^2$ piece, motivated by recent lattice studies [41,42]. Therefore we can conclude that our results (both from lepton widths and spectroscopy) favour the inclusion of such a nonperturbative term into the Cornell potential.

Let us also point out that a value of the gluonic nonperturbative parameter: $\epsilon_3(1\ \text{GeV}) \simeq 1.8^{+1.2}_{-0.7}$ (as found in [40]) is compatible within errors with our analysis on leptonic decays. Actually, one might turn the question round extracting $\epsilon_3(2m_b)$ from Eq.(11) using the experimental data from Table 1 and the Cornell-modified potential, yielding $\epsilon_3(2m_b) = 3.2$ ($\epsilon_3(1\ \text{GeV}) = 0.8$) with an estimated uncertainty of $\sim 30\%$ assuming a $v^2 \sim 10\%$ accuracy in Eq.(7). Additional theoretical uncertainties should increase the allowed range though values of $\epsilon_3(1\ \text{GeV})$ around unity are preferred as a general result of our analysis.

Finally, we want to stress the relevance of further accurate experimental measurements of leptonic widths (among other properties) of heavy quarkonia to carry out precise tests of effective theories of QCD (likely useful to deal with nonperturbative effects showing up at the LHC) and even direct searches for new physics [9,10]. A future Super Flavour Factory would play an invaluable role in this regard.

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