

Do the quark masses run? Extracting $\bar{m}_b(m_Z)$ from LEP data

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We present the first results of next-to-leading order QCD corrections to three jet heavy quark production at LEP including mass effects. Among other applications, this calculation can be used to extract the bottom quark mass from LEP data, and therefore to test the running of masses as predicted by QCD.

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The decay width of the Z gauge boson into three jets has already been computed at the leading order (LO) including complete quark mass effects [1–3] where it has been shown that mass effects could be as large as 1% to 6%, depending on the value of the mass and the jet resolution parameter y_c . In fact, these effects had already been seen in the experimental tests of the flavor independence of the strong coupling constant [4–8]. In view of that we proposed [3], together with the DELPHI collaboration [9], the possibility of using the ratio [3,6,7]

$$R_3^{bd} \equiv \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^d(y_c)/\Gamma^d} \quad (1)$$

as a means to extract the bottom quark mass from LEP data. In this equation $\Gamma_{3j}^q(y_c)/\Gamma^q$ is the three-jet fraction of Z decays into the quark q and y_c is the jet resolution parameter.

Since the measurement of R_3^{bd} is done far away from the threshold of b quark production, it will allow, for the first time, to test the running of a quark mass as predicted by QCD. However, in [3] we also discussed that the leading order calculation does not distinguish among the different definitions of the quark mass, perturbative pole mass, M_b , running mass at M_b , or running mass at m_Z . Therefore in order to correctly take into account mass effects it is necessary to perform a complete next-to-leading order (NLO) calculation of three jet ratios including quark masses [10–12].

In this letter we sketch the main points of this calculation, leaving the details of the complete calculation for other publications [13,14], and we present the results that have been used by the DELPHI collaboration to measure the running mass of the bottom quark at $\mu = m_Z$ [15,16].

In the last years the most popular definitions of jets are based on the so-called jet clustering algorithms. These algorithms can be applied at the parton level in the theoretical calculations and also to the bunch of real particles observed at experiment. In the jet-clustering algorithms jets are defined as follows: starting from a bunch of particles with momenta p_i one computes, for example, a quantity like $y_{ij} = 2 \min(E_i^2, E_j^2)/s (1 - \cos \theta_{ij})$ for all pairs (i, j) of particles. Then one takes the minimum of all y_{ij} and if it satisfies that it is smaller than a given quantity y_c (the resolution parameter, y -cut) the two particles which define this y_{ij} are regarded as belonging to the same jet, therefore, they are recombined into a new pseudoparticle by defining the four-momentum of the pseudoparticle according to some rule, for example, $p_k = p_i + p_j$. After this first step one has a bunch of pseudoparticles and the algorithm can be applied again and again until all the pseudoparticles satisfy $y_{ij} > y_c$. The number of pseudoparticles found in the end is the number of jets in the event. This procedure leads automatically to IR finite quantities because one excludes the regions of phase space that cause trouble. It has been shown that, for some of the algorithms, the passage from partons to hadrons (hadronization) does not change much the behavior of the observables [17], thus allowing to compare theoretical predictions with experimental results.

Although we have studied the four jet-clustering algorithms discussed in [3,11,17,18], here we will present results only for the DURHAM algorithm [19,20] which is the one we just have defined and seems to be the one that behaves better for most of the observables.

The decay width of the Z boson into three jets containing the bottom quark mass can be written as follows [3]

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$$\Gamma_{3j}^b(y_c) = m_Z \frac{g^2}{c_W^2 64\pi} \frac{\alpha_s(m_Z)}{\pi} (g_V^2 H_V(y_c, r_b) + g_A^2 H_A(y_c, r_b)) , \quad (2)$$

where g is the SU(2) gauge coupling constant, $c_W = \cos \theta_W$ and $s_W = \sin \theta_W$ are the cosine and the sine of the weak mixing angle, $g_V = -1 + 4/3s_W^2$ and $g_A = 1$ are the vector and axial coupling of the Z boson to the bottom quark, and $H_V(y_c, r_b)$ and $H_A(y_c, r_b)$ contain all the dependences in the jet resolution parameter, y_c , and the quark mass, $r_b = (M_b/m_Z)^2$, for the vector and axial parts in the different algorithms. These functions can be expanded in α_s , and, if the leading dependence on the quark mass is factorized out we have,

$$H_{V(A)}(y_c, r_b) = A^{(0)}(y_c) + \frac{\alpha_s}{\pi} A^{(1)}(y_c) + r_b \left(B_{V(A)}^{(0)}(y_c, r_b) + \frac{\alpha_s}{\pi} B_{V(A)}^{(1)}(y_c, r_b) \right) + \dots . \quad (3)$$

Here, $A^{(0)}(y_c)$ is the tree level contribution in the case of massless quarks and it is known for the different algorithms in analytic form. It is exactly the same function for the vector and the axial parts. The functions $B_{V(A)}^{(0)}(y_c, r_b)$ take into account tree level mass effects once the leading dependence in r_b has been factorized out. They were calculated numerically in [3] for the different algorithms and results were presented in the form of fits to the numerical results. $A^{(1)}(y_c)$ gives the QCD next-to-leading order correction in the case of massless quarks and to good approximation it is the same for the vector and the axial parts [Because of the triangle anomaly there are one-loop triangle diagrams contributing to $Z \rightarrow \bar{q}qg$ with the top and the bottom quarks running in the loop. Since $m_t \neq m_b$ the anomaly cancellation is not complete. These diagrams contribute to the axial part even for $m_q = 0$ and lead to a deviation from $A_V^{(1)}(y_c) = A_A^{(1)}(y_c)$ [21]. This deviation is, however, small [21] and we are not going to consider its effect here]. This function is also known for the different algorithms [17,18]. Finally, the functions $B_{V(A)}^{(1)}(y_c, r_b)$ were completely unknown and contain the next-to-leading order corrections depending on the quark mass. In the next section we present our results in the form of fits to some combinations of the relevant functions in the case of the DURHAM algorithm, which is the one that gives smaller radiative corrections, and we postpone the presentation of results for the different functions and algorithms for another publication [14]. Note that the way we write $H_{V(A)}(y_c, r_b)$ in eq. (3) is not an expansion for small r_b . We keep the exact dependence on r_b in the functions $B_{V(A)}^{(0)}(y_c, r_b)$ and $B_{V(A)}^{(1)}(y_c, r_b)$. Factoring out r_b makes it easier to analyze the massless limit and the dependence of the results on r_b in the region of interest. This means that our results can also be adapted, by including the photon exchange, to compute the e^+e^- cross section into three jets out of the Z peak at lower energies or at higher energies in top quark production.

At the NLO we have contributions to the three-jet cross section from three and four parton final states. One loop three-parton amplitudes are both IR and UV divergent. Therefore, some regularization procedure is needed. We use dimensional regularization for both IR and UV divergences because dimensional regularization preserves the QCD Ward identities. The UV divergences, however, can be easily removed by renormalization since the appropriate counterterms are very well known. The three-jet cross section is obtained by integrating both contributions, renormalized three-parton and four-parton amplitudes, in the three-jet phase space region defined by the different jet clustering algorithms. This quantity is infrared finite and well defined, although the three and the four parton transition amplitudes independently contain infrared singularities.

The three-parton transition amplitudes can be expressed in terms of a few scalar one-loop integrals. The result contains poles in $\epsilon = (4 - D)/2$, where D is the number of space-time dimensions. Some of the poles come from UV divergences and the other come from IR divergences. After UV renormalization we obtain analytical expressions for the terms proportional to the infrared poles and for the finite contributions. The infrared poles will cancel against the four-parton contributions. The finite contributions are integrated numerically in the three-jet region.

The four-parton transition amplitudes are split into a soft and collinear part in the three-jet region and a hard contribution. The soft terms are integrated analytically in arbitrary D dimensions in the region of the phase space containing the infrared singularities. We obtain analytical expressions for the infrared behavior of the four-parton transition amplitudes and we show how these infrared terms cancel exactly the infrared singularities of the three-parton contributions. The hard terms are calculated in $D = 4$ dimensions. The remaining phase space integrations, giving rise to finite contributions, are performed numerically.

Following Ellis, Ross and Terrano [22] we have classified both, three-parton and four-parton transition probabilities, according to their color factors. It is clear that the cancellation of IR divergences between three-parton and four-parton processes can only occur inside groups of diagrams with the same color factor. The cancellation of IR divergences can be seen more clearly by representing the different amplitudes as the different cuts one can perform in the three-loop bubble diagrams contributing to the Z -boson selfenergy. After summing up the three-parton and four-parton contributions to the three-jet decay width of the Z boson we obtain the functions $H_V(y_c, r_b)$ and $H_A(y_c, r_b)$ in eq. (2)

at order α_s . Details of the calculation, cancellation of divergences and results for the relevant functions will be presented elsewhere [13,14]. Since a large part of the calculation has been done numerically, it is important to have some checks of it. We have performed the following tests:

- We have checked our four parton amplitudes in the massless limit against the amplitudes presented by Ellis-Ross-Terrano (ERT) [22]. The three-parton amplitudes for massive quarks cannot be compared directly with the massless results because collinear poles in ϵ , in the massless case, appear as logarithms of the quark mass when the mass is taken into account.
- The four parton transition amplitudes have also been checked in the case of massive quarks, in four dimensions, by comparing their contribution to four jet processes to the known results [1]
- Of course we have checked that, also in the massive case, all the IR divergences cancel between three parton and four parton contributions [23–25].
- To check the performance of the numerical programs and the overall approach we have applied our method to the massless amplitudes of ERT and obtained the known results for the functions $A^{(1)}$.
- We have checked, independently for each of the groups of diagrams with different color factors, that the final result obtained with massive quarks reduces to the massless result in the limit of very small masses.

The last test is the main check of our calculation. We have calculated the functions $H_V(y_c, r_b)$ and $H_A(y_c, r_b)$ for several small values of r_b , in the range $M_b \sim 1 - 5 \text{ GeV}$, and then we have extrapolated the results for $r_b \rightarrow 0$. In that limit we recover the values for the function $A^{(1)}(y_c)$ in the different algorithms considered and the different groups of diagrams. This check is not trivial at all since the structure of IR divergences for massive quarks is quite different from the case of massless quarks: for massive quarks collinear divergences are regulated by the quark mass, and therefore some of the poles in ϵ that appear in the massless case are softened by $\log r_b$. Although these checks do not constitute a complete test of the massive corrections, we think that all together give some kind of confidence in the final result for massive quarks.

Now, to obtain R_3^{bd} , we can substitute eq. (3) in eq. (2), this into eq. (1) and use the value of Γ^b which is also well known (for mass effects at order α_s see for instance [3,26,27]). Putting everything together and expanding in α_s and r_b we can write

$$R_3^{bd} = 1 + r_b \left(b_0(y_c, r_b) + \frac{\alpha_s}{\pi} b_1(y_c, r_b) \right) , \quad (4)$$

where the functions b_0 and b_1 are an average of the vector and axial parts, weighted by $c_V = g_V^2/(g_V^2 + g_A^2)$ and $c_A = g_A^2/(g_V^2 + g_A^2)$ respectively, and can be written in terms of the different functions introduced before, eq. (3), [3,11]

It is important to note that because the particular normalization we have used in the definition of R_3^{bd} , which is manifested in the final dependence on c_V and c_A , most of the electroweak corrections cancel. Those are about 1% [28] in total rates while in R_3^{bd} are below 0.05%. Therefore for our estimates it is enough to consider tree level values of g_V and g_A . The same argument applies for the passage from decay widths to cross sections. Contributions from photon exchange are small at LEP and can be absorbed in a redefinition of g_V^2 and g_A^2 [29]. They will add a small correction to our observable.

Although intermediate calculations have been performed using the pole mass we can also re-express our results in terms of the running quark mass by using the known perturbative expression

$$M_b^2 = \bar{m}_b^2(\mu) \left[1 + 2 \frac{\alpha_s(\mu)}{\pi} \left(\frac{4}{3} - \log \frac{m_b^2}{\mu^2} \right) \right] . \quad (5)$$

The connection between pole and running masses is known up to order α_s^2 , however consistency of our pure perturbative calculation requires we use only the expression above. We obtain

$$R_3^{bd} = 1 + \bar{r}_b(\mu) \left(b_0(y_c, r_b) + \frac{\alpha_s(\mu)}{\pi} \left(\bar{b}_1(y_c, r_b) - 2b_0(y_c, r_b) \log \frac{m_Z^2}{\mu^2} \right) \right) . \quad (6)$$

Where $\bar{r}_b(\mu) = \bar{m}_b^2(\mu)/m_Z^2$ and

$$\bar{b}_1(y_c, r_b) = b_1(y_c, r_b) + b_0(y_c, r_b) [8/3 - 2 \log(r_b)] . \quad (7)$$

$\bar{r}_b(\mu)$ can be expressed in terms of the running mass of the b quark at $\mu = m_Z$ by using the renormalization group. At the order we are working

$$\bar{r}_b(\mu) = \bar{r}_b(m_Z) \left(\frac{\alpha_s(m_Z)}{\alpha_s(\mu)} \right)^{-4\gamma_0/\beta_0} \quad \text{with} \quad \alpha_s(\mu) = \frac{\alpha_s(m_Z)}{1 + \alpha_s(m_Z)\beta_0 t} \quad (8)$$

and $t = \log(\mu^2/m_Z^2)/(4\pi)$, $\beta_0 = 11 - 2N_f/3$, $N_f = 5$ and $\gamma_0 = 2$.

At the perturbative level eq. (4) and eq. (6) are equivalent. However, since they neglect different higher order terms they lead to different answers. Since the experiment is performed at high energies (the relevant scales are m_Z and $m_Z\sqrt{y_c}$) one would think that the expression in terms of the running mass is more appropriate because the running mass is a true short distance parameter while the pole mass contains in it all the complicated physics at scales $\mu \sim M_b$. Moreover, by using the expression in terms of the running mass we can vary the scale in order to estimate the error due to the neglect of higher order corrections. In any case, if one would push the result of eq. (6) up to scales as low as $\mu = 5 \text{ GeV}$ one would get something closer to the pole mass result. Therefore, we use eq. (6) and vary the scale in an appropriate range to obtain an estimate of the error in the calculation.

The function $b_0(y_c, r_b)$ gives the mass corrections at leading order. As shown in [3] it depends very mildly on the quark mass in the region of interest ($M_b \sim 3 - 5 \text{ GeV}$). Therefore it is appropriate to present our results for $b_0(y_c, r_b)$ as a fit in only y_c : $b_0(y_c, r_b) = \sum_{n=0}^2 k_0^{(n)} \log^n y_c$. For the DURHAM algorithm, in the range $0.01 < y_c < 0.10$ and $3 \text{ GeV} < M_b < 5 \text{ GeV}$ and using $\sin^2 \theta_W = 0.2315$ we obtain $k_0^{(0)} = -10.521$, $k_0^{(1)} = -4.4352$, $k_0^{(2)} = -1.6629$.

The function $\bar{b}_1(y_c, r_b)$ is the main result of this paper. It gives the NLO massive corrections to our observable. It is important to note that \bar{b}_1 contains significant logarithmic corrections depending on the quark mass. They come from different sources: first, the NLO corrections written in terms of the pole mass now contain some residual mass dependence, second the normalization to the total rate induces some additional logarithmic dependences, and finally the passage from the pole mass to the running mass adds also an additional logarithmic dependence. Therefore, we choose to include explicitly this dependence on the quark mass in our fits to the function $\bar{b}_1(y_c, r_b)$. For our purposes a fit of the form $\bar{b}_1(y_c, r_b) = k_1^{(0)} + k_1^{(1)} \log(y_c) + k_m^{(0)} \log(r_b)$ is good enough. The coefficients we obtain for the DURHAM algorithm and in the ranges we just mentioned for y_c and r_b are: $k_1^{(0)} = 297.92$, $k_1^{(1)} = 59.358$, $k_m^{(0)} = 46.238$.

In fig. 1 we present R_3^{bd} for $\mu = m_Z$ (dashed), $\mu = 30 \text{ GeV}$ (dashed-dotted) and $\mu = 10 \text{ GeV}$ (dotted) for $\bar{m}_b(m_Z) = 3 \text{ GeV}$ and $\alpha_s(m_Z) = 0.118$. For comparison we also present the LO results for $M_b = 5 \text{ GeV}$ (lower solid line) which is, roughly, the value of the pole mass obtained at low energies and $\bar{m}_b(m_Z) = 3 \text{ GeV}$ (upper solid line) which is, roughly, the value one obtains for the running mass at the m_Z scale by using the renormalization group [30]. Note that choosing a low value for μ makes the result closer to the LO result written in terms of the pole mass, while choosing a large μ makes the result approach to the LO result written in terms of the running mass at the m_Z scale. If R_3^{bd} is measured to good accuracy one could use eq. (6) and eq. (8) to extract $\bar{m}_b(m_Z)$. However, the extracted result will depend on μ . For illustration, in fig. 2 we represent, as a function of μ , the value one would obtain for $\bar{m}_b(m_Z)$ if $R_{3exp}^{bd} = 0.96$ for $y_c = 0.02$. What is the best scale one should choose in three-jet quantities is a long standing discussion. One would think that if the energy is equally distributed among the three jets one should choose $\mu \sim m_Z/3$. A conservative approach is to vary the scale in an appropriate range and take the spread of the result as an estimate of the error due to higher order corrections. From fig. 2 we see that if we take μ in the range $m_Z/10 - m_Z$ the error due to the scale and α_s in the determination of $\bar{m}_b(m_Z)$ would be of about 0.20 GeV . If scales as low as $\mu = 5 \text{ GeV}$ are accepted the error increases to 0.23 GeV . Whether this error can be reduced by a clever choice of the scale or resummation of leading logs in y_c or r_b remains to be seen.

The calculation presented in this paper has already been used by the DELPHI collaboration [16] to extract $\bar{m}_b(m_Z)$. The preliminary result [we like to thank the DELPHI collaboration for allowing us to use these numbers here] they obtain is

$$\bar{m}_b(m_Z) = 2.85 \pm 0.22 \text{ (stat)} \pm 0.20 \text{ (theo)} \pm 0.36 \text{ (fragmentation)} \text{ GeV} \quad (9)$$

which has to be compared with low energy determinations of the bottom quark mass. The last analysis of the Υ system using QCD sum rules [31] gives $\bar{m}_b(\bar{m}_b) = 4.13 \pm 0.06 \text{ GeV}$ which translates into $\bar{m}_b(m_Z) = 2.83 \pm 0.10 \text{ GeV}$ if one uses three loop renormalization group running and $\alpha_s(m_Z) = 0.118 \pm 0.003$. On the other hand, the last lattice result gives [32] $\bar{m}_b(\bar{m}_b) = 4.15 \pm 0.20 \text{ GeV}$ and $\bar{m}_b(m_Z) = 2.84 \pm 0.21 \text{ GeV}$. Given the errors it is clear that central values agree so well just by chance. In particular the value in eq. (9) has been extracted by using a different central value for α_s .

It is encouraging to see that this preliminary measurement is in full compatibility with low energy data, and, although for the moment it is not competitive with low energy measurements, it is good enough for testing the running of the bottom quark mass from $\mu = M_b$ to $\mu = m_Z$: the result for $\bar{m}_b(m_Z)$ in eq. (9) and the previous values

for $\bar{m}_b(\bar{m}_b)$ differ by more than 2.5 standard deviations. We believe that these results can be substantially improved with more experimental and theoretical work.

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The results in this paper have been previously presented at several conferences [10,15,12,16] and were submitted as part of the Ph.D. requirements for G.R.. In the meanwhile a preprint dealing with the same problem has appeared [33].

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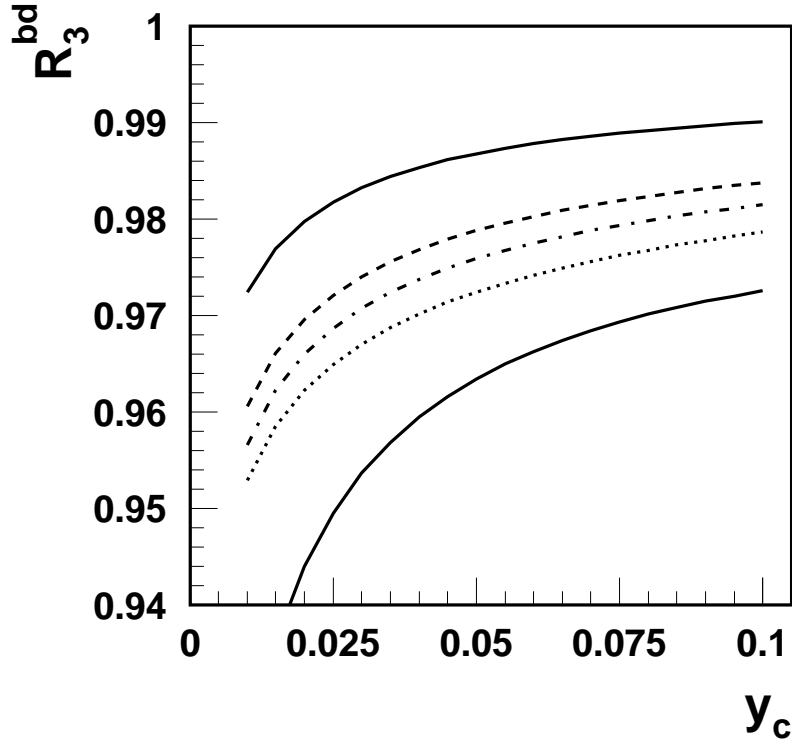


FIG. 1. NLO results for R_3^{bd} for $\mu = m_Z$ (dashed), $\mu = 30 \text{ GeV}$ (dashed-dotted) and $\mu = 10 \text{ GeV}$ (dotted) for $\bar{m}_b(m_Z) = 3 \text{ GeV}$ and $\alpha_s(m_Z) = 0.118$. For comparison we also plot the LO results for $M_b = 5 \text{ GeV}$ (lower solid line) and $\bar{m}_b(m_Z) = 3 \text{ GeV}$ (upper solid line)

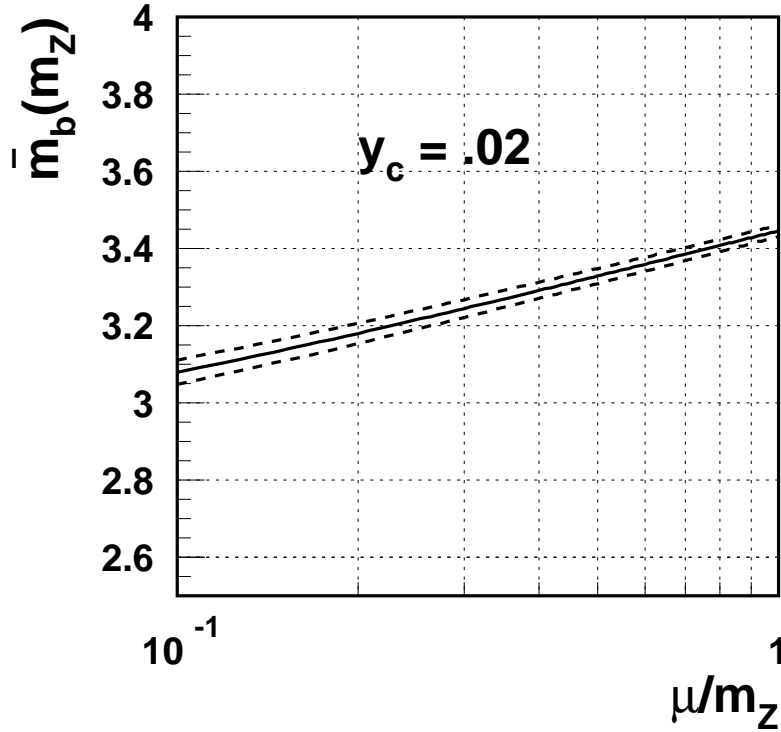


FIG. 2. Extracted value of $\bar{m}_b(m_Z)$ if $R_{3exp}^{bd} = 0.96$ as a function of the scale μ . We take $\alpha_s(m_Z) = 0.118$ (solid) and $\Delta\alpha_s = 0.003$ (dashed).