Proton Stability, Dark Matter and Light Color Octet Scalars in Adjoint SU(5) Unification

Pavel Fileviez Pérez
University of Wisconsin-Madison, Department of Physics
1150 University Avenue, Madison, WI 53706, U.S.A.[*]

Hoernisa Iminniyaz
Physics Department, University of Xinjiang, 830046 Urumqi, P.R. China.[†]

Germán Rodrigo
Instituto de Física Corpuscular, CSIC-Universitat de València,
Apdo. de Correos 22085, E-46071 Valencia, Spain.[‡]

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The unification of gauge interactions in the context of Adjoint SU(5) and its phenomenological consequences are investigated. We show the allowed mass spectrum of the theory which is compatible with proton decay, and discuss the possibility to have a cold dark matter candidate. Due to the upper bounds on the proton decay partial lifetimes, $\tau(p \to K^+ \bar{\nu}) \leq 9.3 \times 10^{36}$ *years and* $\tau(p \to \pi^+ \bar{\nu}) \leq 3.0 \times 10^{35}$ *years, the theory could be tested at future proton decay experiments. The theory predicts also light scalar color octets which could be produced at the Large Hadron Collider.*

I. INTRODUCTION

The possibility to have the unification of the gauge interactions has been a guiding principle for physics beyond the Standard Model (SM) since the paper by Georgi, Quinn and Weinberg in 1974 [1]. The simplest grand unified theory (GUT) was proposed the same year in a seminal paper [2]. As it is well known, this theory is based on the $SU(5)$ gauge symmetry and the SM matter of one family is unified in two different representations. Unfortunately, this simple GUT theory is ruled out by the present values of the coupling constants. At the same time, in the original version of the theory [2] there are no massive neutrinos and a relation between the Yukawa couplings for the charged leptons and down quarks, $Y_E = Y_D^T$, is predicted, which is in disagreement with the experiments.

[*]Electronic address: `fileviez@physics.wisc.edu`
[†]Electronic address: `hoernisa@th.physik.uni-bonn.de`
[‡]Electronic address: `german.rodrigo@ific.uv.es`

One of the main predictions of GUT theories is the decay of the lightest baryon [3]. Thanks to the great effort of many experimental collaborations one has today very impressive lower bounds on the different partial decay lifetimes [4]. In the future, the searches for proton decay will continue and hopefully it will be finally discovered. See for example [5] for a review of future experiments. From the theoretical point of view it is very difficult to make strong predictions for the lifetime of the proton since in most of the theories the grand unified scale needs to be predicted with great precision, the nucleon-meson matrix elements are not well-known, or the supersymmetric character of the theory might led to a large model dependence. See [6] for a review of proton decay predictions in several theoretical frameworks. Since proton decay is a smoking gun signature for GUT theories one should try to make the best predictions in the simplest scenarios.

As we have explained above the simplest, but not realistic, GUT theory was proposed by Georgi and Glashow in Ref. [2]. If one sticks to renormalizability there are, however, three possible ways out to construct realistic theories:

- *Renormalizable $SU(5)$ with Type I seesaw*: As it has been known for a long time in order to have a consistent model for charged fermion masses at the renormalizable level one has to introduce an extra Higgs in the $\mathbf{45}$ representation [7]. At the same time, at least two fermionic singlets are needed to generate neutrino masses through the Type I seesaw mechanism [8]. This scenario has been studied in detail in [9] and [10]. From the results presented in [10] it is possible to conclude that once one imposes the most conservative bound on the mass of the Higgses mediating proton decay this theory is ruled out by proton decay experiments.

- *Renormalizable $SU(5)$ with Type II seesaw*: In this scenario the Higgs sector is composed of $\mathbf{5}_H$, $\mathbf{15}_H$, $\mathbf{24}_H$ and $\mathbf{45}_H$, and the neutrino masses are generated through the Type II seesaw mechanism [11]. One can conclude already that this model is not appealing since the Higgs sector is very complicated. See [10] for a recent discussion of this scenario.

- *Renormalizable Adjoint $SU(5)$*: This is the simplest realistic renormalizable theory based on $SU(5)$ [12]. In this scenario the Higgs sector is composed of $\mathbf{5}_H$, $\mathbf{24}_H$ and $\mathbf{45}_H$ and an extra fermionic representation in the $\mathbf{24}$ adjoint representation is introduced in order to generate the neutrino masses through the Type I [8] and Type III [13] seesaw mechanisms. In this theory, one has a new realization of the Type III seesaw mechanism in the context of a renormalizable GUT theory where only two Higgses, $\mathbf{5}_H$ and $\mathbf{45}_H$, generate all fermion masses. For the supersymmetric version of the theory see [14].

Since the theory proposed in Ref. [12] is the simplest renormalizable $SU(5)$ theory, we study its possible phenomenological predictions in detail. We study for the first time the properties of the full Lagrangian of the theory, computing all fermion masses. We find that there is only one possible scenario for the masses of the fermions in the adjoint representation which is allowed by unification and proton decay. We notice that the neutral component of the real scalar triplet living in the adjoint representation could be a possible candidate for the Cold Dark Matter (CDM) in the Universe. Using the proton decay and dark matter constraints we find that the possible solutions for the mass spectrum in the theory allowed by unification are reduced considerably. Using these results we predict upper bounds on the partial proton decay lifetimes, that can be tested at future proton decay experiments. The possibility to have light exotic fields, like scalar color octets, is discussed.

The outline of the paper is as follows: In Section II we discuss the main properties of *Adjoint* $SU(5)$ and its main predictions. The predictions coming from the unification of gauge interactions are discussed in Section III. The dark matter constraints and the upper bounds on the proton decay partial lifetimes are discussed in Sections IV and V, respectively. The possible light exotic fields are pointed out in Section VI, while in Section VII we summarize our results. In the Appendix A we write down all details of the theory, the matter content and all interactions. Finally, in Appendix B we list all contributions of the fields present in the theory to the running of gauge couplings.

II. ADJOINT $SU(5)$

A new renormalizable GUT theory based on the $SU(5)$ symmetry where the neutrino masses are generated through the Type I and Type III seesaw mechanisms has been recently proposed in Ref. [12]. In this section we discuss the main properties of this theory in order to understand its main phenomenological predictions.

- Matter Unification: As in any theory based on $SU(5)$, the SM matter is unified in the $\mathbf{\overline{5}} = l_L \bigoplus (d^C)_L$ and $\mathbf{10} = (u^C)_L \bigoplus q_L \bigoplus (e^C)_L$ representations. In order to generate the neutrino masses through the Type I [8] and Type III [13] seesaw mechanisms an extra matter field in the adjoint representation $\mathbf{24} = (\rho_8)_L \bigoplus (\rho_3)_L \bigoplus (\rho_{(3,2)})_L \bigoplus (\rho_{(\bar{3},2)})_L \bigoplus (\rho_0)_L$ is introduced. Notice that since the extra matter is in the adjoint representation it does not induce anomalies, making the model very simple and appealing. Here, it is important to mention that the field ρ_8 must be heavier than $10^6 - 10^7$ GeV in order to satisfy the constraints coming from Big Bang Nucleosynthesis, once one assumes that the unification scale is larger than 3×10^{15} GeV, which is consistent with the experimental lower bounds on the proton decay lifetime.

- Higgs Sector: The Higgs sector is composed of the representations $\mathbf{5}_H = H_1 \bigoplus T$, $\mathbf{24}_H = \Sigma_8 \bigoplus \Sigma_3 \bigoplus \Sigma_{(3,2)} \bigoplus \Sigma_{(\overline{3},2)} \bigoplus \Sigma_{24}$, and $\mathbf{45}_H = \Phi_1 \bigoplus \Phi_2 \bigoplus \Phi_3 \bigoplus \Phi_4 \bigoplus \Phi_5 \bigoplus \Phi_6 \bigoplus H_2$. As it is well-known, this is the minimal Higgs sector of any renormalizable model compatible with the spectrum of the charged fermions. The main difference with respect to other theories is that once the $\mathbf{24}$ representation is introduced the $\mathbf{45}_H$ field plays a crucial role to generate neutrino masses. See Appendix A for all interactions in the Higgs sector.

- SM Fermion Masses: In this theory all the fermion masses are generated at the renormalizable level. The relevant Yukawa interactions are given by

$$-S_{\text{Yukawa}} = \int d^4x \left(Y_1 \, 10 \, \bar{5} \, 5_H^* \, + \, Y_2 10 \, \bar{5} \, 45_H^* \, + \, Y_3 \, 10 \, 10 \, 5_H \, + \, Y_4 \, 10 \, 10 \, 45_H \right) \, + \\ + \int d^4x \left(c \, \bar{5} \, 24 \, 5_H \, + \, p \, \bar{5} \, 24 \, 45_H \right) \, + \, \text{h.c.} \tag{1}$$

The masses of the SM charged fermions read

$$M_D = Y_1 \frac{v_5^*}{\sqrt{2}} + 2 \, Y_2 \frac{v_{45}^*}{\sqrt{2}} , \tag{2}$$

$$M_E = Y_1^T \frac{v_5^*}{\sqrt{2}} - 6 \, Y_2^T \frac{v_{45}^*}{\sqrt{2}} , \tag{3}$$

$$M_U = 4 \left(Y_3 + Y_3^T \right) \frac{v_5}{\sqrt{2}} - 8 \, \left(Y_4 - Y_4^T \right) \frac{v_{45}}{\sqrt{2}} , \tag{4}$$

with v_5 and v_{45} being the vacuum expectation values of $\mathbf{5}_H$ and $\mathbf{45}_H$, respectively. See Appendix A for details. The neutrino mass matrix is

$$M_{ij}^{\nu} = \frac{a_i a_j}{M_{\rho_3}} + \frac{b_i b_j}{M_{\rho_0}} , \tag{5}$$

where

$$a_i = \frac{1}{2\sqrt{2}} \left(c_i v_5 - 3 p_i v_{45} \right) \qquad \text{and} \qquad b_i = \frac{\sqrt{15}}{2\sqrt{2}} \left(\frac{c_i v_5}{5} + p_i v_{45} \right) . \tag{6}$$

One of the neutrinos is massless. The additional interactions

$$S_{24} = - \int d^4x \left(M \, \text{Tr} \, 24^2 \, + \, \lambda \, \text{Tr} \left(24^2 \, 24_H \right) \right) \, + \, \text{h.c.} \tag{7}$$

give mass to the fermions living in the adjoint representation, and in particular to ρ_0 and ρ_3, the fields responsible for the seesaw in Eq. (5). Their masses are given by

$$M_{\rho_0} = \left| m - e^{i\alpha} \Lambda \right| , \qquad M_{\rho_3} = \left| m - 3 \, e^{i\alpha} \Lambda \right| , \\ M_{\rho_8} = \left| m + 2 \, e^{i\alpha} \Lambda \right| , \qquad \text{and} \qquad M_{\rho_{(3,2)}} = M_{\rho_{(\bar{3},2)}} = \left| m - \frac{1}{2} \, e^{i\alpha} \Lambda \right| . \tag{8}$$

Here we have used the relations $M_V = v\sqrt{5\pi\alpha_{GUT}/3}$ and $\Lambda = \tilde{\lambda}\, M_{GUT}/\sqrt{\alpha_{GUT}}$ with $\tilde{\lambda} = |\lambda|/\sqrt{50\pi}$, and have chosen the mass of the superheavy gauge boson M_V as the unification scale. The phase α is the relative phase between M and the coupling λ, while $m = |M|$.

- Proton Decay: There are several fields contributing to proton decay. The dimension six gauge contributions are mediated by the superheavy gauge bosons $V \sim (\mathbf{3}, \mathbf{2}, -5/6) \bigoplus (\overline{\mathbf{3}}, \mathbf{2}, 5/6)$, which must be heavier than 3×10^{15} GeV in order to satisfy the experimental lower bound on the proton decay lifetime. The $SU(3)$ triplets $T = (\mathbf{3}, \mathbf{1}, -1/3)$, $\Phi_3 = (\mathbf{3}, \mathbf{3}, -1/3)$, $\Phi_5 = (\mathbf{3}, \mathbf{1}, -1/3)$ and $\Phi_6 = (\overline{\mathbf{3}}, \mathbf{1}, 4/3)$ mediate the dimension six Higgs contributions. The most conservative lower bound on their masses from proton decay is $M_T, M_{\Phi_3}, M_{\Phi_5}, M_{\Phi_6} > 10^{12}$ GeV. These bounds are very important in order to understand the possible solutions for the spectrum which are allowed by unification and proton decay.

III. UNIFICATION OF GAUGE INTERACTIONS

In this section we investigate the main phenomenological predictions due to the unification of gauge couplings. The contribution of all the fields to the running of the gauge couplings are listed in Table I of Appendix B. The fields ρ_3, Σ_3 and Φ_3 can favor unification because they have negative (positive) contribution to $b_1 - b_2 (b_2 - b_3)$. Here, b_i stands for the different beta functions. The field Φ_1 plays also an important role since it has a negative contribution to $b_1 - b_2$, and helps to increase the GUT scale such that it might become compatible with proton decay. In order to simplify our analysis we set all the fields listed in Table I that do not help to achieve unification at the GUT scale, and keep only Σ_3, Φ_1, Φ_3 and the fermionic fields in the $\mathbf{24}$ representation below. Then, we study the possibility to achieve unification in agreement with experiment, and evaluate the maximal allowed GUT scale.

The relevant equations for the running of the gauge couplings at one-loop level are:

$$\alpha_1^{-1}(M_Z) = \alpha_{GUT}^{-1} + \frac{1}{2\pi}\left(b_1^{SM} \ln \frac{M_{GUT}}{M_Z} + \frac{1}{5} \ln \frac{M_{GUT}}{M_{\Phi_3}} + \frac{4}{5} \ln \frac{M_{GUT}}{M_{\Phi_1}} + \frac{10}{3} \ln \frac{M_{GUT}}{M_{\rho_{(3,2)}}} \right),$$

$$\alpha_2^{-1}(M_Z) = \alpha_{GUT}^{-1} + \\ + \frac{1}{2\pi}\left(b_2^{SM} \ln \frac{M_{GUT}}{M_Z} + \frac{4}{3} \ln \frac{M_{GUT}}{M_{\rho_3}} + \frac{1}{3} \ln \frac{M_{GUT}}{M_{\Sigma_3}} + 2 \ln \frac{M_{GUT}}{M_{\Phi_3}} + \frac{4}{3} \ln \frac{M_{GUT}}{M_{\Phi_1}} + 2 \ln \frac{M_{GUT}}{M_{\rho_{(3,2)}}} \right),$$

and

$$\alpha_3^{-1}(M_Z) \; = \; \alpha_{GUT}^{-1} + \frac{1}{2\pi}\left(b_3^{SM} \ln \frac{M_{GUT}}{M_Z} + \frac{1}{2} \ln \frac{M_{GUT}}{M_{\Phi_3}} + 2 \ln \frac{M_{GUT}}{M_{\Phi_1}} + \frac{4}{3} \ln \frac{M_{GUT}}{M_{\rho_{(3,2)}}} + 2 \ln \frac{M_{GUT}}{M_{\rho_8}} \right), \tag{9}$$

where $b_1^{SM} = 41/10$, $b_2^{SM} = -19/6$ and $b_3^{SM} = -7$ are the SM beta functions. The masses of the fields living in the $\mathbf{24}$ representation are linked by Eq. (8). These relations constrain the possible solutions for unification. Indeed, due to Eq. (8) and because we are dealing with a complete representation their contributions to the running of the gauge couplings do not depend on the absolute value of their masses, but only on the mass splitting between them. For different values of m, λ and α we can achieve different benchmark scenarios.

Benchmark I: Consider the general case with $\alpha = 0$ or $\alpha = 2\pi$. If $m > 3\,\Lambda$ the lightest field in $\mathbf{24}$ is ρ_3 and ρ_8 is the heaviest. As explained before, the unification will be determined only by the splitting between the ρ_8 and ρ_3 masses, and not by their absolute value, provided that M_{ρ_8} is not larger than M_{GUT} and M_{ρ_3} is above M_Z. The masses of the fields in the $\mathbf{24}$ multiplet can be written in terms of one single parameter $\hat{m} = M_{\rho_8}/M_{\rho_3}$:

$$M_{\rho_0} = \frac{1}{5}(3 + 2\hat{m})\, M_{\rho_3}\ , \qquad M_{\rho_8} = \hat{m}\, M_{\rho_3}\ , \qquad M_{\rho_{(3,2)}} = M_{\rho_{(\bar{3},2)}} = \frac{1}{2}(1 + \hat{m})\, M_{\rho_3}\ . \tag{10}$$

For $\hat{m} = 1$, or equivalently $\lambda = 0$, all masses in the ρ multiplet are equal. For $\hat{m} \gg 1$ the masses of ρ_0, ρ_8 and $\rho_{(3,2)}$ are of the same order, and there is a gap between them and M_{ρ_3}. The solution of the RGEs (Eq. (9)) is given by

$$\begin{aligned} M_{GUT} &= M_Z \left(\frac{M_{\Sigma_3}\, M_Z^{19}}{M_{\Phi_1}^{20}} \right)^{1/159} \hat{m}^{-12/53} \left(\frac{(1+\hat{m})}{2} \right)^{32/159} \exp\left[\frac{6\pi}{159} \left(5\alpha_1^{-1} + \alpha_2^{-1} - 6\alpha_3^{-1} \right)(M_Z) \right] \\ &= \frac{M_Z^7}{M_{\Sigma_3} M_{\Phi_3}^5} \left(\frac{\hat{m}^2(1+\hat{m})}{2} \right)^{4/3} \exp\left[\frac{2\pi}{3} \left(5\alpha_1^{-1} - 9\alpha_2^{-1} + 4\alpha_3^{-1} \right)(M_Z) \right] \\ &= M_Z \left(\frac{M_Z^5}{M_{\Phi_1}^4\, M_{\Phi_3}} \right)^{1/32} \left(\frac{1+\hat{m}}{2\hat{m}} \right)^{5/24} \exp\left[\frac{5\pi}{24} \left(\alpha_1^{-1} - \alpha_3^{-1} \right)(M_Z) \right] . \end{aligned} \tag{11}$$

The available parameter space is shown in Fig. 1 and Fig. 2. In Fig. 1 we have set Φ_1 at the unification scale, while $M_{\Phi_1} = 200$ GeV in Fig. 2. We can appreciate that it is possible to achieve unification if both Φ_3 and Σ_3 are set at the unification scale when the splitting of masses is about 7 to 10 orders of magnitude depending on M_{Φ_1}. The possible solutions shown in Fig. 1 are, however, not consistent with proton decay ($M_{GUT} > 3 \times 10^{15}$ GeV) since the unification scale is too small. This result constrains Φ_1 to be below the GUT scale. Notice that for $M_{\Phi_1} = 200$ GeV (Fig. 2) the unification scale is larger and consistent with proton decay. Proton decay constrains also the mass of Φ_3: $M_{\Phi_3} > 10^{12}$ GeV. In Fig. 3 we have set M_{Φ_3} to 10^{12} GeV in order to show the dependence of the unification scale in terms of M_{Φ_1}. The maximal grand unified scale is then $M_{GUT}^{max} = 7.8 \times 10^{15}$ GeV. For larger values of M_{Φ_3} the maximal unification scale will be smaller, and the parameter space will be more constrained. Fig. 3 also tells us that in order to be consistent with $M_{GUT} > 3 \times 10^{15}$ GeV the mass of Φ_1 has to be $M_{\Phi_1} < 4.4 \times 10^5$ GeV.

For completeness, we also show in Fig. 4 the parameter space for $\hat{m} = 1$ ($\lambda = 0$). This scenario is equivalent to the case of renormalizable Non-SUSY SU(5) without the contributions of the fermions in the $\mathbf{24}$ representation [10]. All the parameter space is however excluded because it requires Φ_3 to be too light to be consistent with the constraints discussed before.

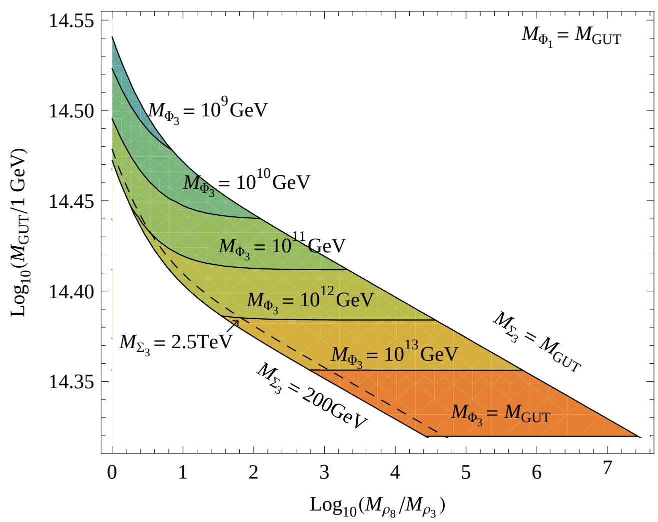

FIG. 1: Parameter space allowed by unification at the one-loop level in Adjoint $SU(5)$ for the Benchmark I scenario when $M_{\Phi_1} = M_{GUT}$.

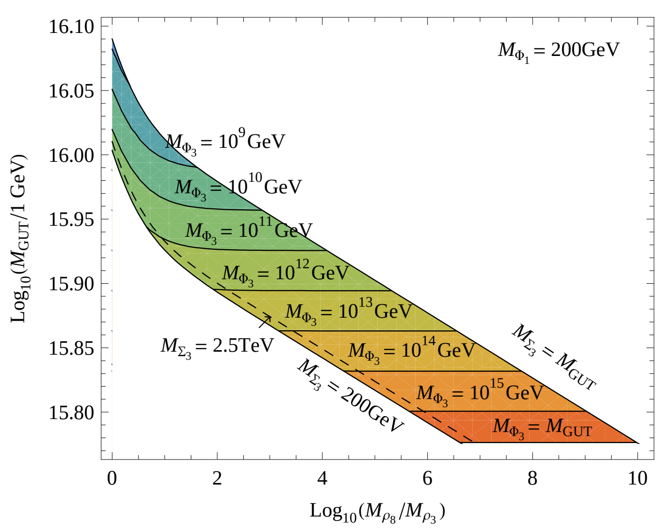

FIG. 2: Parameter space allowed by unification at one-loop level in Adjoint $SU(5)$ for the Benchmark I scenario when $M_{\Phi_1} = 200$ GeV.

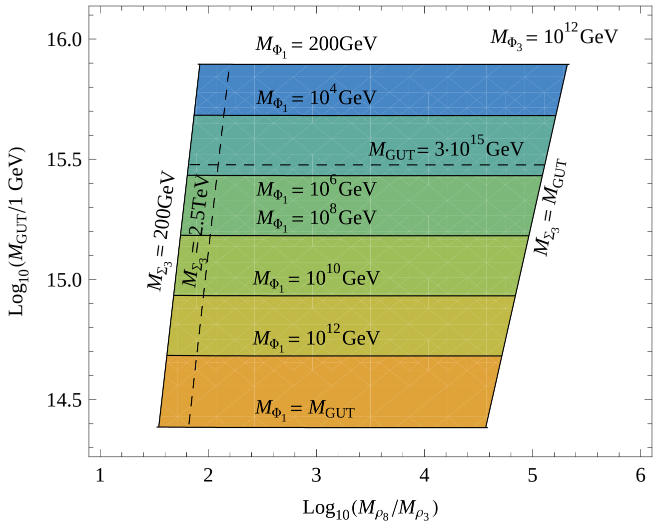

FIG. 3: Parameter space allowed by unification at one-loop level in Adjoint $SU(5)$ for the Benchmark I scenario when $M_{\Phi_3} = 10^{12}$ GeV.

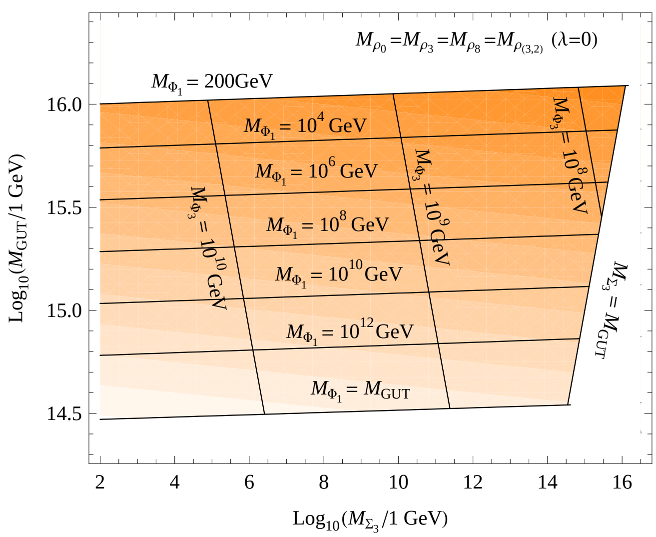

FIG. 4: Parameter space allowed by unification at one-loop level in Adjoint $SU(5)$ when $\lambda = 0$.

Benchmark II: When $m = 0$ the lightest fields in $\mathbf{24}$ are $\rho_{(3,2)}$ and $\rho_{(\bar{3},2)}$, and the following relationship is fulfilled:

$$\frac{M_{\rho_0}}{2} = \frac{M_{\rho_3}}{6} = \frac{M_{\rho_8}}{4} = M_{\rho_{(3,2)}} \ . \tag{12}$$

This scenario is illustrated in Fig. 5. The ρ masses in this scenario are all different but of the same order of magnitude. The unification parameter space is therefore quite similar to the parameter space of Fig. 4.

Since M_{Φ_3} is always below 10^{12} GeV, this scenario is ruled out by proton decay.

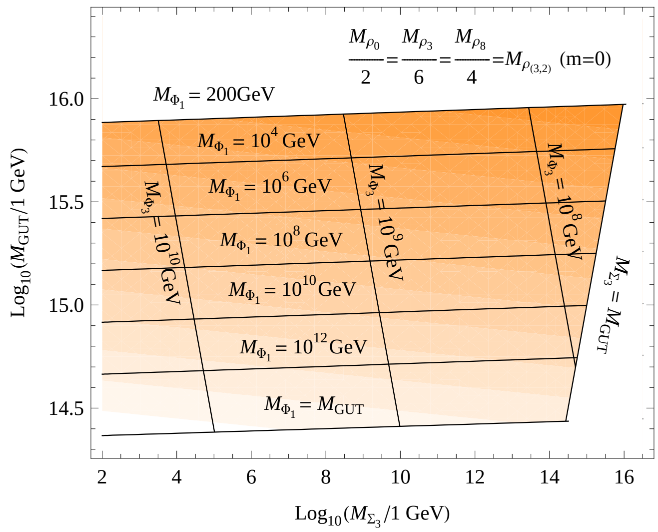

FIG. 5: Parameter space allowed by unification at one-loop level in Adjoint $SU(5)$ when $m = 0$.

Benchmark III: In the scenario $\alpha = \pi/2$ or $\alpha = 3\pi/2$, the lightest fields in $\mathbf{24}$ are $\rho_{(3,2)}$ and $\rho_{(\bar{3},2)}$, while ρ_3 is the heaviest one. The masses are given by:

$$M_{\rho_0} = \sqrt{m^2 + \Lambda^2}\,, \qquad M_{\rho_3} = \sqrt{m^2 + 9\,\Lambda^2}\,,$$
$$M_{\rho_8} = \sqrt{m^2 + 4\,\Lambda^2}\,, \qquad M_{\rho_{(3,2)}} = M_{\rho_{(\bar{3},2)}} = \sqrt{m^2 + \frac{\Lambda^2}{4}}\,, \tag{13}$$

and can be written in terms of the ratio $r_{32} = M_{\rho_3}/M_{\rho_{(3,2)}}$:

$$M_{\rho_0} = \sqrt{\frac{32 + 3\,r_{32}^2}{35}}\,M_{\rho_{(3,2)}}\,, \qquad M_{\rho_3} = r_{32}\,M_{\rho_{(3,2)}}\,, \qquad M_{\rho_8} = \sqrt{\frac{4 + 3\,r_{32}^2}{7}}\,M_{\rho_{(3,2)}}\,. \tag{14}$$

For $r_{32} = 1$ this scenario is equivalent to the case $\lambda = 0$, and for $r_{32} = 6$ to $m = 0$. For a given value of r_{32} in the range $[1, 6]$ the parameter space compatible with unification at one-loop level will interpolate between Fig. 4 and Fig. 5. Therefore, this scenario is ruled out by proton decay.

Benchmark IV: If $\alpha = \pi$ the heaviest field in $\mathbf{24}$ is ρ_3 and ρ_8 is the lightest (for $m > 2\,\Lambda$). Eq. (10) and Eq. (11) are valid provided that $\hat{m} = M_{\rho_8}/M_{\rho_3}$ is smaller than one. In Fig. 6 we show the allowed parameter space for two values of M_{Φ_1}. The parameter space is however rather insensitive to M_{Φ_1}. If we impose proton decay this scenario is also excluded.

Let us comment here about the benchmark I scenario with $0 < m < 3\,\Lambda$. In this region of the parameter space ρ_3 is not necessary the lightest field, but all the ρ masses are of the same order, excluding the points $m \simeq \Lambda$ and $m \simeq \Lambda/2$ where either ρ_0 or $\rho_{(3,2)}$ might be light, respectively. These scenarios are, however,

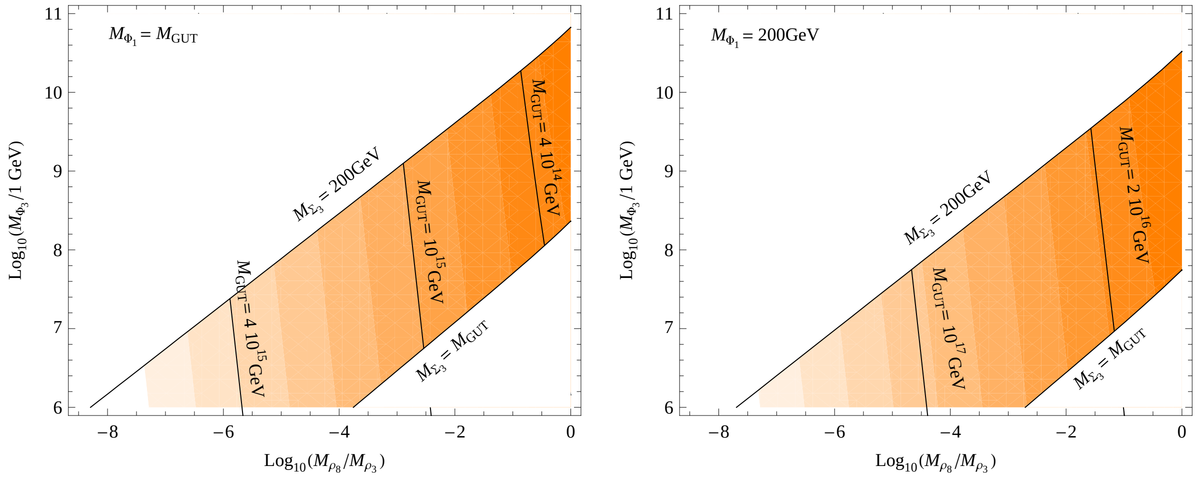

FIG. 6: Parameter space allowed by unification at one-loop level in Adjoint $SU(5)$ in the Benchmark scenario IV.

excluded because they would require Φ_3 to be too light. Similar arguments can be employed to exclude the benchmark IV scenario with $0 < m < 2\,\Lambda$.

In summary, we have studied the parameter space allowed by unification in different scenarios. Once the most conservative bounds from the proton stability are imposed the only scenario which is consistent with the experiments is the Benchmark scenario I where the maximal GUT scale is $M_{GUT}^{max} = 7.8 \times 10^{15}$ GeV for $M_{\Phi_1} = 200$ GeV, and the upper bound on the mass of the scalar color octet is $M_{\Phi_1} < 4.4 \times 10^5$ GeV. These results are crucial in order to understand the possibility to test the theory through proton decay at future experiments.

IV. COLD DARK MATTER CONSTRAINTS

In supersymmetric theories the lightest supersymmetric particle, for example the neutralino, is a natural candidate for the CDM in the Universe once the so-called R-parity is imposed as a symmetry of the theory. In non-supersymmetric theories the neutral component of $\Sigma_3 \sim (\mathbf{1}, \mathbf{3}, 0)$ is also a possible candidate for the CDM, and as we have shown in the previous section it can be light in Adjoint $SU(5)$. In order to understand this issue one can imagine a minimal extension of the SM, where the SM Higgs sector is extended by adding a real triplet Σ_3. The matrix representation of Σ_3 is given by:

$$\Sigma_3 = \frac{1}{2} \begin{pmatrix} \Sigma^0 & \sqrt{2}\Sigma^+ \\ \sqrt{2}\Sigma^- & -\Sigma^0 \end{pmatrix} . \tag{15}$$

The neutral component of Σ_3 can be a CDM candidate if the coupling $H^\dagger \Sigma_3 H$ is not present and its vacuum expectation value is zero. This can be achieved by imposing the symmetry $\Sigma_3 \to -\Sigma_3$. This CDM candidate has been studied in [15]. As it has been explained by the authors in [15] the charged $\Sigma^\pm$ and Σ^0 components have the same mass at tree level. Once radiative corrections are included, however, a mass splitting $\Delta m_\Sigma = 166$ MeV is generated, and the charged component decays mainly through $\Sigma^+ \to \Sigma^0 \pi^+$.

In our renormalizable GUT theory the interaction $H_1^\dagger \Sigma_3 H_1$ can be eliminated at tree level by imposing the following condition:

$$\beta_6 = \frac{3}{5}\beta_8 \sqrt{\frac{2}{\pi \alpha_{GUT}}} M_{GUT} \; , \tag{16}$$

where β_6 and β_8 are, respectively, the couplings of the $5_H^\dagger 24_H 5_H$ and $5_H^\dagger 24_H^2 5_H$ interactions. A similar fine-tuning can be made in order to set the couplings $H_2^\dagger \Sigma_3 H_2$ and $H_2^\dagger \Sigma_3 H_1$ to zero. See Appendix A for all relevant scalar interactions. In this GUT theory, however, the symmetry $\Sigma_3 \to -\Sigma_3$ cannot be realized without embedding in $\mathbf{24}_H \to -\mathbf{24}_H$ at the GUT level, which prevents to achieve unification in agreement with the experiment. In this case all the fields in the fermionic **24** representation would have the same mass and would not contribute to the running of the gauge couplings at one-loop. Also the field Σ_3 would not be neither light in this case. The fine-tuning in Eq. (16) can thus not be made stable under radiative corrections, and the suppression of the undesired interactions is possible only order by order. Although, this fact makes less appealing the idea of Σ^0 as a CDM candidate, in our opinion, it deserves some attention because the fine tuning is always possible.

It has been pointed out in [15] that if the mass of the neutral component of the real triplet is $M_{\Sigma_3} \approx 2.5$ TeV the thermal relic abundance is equal to the observed Dark Matter (DM) abundance $\Omega_{DM} h^2 = 0.110 \pm 0.005$. Therefore, if we stick to the possibility of having Σ^0 as a CDM candidate one could say that only the region of the parameter space consistent with unification shown in the previous section where $M_{\Sigma_3} \approx 2.5$ TeV is allowed by the Dark Matter constraints. We show this case in Fig. 7.

V. UPPER BOUNDS ON THE PARTIAL PROTON DECAY LIFETIMES

As we have mentioned in the previous sections there are five fields in this theory that mediate proton decay. These are the superheavy gauge bosons $V \sim (\mathbf{3}, \mathbf{2}, -5/6) \bigoplus (\overline{\mathbf{3}}, \mathbf{2}, 5/6)$, and the $SU(3)$ triplets T, Φ_3, Φ_5 and Φ_6. The least model dependent and usually the dominant proton decay contribution in non-supersymmetric scenarios comes from gauge boson mediation. It is important to understand the possibility to test this theory at future proton decay experiments [5]. Assuming that the Yukawa matrix for up quarks

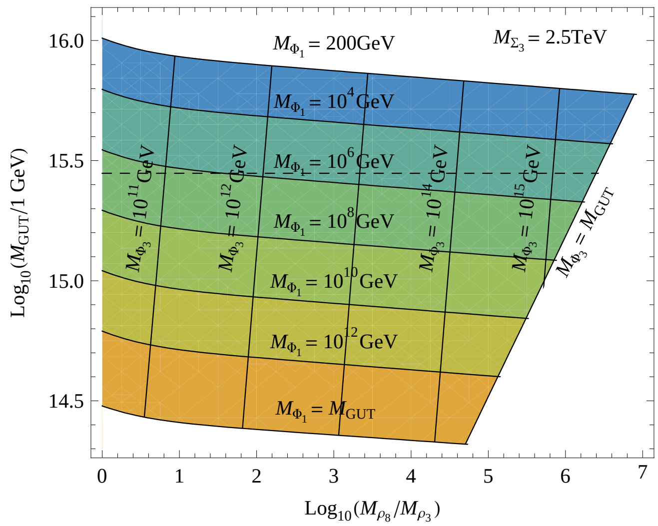

FIG. 7: Parameter space allowed by unification at one-loop level in Adjoint $SU(5)$ for the Benchmark I scenario when $M_{\Sigma_3} = 2.5$ TeV. Dashed line at $M_{GUT} = 3 \times 10^{15}$ GeV.

is symmetric, $Y_U = Y_U^T$, the golden channels to test the theory are [16]:

$$\Gamma(p \to \pi^+ \bar{\nu}) = \sum_{i=1}^{3} \Gamma(p \to \pi^+ \bar{\nu}_i) = \frac{\pi}{2} \frac{\alpha_{GUT}^2}{M_{GUT}^4} \frac{m_p}{f_\pi^2} \, |V_{ud}|^2 \, A_r^2 \, |\alpha|^2 \, (1 + D + F)^2 \ , \tag{17}$$

and

$$\Gamma(p \to K^+ \bar{\nu}) = \sum_{i=1}^{3} \Gamma(p \to K^+ \bar{\nu}_i) = \frac{\pi}{2} \frac{\alpha_{GUT}^2}{M_{GUT}^4} \frac{(m_p^2 - m_K^2)^2}{m_p^3 \, f_\pi^2} \, A_r^2 \, |\alpha|^2 \left(A_1^2 \, |V_{ud}|^2 + A_2^2 \, |V_{us}|^2 \right) \ , \tag{18}$$

where

$$A_1 = \frac{2 m_p}{3 m_B} D, \qquad \text{and} \qquad A_2 = 1 + \frac{m_p}{3 m_B} (D + 3F) \ . \tag{19}$$

In the above expressions D and F are the parameters of the Chiral Lagrangian [6, 17] and $m_B = 1.150$ GeV $\approx m_\Sigma \approx m_\Lambda$ is the averaged Baryon mass. The values $F = 0.463$ and $D = 0.804$ [18] are used as input values for our numerical analysis, α is the matrix element of the three quark states between the proton and the vacuum state: $\langle 0 | \epsilon_{abc} \, \epsilon_{\alpha\beta} \, u_{aR}^\alpha \, d_{bR}^\beta \, u_L^\gamma | p \rangle = \alpha \, u_L^\gamma$ (we use $\alpha = 0.015$ GeV^3 [19] calculated at the scale 2.3 GeV), and $A_r = A_L A_S$ is the renormalization factor. See Ref. [6] for more details. The long-range renormalization factor is given by

$$A_L = \left(\frac{\alpha_3(m_b)}{\alpha_3(M_Z)} \right)^{6/23} \left(\frac{\alpha_3(Q)}{\alpha_3(m_b)} \right)^{6/25} \approx 1.25 \qquad \text{using} \qquad Q = 2.3 \text{ GeV}, \tag{20}$$

while the short distance renormalization factor reads as

$$A_S = \left(\frac{\alpha_3(M_{GUT})}{\alpha_3(M_Z)} \right)^{2/b_3} . \tag{21}$$

Using the maximal allowed value for the GUT scale $M_{GUT}^{max} = 7.8 \times 10^{15}$ GeV, which corresponds to set $M_{\Phi_3} = 10^{12}$ GeV and $M_{\Phi_1} = 200$ GeV (This value is almost independent of $\hat{m}$ and M_{Σ_3}.), we find the following upper bounds on the proton decay partial lifetimes

$$\tau\left(p \to K^+ \bar{\nu}\right) \ \leq \ 9.3 \times 10^{36} \text{ years} \qquad \text{and} \qquad \tau\left(p \to \pi^+ \bar{\nu}\right) \leq 3.0 \times 10^{35} \text{ years}, \tag{22}$$

where $\alpha_{GUT}^{-1} = 33.4$, which corresponds to set $M_{\rho_8} = M_{GUT}$. The present experimental lower bounds are $\tau^{exp}\left(p \to K^+ \bar{\nu}\right) > 2.3 \times 10^{33}$ years [20] and $\tau^{exp}\left(p \to \pi^+ \bar{\nu}\right) > 2.5 \times 10^{31}$ years [4], respectively. Future experiments [5] will improve these bounds by two or more orders of magnitude. We expect therefore that this theory could be tested at the next proton decay experiments. It is important to say that if one neglects the mixing between quarks and leptons one gets similar predictions for all proton decay channels. Particularly, one gets $\tau(p \to e^+ \pi^0) \leq 1.2 \times 10^{35}$ years.

VI. LIGHT EXOTIC FIELDS: COLORED SCALAR OCTETS

The phenomenological aspects of an extension of the SM where the Higgs sector is composed of the SM Higgs and a scalar color octet have been studied recently in Ref. [21]. The authors in [21] have noticed that the color octet has Yukawa couplings to quarks with natural flavour conservation. After the electroweak symmetry breaking there are four physical Higgses in the adjoint of $SU(3)$, S_R^0 (Real component of S^0), S_I^0 (Imaginary component of S^0), and $S^{\pm}$. As expected, the splitting between their masses is of order M_W. The different decays of the octets, the pair and singlet production mechanisms at the LHC have been studied in detail in [21]. For other studies see also [22]. The presence of these light exotic fields will also modify the Higgs production at the LHC and its decays [23].

The Adjoint $SU(5)$ GUT theory predicts the existence of these light exotic fields in a natural way; this is the field $\Phi_1 \sim (\mathbf{8}, \mathbf{2}, 1/2)$. As we have discussed in the previous sections by imposing all relevant constraints from the unification of gauge interactions and proton decay we have found that the upper bound on the mass of this color octet is $M_{\Phi_1} < 4.4 \times 10^5$ GeV. The lightness of Φ_1 is needed for achieving a GUT scale large enough to be in agreement with proton decay. Following the notation of [21]:

$$\Phi_1 = \begin{pmatrix} S^+ \\ S^0 \end{pmatrix} = S^a T^a, \tag{23}$$

where $a = 1, .., 8$ and T^a are the $SU(3)$ generators. This leads to very exciting phenomenological implications and the possibility to test the theory at the LHC. The properties of the light colored octets in this context will be studied in detail in a future publication.

VII. SUMMARY AND OUTLOOK

We have investigated the unification of gauge interactions in the context of Adjoint SU(5). In this simple GUT theory the neutrino masses are generated through the Type I and Type III seesaw mechanisms. We have found the following phenomenological predictions:

- Among all the possible mass spectra of the theory, there is only one scenario for the relation between the masses of the fermionic fields in the adjoint representation which is consistent with unification of gauge couplings and the conservative bounds coming from proton decay. In the allowed parameter space the lightest field is ρ_3. This result is crucial to understand the Baryogenesis via Leptogenesis mechanism in this context.

- In this theory, we have identified the neutral component of the real scalar triplet, $\Sigma_3 \sim (\mathbf{1}, \mathbf{3}, 0)$, as a possible CDM candidate. This possibility requires, however, a delicate fine-tuning to suppress the interaction of Σ_3 to other color singlet Higgses, which cannot be protected by a discrete symmetry because Σ_3, in the GUT theory, is embedded in a larger multiplet.

- Since the predicted upper bounds on the proton decay partial lifetimes are $\tau(p \to K^+ \bar{\nu}) \ \leq \ 9.3 \times 10^{36}$ years and $\tau(p \to \pi^+ \bar{\nu}) \ \leq \ 3.0 \times 10^{35}$ years, future proton decay experiments could test this theory.

- The theory predicts light colored scalar octets, S_R^0, S_I^0, and $S^{\pm}$, which could be produced at the Large Hadron Collider and modify the production and decays of the SM Higgs. The properties of these fields in this context will be studied in a future publication.

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APPENDIX A: FIELD CONTENT AND INTERACTIONS IN ADJOINT SU(5)

In Adjoint $SU(5)$ [12] the matter is unified in

$$\mathbf{\overline{5}} = l_L \bigoplus (d^C)_L = (\mathbf{1}, \mathbf{2}, -1/2) \bigoplus (\mathbf{\overline{3}}, \mathbf{1}, 1/3), \tag{A1}$$

$$\mathbf{10} = (u^C)_L \bigoplus q_L \bigoplus (e^C)_L = (\mathbf{\overline{3}}, \mathbf{1}, -2/3) \bigoplus (\mathbf{3}, \mathbf{2}, 1/6) \bigoplus (\mathbf{1}, \mathbf{1}, 1), \tag{A2}$$

and

$$\begin{aligned} \mathbf{2}4 &= (\rho_8)_L \bigoplus (\rho_3)_L \bigoplus (\rho_{(3,2)})_L \bigoplus (\rho_{(\bar{3},2)})_L \bigoplus (\rho_0)_L \\ &= (\mathbf{8}, \mathbf{1}, 0) \bigoplus (\mathbf{1}, \mathbf{3}, 0) \bigoplus (\mathbf{3}, \mathbf{2}, -5/6) \bigoplus (\mathbf{\overline{3}}, \mathbf{2}, 5/6) \bigoplus (\mathbf{1}, \mathbf{1}, 0), \end{aligned} \tag{A3}$$

while the Higgs sector is composed of

$$\mathbf{5}_H = H_1 \bigoplus T = (\mathbf{1}, \mathbf{2}, 1/2) \bigoplus (\mathbf{3}, \mathbf{1}, -1/3), \tag{A4}$$

$$\begin{aligned} \mathbf{24}_H &= \Sigma_8 \bigoplus \Sigma_3 \bigoplus \Sigma_{(3,2)} \bigoplus \Sigma_{(\bar{3},2)} \bigoplus \Sigma_{24} \\ &= (\mathbf{8}, \mathbf{1}, 0) \bigoplus (\mathbf{1}, \mathbf{3}, 0) \bigoplus (\mathbf{3}, \mathbf{2}, -5/6) \bigoplus (\mathbf{\overline{3}}, \mathbf{2}, 5/6) \bigoplus (\mathbf{1}, \mathbf{1}, 0), \end{aligned} \tag{A5}$$

and

$$\begin{aligned} \mathbf{45}_H &= \Phi_1 \bigoplus \Phi_2 \bigoplus \Phi_3 \bigoplus \Phi_4 \bigoplus \Phi_5 \bigoplus \Phi_6 \bigoplus H_2 \\ &= (\mathbf{8}, \mathbf{2}, 1/2) \bigoplus (\mathbf{\overline{6}}, \mathbf{1}, -1/3) \bigoplus (\mathbf{3}, \mathbf{3}, -1/3) \bigoplus (\mathbf{\overline{3}}, \mathbf{2}, -7/6) \\ &\bigoplus (\mathbf{3}, \mathbf{1}, -1/3) \bigoplus (\mathbf{\overline{3}}, \mathbf{1}, 4/3) \bigoplus (\mathbf{1}, \mathbf{2}, 1/2). \end{aligned} \tag{A6}$$

The Action of this model is given by

$$S = S_{\text{kinetic}} + S_{\text{Yukawa}} + S_{\text{scalar}}, \tag{A7}$$

$$\begin{aligned} S_{\text{kinetic}} &= \int d^4x \, [-\frac{1}{4} \text{Tr} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} (D^\mu 5_H)^\dagger (D_\mu 5_H) + \\ &+ \text{Tr}(D^\mu 24_H)^\dagger (D_\mu 24_H) + \text{Tr}(D^\mu 45_H)^\dagger (D_\mu 45_H) + \\ &+ \bar{5}^\dagger \gamma^0 i \gamma^\mu D_\mu \bar{5} + \text{Tr}\, (\overline{10} i \gamma^\mu D_\mu 10) + \text{Tr}\, (\overline{24} i \gamma^\mu D_\mu 24) \,], \end{aligned} \tag{A8}$$

where

$$D_\mu 5_H \;=\; \partial_\mu 5_H + i g_{GUT} A_\mu 5_H, \quad D_\mu 10 = \partial_\mu 10 \;+\; i g_{GUT} \left(A_\mu 10 \;+\; 10 A_\mu^T \right), \tag{A9}$$

$$D_\mu \bar{5} \;=\; \partial_\mu \bar{5} \;-\; i g_{GUT} \; A_\mu^T \bar{5}, \quad D_\mu 24_H = \partial_\mu 24_H \;+\; i g_{GUT} [A_\mu, 24_H], \tag{A10}$$

$$D_\mu (45_H)^{\alpha\beta}_\gamma \;=\; \partial_\mu (45_H)^{\alpha\beta}_\gamma \;+\; i g_{GUT} \left(A_\mu^{\alpha m} (45_H)^{m\beta}_\gamma \;+\; (45_H)^{\alpha m}_\gamma (A_\mu^T)^{m\beta} \;-\; (A_\mu^T)_{\gamma\delta} (45_H)^{\alpha\beta}_\delta \right), \tag{A11}$$

$$F_{\mu\nu} \;=\; \partial_\mu A_\nu - \partial_\nu A_\mu - i g_{GUT} [A_\mu, A_\nu], \tag{A12}$$

$$\begin{aligned} S_{\text{Yukawa}} \;=\; & -\int d^4x \left(Y_1^{ab} 10_a^{ij} \; \bar{5}_b^i \; (5_H^*)^j \;+\; Y_2^{ab} 10_a^{ij} \; \bar{5}_b^l \; (45_H^*)^{ij}_l \right) \\ & -\; \int d^4x \; \epsilon_{ijklr} \; \left(\; Y_3^{ab} 10_a^{ij} \; 10_b^{kl} \; 5_H^r \;+\; Y_4^{ab} 10_a^{ij} \; 10_b^{pk} \; (45_H)^{lr}_p \right) \\ & -\; \int d^4x \left(c_a \; \bar{5}_{ai} 24^i{}_k 5^k_H \;+\; p_a \; \bar{5}_{ai} 24^j{}_k (45_H)^{ik}_j \right) \;+\; S_{24} \;+\; \text{h.c.} \end{aligned} \tag{A13}$$

The additional terms relevant for the seesaw mechanism are given by

$$S_{24} \;=\; -\int d^4x \left(M \; 24^i{}_j 24^j{}_i \;+\; \lambda \; 24^i_j \; 24^j{}_k (24_H)^k{}_i \right) \;+\; \text{h.c.} \tag{A14}$$

In order to simplify our notation in the next equations we use $5_H = 5$, $24_H = 24$ and $45_H = 45$. The scalar interactions are given by

$$S_{\text{scalar}} = -\int d^4x \; V(5_H, 24_H, 45_H). \tag{A15}$$

The field 45_H satisfies the following conditions: $(45_H)^{ij}_k = -(45_H)^{ji}_k$, $\Sigma_{\imath=1}^5 (45_H)^{ij}_i = 0$, and $v_{45}/\sqrt{2} = \langle 45_H \rangle^{15}_1 = \langle 45_H \rangle^{25}_2 = \langle 45_H \rangle^{35}_3$. The $SU(5)$ invariant Higgs potential for our model is

$$\begin{aligned} V(24_H, 45_H, 5_H) \;=\; & V_1(24_H) + V_2(45_H) + V_3(5_H) + V_4(24_H, 45_H) \\ & +\; V_5(24_H, 5_H) + V_6(5_H, 45_H) + V_7(24_H, 45_H, 5_H), \end{aligned} \tag{A16}$$

where

$$V_1(24_H) \;=\; -\frac{\mu_{24}^2}{2} 24^\alpha{}_\beta 24^\beta{}_\alpha \;+\; \frac{a_1}{2} (24^\alpha{}_\beta 24^\beta{}_\alpha)^2 \;+\; \frac{a_2}{3} 24^\alpha{}_\beta 24^\beta{}_\gamma 24^\gamma{}_\alpha \;+\; \frac{a_3}{2} 24^\alpha{}_\beta 24^\beta{}_\gamma 24^\gamma{}_\delta 24^\delta{}_\alpha, \tag{A17}$$

$$\begin{aligned} V_2(45_H) \;=\; & -\frac{1}{2} \mu_{45}^2 \; (45^{\alpha\beta}_\gamma 45^\gamma_{\alpha\beta}) \;+\; \lambda_1 \; (45^{\alpha\beta}_\gamma 45^\gamma_{\alpha\beta})^2 \;+\; \lambda_2 \; 45^{\alpha\beta}_\gamma 45^\delta_{\alpha\beta} 45^{k\lambda}_\delta 45^\gamma_{k\lambda} \;+ \\ & +\; \lambda_3 \; 45^{\alpha\beta}_\gamma 45^\delta_{\alpha\beta} 45^{k\gamma}_\lambda 45^\lambda_{k\delta} \;+\; \lambda_4 \; 45^{\alpha\delta}_\beta 45^\beta_{\alpha\gamma} 45^{k\gamma}_\lambda 45^\lambda_{k\delta} \;+\; \lambda_5 \; 45^{\alpha\gamma}_\delta 45^\beta_{\gamma\epsilon} 45^{k\delta}_\alpha 45^\epsilon_{k\beta} \;+ \\ & +\; \lambda_6 \; 45^{\alpha\gamma}_\delta 45^\beta_{\gamma\epsilon} 45^{k\epsilon}_\alpha 45^\delta_{k\beta} \;+\; \lambda_7 \; 45^{\alpha\gamma}_\delta 45^\beta_{\gamma\epsilon} 45^{k\delta}_\beta 45^\epsilon_{k\alpha} \;+\; \lambda_8 \; 45^{\alpha\gamma}_\delta 45^\beta_{\gamma\epsilon} 45^{k\epsilon}_\beta 45^\delta_{k\alpha}. \end{aligned} \tag{A18}$$

See reference [24]. The rest of the scalar interactions are given by

$$V_3(5_H) = -\frac{\mu_5^2}{2}\, 5^*_\alpha 5^\alpha \;+\; \frac{a_4}{4}\; (5^*_\alpha 5^\alpha)^2\,, \tag{A19}$$

$$\begin{aligned} V_4(24_H, 45_H) \;=\; & a_5\; 45^{\alpha\beta}_\gamma 24^\gamma{}_\delta 45^\delta_{\alpha\beta} \;+\; a_6\; (45^{\alpha\beta}_\gamma 45^\gamma_{\alpha\beta}) 24^\delta{}_\epsilon 24^\epsilon{}_\delta \;+\; \beta_1\; 45^{\alpha\beta}_\gamma 24^\delta{}_\alpha 24^\epsilon{}_\beta 45^\gamma_{\delta\epsilon} \;+ \\ & +\; \beta_2\; 45^{\alpha\beta}_\gamma 24^\gamma{}_\beta 24^\delta{}_\epsilon 45^\epsilon_{\alpha\delta} \;+\; \beta_3\; 45^{\alpha\beta}_\gamma 24^\gamma{}_\epsilon 24^\delta{}_\beta 45^\epsilon_{\alpha\delta} \;+\; \beta_4\; 45^{\alpha\beta}_\gamma 24^k{}_\alpha 24^\lambda{}_k 45^\gamma_{\lambda\beta} \;+ \\ & +\; \beta_5\; 45^{\alpha\beta}_\gamma 24^\gamma{}_k 24^k{}_\lambda 45^\lambda_{\alpha\beta}, \end{aligned} \tag{A20}$$

$$V_5(24_H, 5_H) = \beta_6\; 5^*_\alpha 24^\alpha{}_\beta 5^\beta \;+\; \beta_7\; 5^*_\alpha 5^\alpha 24^\beta{}_\gamma 24^\gamma{}_\beta \;+\; \beta_8\; 5^*_\alpha 24^\alpha{}_\beta 24^\beta{}_\gamma 5^\gamma, \tag{A21}$$

$$V_6(5_H, 45_H) = c_1\; (45^{\alpha\beta}_\gamma 45^\gamma_{\alpha\beta}) 5^*_\delta 5^\delta \;+\; c_2\; 45^{\alpha\beta}_\delta\; 5^*_\gamma\; 45^\gamma_{\alpha\beta}\; 5^\delta \;+\; c_3\; 45^{\alpha\beta}_\gamma\; 45^\gamma_{\alpha\delta}\; 5^*_\beta\; 5^\delta, \tag{A22}$$

and

$$V_7(24_H, 45_H, 5_H) = c_4\; 5^*_\alpha 24^\gamma{}_\beta 45^{\alpha\beta}_\gamma \;+\; c_5\; 5^*_\alpha 24^\gamma{}_\delta 24^\delta{}_\beta 45^{\alpha\beta}_\gamma \;+\; c_6\; 5^*_\alpha 24^\alpha{}_\beta 24^\gamma{}_\delta 45^{\beta\delta}_\gamma \;+\; \text{h.c.} \tag{A23}$$

Notice that we have generalized the results for the scalar potential presented in references [25] and [26].

APPENDIX B: RUNNING OF GAUGE COUPLINGS

In order to understand the predictions coming from the unification of gauge couplings at the scale $\Lambda = M_{GUT}$ one uses the RGEs:

$$\frac{1}{\alpha_i(M_Z)} - \frac{1}{\alpha_i(\Lambda)} \;=\; \frac{1}{2\pi}\, b_i^{SM} \ln\frac{\Lambda}{M_Z} \;+\; \frac{1}{2\pi}\sum_I\; b_{iI}\; \Theta\,(\Lambda - M_I) \ln\frac{\Lambda}{M_I}, \tag{B1}$$

where the function $\Theta(x)$ is one for $x > 0$ and zero for $x \leq 0$. The different contributions to the running of the gauge couplings are listed in Table I. $b_1^{SM} = 41/10$, $b_2^{SM} = -19/6$ and $b_3^{SM} = -7$.

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TABLE I: Contributions to the running of gauge couplings.

Fields	b_1	b_2	b_3
H_1	1/10	1/6	0
T	1/15	0	1/6
Σ_8	0	0	1/2
Σ_3	0	1/3	0
Φ_1	4/5	4/3	2
Φ_2	2/15	0	5/6
Φ_3	1/5	2	1/2
Φ_4	49/30	1/2	1/3
Φ_5	1/15	0	1/6
Φ_6	16/15	0	1/6
H_2	1/10	1/6	0
ρ_8	0	0	2
ρ_3	0	4/3	0
$\rho_{(3,2)}$, $\rho_{(\bar{3},2)}$	5/3	1	2/3

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