The running of the b-quark mass from LEP data *

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Next-to-leading order QCD corrections to three jet heavy quark production in e^+e^- collisions, including quark mass effects, are presented. The extraction of the b-quark mass from LEP data is considered and the first experimental evidence for the running of a quark mass is discussed.

1. Introduction

The question of the origin of the masses of quarks and leptons is one of the unresolved puzzles in present high energy physics. To answer this question one needs to know precisely their value. However, quarks are not free and their mass has to be interpreted more like a coupling than an inertial parameter and it can run if measured at different scales. Moreover, in the standard model (SM) all fermion masses come from Yukawa couplings and those also run with the energy. To test fermion mass models one has to run masses extracted at quite different scales to the same scale and compare them with the same "ruler". This way, for instance, one can check that in some unified models the b-quark mass and the τ-lepton mass, although different at threshold energies they could be equal at the unification scale.

However, the running of fermion masses, although predicted by quantum field theory, has not been tested experimentally until now. The reason being that for energies $\sqrt{Q^2}$ much higher than the fermion mass, m_q, the mass effects become negligible since usually they are suppressed by m_q^2/Q^2.

While this argument is correct for total cross sections for production of heavy quarks, it is not completely true for quantities that depend on other variables. In particular, it is not true for jet cross sections which depend on a new variable, y_c (the jet-resolution parameter that defines the jet multiplicity) and which introduces a new scale in the analysis, $E_c = \sqrt{Q^2\, y_c}$. Then, for small values of y_c there could be contributions coming like $m_q^2/E_c^2 = (m_q^2/Q^2)/y_c$ which could enhance the mass effect considerably. In addition mass effects could also be enhanced by logarithms of the mass. For instance, the ratio of the phase space for two massive quarks and a gluon to the phase space for three massless particles is $1 + 8(m_q^2/Q^2)\log(m_q/Q)$. At $Q^2 = m_Z^2$ and for the bottom quark this gives a 7% effect, for $m_b = 5$ GeV and a 3% effect for $m_b = 3$ GeV.

The high precision achieved at LEP makes heavy quark mass effects relevant. In fact, they have to be taken into account in the tests of the flavour independence of $\alpha_s(m_Z)$ [1,2]. This in turn means that mass effects have already been seen. One can reverse the question and ask about the possibility of measuring the mass of the bottom quark, m_b, at LEP by assuming the flavour universality of the strong interactions.

In [3] we showed that mass effects in three-jet production at LEP are large enough to be mea-

*Supported in part by CICYT, Spain, under grant AEN-96/1718 and in part by DFG Projekt No. Ku 502/8-1, Germany.

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sured. The observable proposed as a means to extract the bottom-quark mass from LEP data was the ratio [1,3]

$$R_3^{bd} \equiv \frac{\Gamma_{3j}^b(y_c)/\Gamma^b}{\Gamma_{3j}^d(y_c)/\Gamma^d} \ . \qquad (1)$$

In this equation $\Gamma_{3j}^q(y_c)/\Gamma^q$ is the three-jet fraction and q denotes the quark flavor. In this ratio and at the leading order (LO) the quark mass effects can be as large as 1% to 6%, depending on the value of the mass and on the jet-resolution parameter, y_c.

The three jet decay width is defined by the jet-clustering algorithms (see e.g. [4]). The effect of hadronization is, in principle, small and has been estimated using the Montecarlo approach [5,6].

Since the measurement of R_3^{bd} is done far away from the threshold of b-quark production it can be used to test the running of a quark mass as predicted by QCD.

However, as we discussed in [3], the leading order calculation does not distinguish among the different definitions of the quark mass: perturbative pole mass, M_b, running mass at M_b-scale, or running mass at m_Z-scale. As the numerical difference is significant when the different definitions of masses are used in LO calculations, in order to correctly take into account mass effects, it is necessary to perform a complete next–to–leading order (NLO) calculation of three-jet ratios including quark masses.

Although, heavy quark production has been considered in a large variety of processes and, in particular, in Z-boson decays [7–10], there are very few NLO calculations of heavy quark jet production taking into account complete mass effects (In [11] and [12] this was done for gluon-gluon fusion and virtual-photon production of heavy quarks). Only very recently NLO calculations of heavy quark jet production in e^+e^- collisions have become available [13–17]. Here we overview our calculation. Its final results were presented in [14] and have been used by the DELPHI Collaboration to extract the b-quark mass at the m_Z scale [5,6] showing clearly that indeed the b-quark mass runs from $\mu = m_b$ to $\mu = m_Z$ as predicted by QCD.

2. Jet ratios with heavy quarks at NLO

The decay width of the Z-boson into three jets with a heavy quark can be written as follows

$$\Gamma_{3j}^b = \frac{m_Z g^2 \alpha_s}{c_W^2 64\pi^2} \left[g_V^2 H_V(y_c, r_b) + g_A^2 H_A(y_c, r_b) \right] \ , \qquad (2)$$

where g is the SU(2) gauge coupling constant, c_W and s_W are the cosine and the sine of the weak mixing angle, $g_V = -1 + 4/3 s_W^2$ and $g_A = 1$ are the vector and axial-vector coupling of the Z-boson to the bottom quark and α_s is the strong coupling constant. Functions $H_{V(A)}(y_c, r_b)$ contain all the dependences on y_c and the quark mass, $r_b = (M_b/m_Z)^2$, for the different algorithms. These functions can be expanded in α_s as

$$\begin{aligned} H_{V(A)} &= A^{(0)}(y_c) + \frac{\alpha_s}{\pi} A_{V(A)}^{(1)}(y_c) \qquad (3) \\ &+ r_b \left[B_{V(A)}^{(0)}(y_c, r_b) + \frac{\alpha_s}{\pi} B_{V(A)}^{(1)}(y_c, r_b) \right] + \cdots \ . \end{aligned}$$

Here, $A^{(0)}$, is the tree-level contribution in the massless limit. It is the same function for the vector and the axial-vector parts and it is known for the different jet-clustering algorithms in analytic form. The function $A_{V(A)}^{(1)}$ gives the QCD NLO correction for massless quark. This function is also known for the different jet-clustering algorithms [4] §. Note that even in the chiral limit these corrections are different for the vector and axial-vector parts. However, the difference, which is due to the one-loop triangle diagrams [18] is rather small. The net effect of these triangle diagrams is smaller than 10^{-4} in R_3^{bd}. In the ratio R_3^{bl}, which is similar to the one defined in eq. (1) but normalized to the sum of three light flavours, l=(u,d,s), the triangle anomaly produces a shift of $+2 \cdot 10^{-3}$. This has been taken into account in the experimental analysis [6]. Functions $B_{V(A)}$ take into account residual mass effects, once the leading dependence in r_b has been factorized. The tree-level contributions, $B_{V(A)}^{(0)}$, were calculated numerically in [3] for the different algorithms and results were presented in the form of simple fits

§With our choice of the normalization $A^{(0)}(y_c) = A(y_c)/2$ and $A_V^{(1)}(y_c) = B(y_c)/4$, where $A(y_c)$ and $B(y_c)$ are defined in [4].

to the numerical results. Finally, the functions $B^{(1)}_{V(A)}$, contain the NLO corrections depending on the quark mass.

Note that the way we write $H_{V(A)}$ in eq. (3) is not an expansion for small r_b. We keep the exact dependence on r_b in the functions $B_{V(A)}$. Factoring out r_b makes it easier to analyze the massless limit and the dependence of the results on r_b in the region of interest. This means that our results can also be adapted, by including the photon exchange, to compute the e^+e^--cross section into three jets outside the Z-peak at lower energies or at higher energies for top quark production.

At the NLO we have contributions to the three-jet cross section from three- and four-parton final states. For diagrams with emission of four real quarks, that can mix different flavours, we take the convention that the flavour is defined by the quark coupled directly to the Z-boson. Therefore, events with emission of a heavy quark pair radiated of a light $q\bar{q}$ are classified as light events despite the fact that b quarks are present in this four-fermion final state. From the theoretical point of view this avoids the appearance of large logarithms on the quark mass. The same convention was considered in the experimental analysis [6] therefore allowing for a consistent comparison.

One-loop three-parton amplitudes are both infrared (IR) and ultraviolet (UV) divergent. Therefore, some regularization procedure is needed. We use dimensional regularization for both IR and UV divergences, because it preserves the QCD Ward identities. The three-parton transition amplitudes can be expressed in terms of a few scalar one-loop integrals [15]. The result contains poles in $\epsilon = (4 - D)/2$, where D is the number of space-time dimensions. Some of the poles come from UV divergences and the other come from IR divergences. The UV divergences, however, are removed after the renormalization of the parameters of the QCD lagrangian. After that we obtain analytical expressions, which contain terms proportional to the IR poles and finite contributions.

The four-parton transition amplitudes are also IR divergent. These IR divergences cancel the corresponding IR poles coming from the virtual corrections according to [19]. Two different methods of analytic cancelation of IR singularities have been developed: the *phase space slicing method* [20] and the *subtraction method* [4,21,22]. We follow the first approach. The four-parton transition probabilities for $Z \to b\bar{b}gg(b\bar{b}q\bar{q})$ are split in two parts. The first part contains the terms which are divergent when one gluon is soft or two gluons (or light quarks) are collinear. These terms are integrated analytically in arbitrary D dimensions in the soft and collinear regions of phase space. This way we obtain the IR singular contributions of four partons in the three-jet region and show that they are canceled exactly by the tree-parton contribution. The second part, corresponding to the radiation of hard gluons, gives rise to finite contributions and can be calculated in $D = 4$ dimensions. The three-jet Z-width is obtained by integrating both, renormalized three-parton contribution and four-parton transition probabilities, in the three-jet phase-space region defined by the different jet-clustering algorithms. This quantity is infrared finite and well defined.

Following Ellis, Ross and Terrano [21] (ERT) we have classified both, three-parton and four-parton transition probabilities, according to their color factors. It is clear that the cancelation of IR divergences between three-parton and four-parton processes can only occur inside groups of diagrams with the same color factor. The cancelation of IR divergences can be seen more clearly by representing the different amplitudes as the different cuts one can perform in the three-loop bubble diagrams contributing to the Z-boson self-energy. After summing up the three-parton and four-parton contributions to the three-jet decay width of the Z-boson we obtain the functions $H_{V(A)}$ in eq. (2) at order α_s. Since a large part of the calculation has been done numerically, it is important to have some checks of it. We have performed the following tests: i) We have checked our four-parton probabilities in the massless limit against the amplitudes presented in ref. [21]. The three-parton amplitudes for massive quarks cannot be compared directly with the corresponding massless result as they have different structure of

IR singularities. ii) The four-parton transition amplitudes have also been checked in the case of massive quarks by comparing their contribution to four-jet processes with the known results [10]. iii) To check the performance of the numerical programs we have applied our method to the massless amplitudes of ERT and obtained the known results for the functions $A^{(1)}$. iv) We have checked, independently for each of the groups of diagrams with different color factors, that the final result obtained with massive quarks reduces to the massless result in the limit of very small masses.

The last test is the main check of our calculation. We have calculated the functions $H_{V(A)}$ for several values of r_b, in the range $M_b \sim 1-5\ GeV$, and then we have extrapolated the results for $r_b \to 0$. In that limit we reproduce the values for the function $A^{(1)}$ in the different algorithms considered and the different groups of diagrams. This check is not trivial at all since the structure of IR divergences for massive quarks is quite different from the case of massless quarks: for massive quarks collinear divergences are regulated by the quark mass, and therefore some of the poles in ϵ that appear in the massless case are softened by $\log r_b$.

Combining eq. (1), eq. (2) and eq. (3) and using the known expression for Γ^b [3,8] we write R_3^{bd} as the following expansion in α_s

$$R_3^{bd} = 1 + r_b \left(b_0 + \frac{\alpha_s}{\pi} b_1 \right) \ , \qquad (4)$$

where the functions b_0 and b_1 are an average of the vector and axial-vector parts, weighted by $c_V = g_V^2/(g_V^2+g_A^2)$ and $c_A = g_A^2/(g_V^2+g_A^2)$ respectively. They can be written in terms of the different functions introduced before, eq. (3), [3,13] and also depend on y_c and r_b.

It is important to note that because the particular normalization we have used in the definition of R_3^{bd}, which is manifested in the final dependence on c_V and c_A, most of the electroweak corrections cancel. Those are about 1% [23] in total rates, while in R_3^{bd} are below 0.05%. Therefore, for our estimates it is enough to consider tree-level values of g_V and g_A. The same argument applies for the passage from decay widths to cross sections. Contributions from photon exchange are small at LEP and can be absorbed in a redefinition of g_V^2 and g_A^2 [9]. They will add a small correction to our observable.

Although intermediate calculations have been performed using the pole mass, we can also reexpress our results in terms of the running quark mass by using the known perturbative expression $M_b^2 = \bar{m}_b^2(\mu)[1 + 2\alpha_s(\mu)/\pi\ (4/3 - \log(m_b^2/\mu^2))]$. The connection between pole and running masses is known up to order α_s^2, however consistency of our pure perturbative calculation requires we use only the expression above. We obtain

$$R_3^{bd} = 1+\bar{r}_b(\mu) \left(b_0 + \frac{\alpha_s(\mu)}{\pi} \left[\bar{b}_1 - 2b_0 \log \frac{m_Z^2}{\mu^2} \right] \right) \ , \qquad (5)$$

where $\bar{r}_b(\mu) = \bar{m}_b^2(\mu)/m_Z^2$ and

$$\bar{b}_1 = b_1 + b_0 \left[8/3 - 2\log(r_b) \right] \ . \qquad (6)$$

$\bar{r}_b(\mu)$ can be expressed in terms of the running mass of the b-quark at $\mu = m_Z$ by using the renormalization group. At the order we are working $\bar{r}_b(\mu) = \bar{r}_b(m_Z) \left(\alpha_s(m_Z)/\alpha_s(\mu)\right)^{-4\gamma_0/\beta_0}$ with $\alpha_s(\mu) = \alpha_s(m_Z)/(1 + \alpha_s(m_Z)\beta_0 t)$ and $t = \log(\mu^2/m_Z^2)/(4\pi)$, $\beta_0 = 11 - 2N_f/3$, $N_f = 5$ and $\gamma_0 = 2$.

At the perturbative level eq. (4) and eq. (5) are equivalent. However, they neglect different higher order terms and lead to different answers. Since the experiment is performed at high energies (the relevant scales are m_Z and $m_Z\sqrt{y_c}$) one would think that the expression in terms of the running mass is more appropriate because the running mass is a true short distance parameter, while the pole mass contains in it all the complicated physics at scales $\mu \sim M_b$. Moreover, by using the expression in terms of the running mass we can vary the scale in order to estimate the error due to the neglect of higher order corrections. In any case, if one would use in eq. (5) scales as low as $\mu = 5\ GeV$, one would get something closer to the pole mass result.

Although we have studied the observable eq. (1) for the four jet-clustering algorithms discussed in [3,4,13], in the following we concentrate only on the DURHAM algorithm [24], which gives smaller radiative corrections and was the one used by the DELPHI collaboration in its analysis.

The function b_0 gives the mass corrections at the leading order. As shown in [3] it depends very mildly on the quark mass in the region of interest ($M_b \sim 3-5\ GeV$). Therefore it is appropriate to present our results for b_0 as a fit in only y_c: $b_0 = \sum_{n=0}^{2} k_0^{(n)} \log^n y_c$. For the DURHAM algorithm, in the range $0.01 < y_c < 0.10$ and $3\ GeV < M_b < 5\ GeV$, using $s_W^2 = 0.2315$, we obtain $k_0^{(0)} = -10.521$, $k_0^{(1)} = -4.4352$, $k_0^{(2)} = -1.6629$.

The function $\bar{b}_1$ [14] gives the NLO massive corrections to R_3^{bd}. It is important to note that $\bar{b}_1$ contains significant logarithmic corrections depending on the quark mass. We take them into account by using the form $\bar{b}_1 = k_1^{(0)} + k_1^{(1)} \log(y_c) + k_m^{(0)} \log(r_b)$ in the fit. The coefficients we obtain, for the DURHAM scheme and ranges for y_c and r_b mentioned above, are: $k_1^{(0)} = 297.92$, $k_1^{(1)} = 59.358$, $k_m^{(0)} = 46.238$.

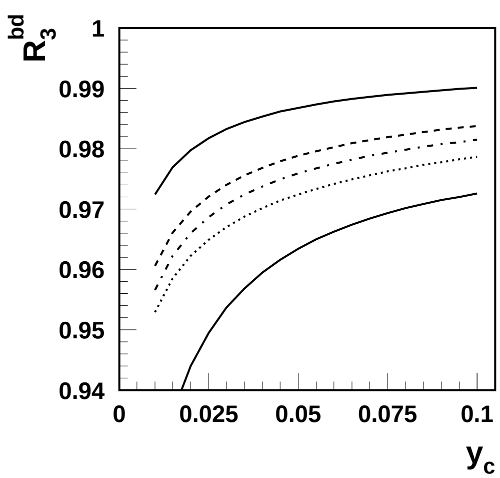

Figure 1. NLO results for R_3^{bd} (DURHAM) for $\mu = m_Z$ (dashed), $\mu = 30\ GeV$ (dashed-dotted) and $\mu = 10\ GeV$ (dotted) for $\bar{m}_b(m_Z) = 3\ GeV$ and $\alpha_s(m_Z) = 0.118$. For comparison we also plot the LO results for $M_b = 5\ GeV$ (lower solid line) and $\bar{m}_b(m_Z) = 3\ GeV$ (upper solid line)

In fig. 1 we present R_3^{bd} for $\mu = m_Z$ (dashed), $\mu = 30\ GeV$ (dashed-dotted) and $\mu = 10\ GeV$ (dotted) for $\bar{m}_b(m_Z) = 3\ GeV$ and $\alpha_s(m_Z) = 0.118$. For comparison we also present the LO results for the quark mass equal to 5 GeV (lower solid line) which is, roughly, the value of the pole mass obtained at low energies and 3 GeV (upper solid line) which is, roughly, the value one obtains for the running mass at the m_Z scale by using the renormalization group. Note that choosing a low value for μ makes the result closer to the LO result written in terms of the pole mass, while choosing a large μ makes the result approach to the LO result written in terms of the running mass at the m_Z scale.

3. $\bar{m}_b(m_Z)$ from LEP data

If R_3^{bd} is measured to good accuracy one could use eq. (5) and the relationship between $\bar{m}_b(\mu)$ and $\bar{m}_b(m_Z)$ to extract $\bar{m}_b(m_Z)$. However, the extracted result will depend on the scale μ. For illustration, in fig. 2 we show the value for $\bar{m}_b(m_Z)$ which one would obtain from $R_{3\,exp}^{bd} = 0.96$ ($y_c = 0.02$) as a function of the scale μ. Although one would naturally think that the scale μ has to be taken of the order $\sim m_Z/3$ if the energy is equally distributed among the three jets, strictly speaking the precise value of μ is undefined. The spread of the result due to the variation of the scale in an appropriate range gives an estimate of the uncertainty due to higher order corrections. From fig. 2 we see that if we vary μ in the range $m_Z/10 - m_Z$ the uncertainty in the determination of $\bar{m}_b(m_Z)$ would be of about 0.20 GeV for the DURHAM algorithm. In the same range and for the JADE and EM algorithms the obtained error would be [13] of about 0.25 GeV, while for the E algorithm we would obtain an error bigger than 0.50 GeV.

The DELPHI collaboration has used a slightly different approach to extract the value of $\bar{m}_b(m_Z)$ from R_3^{bd} [6]. Using eq. (4) written in terms of the pole mass, M_b, and, exploiting the perturbative relation between the pole mass and the running mass, they obtained the value of $\bar{m}_b(m_Z)$ from M_b extracted from the experimentally measured R_3^{bl}. The difference between $\bar{m}_b(m_Z)$ obtained in this way and the running mass obtained di-

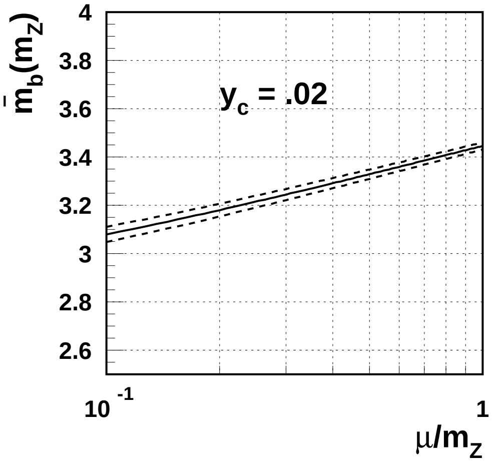

Figure 2. Extracted value of $\bar{m}_b(m_Z)$ if $R^{bd}_{3\,exp} = 0.96$ as a function of the scale μ. We take $\alpha_s(m_Z) = 0.118$ (solid) and $\Delta\alpha_s = 0.003$ (dashed).

rectly using eq. (5), which is due to the different treatments of the higher order terms, was considered as a theoretical uncertainty. Accounting in addition for the uncertainty due to the variation of the scale μ they get a more conservative estimate for the theoretical error, 0.27 GeV, for the DURHAM algorithm. We would like to note, however, that the central value for $\bar{m}_b(m_Z)$ reported in [6] is fully compatible with the one obtained by using only eq. (5). The comparison of the two methods gives a check of consistency. Furthermore, the mass of the bottom-quark, $\bar{m}_b(m_Z)$, measured from the three-jet decay of the Z-boson [6] is also fully compatible with the value obtained from low energy determinations [25] after using the renormalization group. This provides, for the first time, a nice check of the running of a quark mass.

Acknowledgements. We are indebted with S. Cabrera, J. Fuster and S. Martí for a very enjoyable collaboration. G.R. acknowledges O. Biebel for very useful comments and S. Narison for the very kind atmosphere created at Montpellier.

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